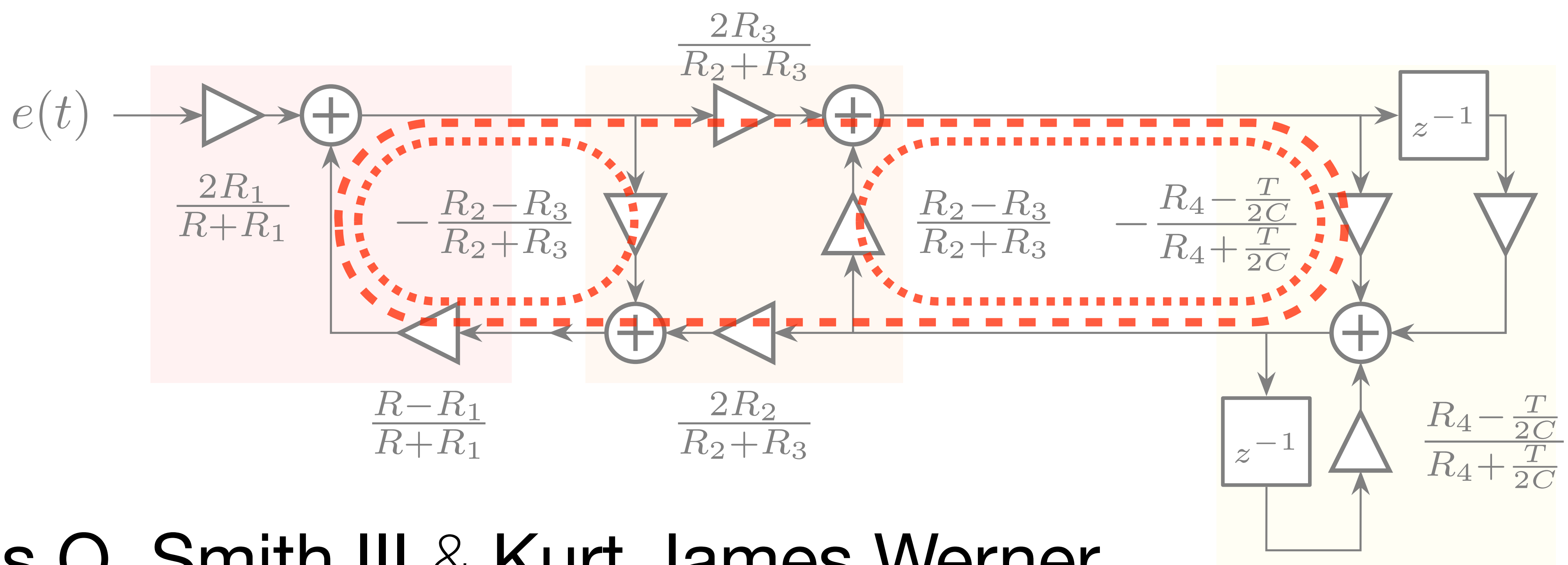
recent progress in
WAVE DIGITAL AUDIO EFFECTS



Julius O. Smith III & Kurt James Werner

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@ Stanford University, California, USA
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keynote talk, international conf. on digital audio effects (DAFx-15)
Trondheim, Norway, 2 December 2015

THANKS

- DAFx organizing committee

- @ CCRMA
  - Vaibhav Nangia & Jonathan Abel
  - Ross Dunkel & Max Rest & Michael Olsen
  - François Germain

- @ Politecnico di Milano
  - Alberto Bernardini & Augusto Sarti

Musicians like vintage stuff.

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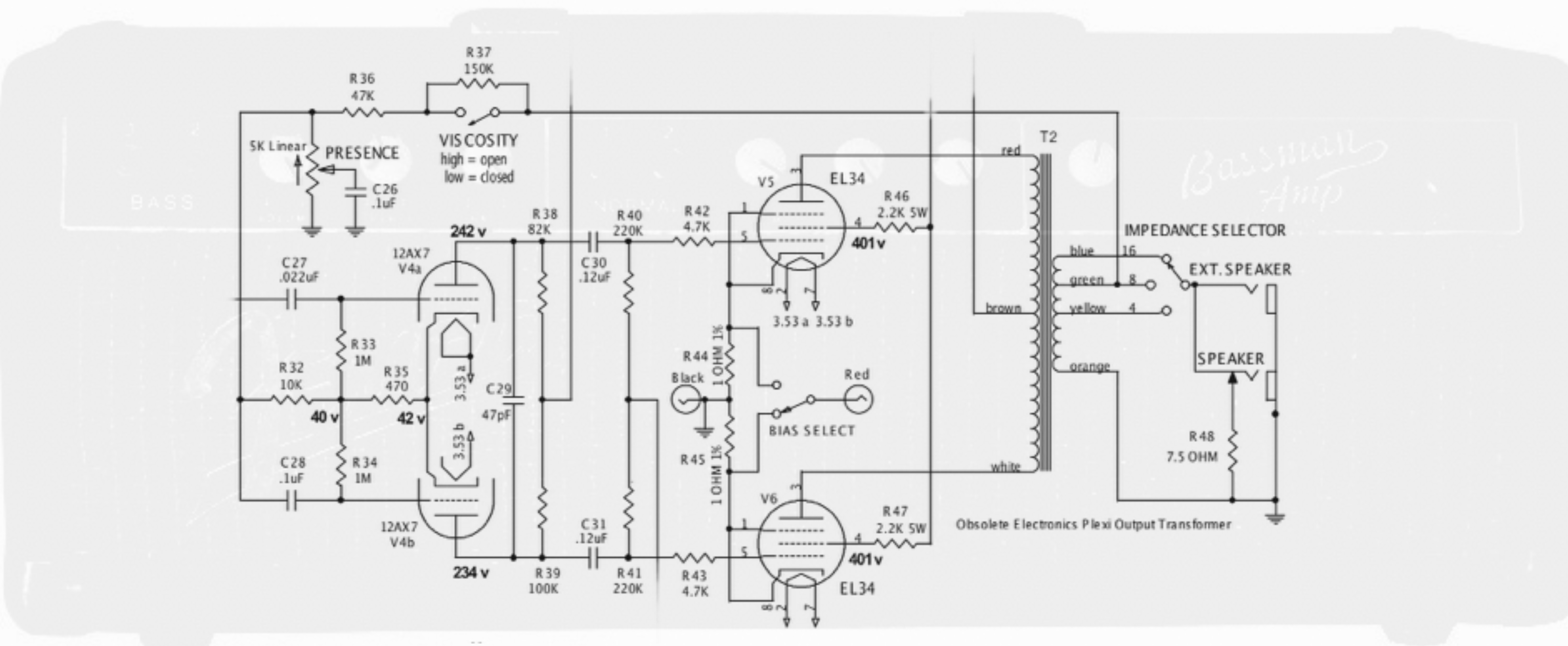
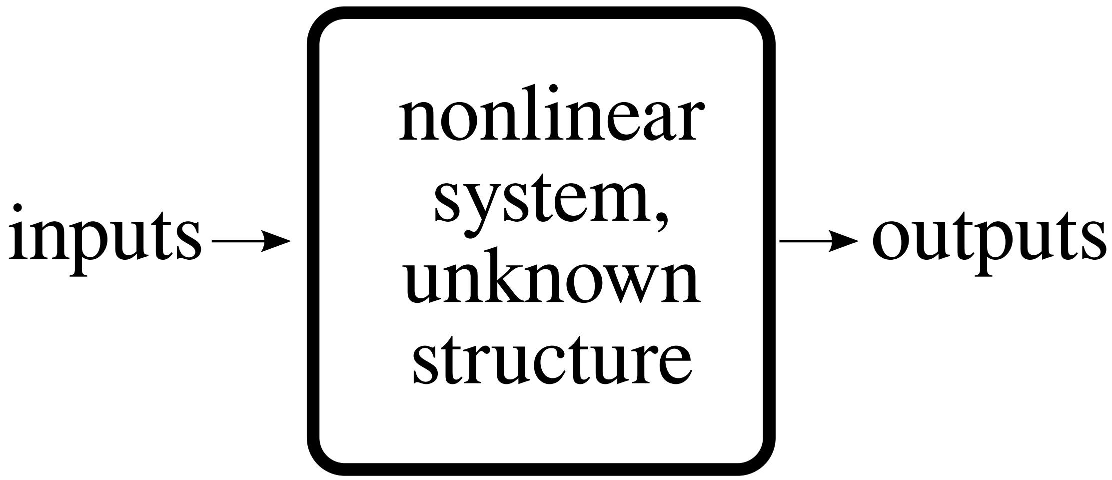


Two Approaches to Modeling Vintage Gear

Nonlinear System Identification ("black box")



- No knowledge of circuit required
- Run test signals to characterize model
- Non-parametric model

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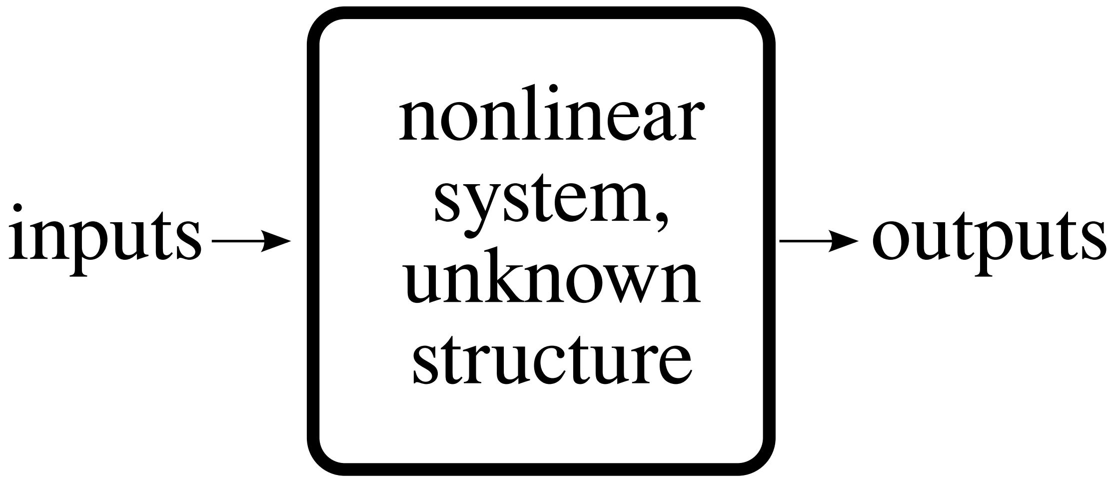

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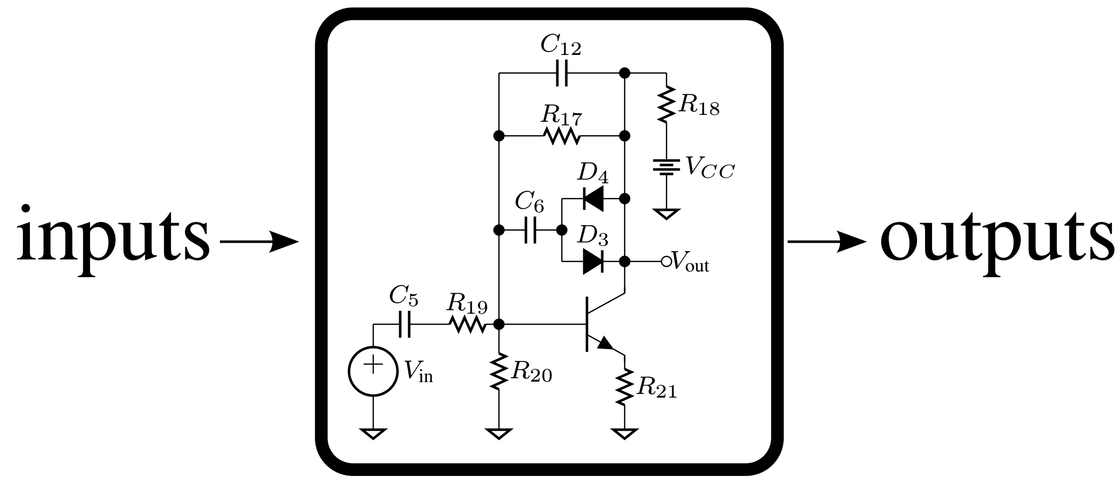
Physical Modeling ("white box")



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TWO APPROACHES TO MODELING VINTAGE GEAR

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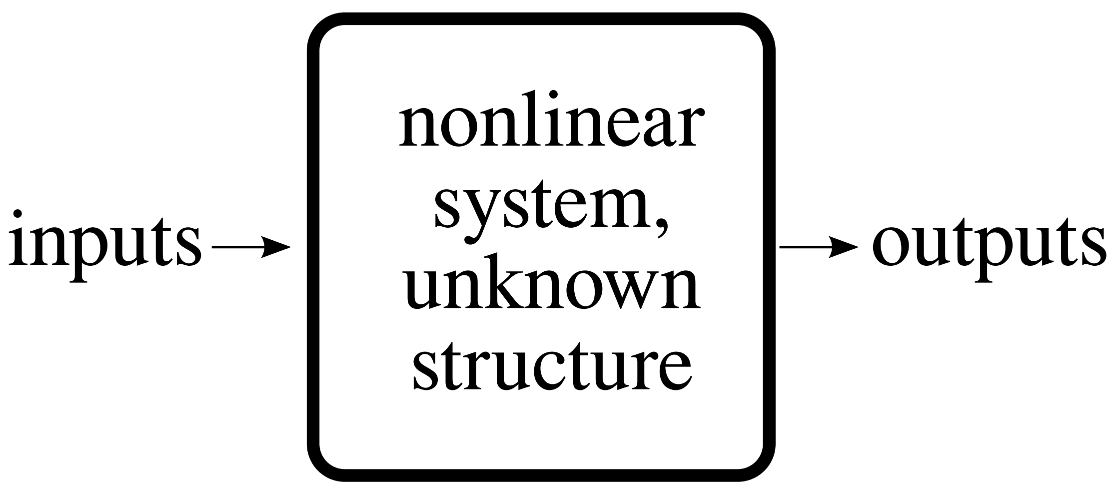

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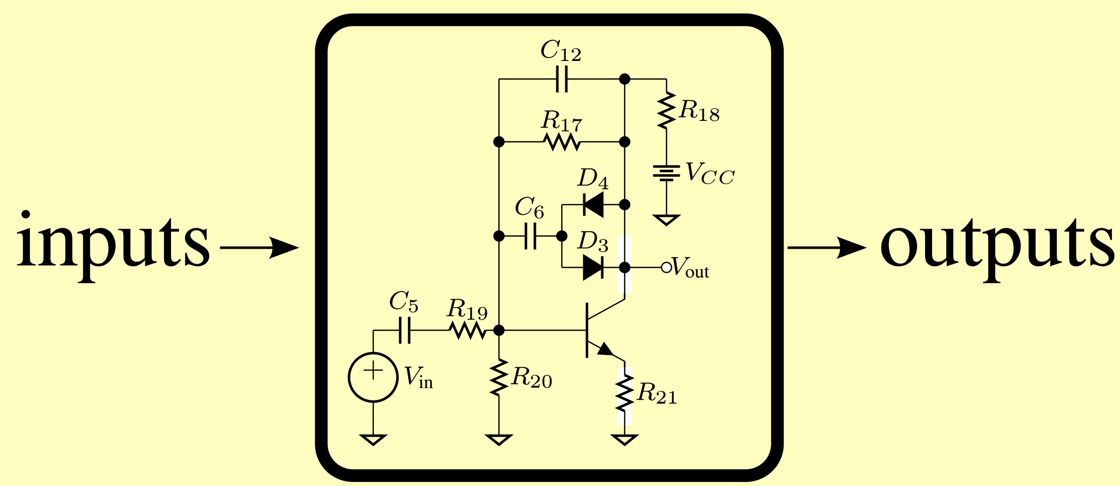

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INTRODUCTION

1. tutorial review of WDF principles
2. recent theoretical progress in WDFs
3. WDF software overview and demo

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 (*But Were Afraid to Ask)
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(and new research intro by Kurt *et al.* @ CCRMA)
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INTRODUCTION

1. Everything You Always Wanted to Know About WDFs*
 (*But Were Afraid to Ask)
2. research by DAFx folks
(and new research intro by Kurt *et al.* @ CCRMA)
3. "Please, no more math!!!"
"Just show us how to code it up…"

WAVE DIGITAL FILTER HISTORY

- 1970–1986 : early research (Alfred Fettweis *et al.*)
- 1989–present : nonlinear theory
- 1996–present : virtual analog / physical modeling applications

Wave Digital Filter Basics

WDF approach involves:

- introduction of free parameter (port resistance) at each port:

$$R_n > 0 \text{ , for each port } n$$

- introduction of wave variables:

$$a_n = v_n + R_n i_n$$
$$b_n = v_n - R_n i_n$$

- discretization of reactive elements (capacitors, inductors) using the Bilinear transformation:

$$s \leftarrow c\frac{1 - z^{-1}}{1 + z^{-1}}, \quad c = 2/T \text{ (typically)}$$

- scattering at impedance mismatches
- resolve delay-free loops by tuning port impedances

WAVE DIGITAL FILTER BASICS

closely related to **Digital Waveguides** (DWG), where:

- wave propagation characterized by *physical* transmission impedance

$$R_n > 0 \text{ , for each port } n$$

- introduction of wave variables:

$$v_n^+ = (1/2)v_n + (R_n/2)i_n$$
$$v_n^- = (1/2)v_n - (R_n/2)i_n$$

- discretization of lumped impedances (bridge, nut, etc.) using the Bilinear transformation:

$$s \leftarrow c\frac{1 - z^{-1}}{1 + z^{-1}}, \quad c = 2/T \text{ (typically)}$$

- scattering at impedance mismatches
- propagation delay decouples elements

WAVE DIGITAL FILTER BASICS

difference between **WDF**s and **DWG**s???

- *abstract* vs. *physical* meaning of port impedances
- slight different in variable definition and notation
- WDFs have an extra layer of realizability issues—they can be considered DWGs with *length-zero* transmission lines
- basic DWG formulation is *distributed*—waves are observed
- basic WDF formulation is *lumped*…why wave variables then?

Lumped Systems

"A **lumped system** is one in which the **dependent variables** of interest are a **function of time alone**. In general, this will mean solving a set of ordinary differential equations (**ODE**s)."

Stefan Bilbao and Julius O. Smith III, "MUS420/EE367A Lecture 7D: Discrete-Time Lumped Models," URL: https://ccrma.stanford.edu/~jos/NumericalInt/NumericalInt.html

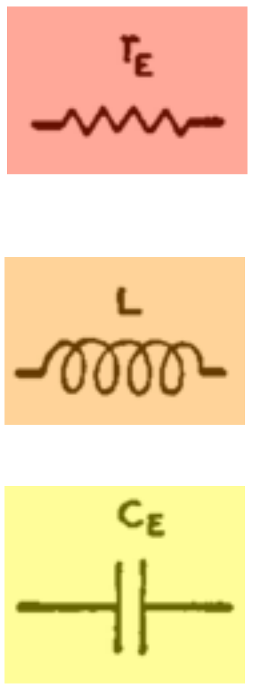
LUMPED SYSTEMS

"A **lumped system** is one in which the **dependent variables** of interest are a **function of time alone**. In general, this will mean solving a set of ordinary differential equations (**ODE**s)."

…as opposed to **distributed systems** where dependent variables are also a **function of space** (**PDE**s)…

Stefan Bilbao and Julius O. Smith III, "MUS420/EE367A Lecture 7D: Discrete-Time Lumped Models," URL: https://ccrma.stanford.edu/~jos/NumericalInt/NumericalInt.html

Lumped Elements (electrical)



ELECTRICAL

Graphical representation of the three basic elements in electrical systems.

