Usage of concentrated spectrogram for analysis of acoustical signals

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Abstract: *A novel precise method of signal analysis in the time-frequency domain is presented. A signal energy distribution is estimated by discard and displacement of energy parts of the classical spectrogram. A channelized instantaneous frequency and a local group delay are used in order to energy replacement. Additionally, newly introduced representations such as: a channelized instantaneous bandwidth and a local group duration are used for remove some part of irrelevant energy. An obtained energy distribution called attractogram is highly concentrated. And it causes that mono-components of an analyzed signal are precisely localized in the time-frequency domain. The presented method is utilised in order to acustical signals analysis.*

Keywords: *time-frequency representation, cross-spectral method, short-time Fourier transform, STFT, channelized instantaneous complex frequency, complex local group delay*

1. INTRODUCTION

In 1976 Kodera *et al.* proposed a new method of energy distribution estimation in the joint time-frequency domain using the channelized instantaneous frequency (CIF) and the local group delay (LGD). The approach is known under many names including: the modified moving window method [1, 2], the cross-spectral method [3], the reassignment method [4, 5], relocation, displacement method, etc. These variants of the method are distinguished mainly by usage of different estimators of CIF and LGD. But the main concept is the same: signal analysis that leads to high concentrated energy distribution in the time-frequency domain due to relocation executed according to CIF and LGD values.

Both CIF and LGD are parts of the gradient of the STFT complex phase. Beyond them, other parts are so-called signed channelized instantaneous bandwidth and signed local group duration [6, 7]. In the presented approach, all mentioned parts of the gradient of the STFT complex phase are used. Firstly, the short-time Fourier transform is derived in the following manner:

$$U(t,\omega) = A(t,\omega)\exp\big(j\phi(t,\omega)\big) = \int_{-\infty}^{\infty} u(\tau+t)h^*(-\tau)\exp(-j\omega\tau)d\tau \tag{1}$$

where complex conjugation is denoted by an asterisk,

$$A(t,\omega) = \big|U(t,\omega)\big| \quad \text{and} \quad \phi(t,\omega) = \arg\{U(t,\omega)\}\,, \quad A(t,\omega),\, \phi(t,\omega) \in \mathbb{R} \tag{2}$$

The complex waveform denoted by $u(t)$ should have non-zero values and has to be differentiable in every instant, $U(t,\omega)$ means resultant STFT and $h(t)$ represents an analyzing window function. $A(t,\omega)$ and $\phi(t,\omega)$ denote accordingly amplitude and phase instantaneous spectra.

In the presented method, subsequently for each locus (t,ω) of STFT corrected localization is estimated in the time-frequency plain by CIF and LGD [1, 2]. They are expressed respectively:

$$\Omega(t,\omega) = \frac{\partial}{\partial t}\phi(t,\omega) \tag{3}$$

$$\Theta(t,\omega) = -\frac{\partial}{\partial \omega}\phi(t,\omega) \tag{4}$$

and are used for obtained new localizations as follows:

$$(t,\omega) \to \big(t + \Theta(t,\omega)/(2\pi), \Omega(t,\omega)\big) \tag{5}$$

where t and ω mean accordingly time and angular frequency. CIF is denoted by $\Omega(t,\omega)$ and LGD is referred to as $\Theta(t,\omega)$. The new distribution of energy is called concentrated spectrogram [8, 9]. In Fig. 1 classical and concentrated spectrograms of a synthetic FM two-monocomponent signal are presented. The test signal is expressed by the following formula:

$$\begin{aligned} u_{\mathrm{s1}}(t) = &\exp\Big(j\big(2\pi t f_1 + 0.25 f_{\mathrm{d}}\sin(4\pi t)/\pi\big)\Big) + \\ &+ \exp\Big(j\big(2\pi t f_2 - 0.25 f_{\mathrm{d}}\sin(4\pi t)/\pi\big)\Big) \end{aligned} \tag{6}$$

where $f_1 = 300$ Hz, $f_2 = 700$ Hz and $f_{\mathrm{d}} = 150$ Hz. In general, the multicomponent complex waveform can be represented by the following model:

$$u(t) = \sum_{n=1}^{N} a_n(t)\exp\big(j\varphi_n(t)\big) \tag{7}$$

where N is a number of monocomponents, $a_n(t)$ and $\varphi_n(t)$ represent envelope and instantaneous phase of n-th monocomponent. In this paper understanding of a single monocomponent is related to each n-th $a_n(t)\exp\big(j\varphi_n(t)\big)$ waveform [10].

In classical and concentrated spectrograms two monocomponents interfere significantly, if they are located close to each other on the time-frequency plain. The phenomenon is described by the uncertainty principle. Thus the interferences of components occurs locally. They are dependent on the time-frequency range of the analyzing window that is represented by the unambiguity function [11, 12].

