RUHR-UNIVERSITÄT BOCHUM

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RUB

Hydraulic fracturing:

Towards a numerical modeling approach

A study on the hydromechanics of static fractures

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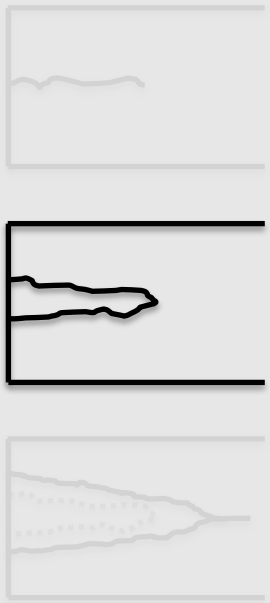
INTRODUCTION

Hydromechanics of “static” fractures

Investigation of a **fluid-filled fracture** embedded in a porous rock matrix intersecting a borehole

Pressure induced by fluid injection:

- **opens** new fractures
- **dilates** (pre-existing) fractures
- **propagates** existing fractures



→ Investigation and modeling of **deformable fractures** (no propagation)

➢ AIM:

- solve the **coupled hydro-mechanical problem** between fluid-filled joint and rock matrix → numerically capture hydraulic transport and other time dependent physical phenomena

MOTIVATION

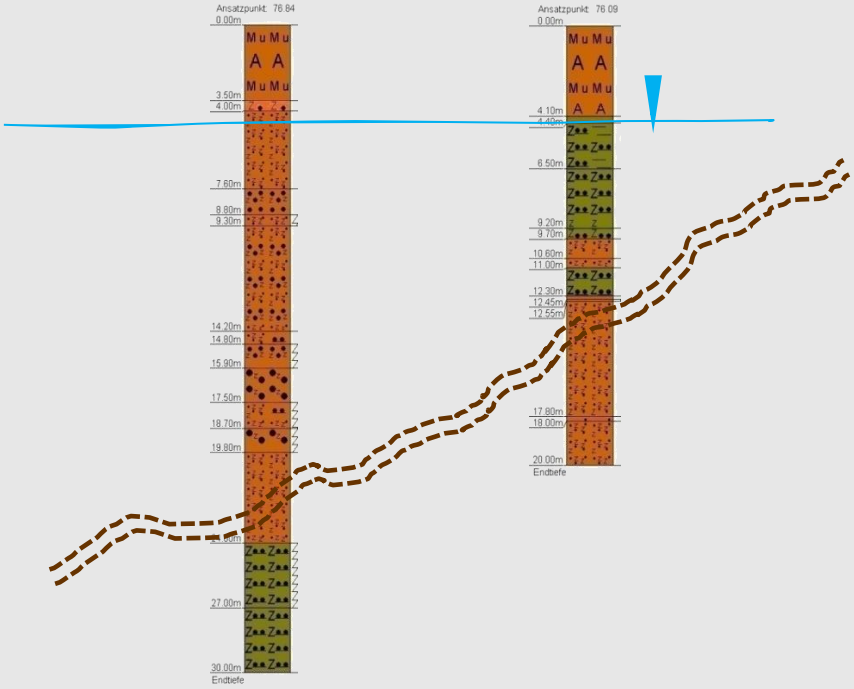
Capture physical phenomena

- Explain data recorded during **field experiments**, e.g.:
  - Borehole injection tests
  - Slug tests