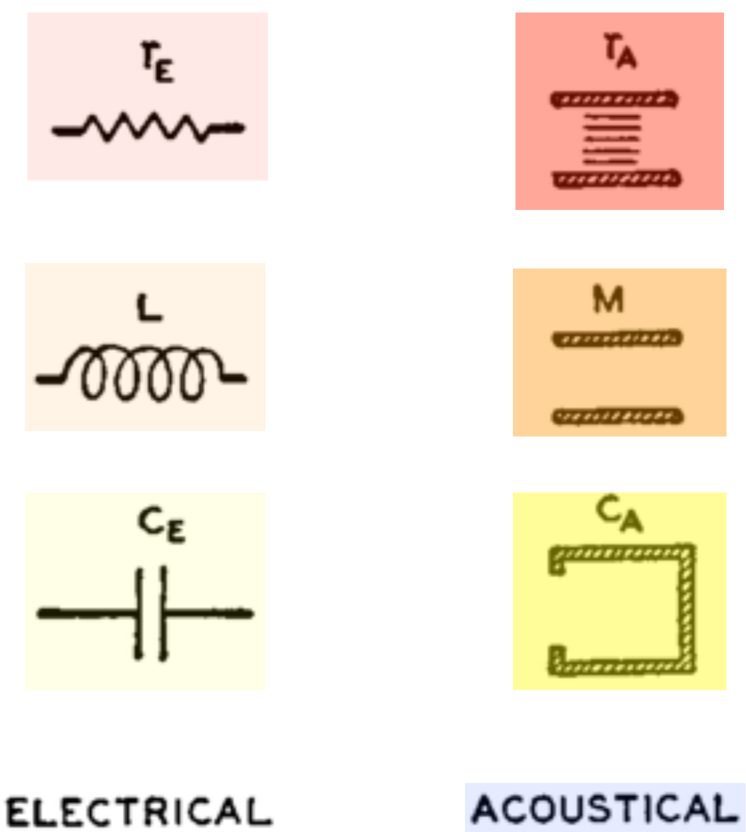
r_E = electrical resistance

L = inductance

C_E = electrical capacitance

Harry F. Olson, *Dynamical Analogies*, New York: D. Van Nostrand Company, Inc., 1943, p. 20

LUMPED ELEMENTS (acoustical)



Graphical representation of the three basic elements in electrical and acoustical systems.

r_E = electrical resistance	r_A = acoustical resistance
L = inductance	M = inertance
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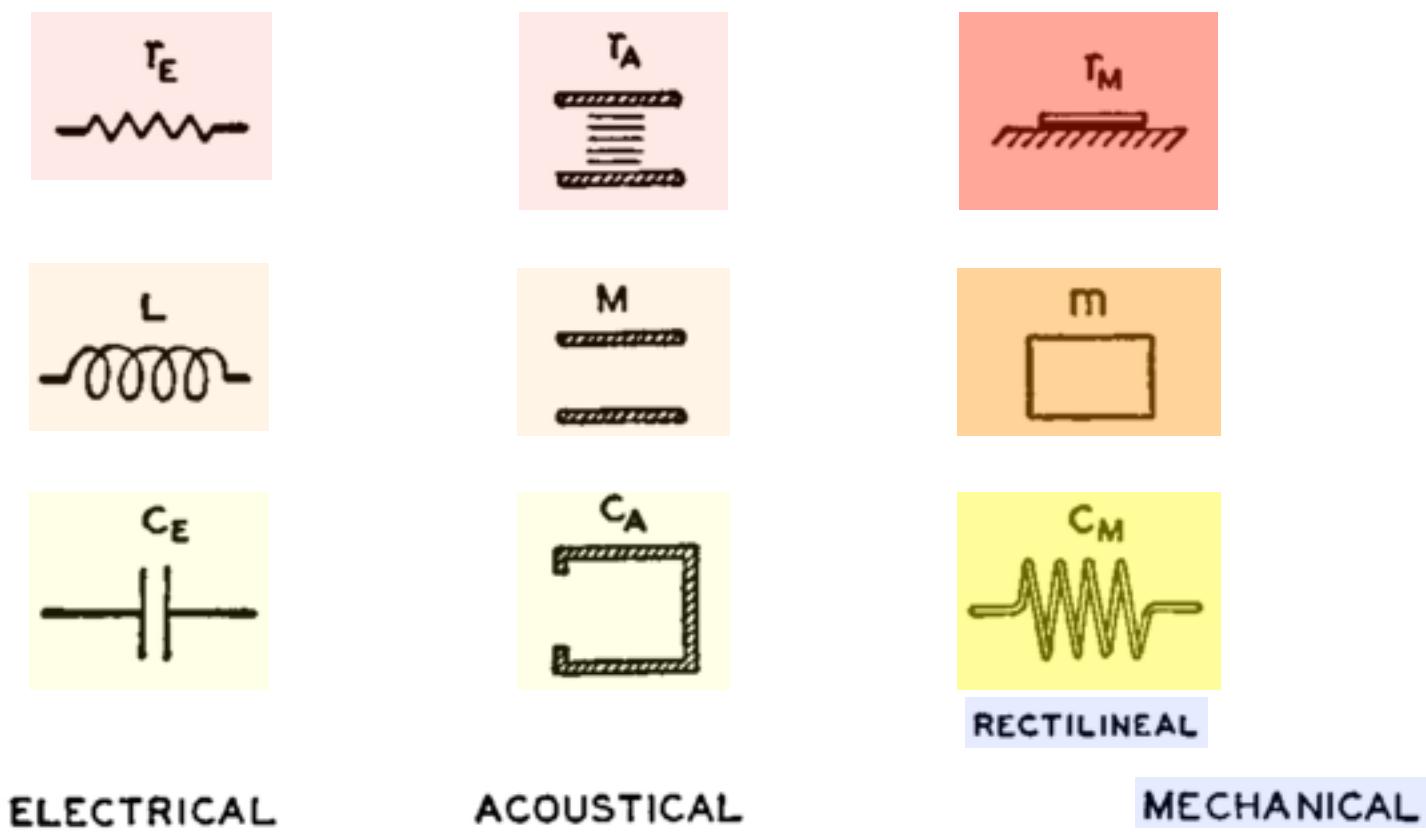
LUMPED ELEMENTS (mechanical rectilinear)

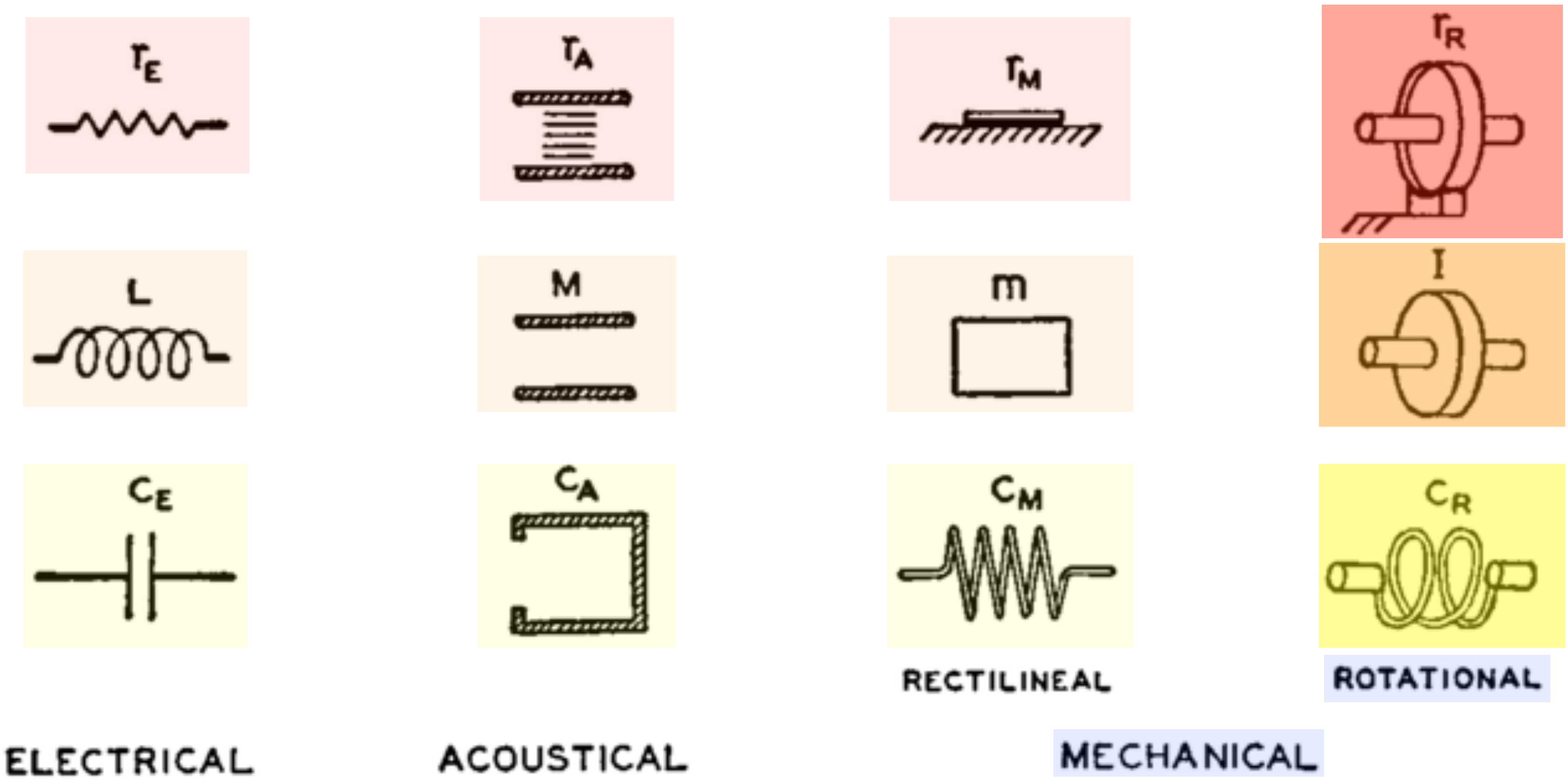


Graphical representation of the three basic elements in electrical, mechanical rectilineal, and acoustical systems.

r_E = electrical resistance	r_A = acoustical resistance	r_M = mechanical rectilineal resistance
L = inductance	M = inertance	m = mass
C_E = electrical capacitance	C_A = acoustical capacitance	C_M = compliance

Harry F. Olson, *Dynamical Analogies*, New York: D. Van Nostrand Company, Inc., 1943, p. 20

LUMPED ELEMENTS (mechanical rotational)

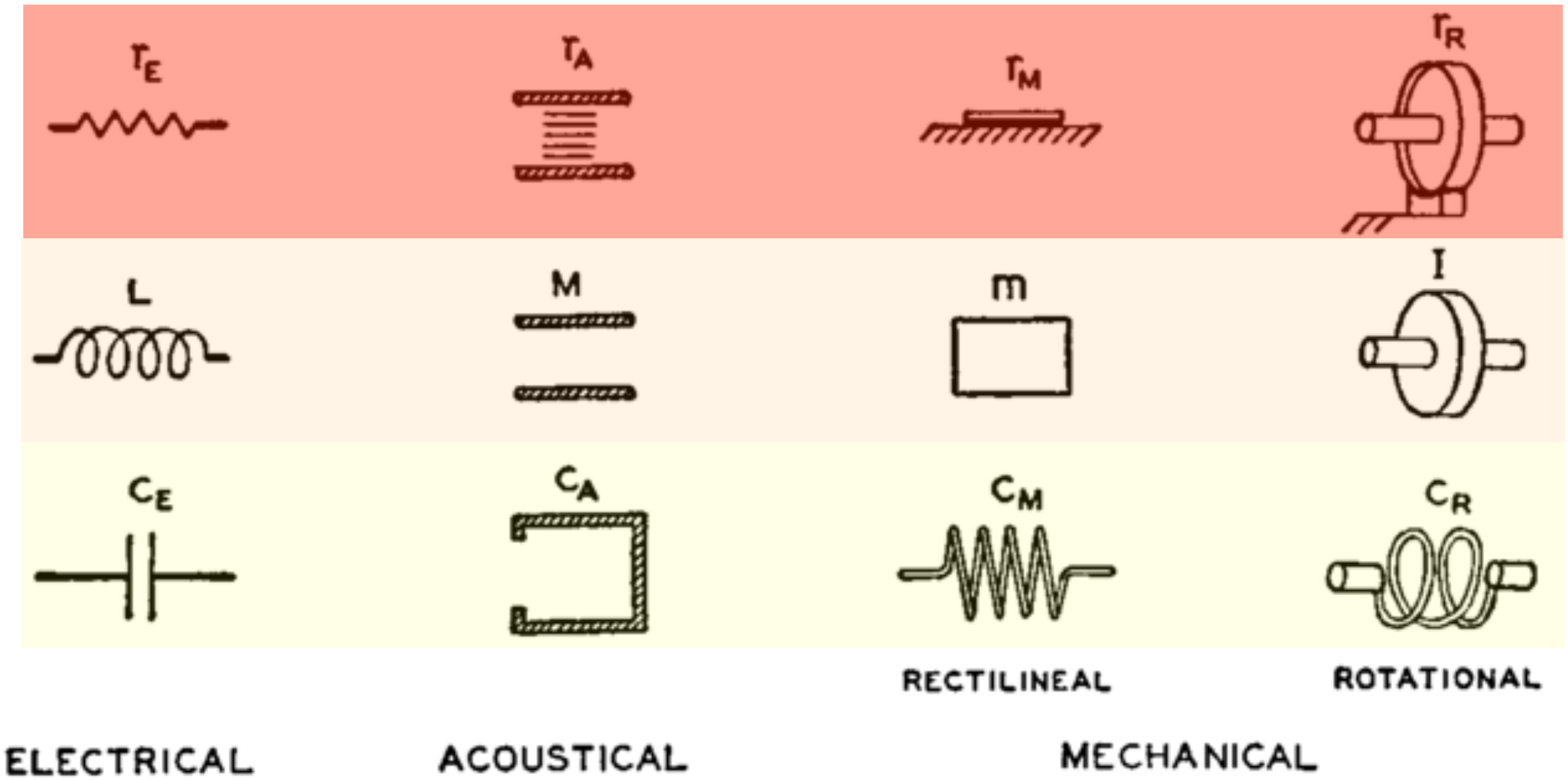


Graphical representation of the three basic elements in electrical, mechanical rectilineal, mechanical rotational and acoustical systems.

r_E = electrical resistance	r_A = acoustical resistance	r_M = mechanical rectilineal resistance	r_R = mechanical rotational resistance
L = inductance	M = inertance	m = mass	I = moment of inertia
C_E = electrical capacitance	C_A = acoustical capacitance	C_M = compliance	C_R = rotational compliance

Harry F. Olson, *Dynamical Analogies*, New York: D. Van Nostrand Company, Inc., 1943, p. 20

LUMPED ELEMENTS (equivalence across domains)



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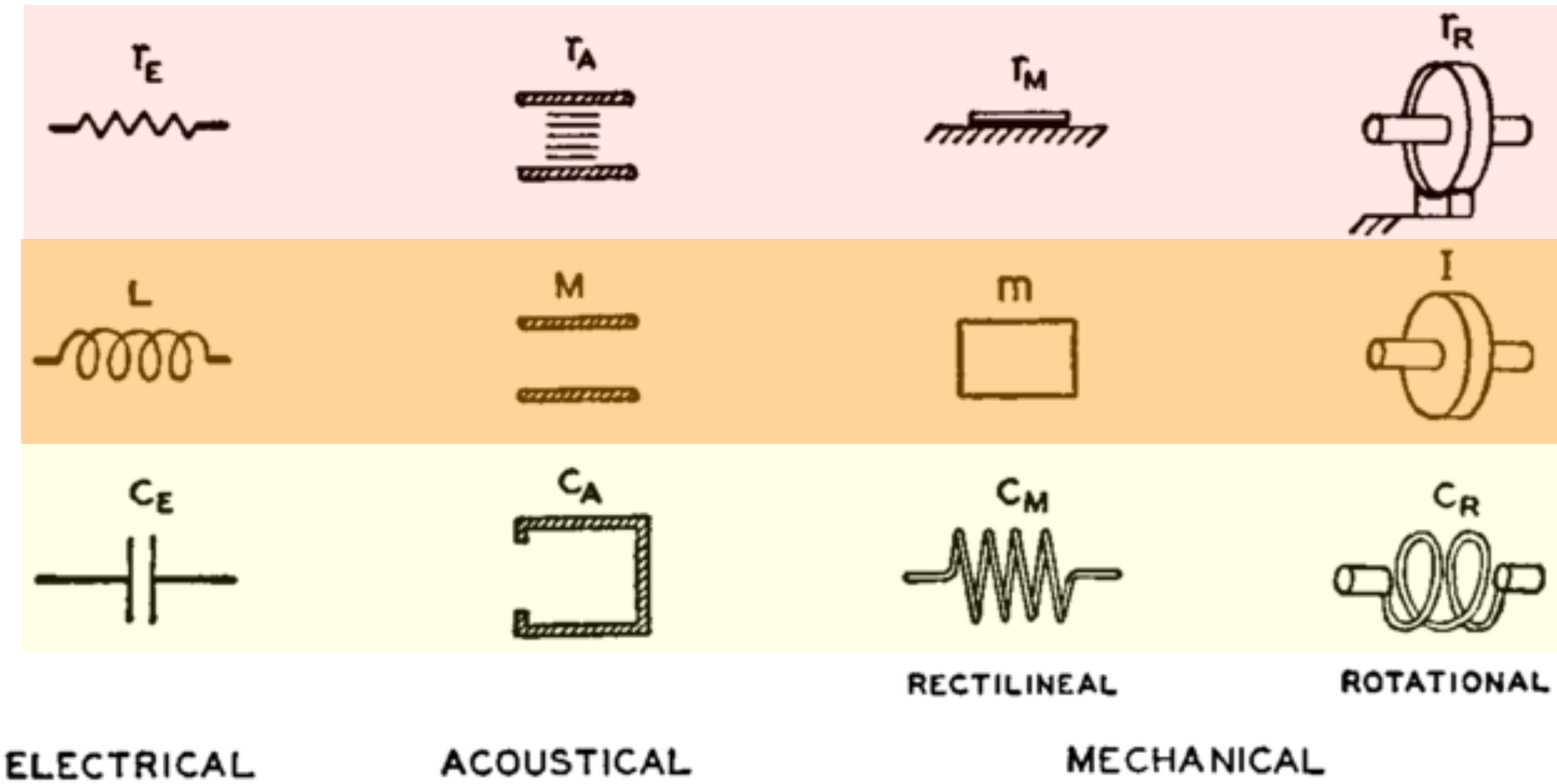

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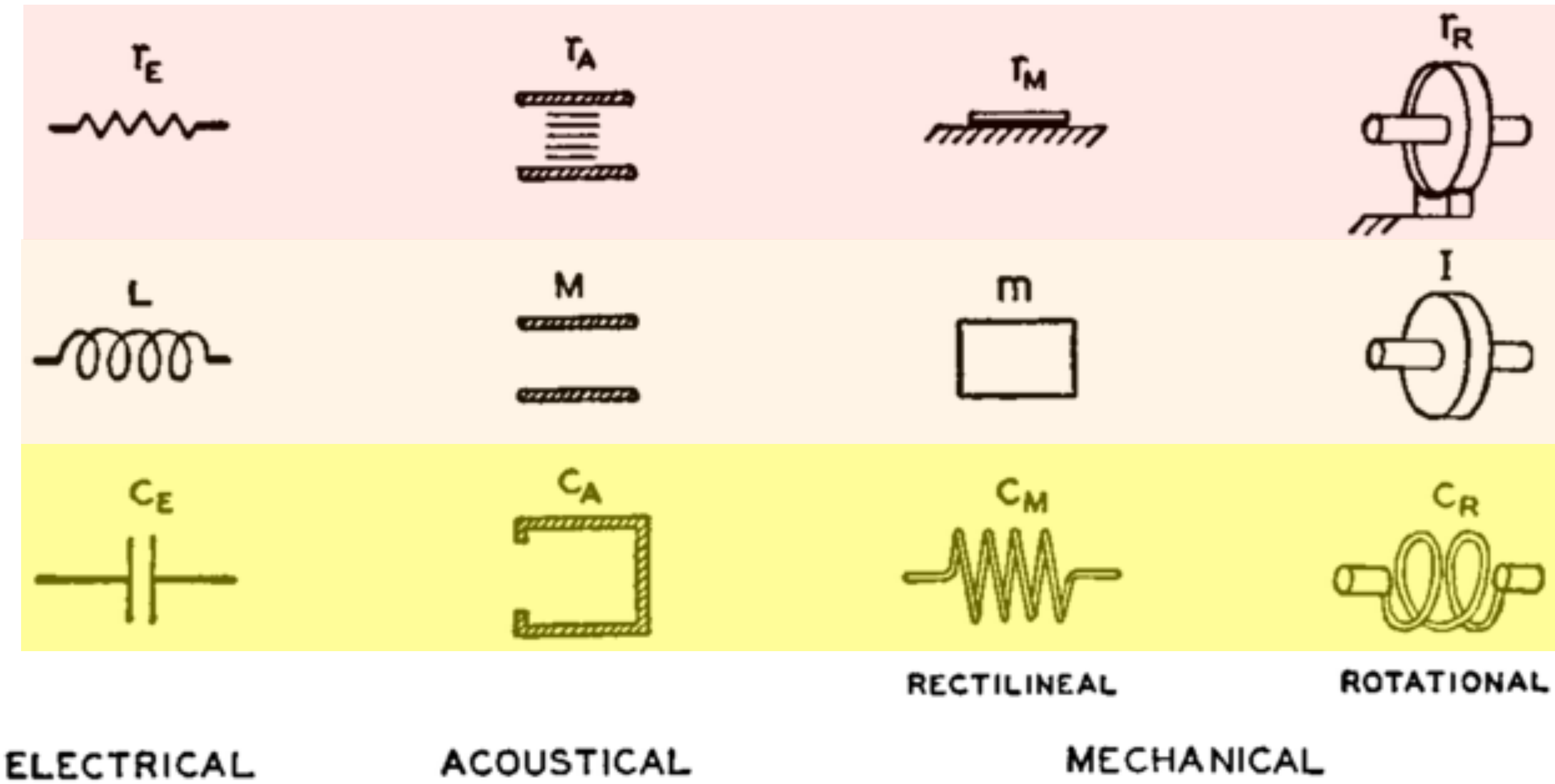

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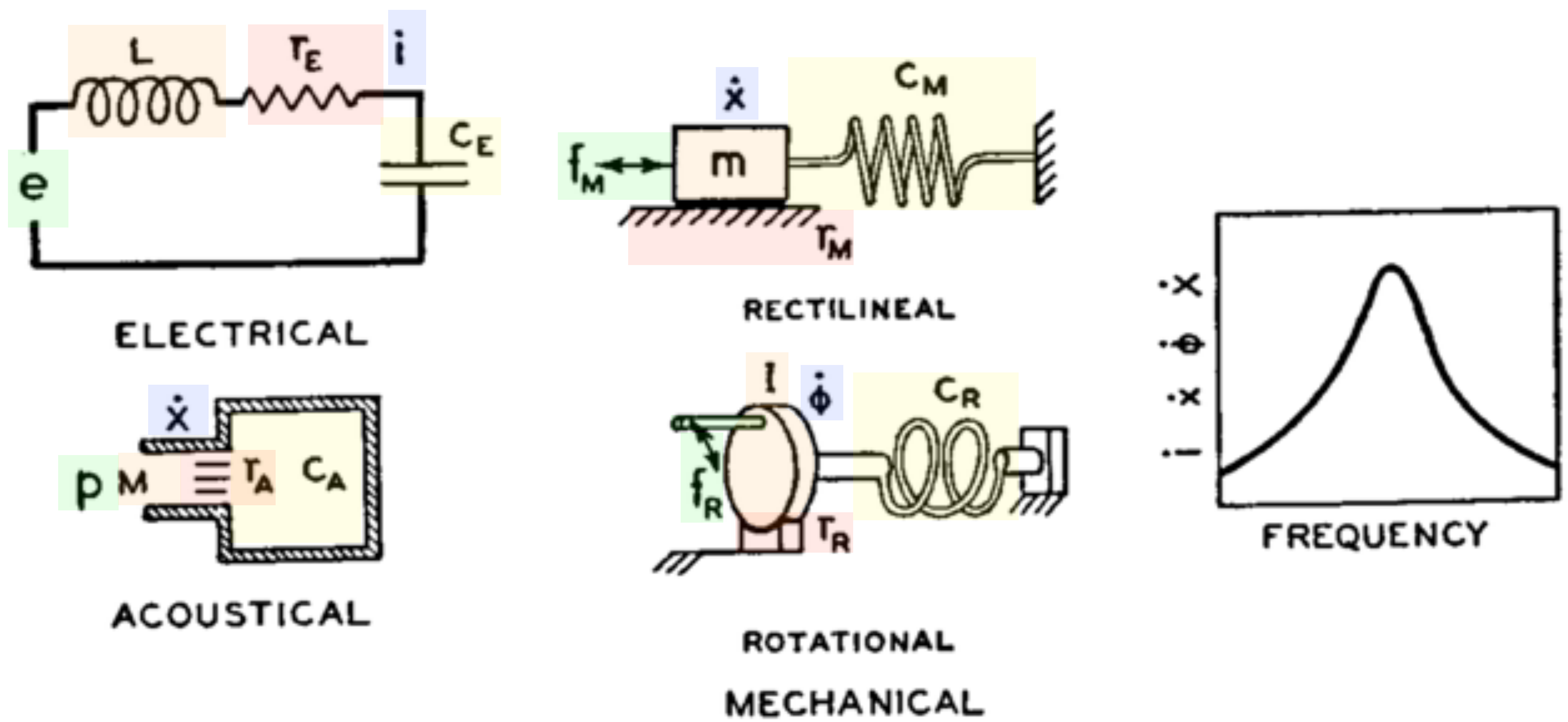
A LUMPED SYSTEM (mechanical rotational)



$$\frac{I(s)}{E(s)} = \frac{C_E s}{L C_E s^2 + C_E r_E s + 1}$$

$$\frac{\dot{X}(s)}{P(s)} = \frac{C_A s}{M C_A s^2 + C_A r_A s + 1}$$

$$\frac{\dot{X}(s)}{F_M(s)} = \frac{C_M s}{m C_M s^2 + C_M r_M s + 1}$$

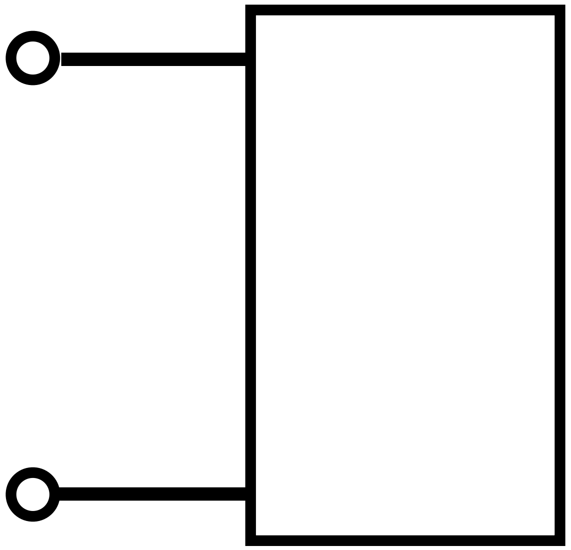
$$\frac{\dot{\Phi}(s)}{F_R(s)} = \frac{C_R s}{I C_M s^2 + C_R r_R s + 1}$$

Electrical, mechanical rectilineal, mechanical rotational and acoustical systems of one degree of freedom and the current, velocity, angular velocity and volume current response characteristics.