In the next sections of the paper, a number of degrees of freedom distribution estimate as a product of the channelized instantaneous bandwidth (CIBW) and a local group duration (LGDR) is introduced. This representation is used in order to select the areas where energy is originated mainly from a single mono-component of a signal. The energy is extracted from the spectrogram and located according to the Kodera's *et al.* approach [1, 2].

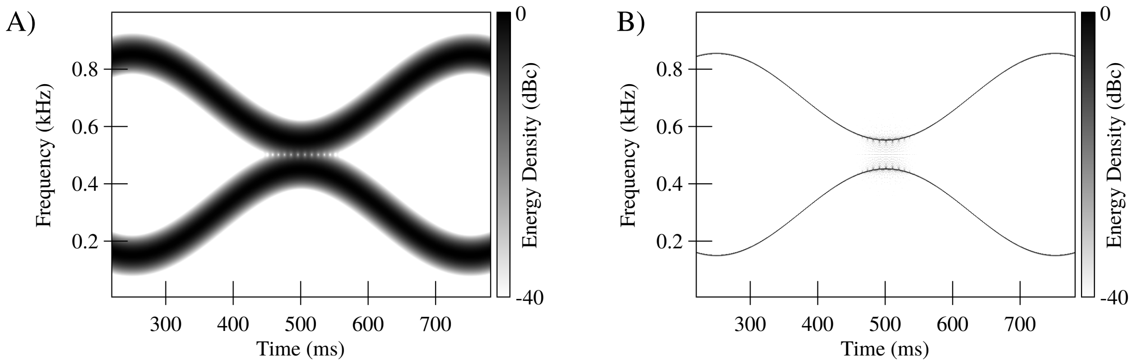

Fig.1: Energy distributions of test signal in the time-frequency domain: A) classical spectrogram; B) concentrated spectrogram. Blackman-Harris window type with an effective width equal to approx. 4.5 ms is used.

2. DEGREES OF FREEDOM IN SIGNAL THEORY

A number of degrees of freedom for any signal denoted here by ξ_e can be calculated as a product of its effective bandwidth and its effective duration as follows [13]:

$$\xi_\mathrm{e} = B_\mathrm{e} T_\mathrm{e} \tag{8}$$

where B_e and T_e represent respectively the effective bandwidth:

$$B_\mathrm{e}^2 = \int\limits_{-\infty}^{\infty} (\omega - \omega_o)^2 E(\omega)\mathrm{d}\omega \Big/ \int\limits_{-\infty}^{\infty} E(\omega)\mathrm{d}\omega\,, \quad \omega_o = \int\limits_{-\infty}^{\infty} \omega E(\omega)\mathrm{d}\omega \Big/ \int\limits_{-\infty}^{\infty} E(\omega)\mathrm{d}\omega \tag{9}$$

and the effective duration of the signal:

$$T_\mathrm{e}^2 = \int\limits_{-\infty}^{\infty} (t - t_o)^2 E(t)\mathrm{d}t \Big/ \int\limits_{-\infty}^{\infty} E(t)\mathrm{d}t\,, \quad t_o = \int\limits_{-\infty}^{\infty} tE(t)\mathrm{d}t \Big/ \int\limits_{-\infty}^{\infty} E(t)\mathrm{d}t \tag{10}$$

Both B_e and T_e are always positive and ξ_e limited: $0 < B_\mathrm{e}, 0 < T_\mathrm{e}$ and $\xi_\mathrm{e} < \infty$. $E(t)$ denotes a distribution of signal energy in the time domain and $E(\omega)$ represents a distribution of spectrum energy in the frequency domain. The number of degrees of freedom (8) is wider known as the time-bandwidth product and is used in order to evaluate of analyzing windows.

3. NUMBER OF DEGREES OF FREEDOM DENSITY

The local bandwidth can be assigned in every instant and in every output channel from the short-time Fourier transformer, similarly as the channelized instantaneous frequency [14]. Then for both continues time and frequency it is called the channelized instantaneous bandwidth and can be obtain as follows:

$$B(t,\omega) = \frac{1}{2\pi}\left|\frac{\partial}{\partial t}\Lambda(t,\omega)\right| \tag{11}$$

where $\Lambda(t,\omega) = \ln\big(A(t,\omega)\big)$ and $\ln()$ is the complex natural logarithm functor. Dually, a local group duration can be defined in the time-frequency domain:

$$T(t,\omega) = \frac{1}{2\pi}\left|\frac{\partial}{\partial \omega}\Lambda(t,\omega)\right| \tag{12}$$

The channelized instantaneous bandwidth and the local group duration express a local stretching of the signal respectively in frequency and in time. Thus the scalar product $B_e T_e$ can be extendet into a multiplication (point-by-point) of these representations. Then a number of degrees of freedom density (distribution; NDFD) can be estimated by the following formula [7, 15]:

$$\chi(t,\omega) = B(t,\omega)T(t,\omega) \tag{13}$$

where $\chi(t,\omega)$ is a number of degrees of freedom density distributed in the joint time-frequency domain. In order to distinguish from the global number of degrees of freedom (8) NDFD is denoted by $\chi(t,\omega)$ without any subscript. NDFD for the test FM chirp signal is presented in Fig. 2.