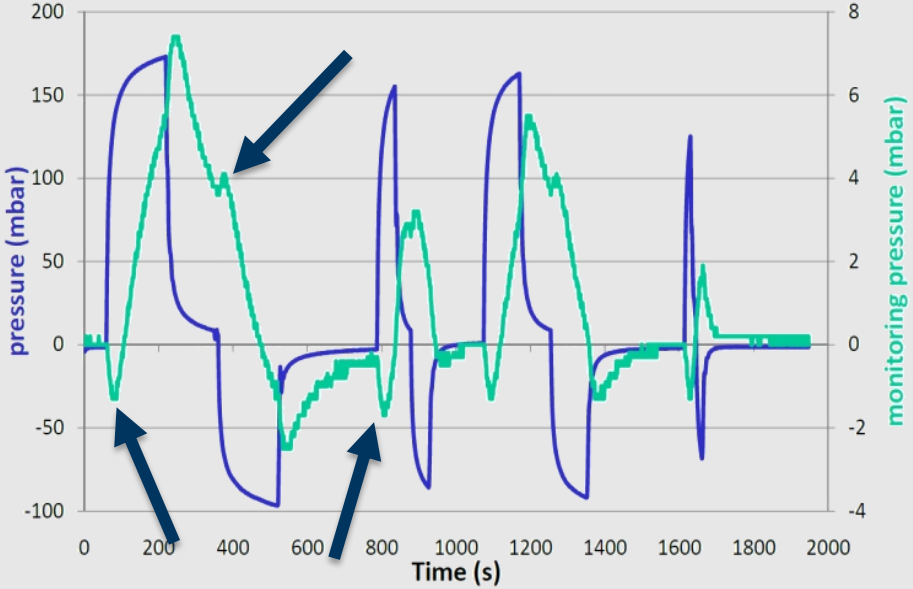
Kemnader See, Bochum

Borehole 1
Injection

Borehole 2
Monitoring



Capture physical effects e.g.:
„inverse“ response to pumping



MOTIVATION

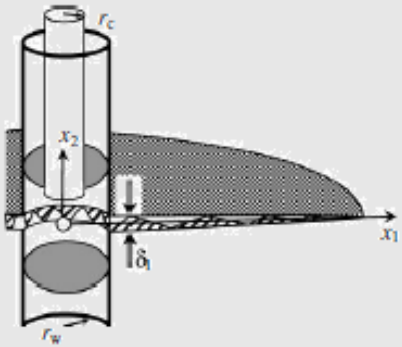
Previous Investigations

- **Murdoch and Germanovich** (2006),

  "Modeling of a deformable fracture"

- **Zimmerman and Yeo** (2000),

  "Fluid Flow in Rock Fractures:
  from the Navier-Stokes Equations to the Cubic Law"



Murdoch & Germanovich 2006

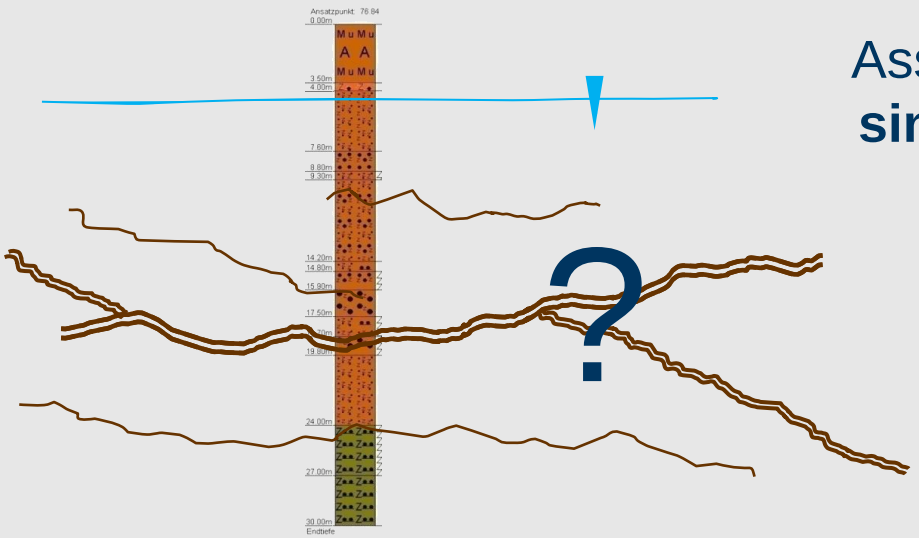
- The following investigation focuses on:

  - **numerical** approaches → solve with FEM
  - describing relevant local physical mechanisms
  - analyzing the time dependent non-equilibrium configurations of the joint and the corresponding pressure profiles

MODELING

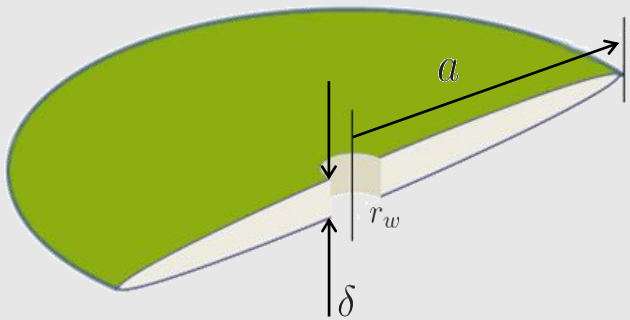
Fracture Geometry

Real geometry



Assumption of a
single fracture

Modeled geometry



Unknown fracture geometry

- Single fracture ?
- Network of fractures ?