Harry F. Olson, *Dynamical Analogies*, New York: D. Van Nostrand Company, Inc., 1943, p. 26

NETWORK THEORY (port definition)

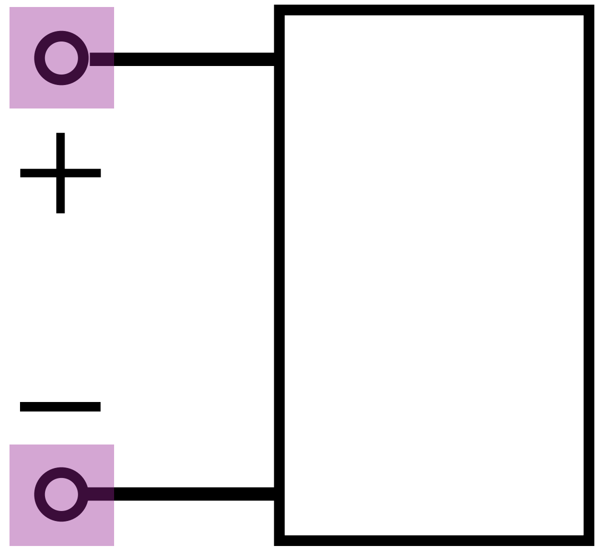
Ports have:



NETWORK THEORY (port definition)

Ports have:

- two terminals, + and –



NETWORK THEORY (port definition)

Ports have:

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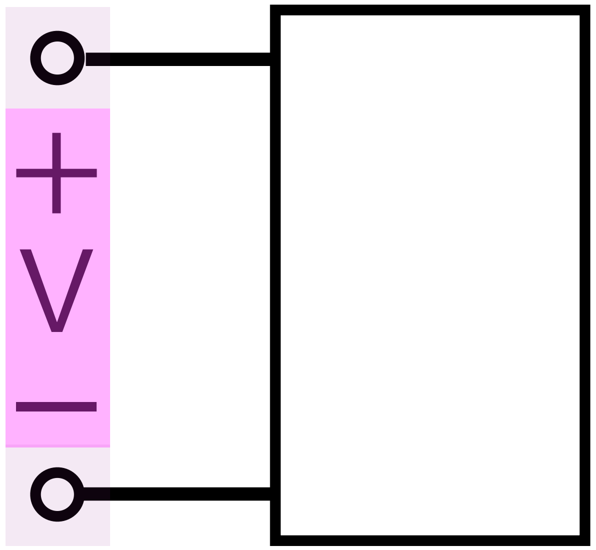
- a voltage v across the terminals



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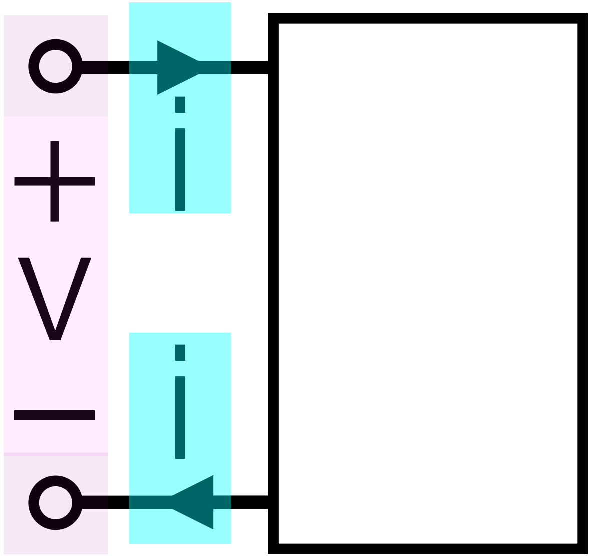

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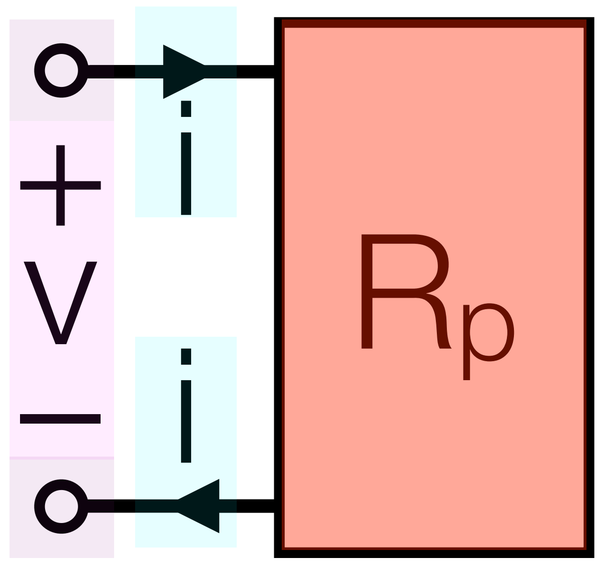

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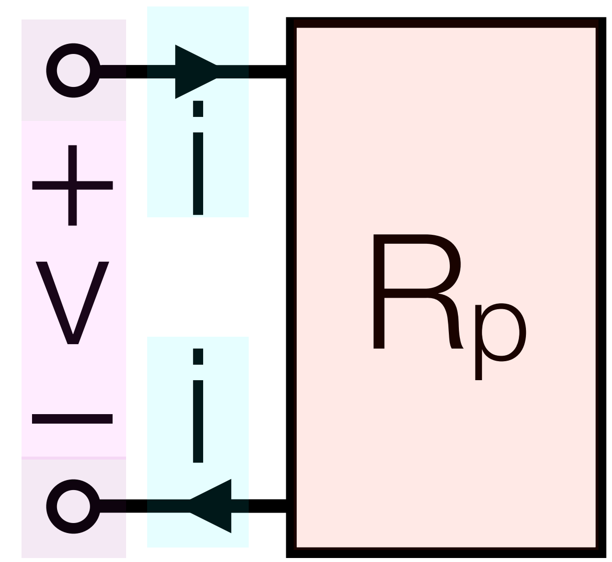
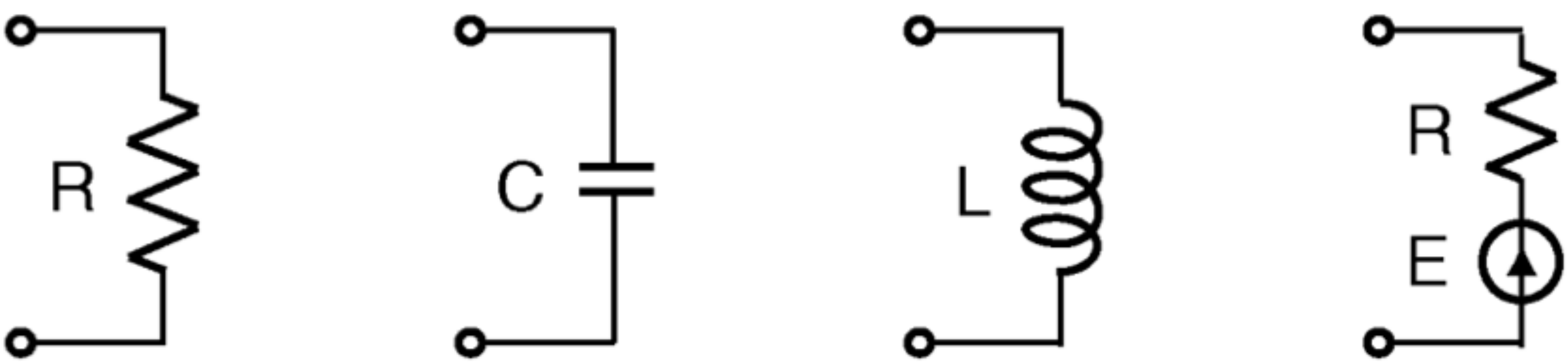


linear **One-Ports** include:



Matti Karjalainen, “Efficient realization of wave digital components for physical modeling and sound synthesis,” IEEE Transactions on Audio, Speech, and language Processing, July 2008.

NETWORK THEORY (n-ports)

- connected ports have equal port resistance

- 2-ports (e.g. transformers, parallel/series connections)
- 3-ports (e.g. parallel/series connections)
- 4+ ports, etc.

- mismatches of port resistance and topological aspects handled by “adaptors”, where “scattering” of wave variables occurs

DISCRETIZATION

replace all continuous-time derivatives **s** on Laplace plane *with discrete-time approximations (in delays* $\mathbf{z^{-1}}$*)*

forward Euler $s \leftarrow \frac{1 - z^{-1}}{T z^{-1}}$

backward Euler $s \leftarrow \frac{1 - z^{-1}}{T}$

bilinear transform $s \leftarrow c\frac{1 - z^{-1}}{1 + z^{-1}}, \; c = 2/T \text{ (typically)}$

François Germain and Kurt James Werner, "Design Principles for Lumped Model Discretisation Using Möbius Transforms," in proc. Int. Conf. on Digital Audio Effects (DAFx-15), Trondheim, Norway, Nov. 30 – Dec. 3, 2015.

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specific conformal maps / Möbius transform

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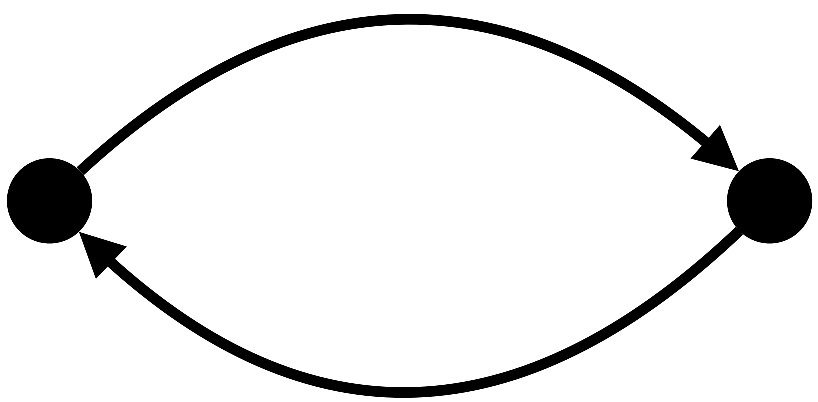
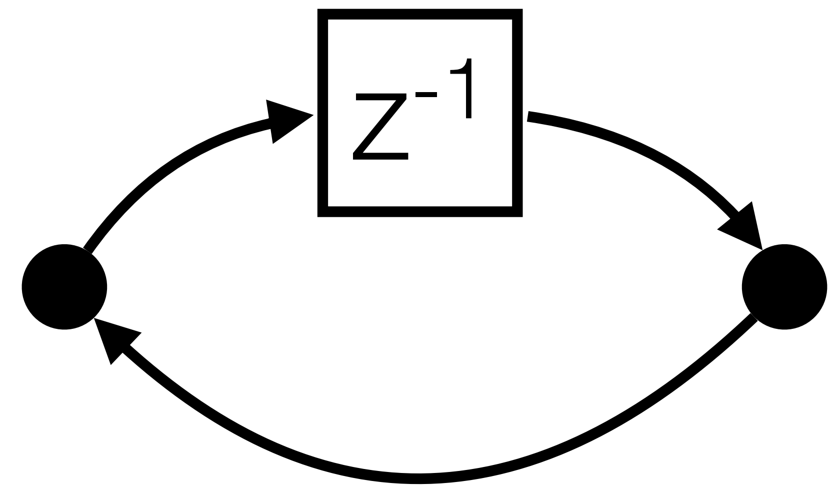
DELAY-FREE LOOPS

mutual, instantaneous dependence
or "delay-free loop" (**implicit**)



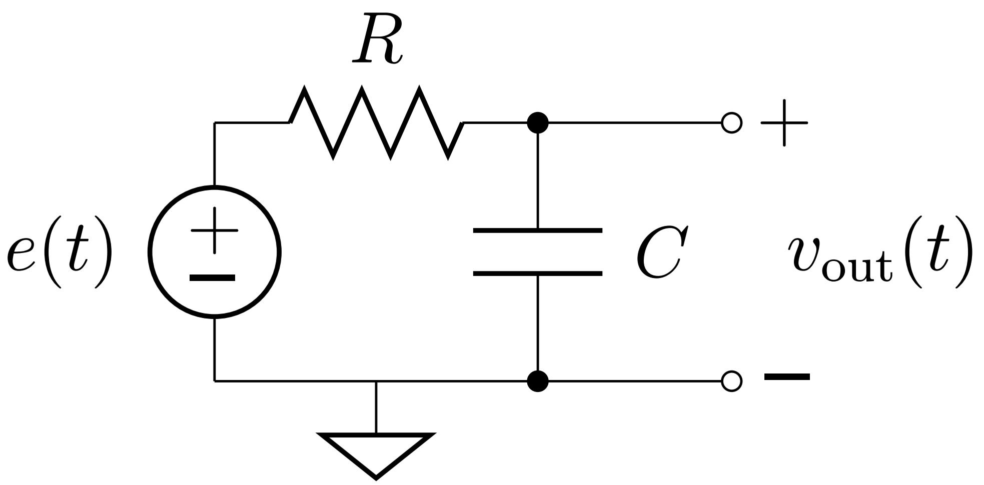
not OK

delay makes things
computable (**explicit**)



OK

DISCRETIZE RC NETWORK (traditional approach)



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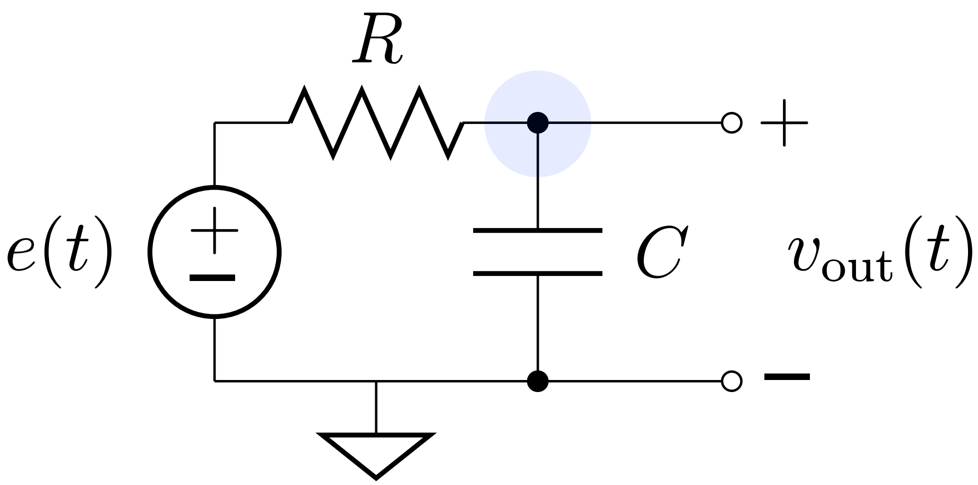

KCL at "+" node

$$\frac{V_{\text{out}}(s)}{\frac{1}{sC}} = \frac{E(s) - V_{\text{out}}(s)}{R}$$

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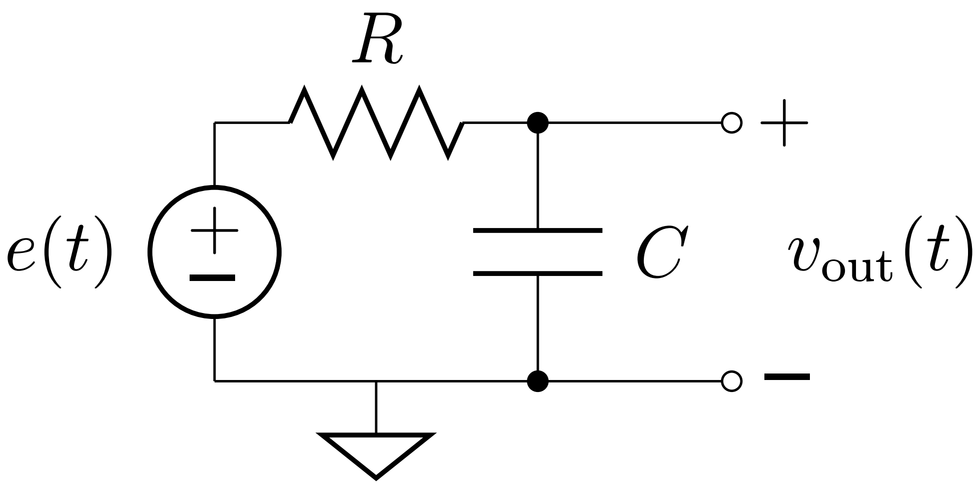

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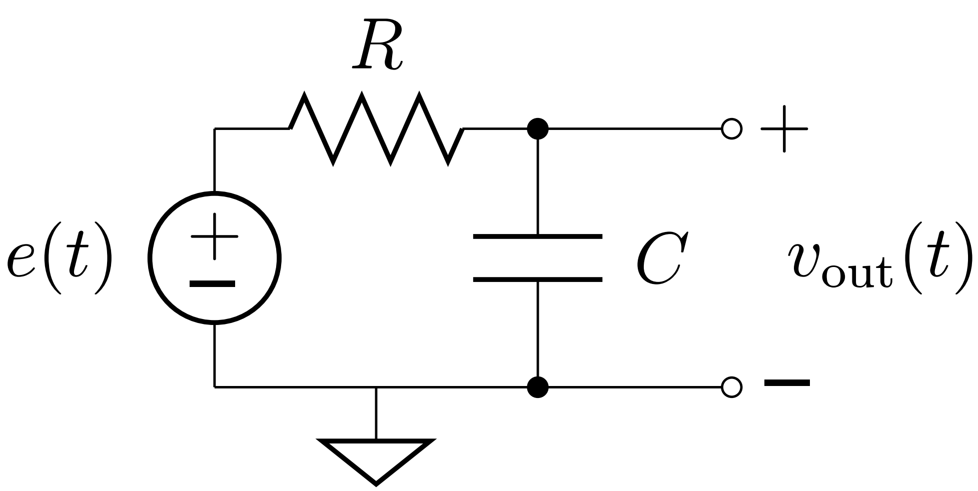
$$V_{\text{out}}(s)\,(sRC + 1) = E(s)$$

s-plane transfer function

$$H(s) = \frac{V_{\text{out}}(s)}{E(s)} = \frac{1}{sRC + 1}$$

Discretize RC Network (traditional approach)



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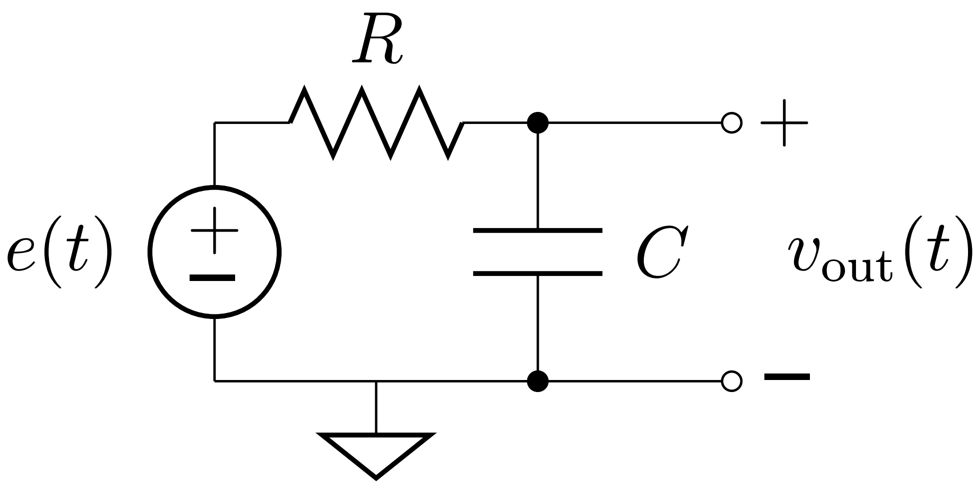

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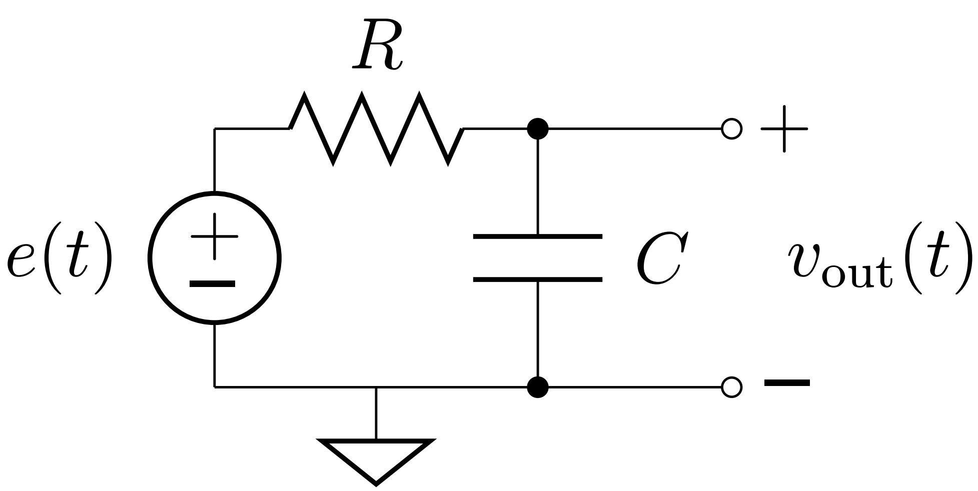