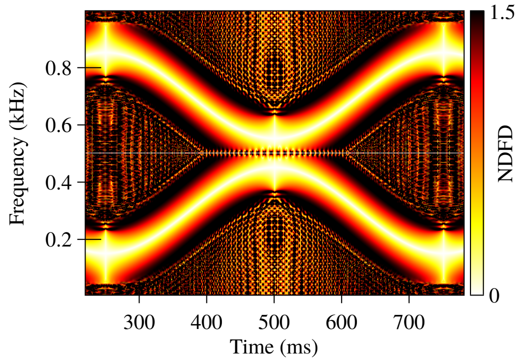

Fig.2: Number of degrees of freedom density obtain for the test signal.

4. ENERGY EXTRACTION FROM THE CLASSICAL SPECTROGRAM

Both classical and concentrated spectrograms are some estimates of an energy distribution in the time-frequency domain. The main purpose of the proposed method is a classification of the energy in order to extract unambiguous not blurred part. It is assumed that the not smeared energy expresses localizations of signal components in the time-frequency plain. The blurred energy can be detected using NDFD. If local number of degrees of freedom is large, a time-frequency representation (TFR) of a signal is chaotic and it is rapidly changing. The NDFD estimated in a point concerns area that can be indicated by the unambiguity

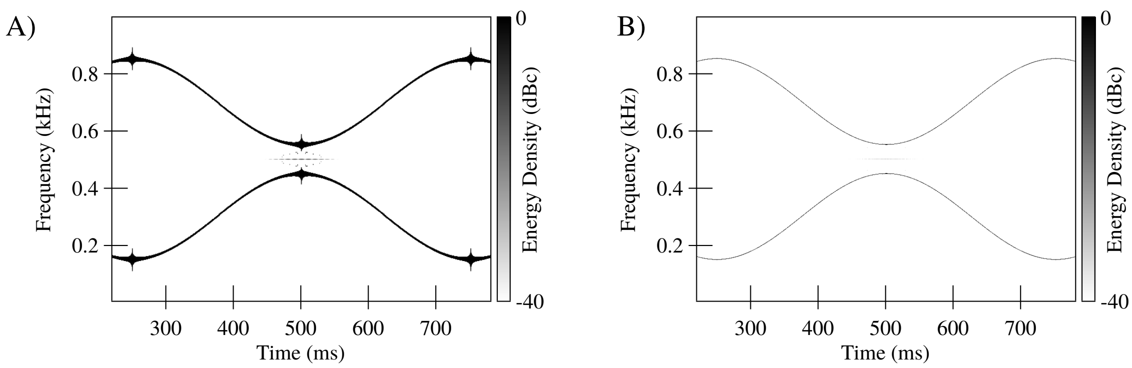

Fig.3: Spectrograms after discard of an energy part: A) before replacement B) after replacement. The threshold α_χ is assumed as 0.05.

function of an analyzing window near the point. In contrast, if NDFD achieves small value, then TFR is locally orderly and slowly variable.

A threshold of NDFD can be arbitrarily assumed in order to an energy separation. Let the threshold be denoted as α_χ. Then, if NDFD is greater than α_χ, an energy of a classical spectrogram is removed from this point of time-frequency plain. Otherwise an energy is preserved. The remaining energy is subsequently redistributed according to the Kodera's *et al.* approach [1, 2]. The resultant energy distribution is called **attractogram**. Attractograms of the test chirp signal are illustrated by Fig. 3. What causes a great impression is a high concentration (meassured by energy concentration index [16]) of the energy for two-component signal, even where the two components are relatively close to each other.

5. ACOUSTICAL SIGNALS ANALYSIS

The proposed method of time-frequency analysis can be used in order to acoustical signals investigation. In this section results of the analysis are presented. Firstly ..