Circular - ”penny-shaped” fracture

Ellipsoidal joint intersected by a borehole

MODELING

Strategies

Two approaches to model and solve the coupled hydro-mechanical problem

BALANCE EQUATIONS

Derive PDEs describing the fluid flow in a deformable fracture

+

Solve the elastic deformation of the surrounding matrix

↓

Staggered solution scheme

POROELASTIC APPROACH

Numerical setup in the framework of **poroelasticity**

• PDEs from Biot´s Theory of poroelasticity

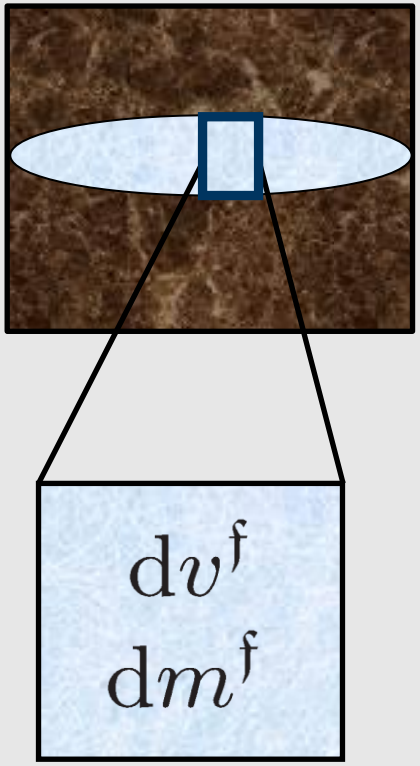
↓

Fully coupled solution scheme

BALANCE EQUATIONS

Fluid flow

Simplification to a 1-dimensional flow problem



- **Balance of mass** of the fluid

$$\frac{\partial}{\partial t}\left(\rho^{\mathfrak{f}R}\,\delta\right) + \frac{\partial}{\partial x}\left(w_{\mathfrak{f}}\,\rho^{\mathfrak{f}R}\,\delta\right) + \frac{w_{\mathfrak{f}}\,\rho^{\mathfrak{f}R}\,\delta}{x} = 0$$

δ … effective aperture of the fracture

$\rho^{\mathfrak{f}R}$ … effective density of the fluid

$w^{\mathfrak{f}}$ … fluid velocity

Deformable fracture

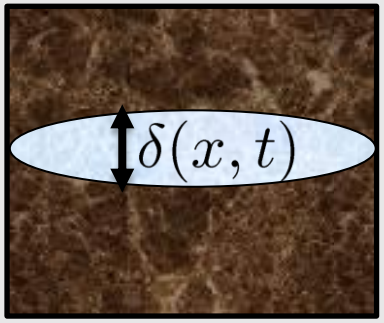
Compressible fluid

BALANCE EQUATIONS

Fluid flow

- **Balance of momentum** (Poiseuille flow assumption)

$$w_{\mathfrak{f}} = -\frac{\delta^2}{12\,\eta^{\mathfrak{f}R}}\frac{\partial p}{\partial x} \quad \Rightarrow \quad w_{\mathfrak{f}} \propto \frac{\partial p}{\partial x}$$



Strong formulation of the hydraulic problem

$$\frac{\partial p}{\partial t} - \boxed{\frac{\delta^2}{\beta_{\mathfrak{f}}\,\alpha}\frac{\partial^2 p}{\partial x^2}} - \boxed{\frac{\delta}{\beta_{\mathfrak{f}}\,\alpha}\left(3\frac{\partial \delta}{\partial x} + \frac{\delta}{x}\right)\frac{\partial p}{\partial x}} - \boxed{\frac{\delta^2}{\alpha}\left(\frac{\partial p}{\partial x}\right)^2} + \frac{1}{\delta\,\beta_{\mathfrak{f}}}\frac{\partial \delta}{\partial t} = 0$$