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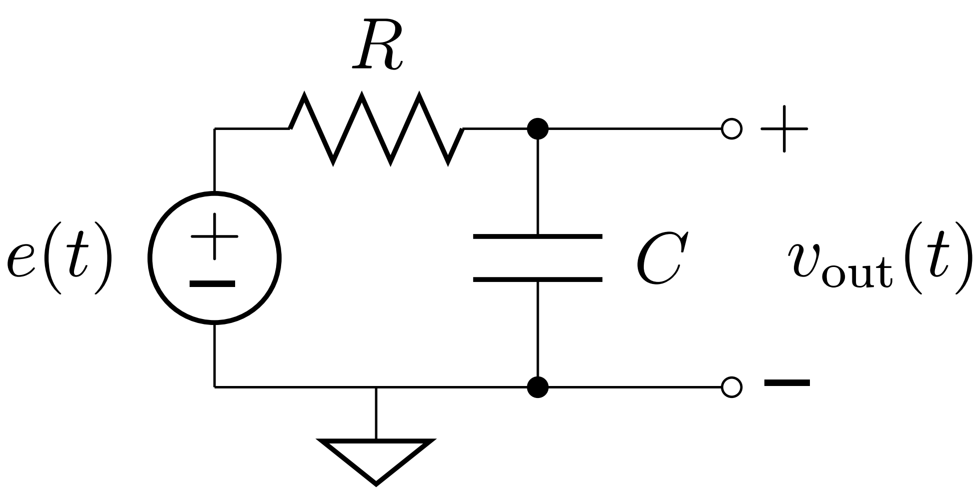
$$\frac{V_{\text{out}}(z)}{E(z)} = \frac{1 + z^{-1}}{\frac{2RC}{T} + 1 + \left(1 - \frac{2RC}{T}\right) z^{-1}}$$

z-plane transfer function

$$H(z) = \frac{V_{\text{out}}(z)}{E(z)} = \frac{\frac{T}{T+2RC} + \frac{T}{T+2RC} z^{-1}}{1 + \frac{T-2RC}{T+2RC} z^{-1}}$$

Julius O. Smith III, *Physical Audio Signal Processing*, URL: https://ccrma.stanford.edu/~jos/pasp/Bilinear_Transformation.html

DISCRETIZE RC NETWORK (traditional approach)



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$$\frac{V_{\text{out}}(z)}{E(z)} = \frac{1}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}RC + 1}$$

$$\frac{V_{\text{out}}(z)}{E(z)} = \frac{1 + z^{-1}}{\frac{2RC}{T} + 1 + \left(1 - \frac{2RC}{T}\right) z^{-1}}$$

z-plane transfer function

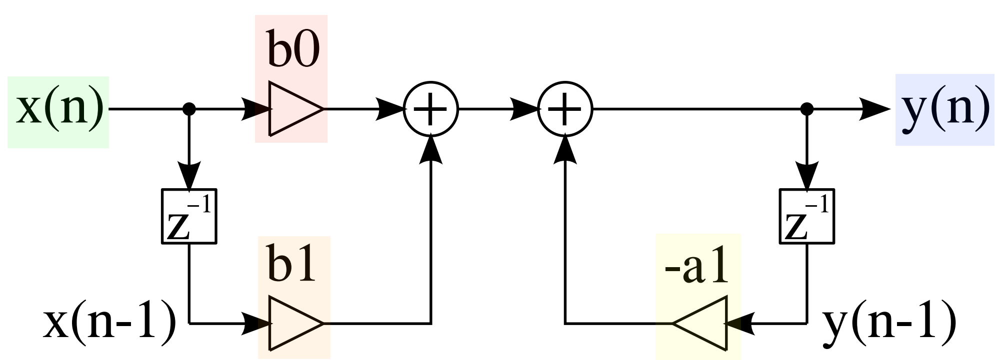
$$H(z) = \frac{V_{\text{out}}(z)}{E(z)} = \frac{\frac{T}{T+2RC} + \frac{T}{T+2RC} z^{-1}}{1 + \frac{T-2RC}{T+2RC} z^{-1}}$$

how can we realize this transfer function in code???

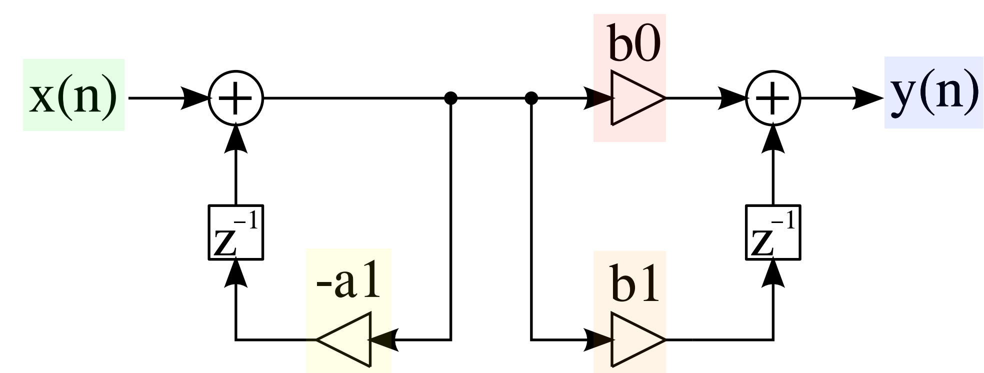
Julius O. Smith III, *Physical Audio Signal Processing*, URL: https://ccrma.stanford.edu/~jos/pasp/Bilinear_Transformation.html

DISCRETIZE RC NETWORK (traditional approach)

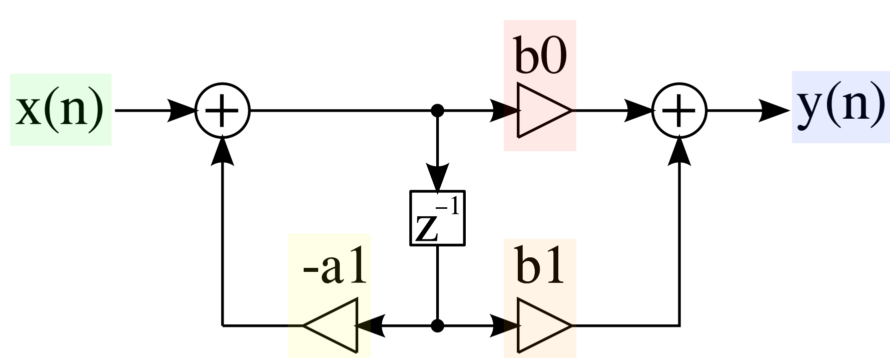
$$H(z) = \frac{V_{\text{out}}(z)}{E(z)} = \frac{\frac{T}{T+2RC} + \frac{T}{T+2RC} z^{-1}}{1 + \frac{T-2RC}{T+2RC} z^{-1}} = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}} \text{ with } \begin{cases} b_0 = \frac{T}{T+2RC} \\ b_1 = \frac{T}{T+2RC} \\ a_1 = \frac{T-2RC}{T+2RC} \end{cases}$$



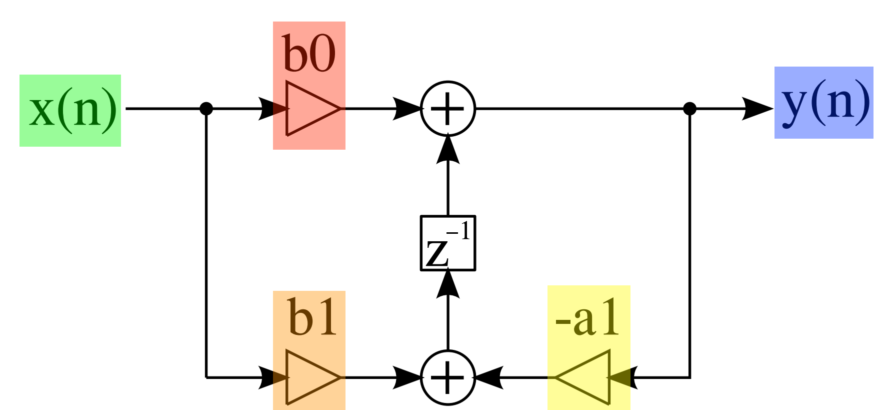
direct form I



transposed direct form I



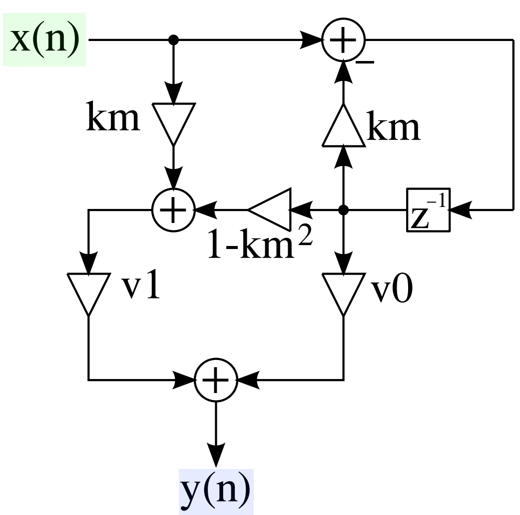
direct form II



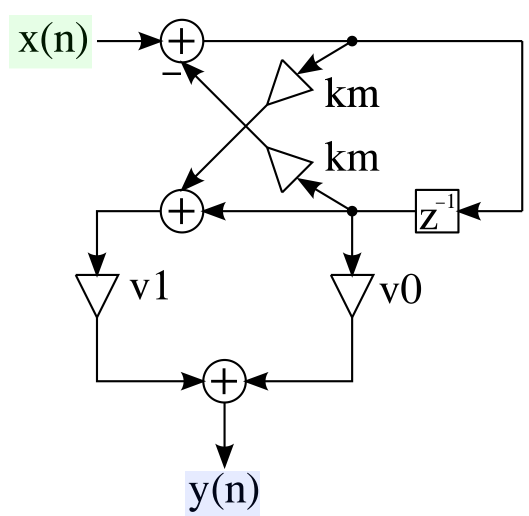
transposed direct form II

DISCRETIZE RC NETWORK (traditional approach)

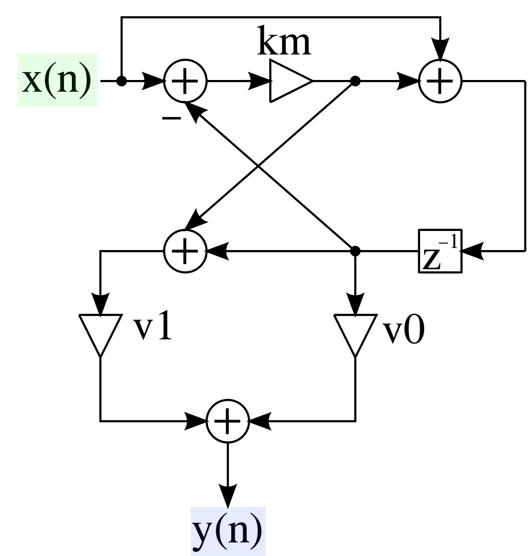
$$H(z) = \frac{V_{\text{out}}(z)}{E(z)} = \frac{\frac{T}{T+2RC} + \frac{T}{T+2RC} z^{-1}}{1 + \frac{T-2RC}{T+2RC} z^{-1}} = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}} \quad \text{with} \quad \begin{cases} b_0 = \frac{T}{T+2RC} \\ b_1 = \frac{T}{T+2RC} \\ a_1 = \frac{T-2RC}{T+2RC} \end{cases}$$



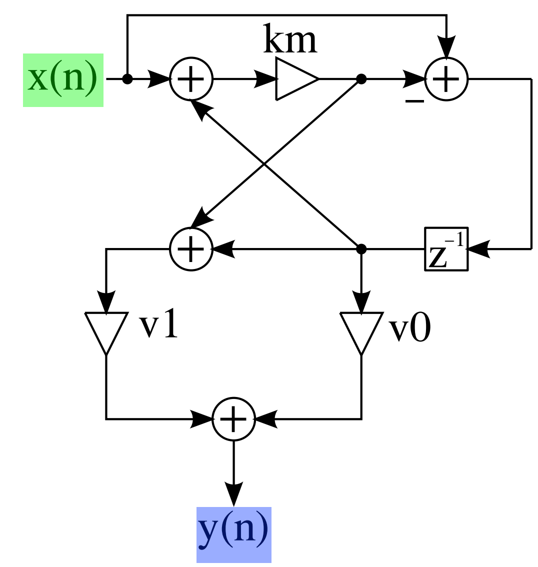
three multiplier form



two multiplier form



one multiplier form (A)



one multiplier form (B)

A. H. Gray, Jr. and John D. Markel, “Digital Lattice and Ladder Filter Synthesis,” IEEE Transactions on Audio and Electroacoustics, Vol. AU-21, No. 6, December 1975

DISCRETIZE RC NETWORK (traditional approach)

- need transfer function
- factor into biquads if high order
- have to choose form for desired properties

DISCRETIZE RC NETWORK (traditional approach)

- need transfer function
- factor into biquads if high order
- have to choose form for desired properties
- issues with time-varying circuits

Aaron Wishnick, “Time-Varying Filters for Musical Applications,” in proc. Int. Conf. on Digital Audio Effects (DAFx-14), Erlangen, Germany, September 1–5, 2014.

DISCRETIZE RC NETWORK (attempt modular)

what if…

- modular / topology-preserving?
- reusable?
- skip transfer function representation?

DISCRETIZE RC NETWORK (attempt modular)

what if…

- modular / topology-preserving?
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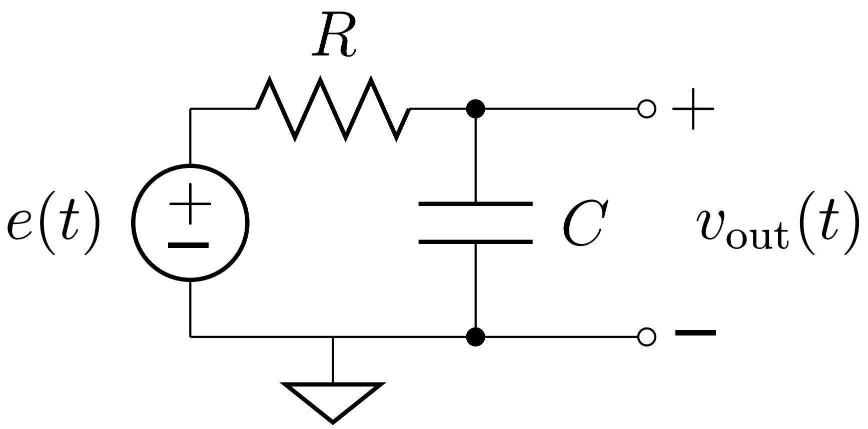
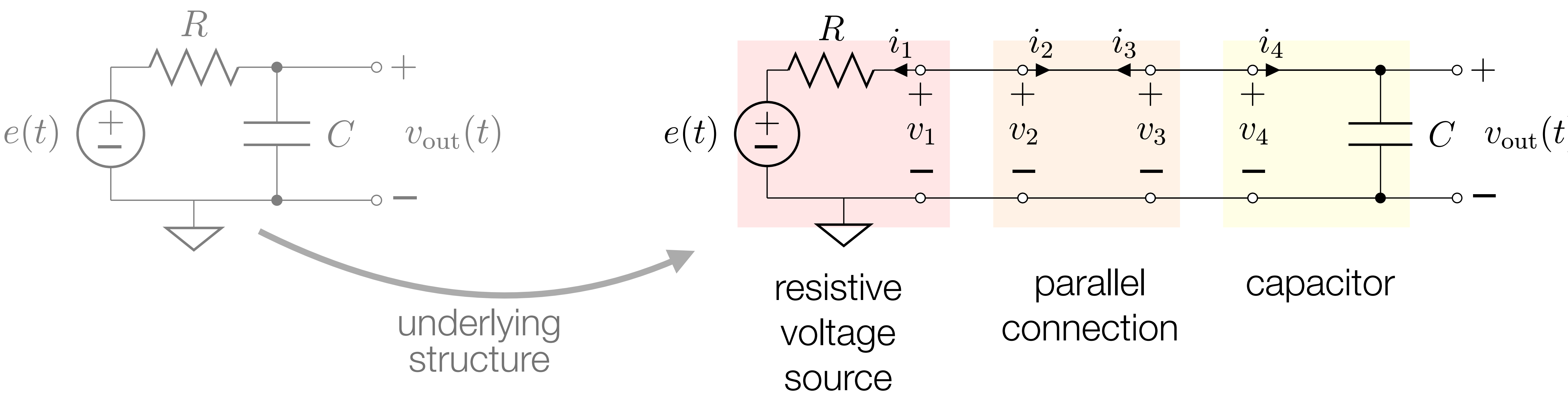
spoiler alert : **this <u>won't</u> work** in the Kirchhoff domain…

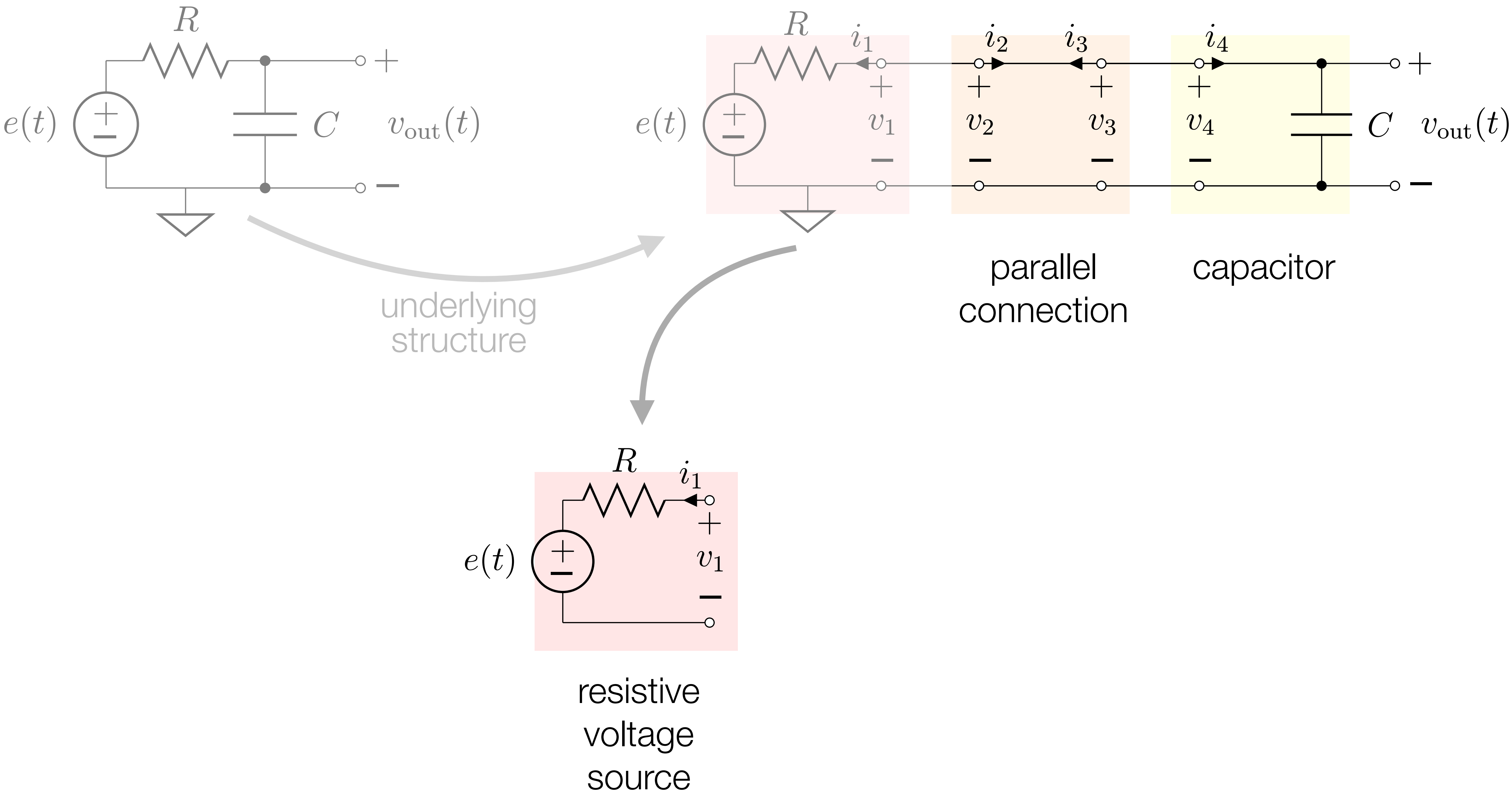
DISCRETIZE RC NETWORK (attempt modular)



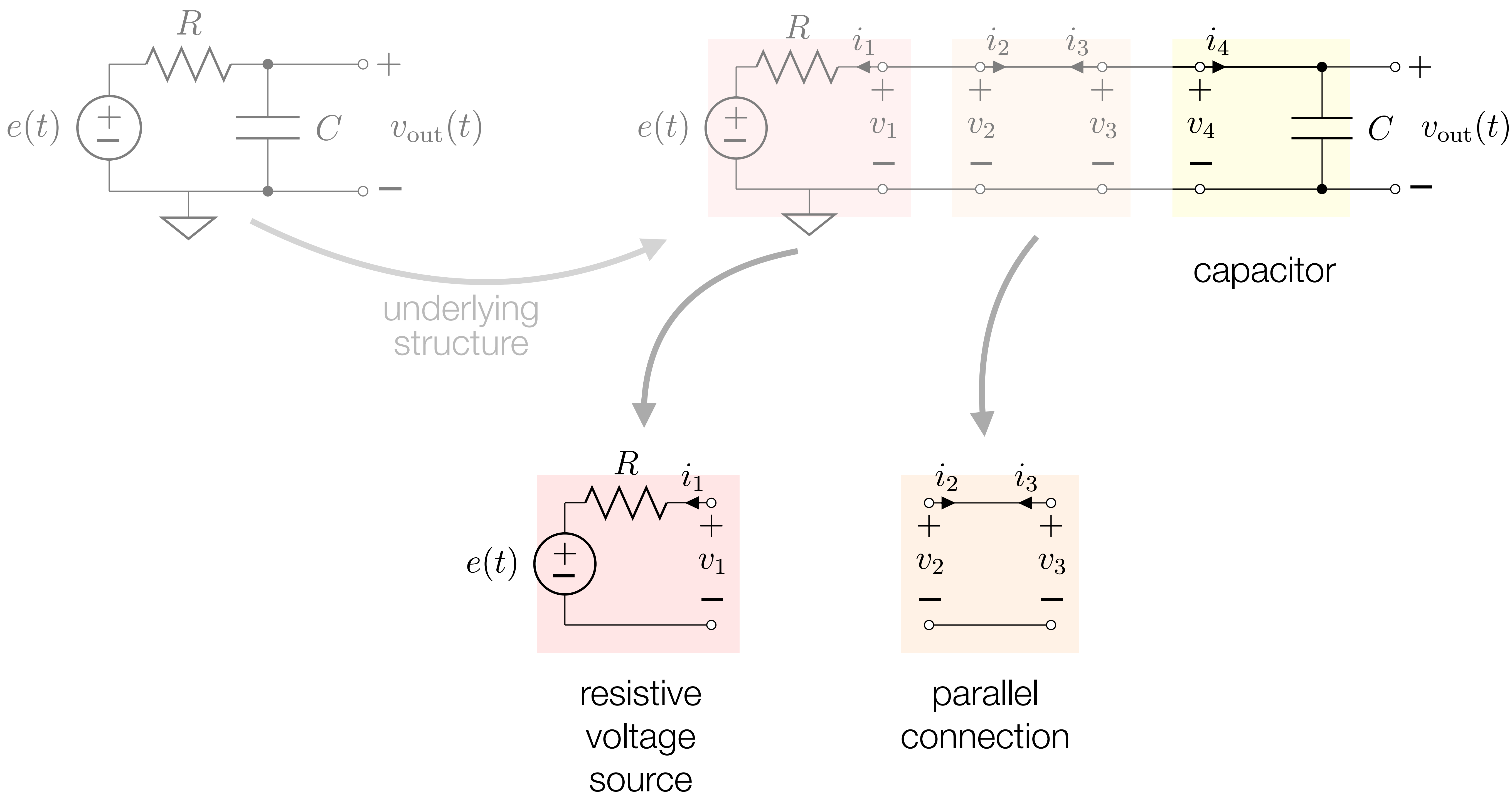
DISCRETIZE RC NETWORK (attempt modular)