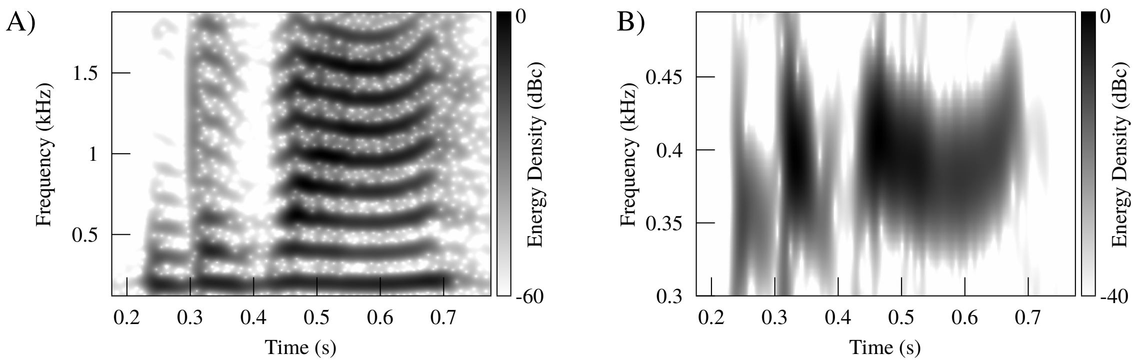

Fig.4: Essential spectrograms: A) before energy replacement B) after energy replacement. The threshold α_χ is assumed as 0.05.

6. CONCLUSION

In the paper, the novel method of an energy distribution estimate is presented. The resultant energy distribution is called **attractogram**. A new aspect of the method is the usage of the number of degrees of freedom density (NDFD). That is distributed in the joint time-frequency domain and allows for separation of energy into two parts. NDFD distribution is obtain as a product of the channelized instantaneous bandwidth and a local group duration. The part of energy, where NDFD values are small, is referred to as the attractogram. The localization of the energy is calculated according to the Kodera *et al.* approach. The second part is treated as an irrelevant effect of the short-time Fourier transformation and it is strongly dependent on the Heisenberg-Gabor principle. The spectrograms in Fig. 3. prove that proposed energy distributions are high concentrated and accurate in the time-frequency domain more than classical and concentrated spectrograms.

7. ACKNOWLEDGMENTS

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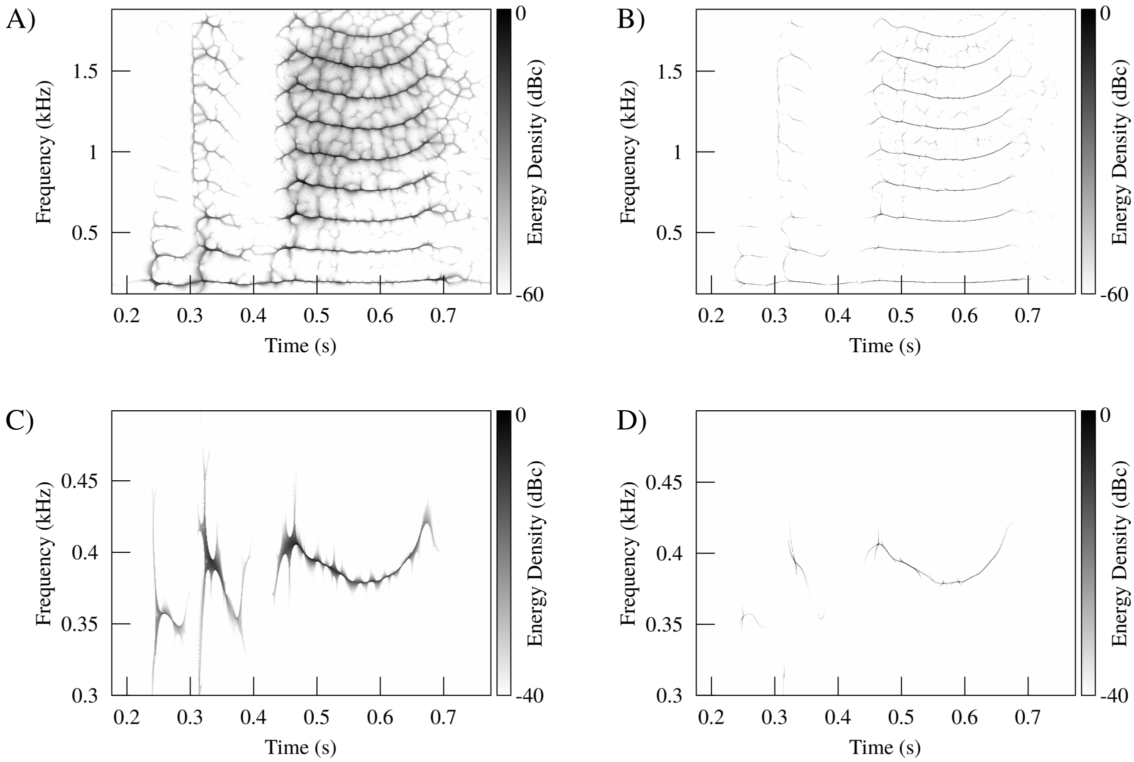

Fig.5: Results of speech analysis: concentrated spectrograms AC) and attractograms BD). The threshold α_χ is assumed as 0.01.

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