Diffusion Term

ConvectionTerm

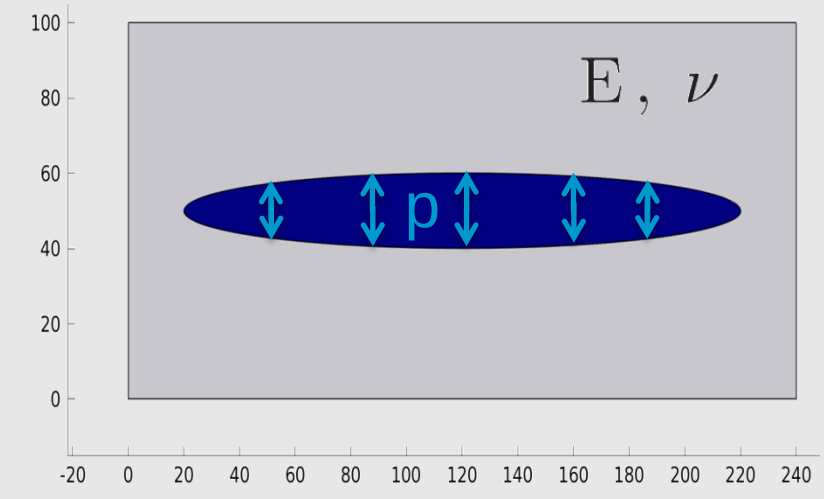
Quadratic Term

BALANCE EQUATIONS

Elastic deformation

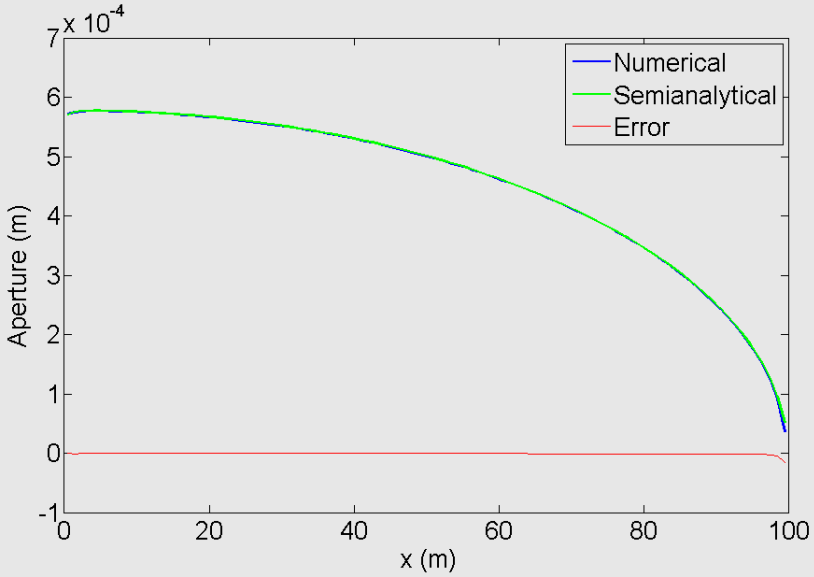
2-dimensional elastic deformation problem

- Fracture embedded in an elastic matrix
- Fluid pressure p acting on the fracture surface



Solution strategies:

- Semi analytical (Sneddon)
- Numerical (FEM)



BALANCE EQUATIONS

Challenges

Hydro-Mechanical coupling

The hydraulic problem and the matrix deformation problem are coupled

$$\partial_t\, p - \frac{\delta^2}{\beta_{\mathfrak{f}}\,\alpha}\frac{\partial^2 p}{\partial x^2} - \frac{\delta}{\beta_{\mathfrak{f}}\,\alpha}\left(3\,\partial_x\,\delta + \frac{\delta}{x}\right)\partial_x\, p - \frac{\delta^2}{\alpha}\left(\partial_x\, p\right)^2 + \frac{\partial_t\,\delta}{\delta\,\beta_{\mathfrak{f}}} = 0$$

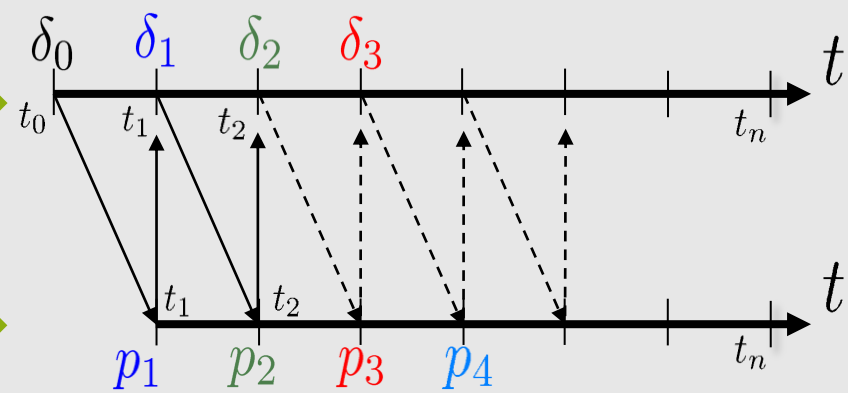
Coupling term δ obtained from:

- Semi analytical solution
- Numerical FEM solution

Staggered solution scheme

Aperture $\delta\,(x,\, p(x,t),\, t_i)$

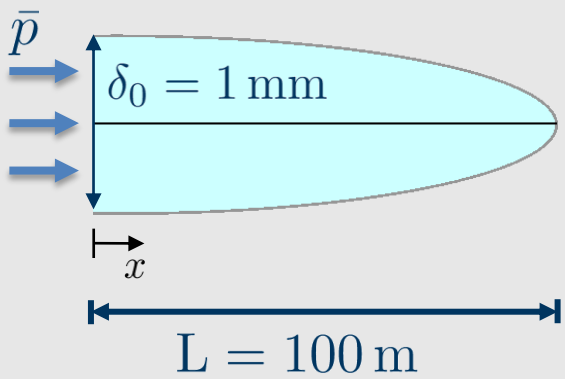
Pressure $p(x,\, t_{i+1})$

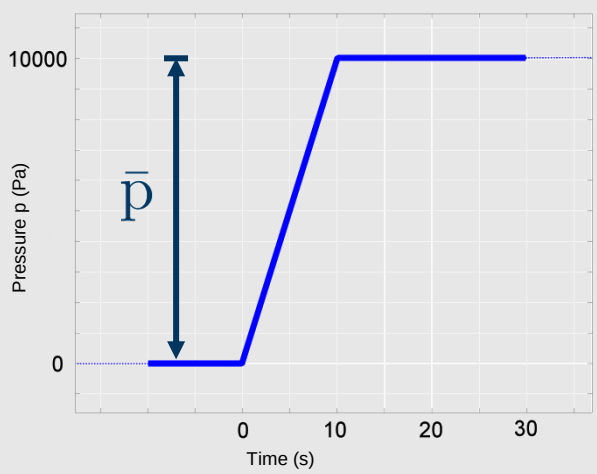


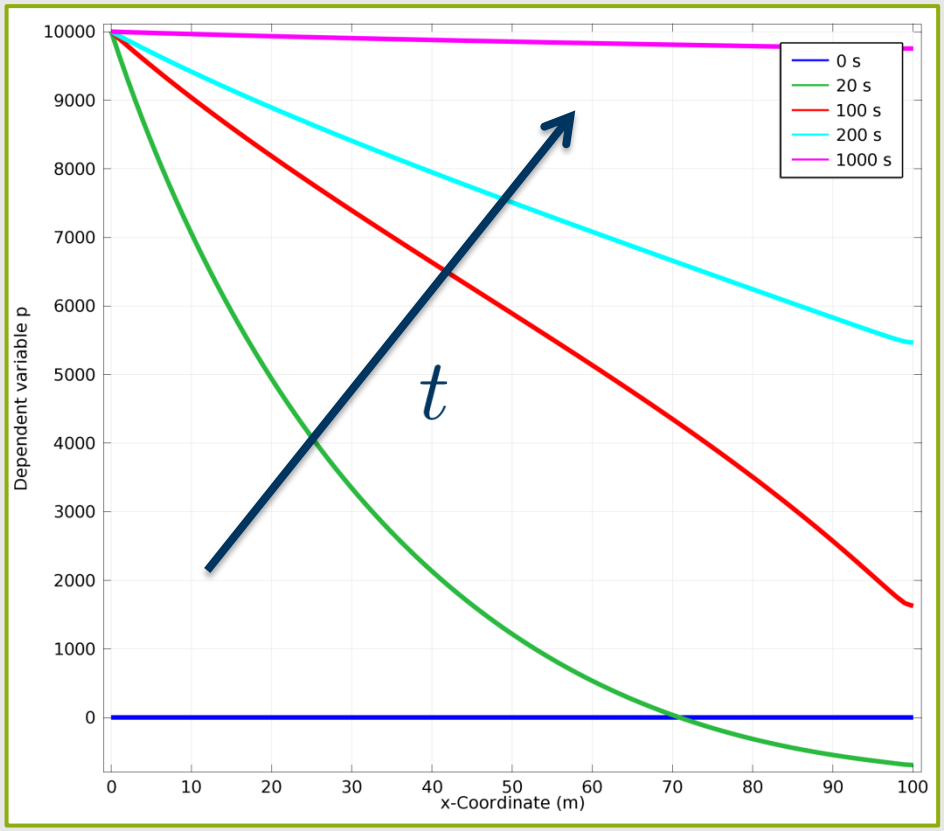
NUMERICAL RESULTS

Balance equations

Boundary Value Problem







“inverse” response to pumping numerically captured by the model

MODELING

Strategies

Two approaches to model and solve the coupled hydro-mechanical problem

BALANCE EQUATIONS

Derive PDEs describing the fluid flow in a deformable fracture

+

Solve the elastic deformation of the surrounding matrix

Staggered solution scheme

POROELASTIC APPROACH

Numerical setup in the framework of **poroelasticity**

- PDEs from Biot´s Theory of poroelasticity

Fully coupled solution scheme

POROELASTIC MODEL

Fluid flow

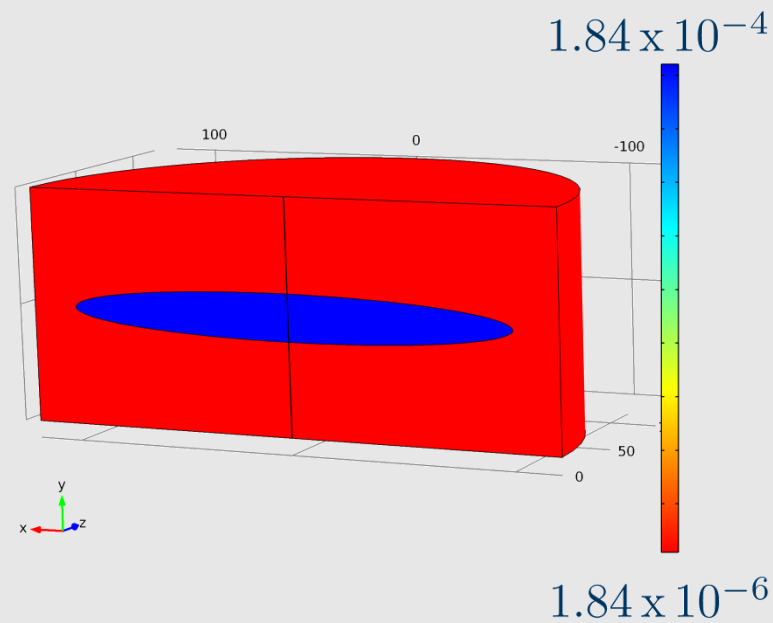
2-dimensional poroelastic model

Fracture and surrounding matrix modeled as **two different porous media**

Strong form of **Biot´s poroelasticity** equations

$$-\operatorname{div}(\sigma^{\mathfrak{s}}_{\mathbf{E}} - \alpha\, p\, \mathbf{I}) = \rho\, \mathbf{b}$$