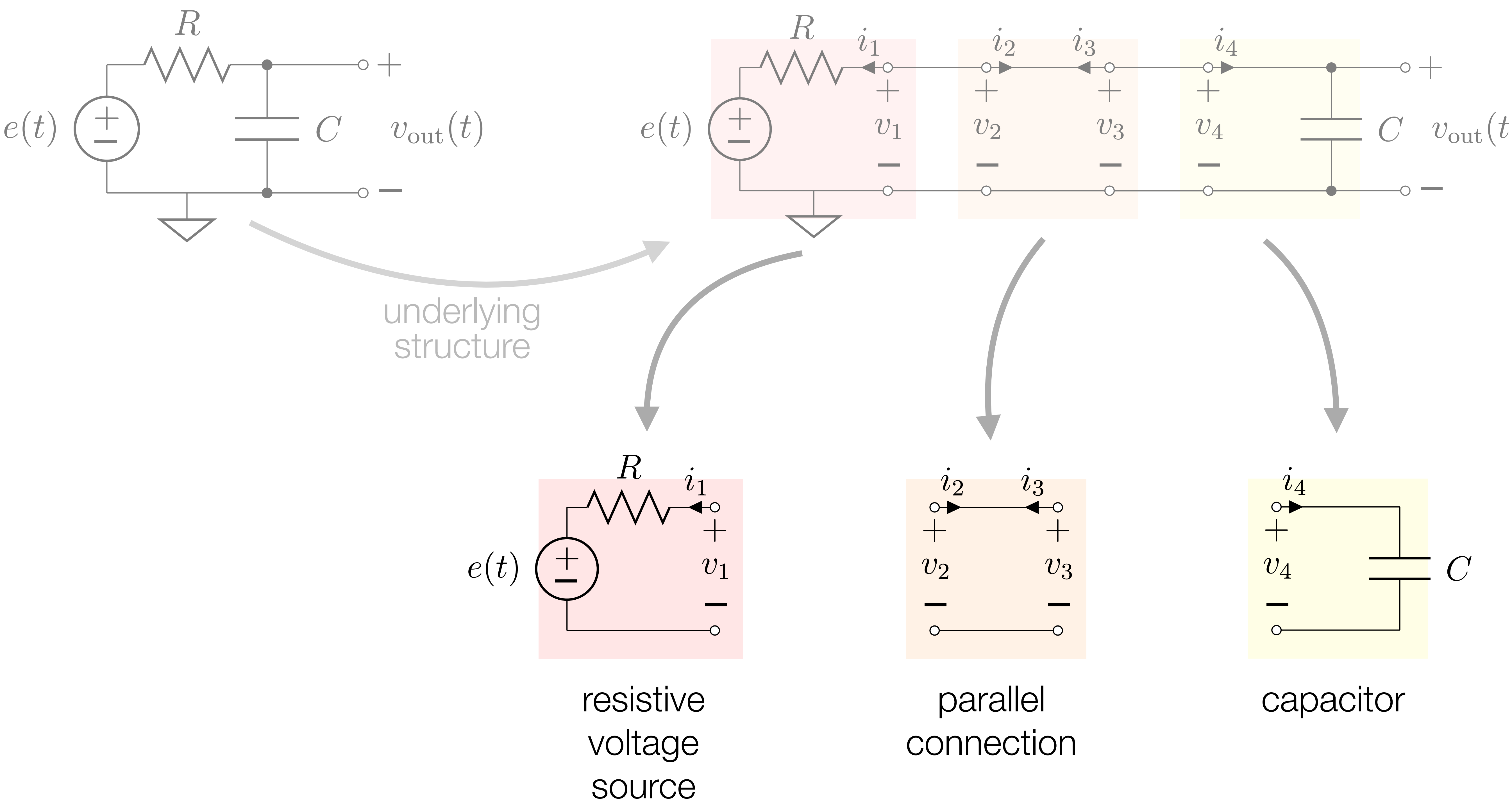
DISCRETIZE RC NETWORK (attempt modular)



Discretize RC Network (attempt modular)

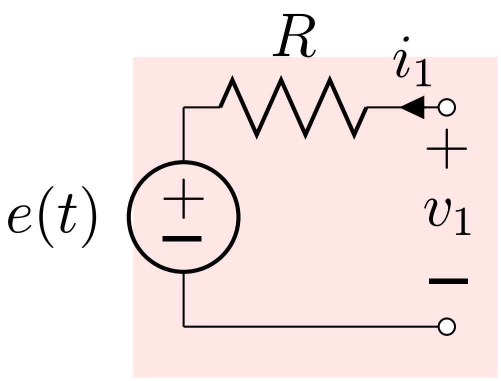


DISCRETIZE RC NETWORK (attempt modular)

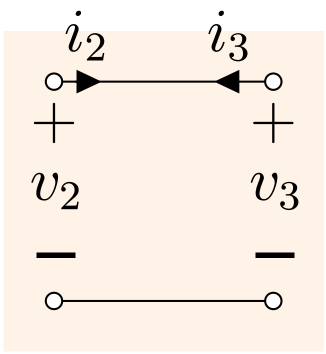


DISCRETIZE RC NETWORK (attempt modular)

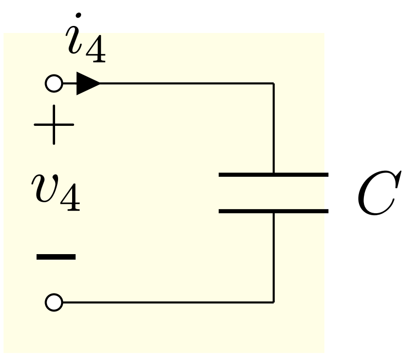
resistive
voltage
source



parallel
connection

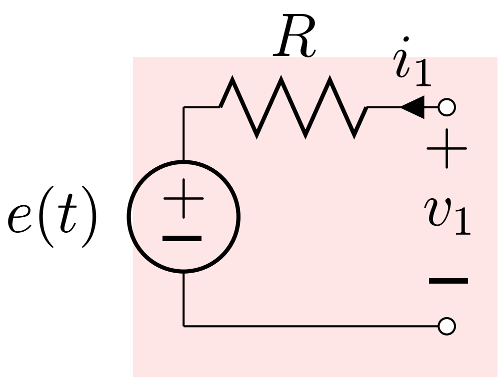


capacitor

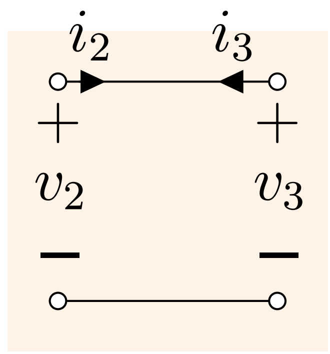


DISCRETIZE RC NETWORK (attempt modular)

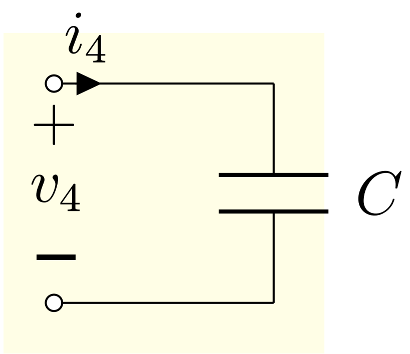
resistive voltage source



parallel connection



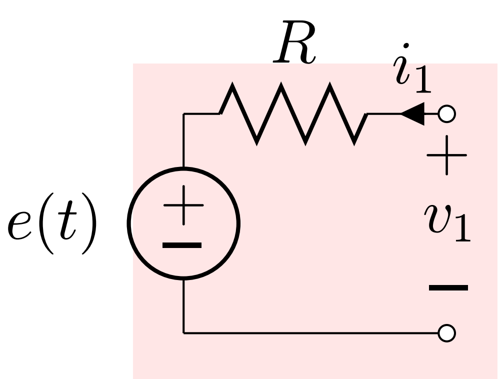
capacitor



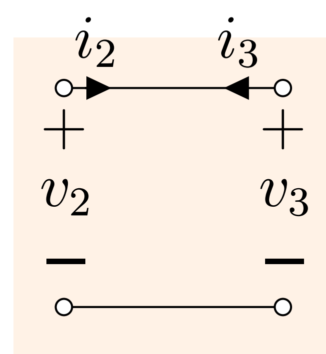
$$v_1 = e(t) + R \cdot i_1$$

DISCRETIZE RC NETWORK (attempt modular)

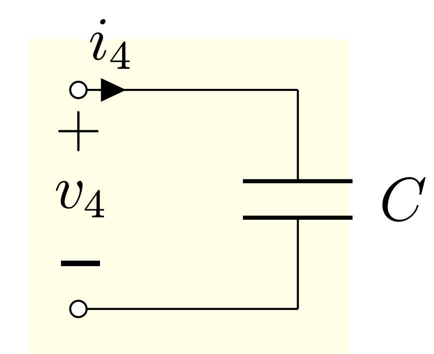
resistive
voltage
source



parallel
connection



capacitor



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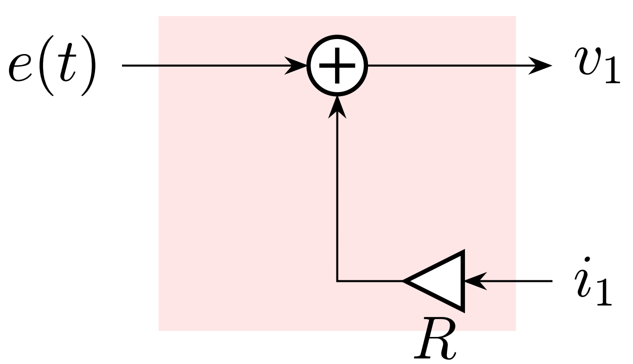
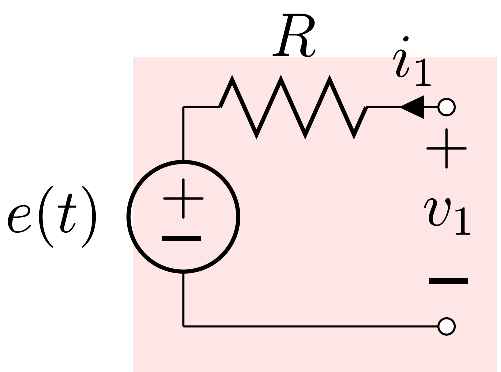
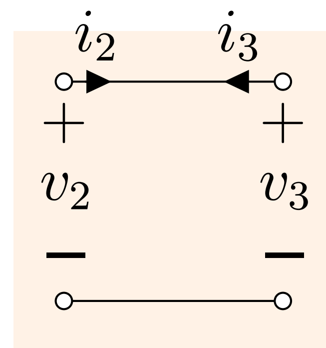


DISCRETIZE RC NETWORK (attempt modular)

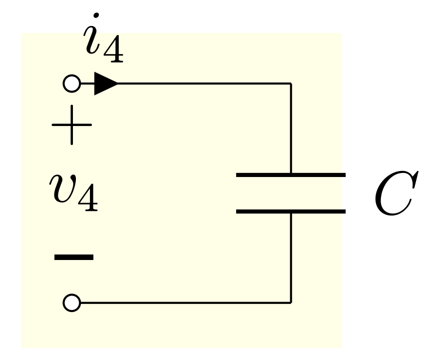
resistive voltage source



parallel connection



capacitor



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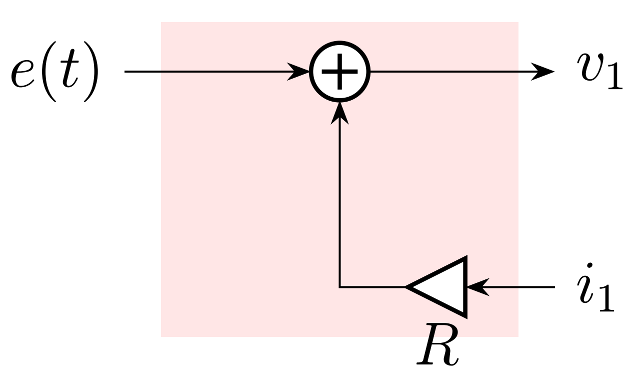
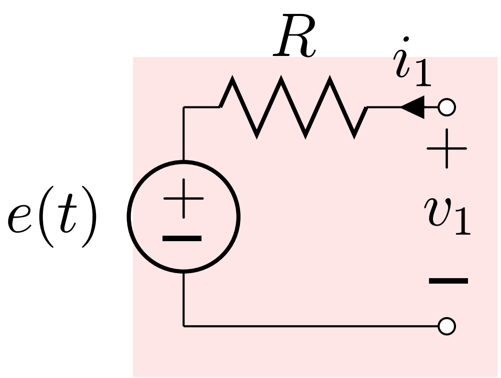


$$v_3 = v_2$$

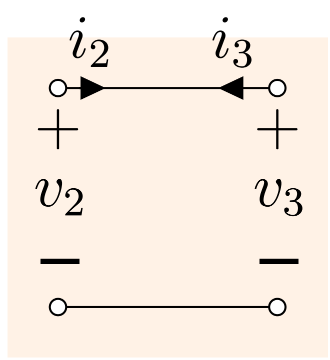
$$i_2 = -i_3$$

DISCRETIZE RC NETWORK (attempt modular)

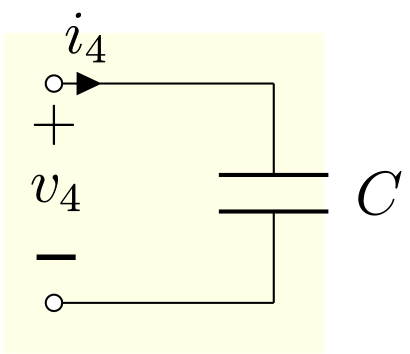
resistive voltage source



parallel connection



capacitor



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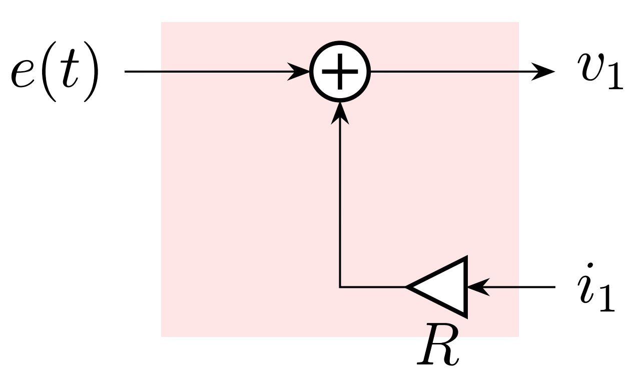
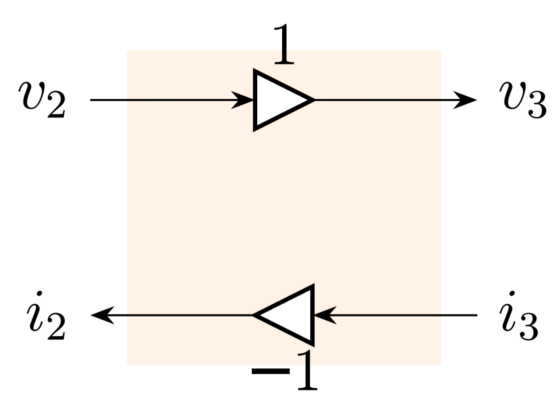
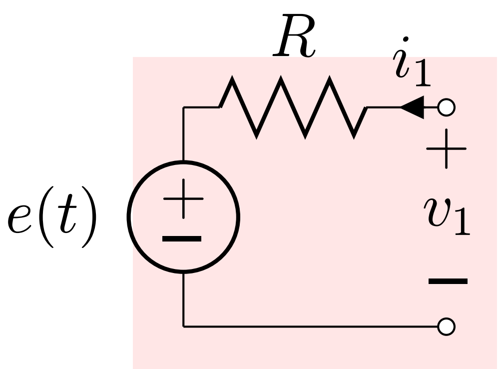




DISCRETIZE RC NETWORK (attempt modular)

resistive voltage source



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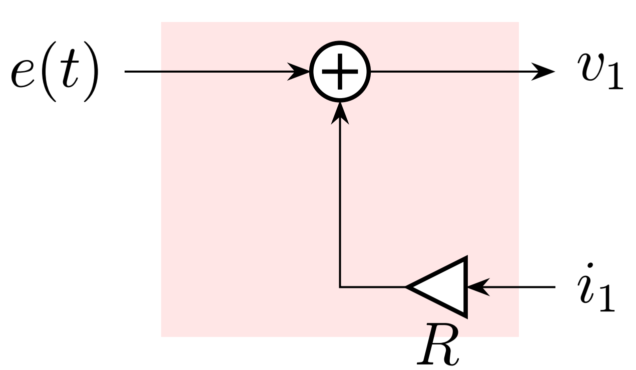
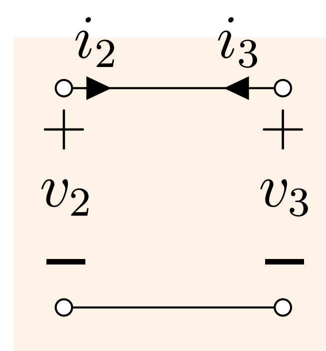


parallel connection



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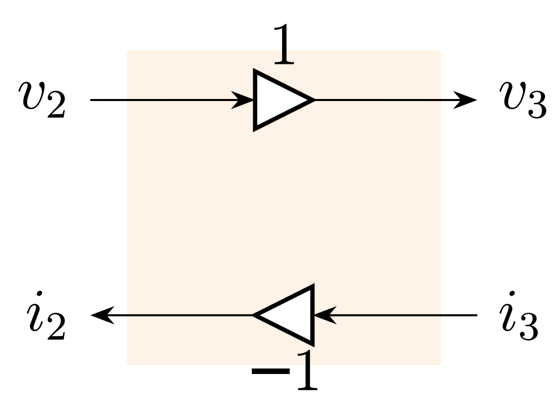
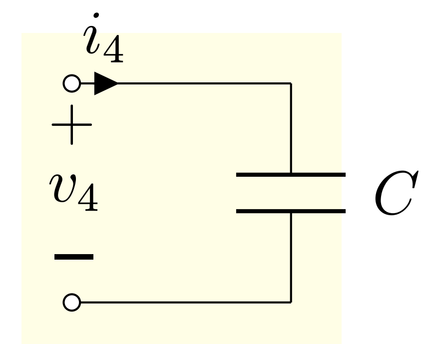


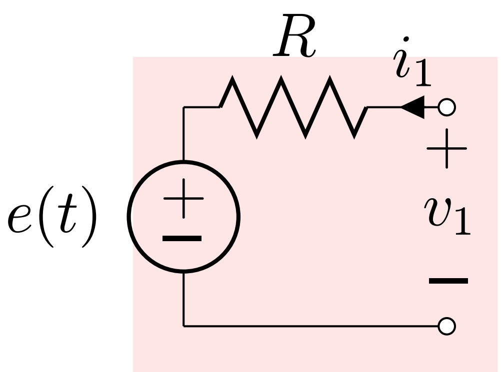
capacitor



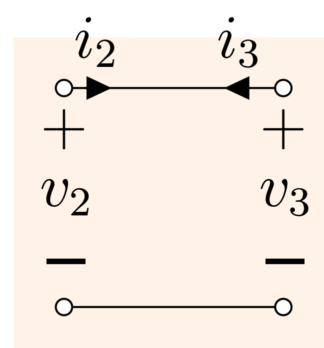
$$i_4 = sC \cdot v_4$$

DISCRETIZE RC NETWORK (attempt modular)

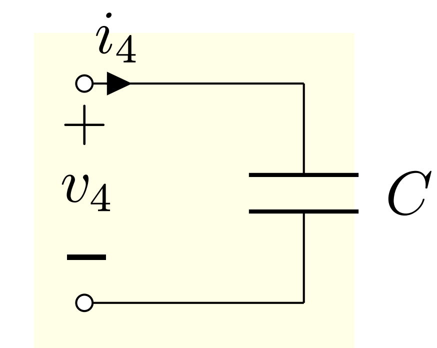
resistive voltage source



parallel connection



capacitor



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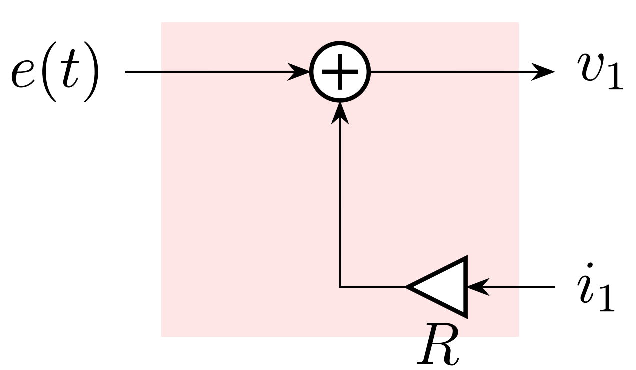
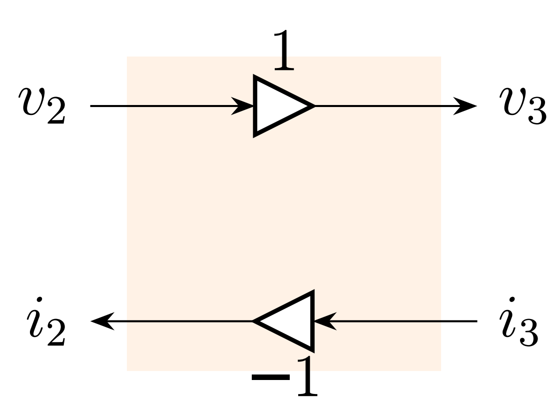
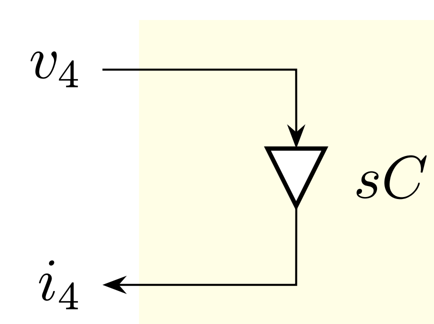
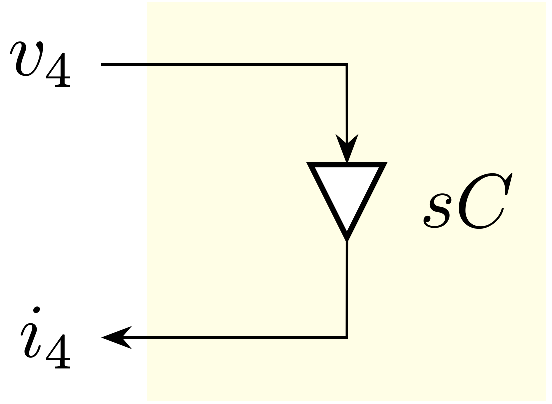