$$\frac{\dot{p}}{M} - \frac{k^{\mathfrak{f}}}{\gamma^{\mathfrak{f}R}} \operatorname{div} \operatorname{grad} p + \alpha \operatorname{div} \mathbf{v}_{\mathfrak{s}} = 0$$



$k^{\mathfrak{f}}$ … Darcy's permeability

The poroelastic equations can be solved using a **fully coupled scheme**

POROELASTIC MODEL

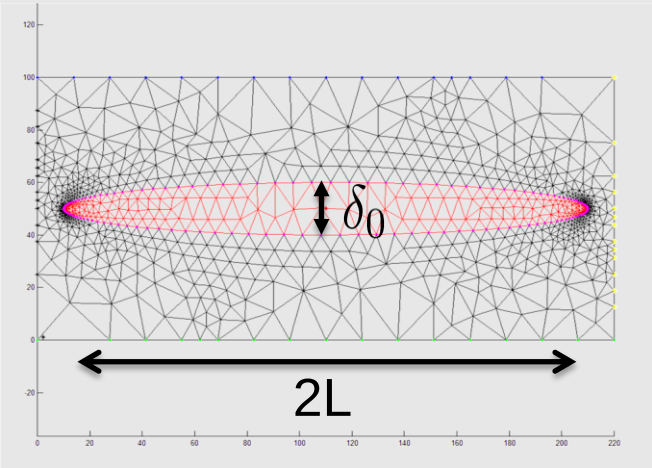
Challenges

Spatial discretization → need to **mesh thin geometry** of the crack

High aspect ratio, e.g.:

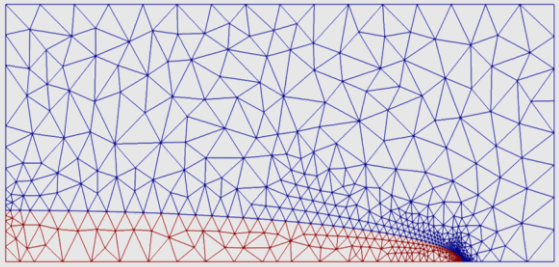
$$\mathrm{L} = 10^2\,\mathrm{m}$$

$$\delta_0 = 10^{-4}\,\mathrm{m}$$



- Use a high aspect ratio mesh generator
- Exploit symmetry conditions

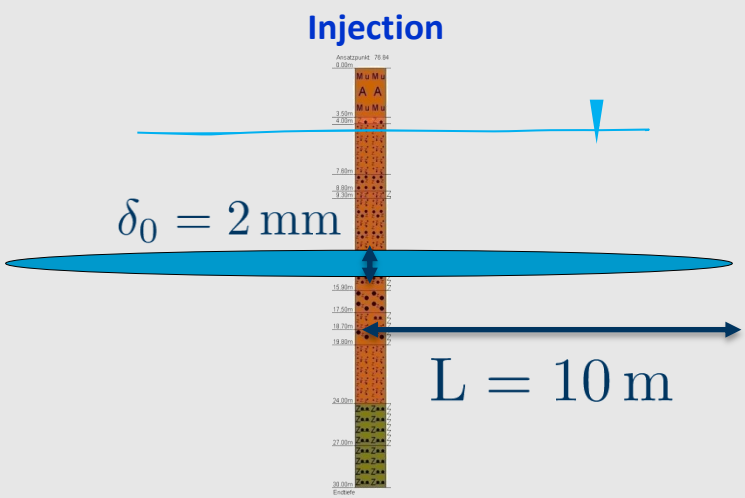
→

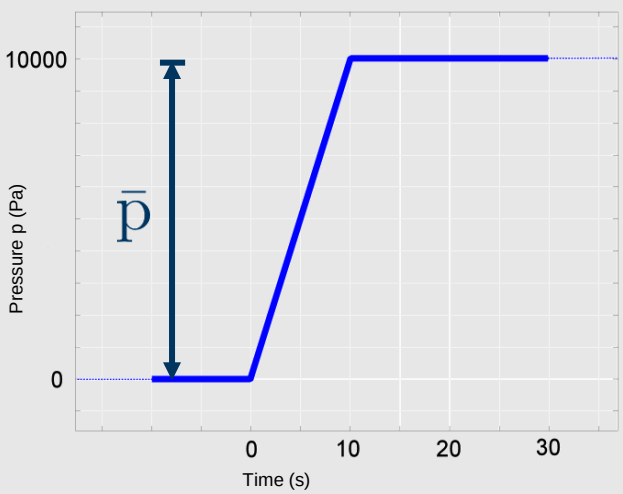


NUMERICAL RESULTS

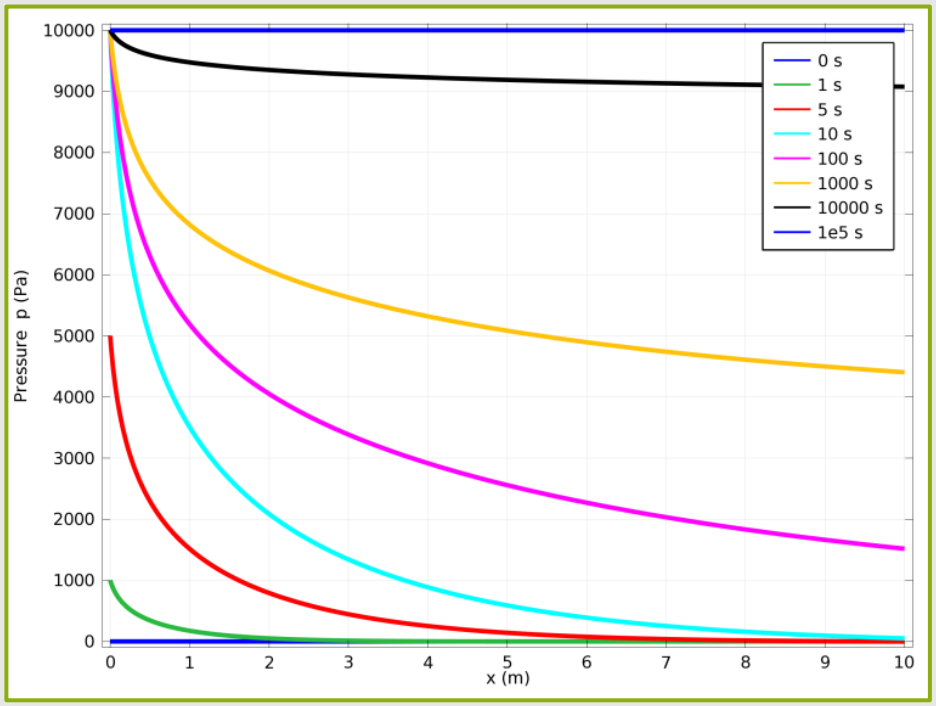
Poroelastic model

Numerical simulation of borehole **fluid injection** in a single fracture





until equilibrium



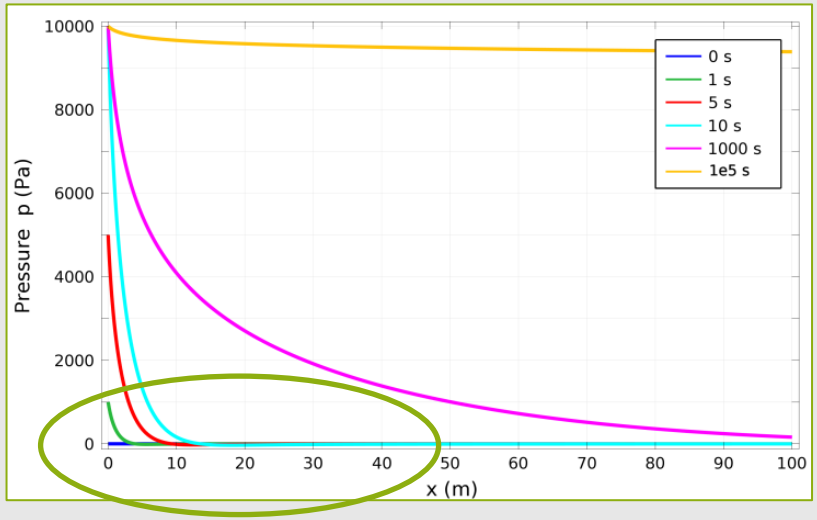
NUMERICAL RESULTS

Poroelastic model

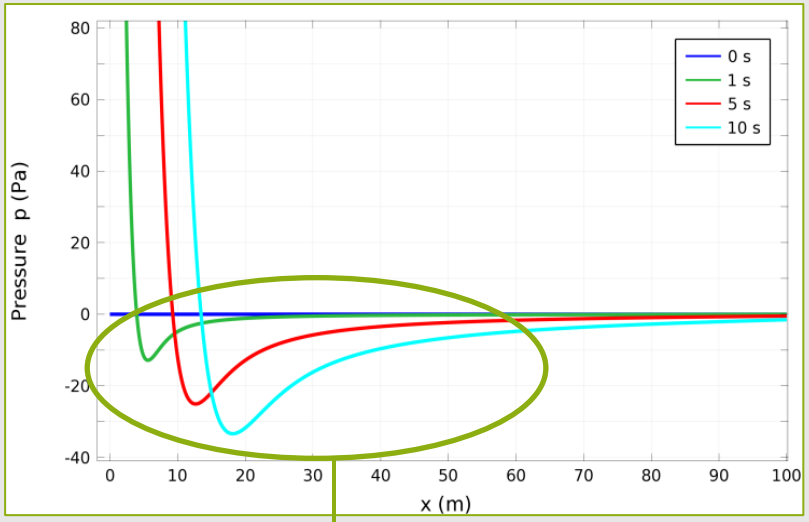
In order to capture the **“inverse” response** effect