DISCRETIZE RC NETWORK (attempt modular)

capacitor
(continuous time)



DISCRETIZE RC NETWORK (attempt modular)

capacitor
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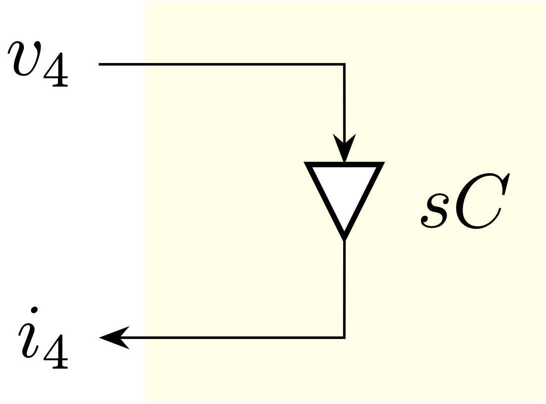
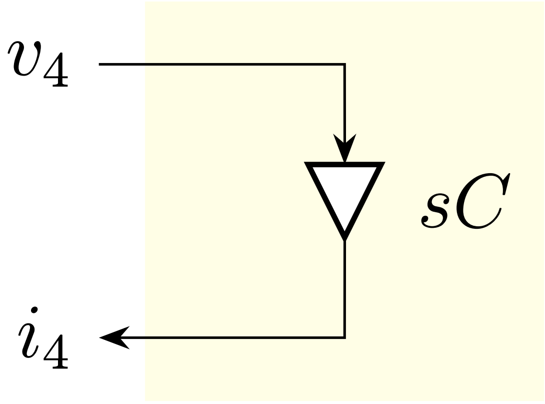


$$V_4(s) = I_4(s)\frac{1}{sC}$$

DISCRETIZE RC NETWORK (attempt modular)

capacitor
(continuous time)



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bilinear transform

$$s \leftarrow c\frac{1-z^{-1}}{1+z^{-1}}, \quad c = 2/T \text{ (typically)}$$

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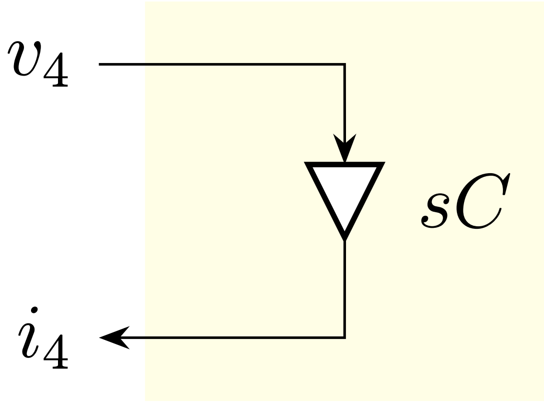
DISCRETIZE RC NETWORK (attempt modular)

capacitor
(continuous time)



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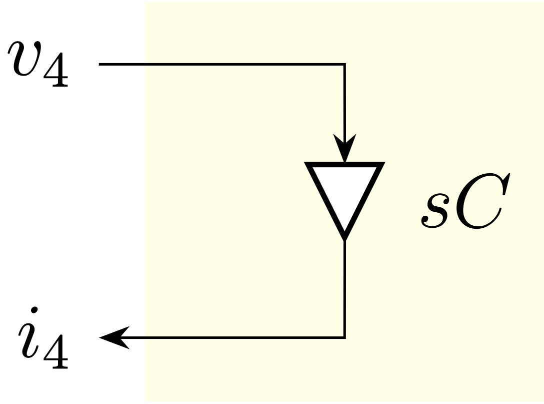
$$V_4(z) = I_4(z)\frac{T}{2C}\frac{1+z^{-1}}{1-z^{-1}}$$

DISCRETIZE RC NETWORK (attempt modular)

capacitor
(continuous time)



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$$V_4(z) = I_4(z)\frac{T}{2C}\frac{1+z^{-1}}{1-z^{-1}}$$

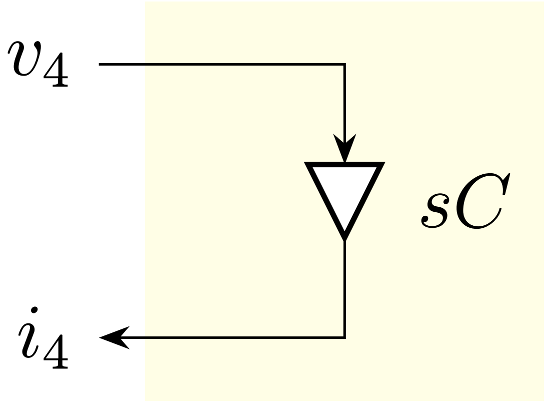
inverse z transform

$$x[n] = \mathcal{Z}^{-1}\{X(z)\}$$

$$v_4[n] - v_4[n-1] = \frac{T}{2C}i_4[n] + \frac{T}{2C}i_4[n-1]$$

DISCRETIZE RC NETWORK (attempt modular)

capacitor
(continuous time)



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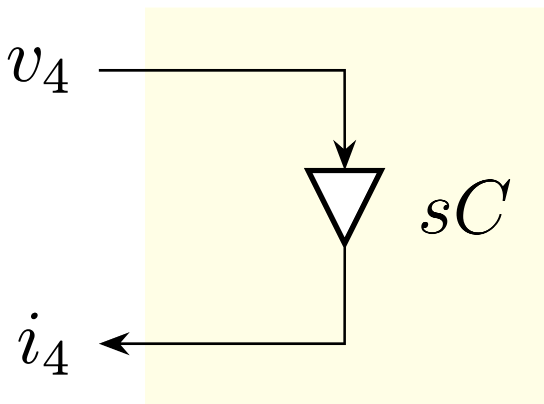
difference equation

$$i_4[n] = \frac{2C}{T}v_4[n] - \frac{2C}{T}v_4[n-1] - i_4[n-1]$$

Discretize RC Network (attempt modular)

capacitor
(continuous time)



$$V_4(s) = I_4(s)\frac{1}{sC}$$

bilinear transform $s \leftarrow c\frac{1 - z^{-1}}{1 + z^{-1}}, \; c = 2/T \text{ (typically)}$

$$V_4(z) = I_4(z)\frac{1}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}C}$$

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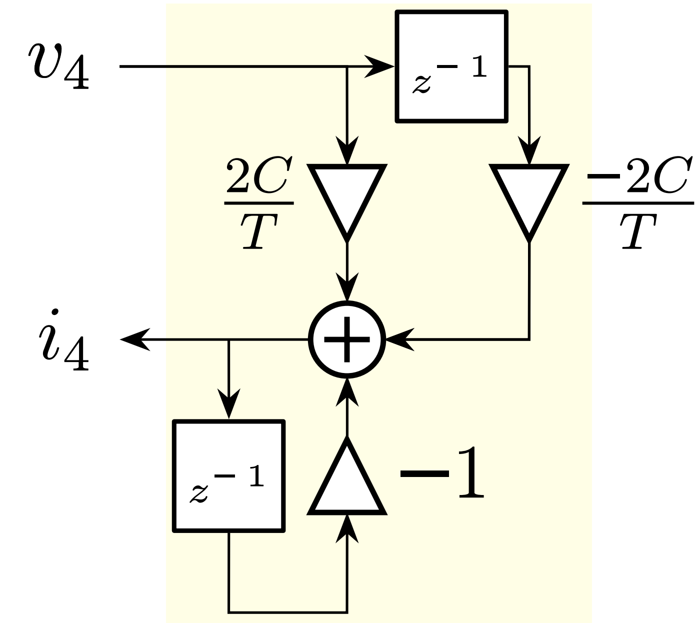
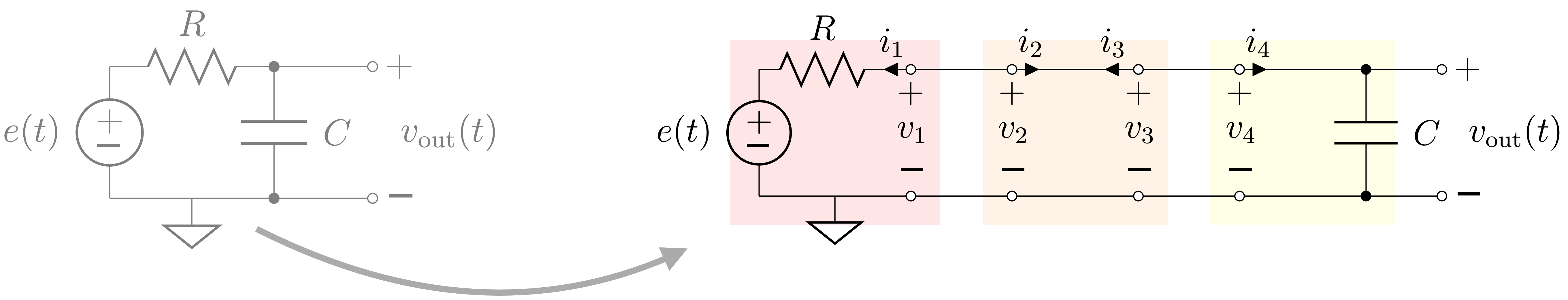
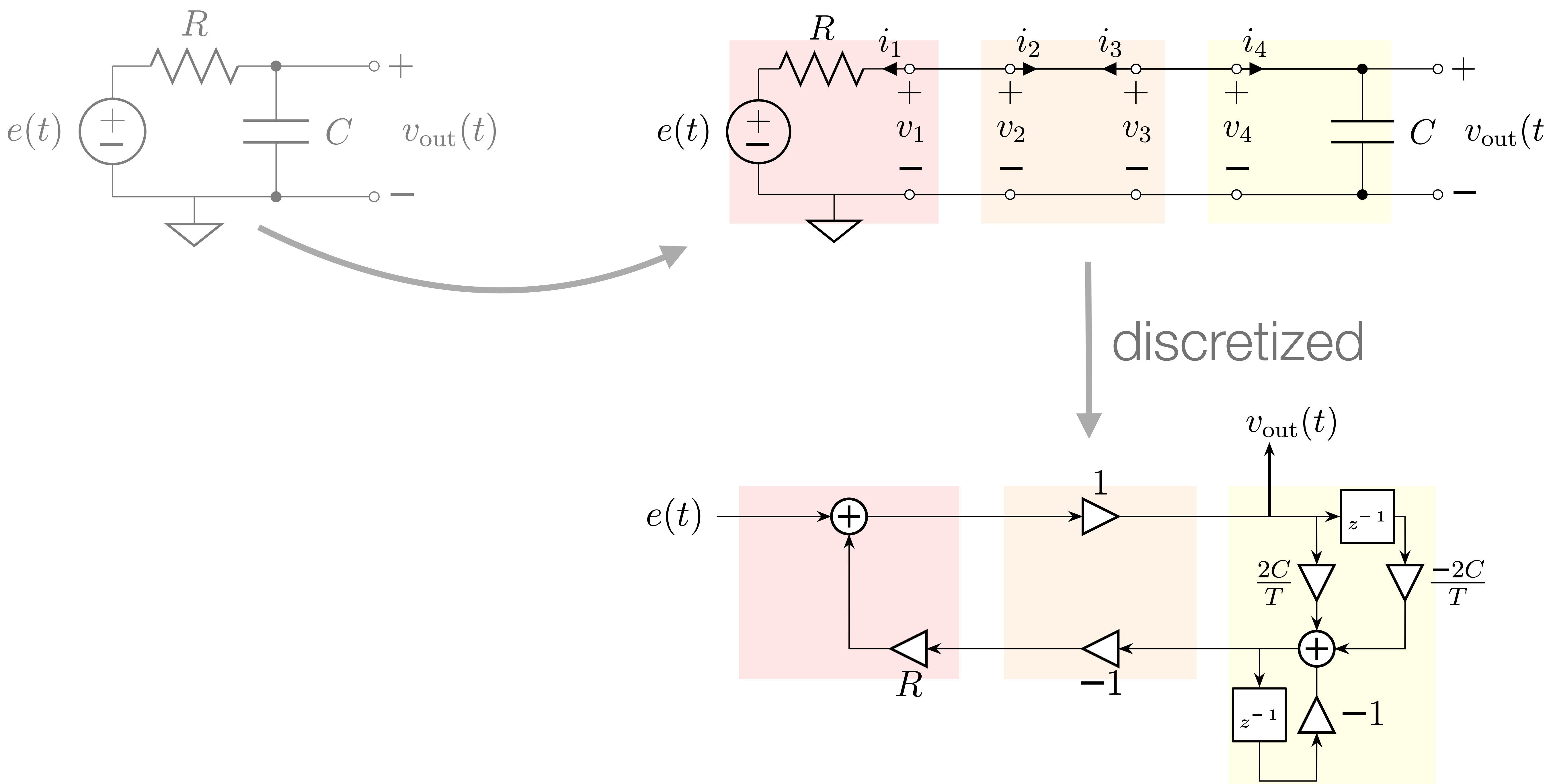


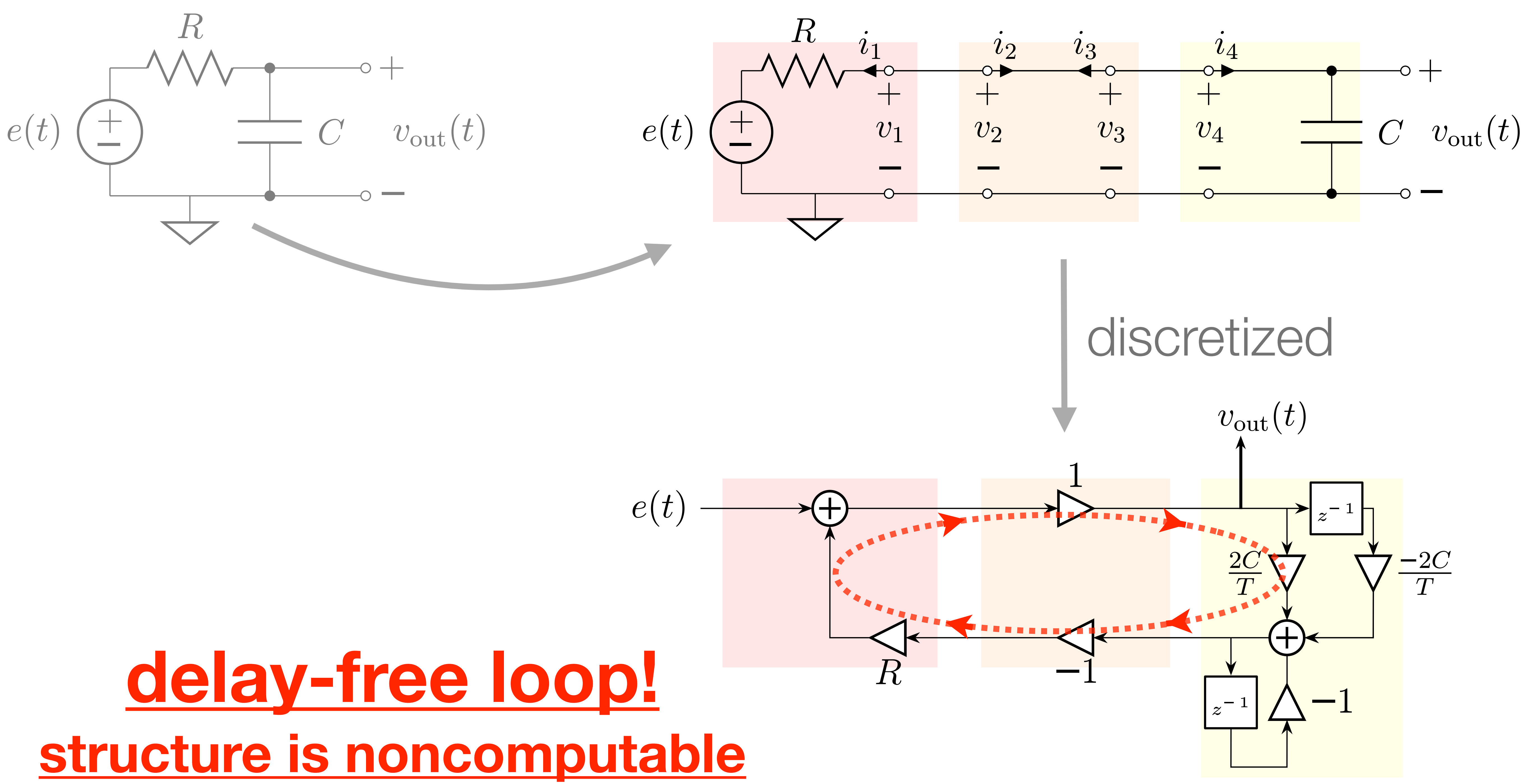
DISCRETIZE RC NETWORK (attempt modular)



DISCRETIZE RC NETWORK (attempt modular)



DISCRETIZE RC NETWORK (attempt modular)



delay-free loop!

structure is noncomputable

DISCRETIZE RC NETWORK (**WDF approach**)

WDF approach involves:

- introduction of free parameter (port resistance) at each port:

$$R_n > 0 \text{ , for each port } n$$

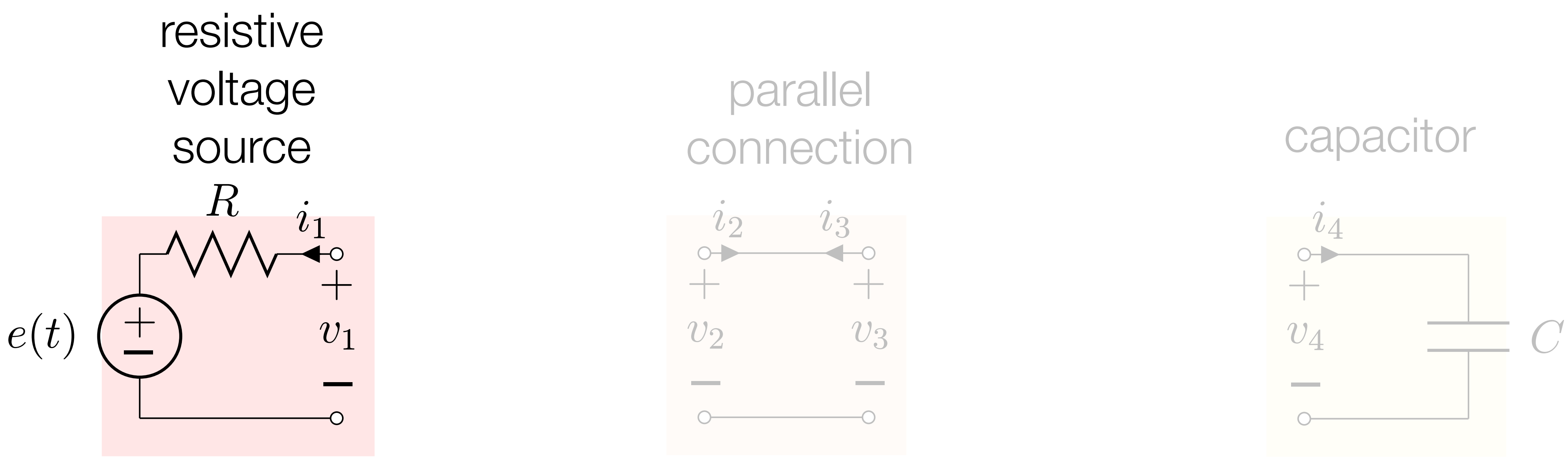
- introduction of wave variables:

$$a_n = v_n + R_n i_n$$
$$b_n = v_n - R_n i_n$$

- discretization of reactive elements (capacitors, inductors) using the Bilinear transformation:

$$s \leftarrow c\frac{1 - z^{-1}}{1 + z^{-1}}, \quad c = 2/T \text{ (typically)}$$

DISCRETIZE RC NETWORK (WDF approach)



port resistance:

incident wave:

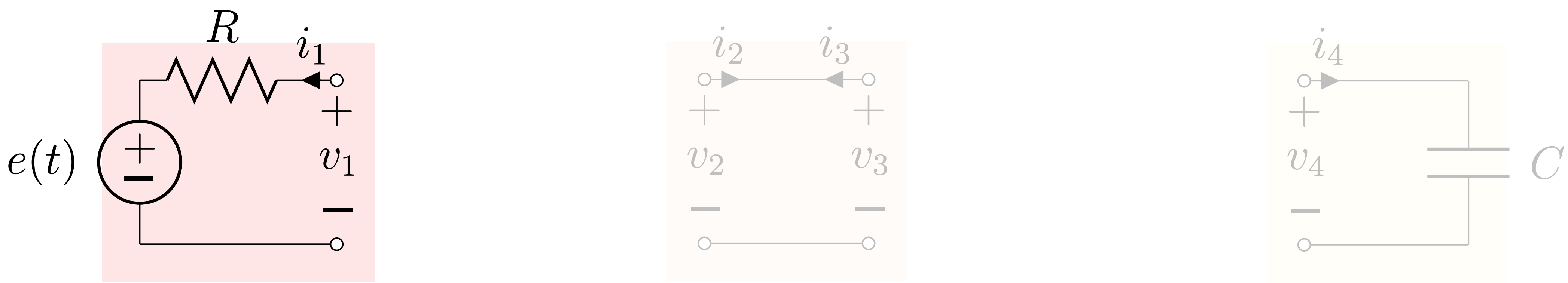
reflected wave:

Discretize RC Network (WDF approach)

resistive voltage source

parallel connection

capacitor



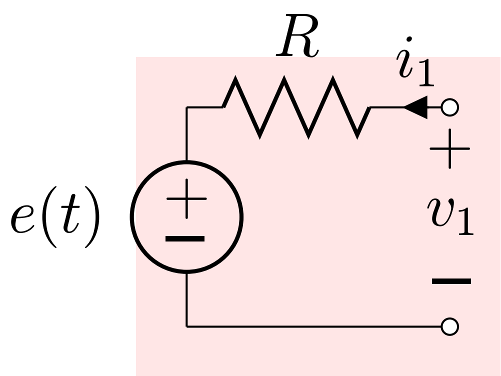
port resistance: R_1

incident wave: $a_1 = v_1 + R_1 i_1$

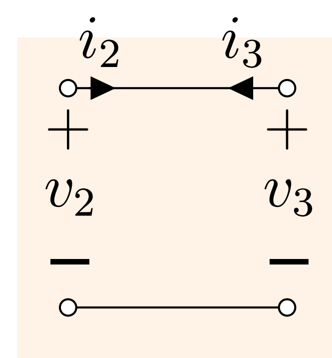
reflected wave: $b_1 = v_1 - R_1 i_1$

DISCRETIZE RC NETWORK (WDF approach)

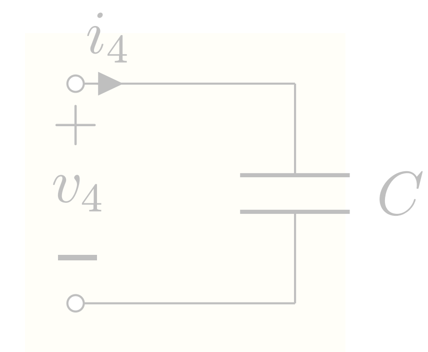
resistive voltage source



parallel connection



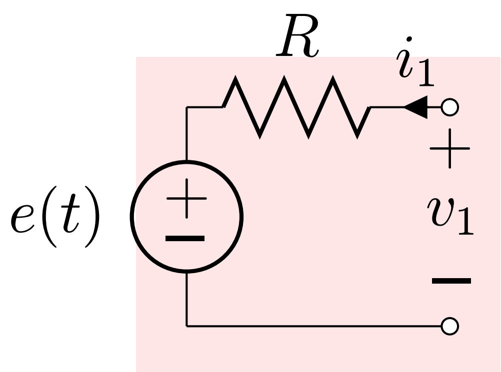
capacitor



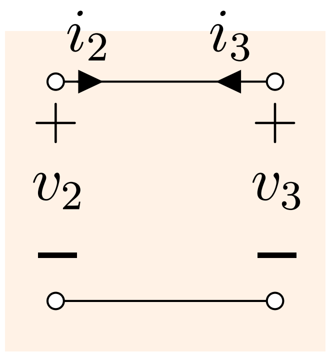
	resistive voltage source	parallel connection
port resistance:	R_1	R_2
incident wave:	$a_1 = v_1 + R_1 i_1$	$a_2 = v_2 + R_2 i_2$
reflected wave:	$b_1 = v_1 - R_1 i_1$	$b_2 = v_2 - R_2 i_2$

DISCRETIZE RC NETWORK (WDF approach)

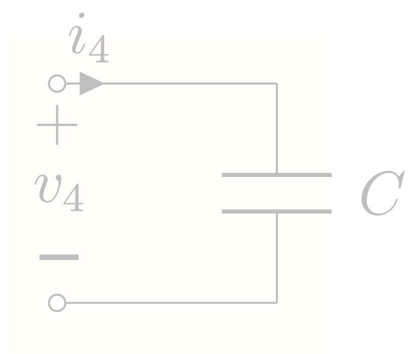
resistive voltage source



parallel connection



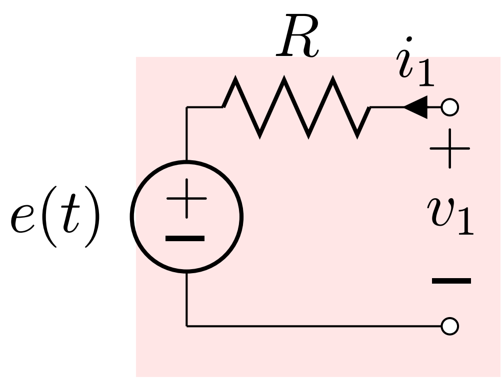
capacitor



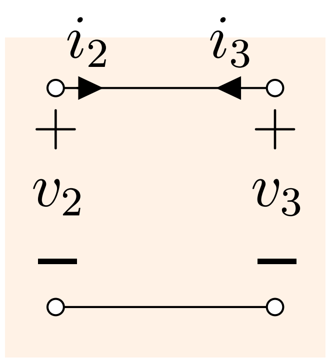
port resistance:	R_1	R_2	R_3
incident wave:	$a_1 = v_1 + R_1 i_1$	$a_2 = v_2 + R_2 i_2$	$a_3 = v_3 + R_3 i_3$
reflected wave:	$b_1 = v_1 - R_1 i_1$	$b_2 = v_2 - R_2 i_2$	$b_3 = v_3 - R_3 i_3$

DISCRETIZE RC NETWORK (WDF approach)

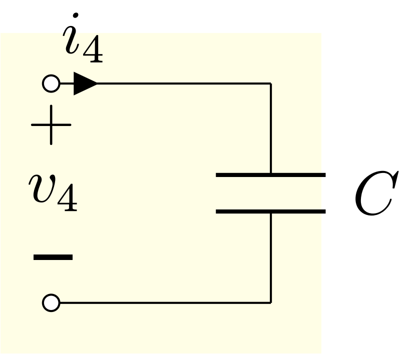
resistive voltage source



parallel connection



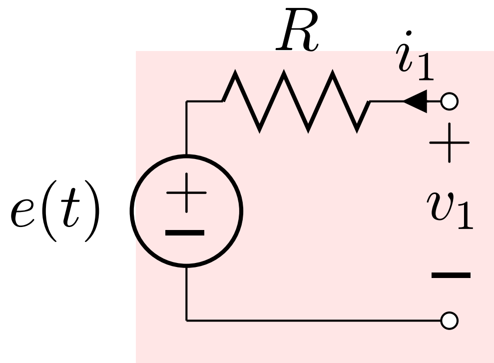
capacitor



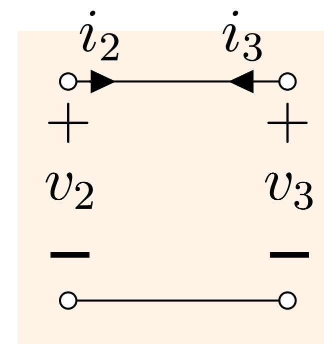
port resistance:	R_1	R_2	R_3	R_4
incident wave:	$a_1 = v_1 + R_1 i_1$	$a_2 = v_2 + R_2 i_2$	$a_3 = v_3 + R_3 i_3$	$a_4 = v_4 + R_4 i_4$
reflected wave:	$b_1 = v_1 - R_1 i_1$	$b_2 = v_2 - R_2 i_2$	$b_3 = v_3 - R_3 i_3$	$b_4 = v_4 - R_4 i_4$

DISCRETIZE RC NETWORK (WDF approach)

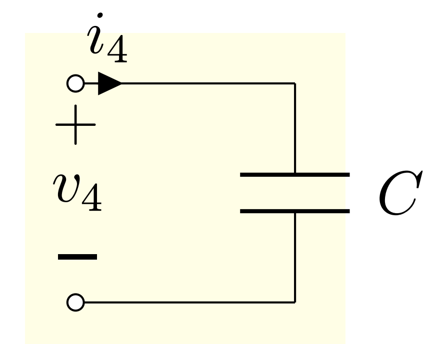
resistive voltage source



parallel connection



capacitor



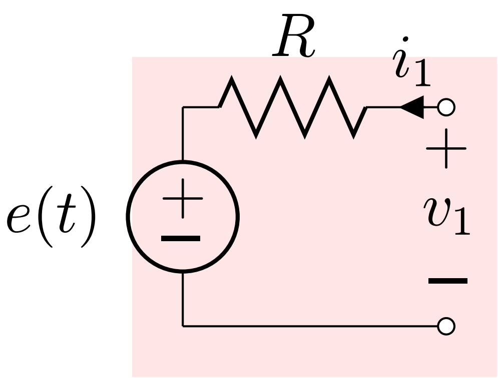
port resistance:	R_1	R_2	R_3	R_4
incident wave:	$a_1 = v_1 + R_1 i_1$	$a_2 = v_2 + R_2 i_2$	$a_3 = v_3 + R_3 i_3$	$a_4 = v_4 + R_4 i_4$
reflected wave:	$b_1 = v_1 - R_1 i_1$	$b_2 = v_2 - R_2 i_2$	$b_3 = v_3 - R_3 i_3$	$b_4 = v_4 - R_4 i_4$

we gained four tunable **degrees of freedom**:

R_1, R_2, R_3, R_4

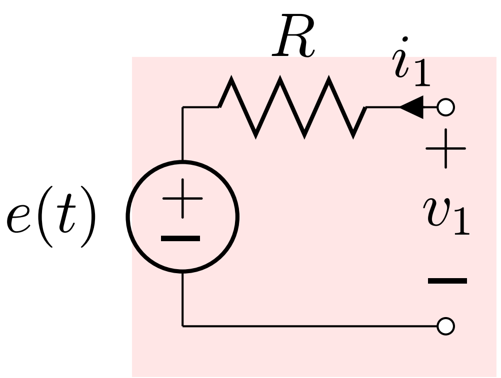
DISCRETIZE RC NETWORK (WDF approach)

resistive
voltage
source



DISCRETIZE RC NETWORK (WDF approach)

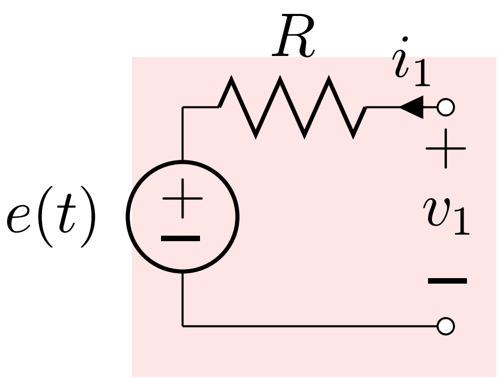
resistive
voltage
source



$$v_1 = e(t) + Ri_1$$

DISCRETIZE RC NETWORK (WDF approach)

resistive
voltage
source

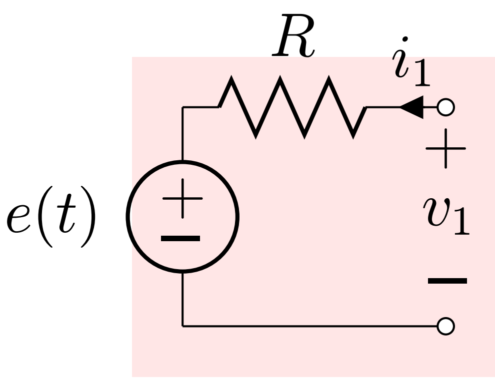


$$v_1 = e(t) + Ri_1$$

$$\frac{a_1}{2} + \frac{b_1}{2} = e(t) + R\frac{a_1}{2R_1} - R\frac{b_1}{2R_1}$$

DISCRETIZE RC NETWORK (WDF approach)

resistive voltage source



$$v_1 = e(t) + Ri_1$$

$$\frac{a_1}{2} + \frac{b_1}{2} = e(t) + R\frac{a_1}{2R_1} - R\frac{b_1}{2R_1}$$

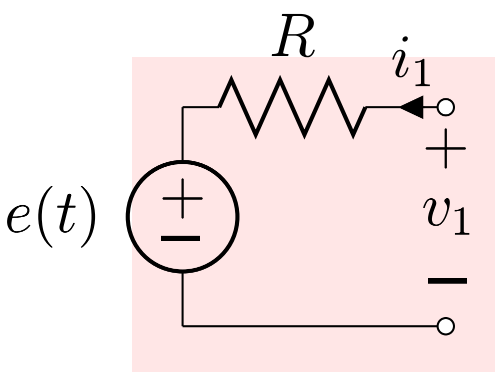
$$b_1(R + R_1) = 2R_1 e(t) + a_1(R - R_1)$$

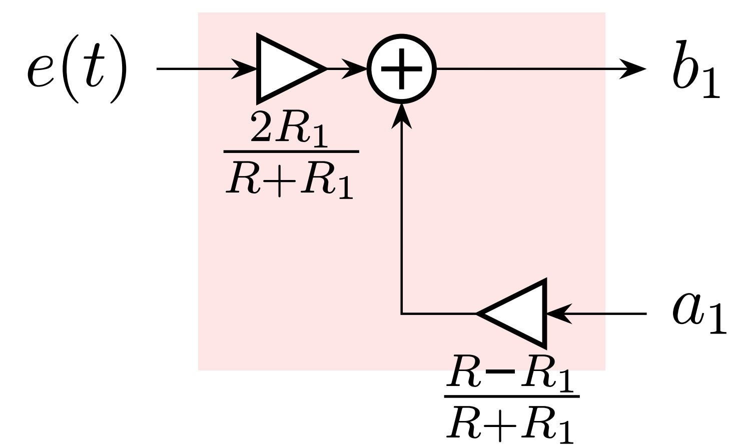
difference equation

$$b_1 = \frac{2R_1}{R + R_1}e(t) + \frac{R - R_1}{R + R_1}a_1$$

Discretize RC Network (WDF approach)

resistive
voltage
source





$$v_1 = e(t) + Ri_1$$

$$\frac{a_1}{2} + \frac{b_1}{2} = e(t) + R\frac{a_1}{2R_1} - R\frac{b_1}{2R_1}$$

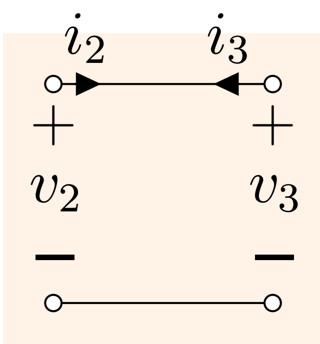
$$b_1(R + R_1) = 2R_1 e(t) + a_1(R - R_1)$$

difference
equation

$$b_1 = \frac{2R_1}{R + R_1}e(t) + \frac{R - R_1}{R + R_1}a_1$$

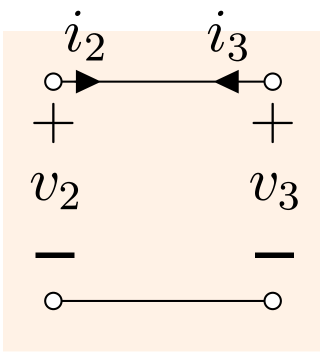
DISCRETIZE RC NETWORK (WDF approach)

parallel
connection



DISCRETIZE RC NETWORK (WDF approach)

parallel connection



$$i_2 = -i_3$$

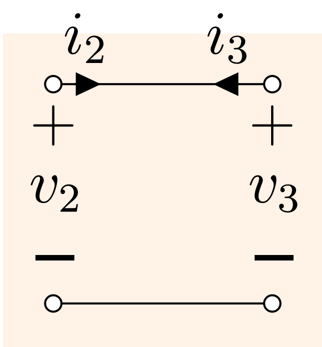
$$v_2 = v_3$$

DISCRETIZE RC NETWORK (WDF approach)

$$i_2 = -i_3$$

$$v_2 = v_3$$

parallel connection

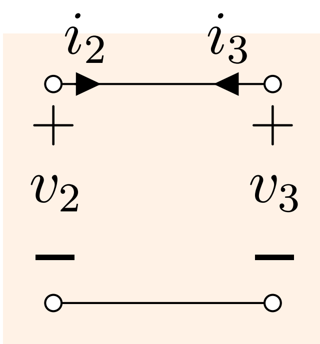


$$\frac{1}{2R_2}a_2 - \frac{1}{2R_2}b_2 = -\frac{1}{2R_3}a_3 + \frac{1}{2R_3}b_3$$

$$\frac{1}{2}a_2 + \frac{1}{2}b_2 = \frac{1}{2}a_3 + \frac{1}{2}b_3$$

Discretize RC Network (WDF approach)

parallel connection



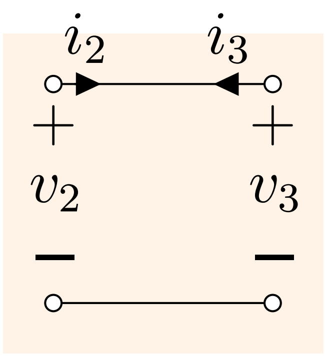
$$i_2 = -i_3$$
$$v_2 = v_3$$

$$\frac{1}{2R_2}a_2 - \frac{1}{2R_2}b_2 = -\frac{1}{2R_3}a_3 + \frac{1}{2R_3}b_3$$
$$\frac{1}{2}a_2 + \frac{1}{2}b_2 = \frac{1}{2}a_3 + \frac{1}{2}b_3$$

$$R_3a_2 - R_3b_2 = -R_2a_3 + R_2b_3$$
$$a_2 + b_2 = a_3 + b_3$$

DISCRETIZE RC NETWORK (WDF approach)

parallel connection



$$i_2 = -i_3$$
$$v_2 = v_3$$

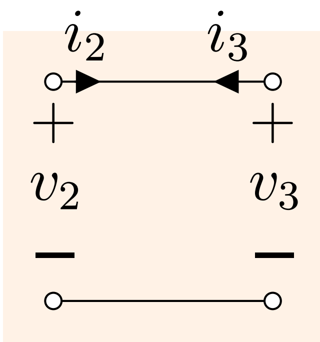
$$\frac{1}{2R_2}a_2 - \frac{1}{2R_2}b_2 = -\frac{1}{2R_3}a_3 + \frac{1}{2R_3}b_3$$
$$\frac{1}{2}a_2 + \frac{1}{2}b_2 = \frac{1}{2}a_3 + \frac{1}{2}b_3$$

$$R_3a_2 - R_3b_2 = -R_2a_3 + R_2b_3$$
$$a_2 + b_2 = a_3 + b_3$$

$$\begin{bmatrix} R_3 & R_2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} R_3 & R_2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

Discretize RC Network (WDF approach)

parallel connection



$$i_2 = -i_3$$
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$$\frac{1}{2R_2}a_2 - \frac{1}{2R_2}b_2 = -\frac{1}{2R_3}a_3 + \frac{1}{2R_3}b_3$$
$$\frac{1}{2}a_2 + \frac{1}{2}b_2 = \frac{1}{2}a_3 + \frac{1}{2}b_3$$

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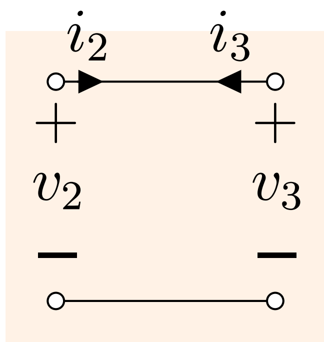
$$\begin{bmatrix} R_3 & R_2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} R_3 & R_2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

$$\begin{bmatrix} b_2 \\ b_3 \end{bmatrix} = \frac{1}{R_2 + R_3} \begin{bmatrix} 1 & -R_2 \\ 1 & R_3 \end{bmatrix} \begin{bmatrix} R_3 & R_2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

DISCRETIZE RC NETWORK (WDF approach)

parallel connection



$$i_2 = -i_3$$
$$v_2 = v_3$$

$$\frac{1}{2R_2}a_2 - \frac{1}{2R_2}b_2 = -\frac{1}{2R_3}a_3 + \frac{1}{2R_3}b_3$$
$$\frac{1}{2}a_2 + \frac{1}{2}b_2 = \frac{1}{2}a_3 + \frac{1}{2}b_3$$

$$R_3 a_2 - R_3 b_2 = -R_2 a_3 + R_2 b_3$$
$$a_2 + b_2 = a_3 + b_3$$

$$\begin{bmatrix} R_3 & R_2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} R_3 & R_2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

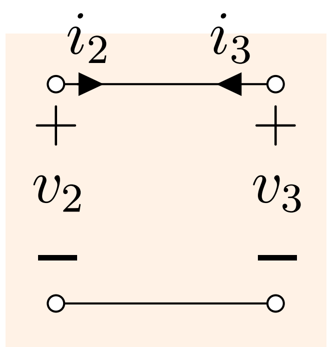
$$\begin{bmatrix} b_2 \\ b_3 \end{bmatrix} = \frac{1}{R_2 + R_3} \begin{bmatrix} 1 & -R_2 \\ 1 & R_3 \end{bmatrix} \begin{bmatrix} R_3 & R_2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

scattering equation

$$\begin{bmatrix} b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_2 - R_3}{R_2 + R_3} & \frac{2R_2}{R_2 + R_3} \\ \frac{2R_3}{R_2 + R_3} & \frac{R_2 - R_3}{R_2 + R_3} \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

DISCRETIZE RC NETWORK (WDF approach)

parallel connection



$$i_2 = -i_3$$
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$$\frac{1}{2R_2}a_2 - \frac{1}{2R_2}b_2 = -\frac{1}{2R_3}a_3 + \frac{1}{2R_3}b_3$$
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$$R_3a_2 - R_3b_2 = -R_2a_3 + R_2b_3$$
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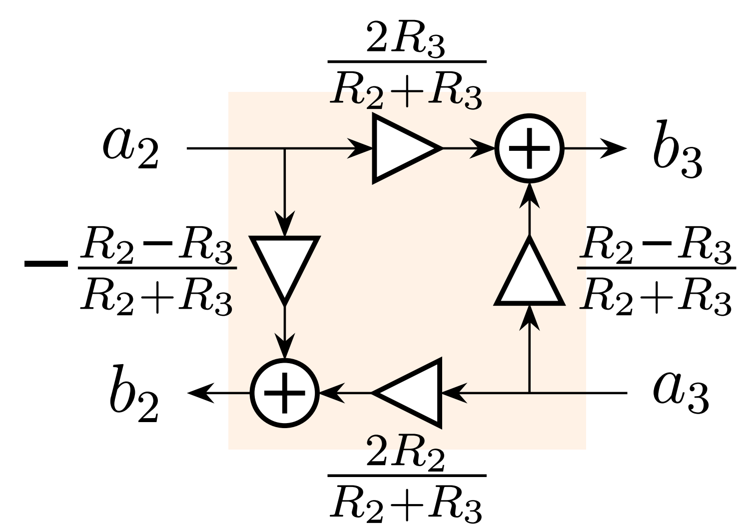
$$\begin{bmatrix} b_2 \\ b_3 \end{bmatrix} = \frac{1}{R_2 + R_3} \begin{bmatrix} 1 & -R_2 \\ 1 & R_3 \end{bmatrix} \begin{bmatrix} R_3 & R_2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$

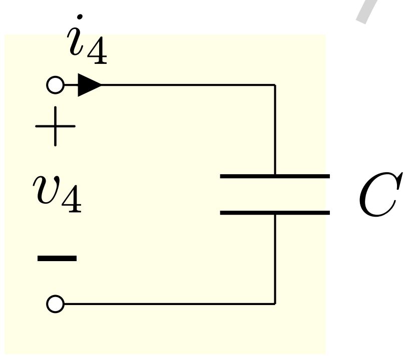
scattering equation

$$\begin{bmatrix} b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_2-R_3}{R_2+R_3} & \frac{2R_2}{R_2+R_3} \\ \frac{2R_3}{R_2+R_3} & \frac{R_2-R_3}{R_2+R_3} \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}$$