Analyze non-equilibrium pressure distribution **just after injection**

Example computed using: $\delta_0 = 2\,\mathrm{cm}$ $\mathrm{L} = 100\,\mathrm{m}$



Zoom in at early time steps

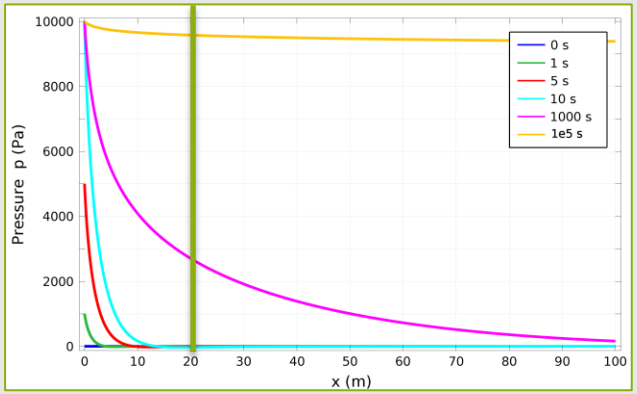


“inverse” response

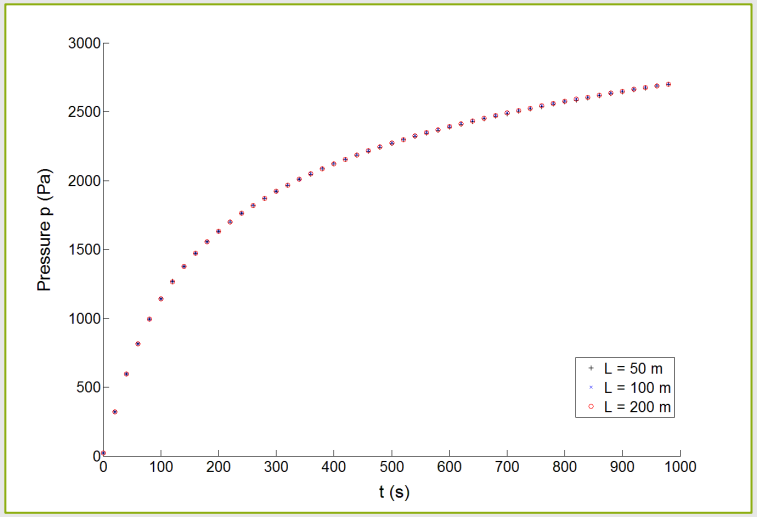
NUMERICAL RESULTS

Case studies

- Varying fracture **half length L (50 – 100 – 200 m)**

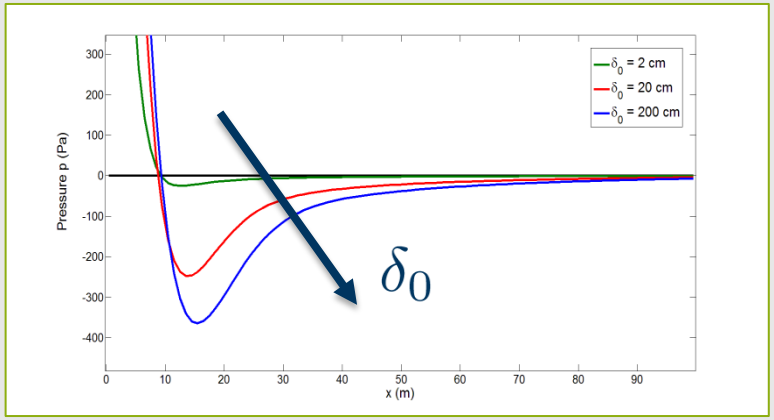


Common observation point at $x = 20\,\mathrm{m}$



Fracture length does not influence the pressure distribution

- Varying initial **effective aperture** δ_0



"inverse" response effect

- increases with increasing δ_0
- earlier response for smaller δ_0

DISCUSSION

BALANCE EQUATIONS	POROELASTIC MODEL
+ Simplified **1-dimensional flow**	+ **Leak-off** effects intrinsically modeled
- **Highly nonlinear** problem	- **High aspect ratio** geometry meshing

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THANK YOU FOR YOUR ATTENTION

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