DISCRETIZE RC NETWORK (WDF approach)

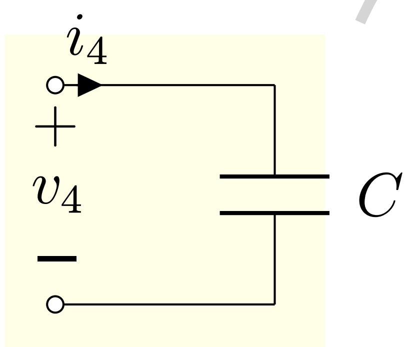
capacitor



$$V_4(s) = \frac{1}{sC} I_4(s)$$

DISCRETIZE RC NETWORK (WDF approach)

capacitor

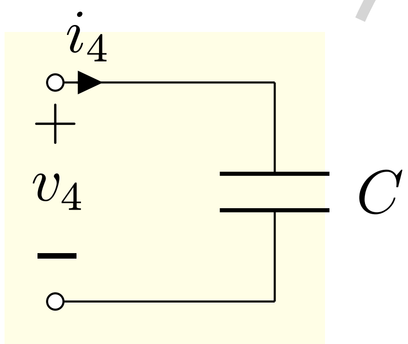


$$V_4(s) = \frac{1}{sC} I_4(s)$$

$$V_4(z)(1 - z^{-1}) = \frac{T}{2C} I_4(z)(1 + z^{-1})$$

Discretize RC Network (WDF approach)

capacitor



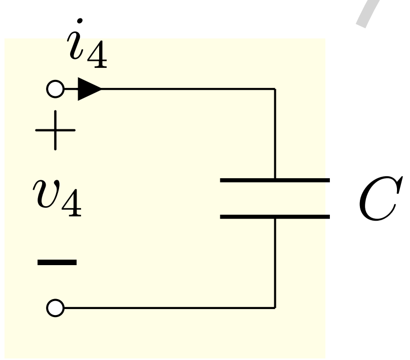
$$V_4(s) = \frac{1}{sC} I_4(s)$$

$$V_4(z)(1 - z^{-1}) = \frac{T}{2C} I_4(z)(1 + z^{-1})$$

$$\left(\frac{1}{2} A_4(z) + \frac{1}{2} B_4(z)\right)(1 - z^{-1}) = \frac{T}{2C}\left(\frac{1}{2R_4} A_4(z) - \frac{1}{2R_4} B_4(z)\right)(1 + z^{-1})$$

DISCRETIZE RC NETWORK (WDF approach)

capacitor



$$V_4(s) = \frac{1}{sC} I_4(s)$$

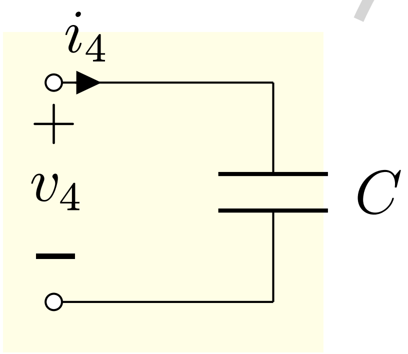
$$V_4(z)(1 - z^{-1}) = \frac{T}{2C} I_4(z)(1 + z^{-1})$$

$$\left(\frac{1}{2}A_4(z) + \frac{1}{2}B_4(z)\right)(1 - z^{-1}) = \frac{T}{2C}\left(\frac{1}{2R_4}A_4(z) - \frac{1}{2R_4}B_4(z)\right)(1 + z^{-1})$$

$$R_4\left(A_4(z) + B_4(z)\right)(1 - z^{-1}) = \frac{T}{2C}\left(A_4(z) - B_4(z)\right)(1 + z^{-1})$$

DISCRETIZE RC NETWORK (WDF approach)

capacitor



$$V_4(s) = \frac{1}{sC} I_4(s)$$

$$V_4(z)(1 - z^{-1}) = \frac{T}{2C} I_4(z)(1 + z^{-1})$$

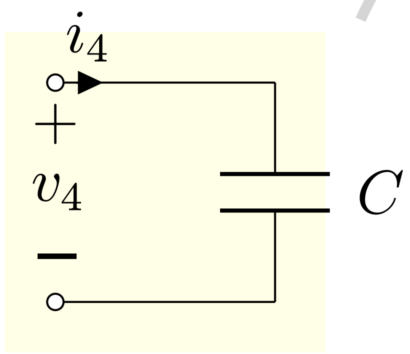
$$\left(\frac{1}{2}A_4(z) + \frac{1}{2}B_4(z)\right)(1 - z^{-1}) = \frac{T}{2C}\left(\frac{1}{2R_4}A_4(z) - \frac{1}{2R_4}B_4(z)\right)(1 + z^{-1})$$

$$R_4\left(A_4(z) + B_4(z)\right)(1 - z^{-1}) = \frac{T}{2C}\left(A_4(z) - B_4(z)\right)(1 + z^{-1})$$

$$\left(R_4 + \frac{T}{2C}\right)B_4(z) = \left(R_4 - \frac{T}{2C}\right)B_4(z)z^{-1} + \left(-R_4 + \frac{T}{2C}\right)A_4(z) + \left(R_4 + \frac{T}{2C}\right)A_4(z)z^{-1}$$

DISCRETIZE RC NETWORK (WDF approach)

capacitor



$$V_4(s) = \frac{1}{sC} I_4(s)$$

$$V_4(z)(1 - z^{-1}) = \frac{T}{2C} I_4(z)(1 + z^{-1})$$

$$\left(\frac{1}{2}A_4(z) + \frac{1}{2}B_4(z)\right)(1 - z^{-1}) = \frac{T}{2C}\left(\frac{1}{2R_4}A_4(z) - \frac{1}{2R_4}B_4(z)\right)(1 + z^{-1})$$

$$R_4\left(A_4(z) + B_4(z)\right)(1 - z^{-1}) = \frac{T}{2C}\left(A_4(z) - B_4(z)\right)(1 + z^{-1})$$

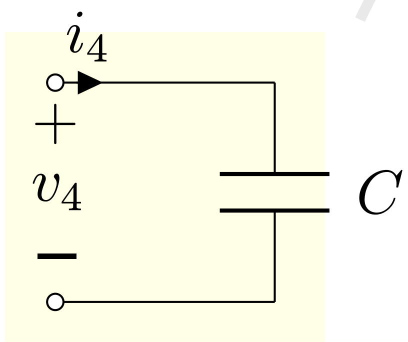
$$\left(R_4 + \frac{T}{2C}\right)B_4(z) = \left(R_4 - \frac{T}{2C}\right)B_4(z)z^{-1} + \left(-R_4 + \frac{T}{2C}\right)A_4(z) + \left(R_4 + \frac{T}{2C}\right)A_4(z)z^{-1}$$

difference equation

$$b_4[n] = \frac{R_4 - \frac{T}{2C}}{R_4 + \frac{T}{2C}} b[n-1] - \frac{R_4 - \frac{T}{2C}}{R_4 + \frac{T}{2C}} a_4[n] + a_4[n-1]$$

DISCRETIZE RC NETWORK (WDF approach)

capacitor



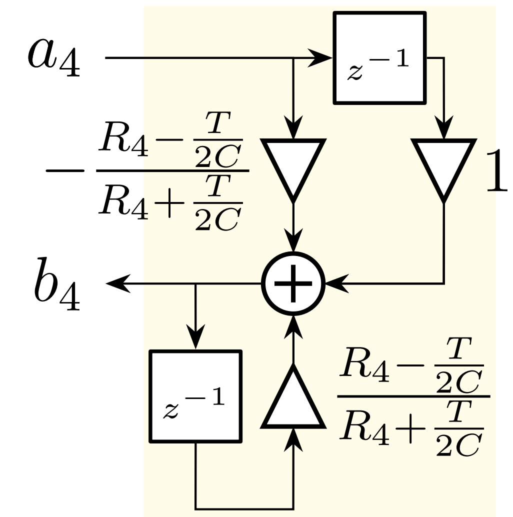
$$V_4(s) = \frac{1}{sC} I_4(s)$$

$$V_4(z)(1 - z^{-1}) = \frac{T}{2C} I_4(z)(1 + z^{-1})$$

$$\left(\frac{1}{2}A_4(z) + \frac{1}{2}B_4(z)\right)(1 - z^{-1}) = \frac{T}{2C}\left(\frac{1}{2R_4}A_4(z) - \frac{1}{2R_4}B_4(z)\right)(1 + z^{-1})$$

$$R_4\left(A_4(z) + B_4(z)\right)(1 - z^{-1}) = \frac{T}{2C}\left(A_4(z) - B_4(z)\right)(1 + z^{-1})$$

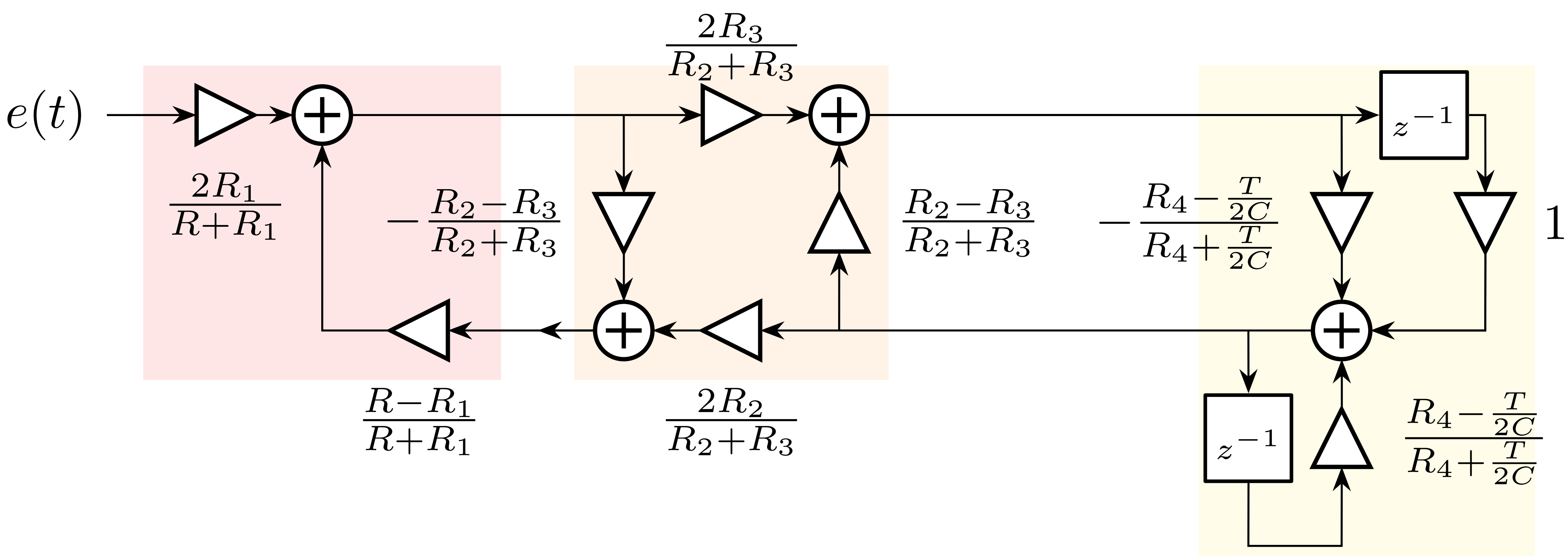
$$\left(R_4 + \frac{T}{2C}\right)B_4(z) = \left(R_4 - \frac{T}{2C}\right)B_4(z)z^{-1} + \left(-R_4 + \frac{T}{2C}\right)A_4(z) + \left(R_4 + \frac{T}{2C}\right)A_4(z)z^{-1}$$



difference equation

$$b_4[n] = \frac{R_4 - \frac{T}{2C}}{R_4 + \frac{T}{2C}} b[n-1] - \frac{R_4 - \frac{T}{2C}}{R_4 + \frac{T}{2C}} a_4[n] + a_4[n-1]$$

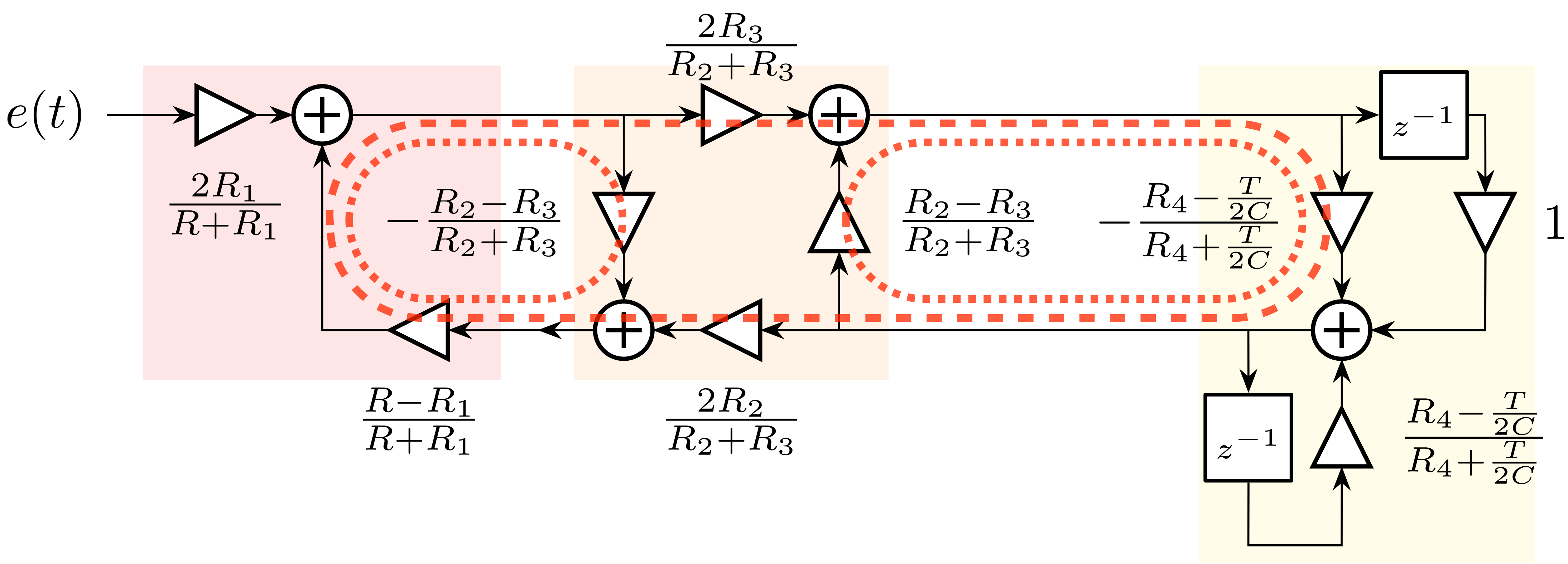
DISCRETIZE RC NETWORK (WDF approach)



DISCRETIZE RC NETWORK (WDF approach)

delay-free loops!

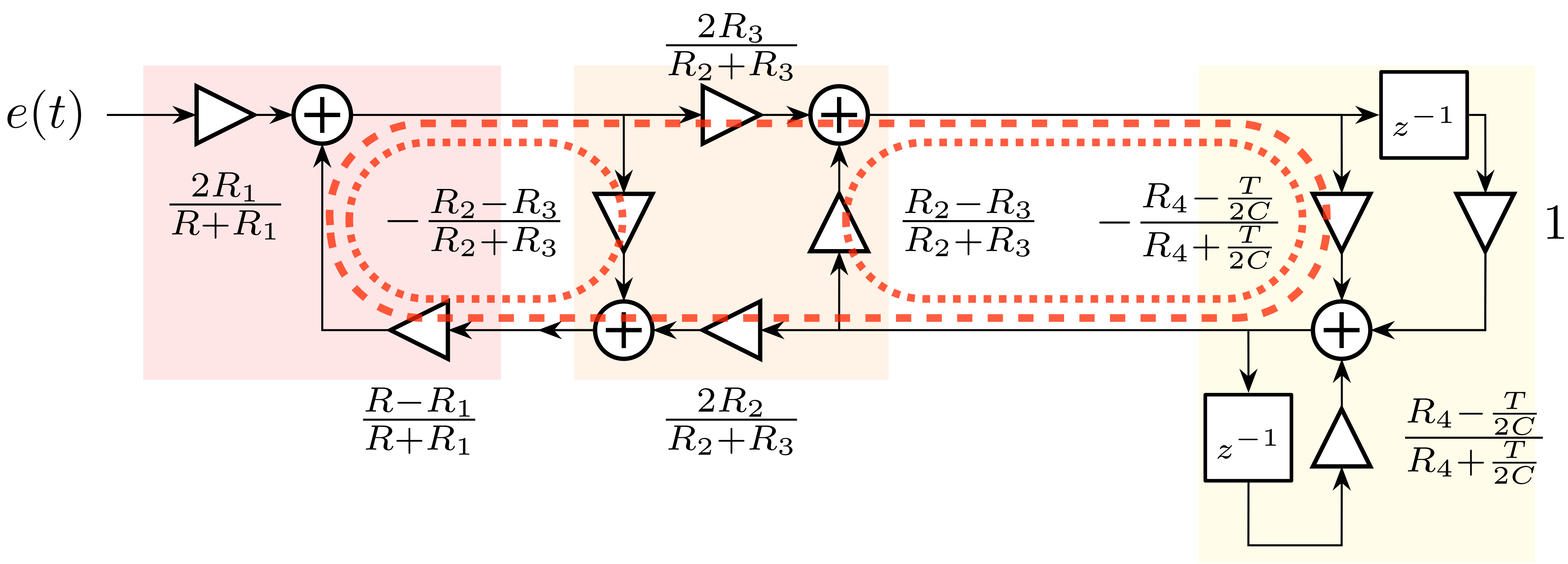
structure is noncomputable



DISCRETIZE RC NETWORK (WDF approach)

delay-free loops!

structure is noncomputable

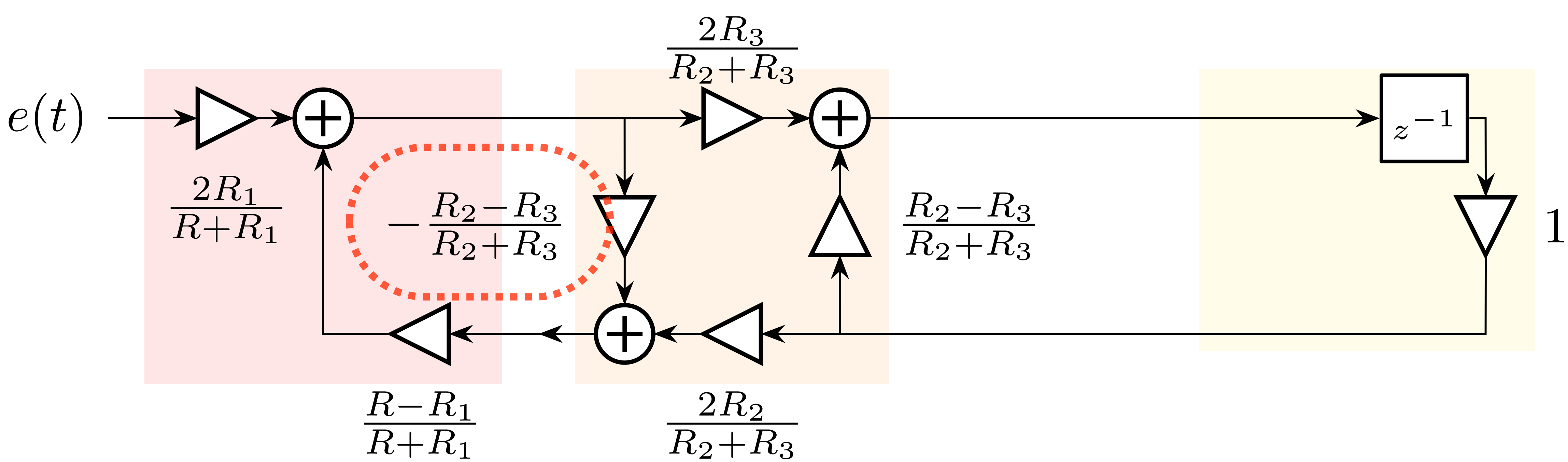


but, this time, we can fix things!

by tuning R_1, R_2, R_3, R_4

DISCRETIZE RC NETWORK (WDF approach)

$$R_4 = \frac{T}{2C}$$



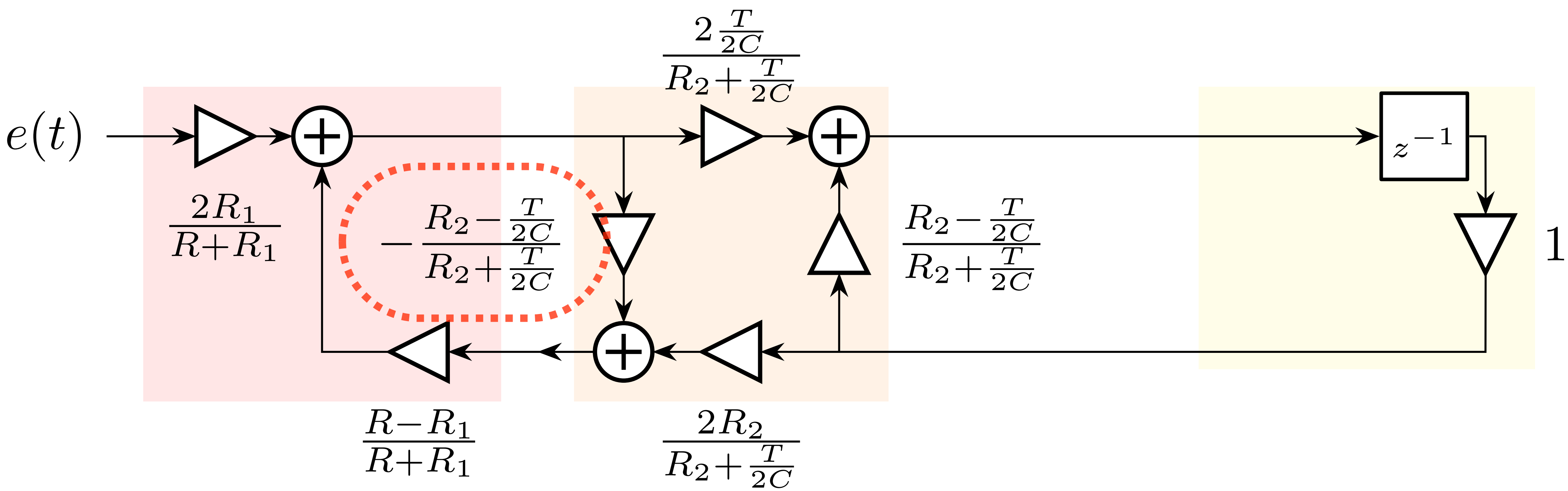
but, this time, we can fix things!

by tuning R

DISCRETIZE RC NETWORK (WDF approach)

$R_3 = R_4$ (by def.)

$R_4 = \frac{T}{2C}$

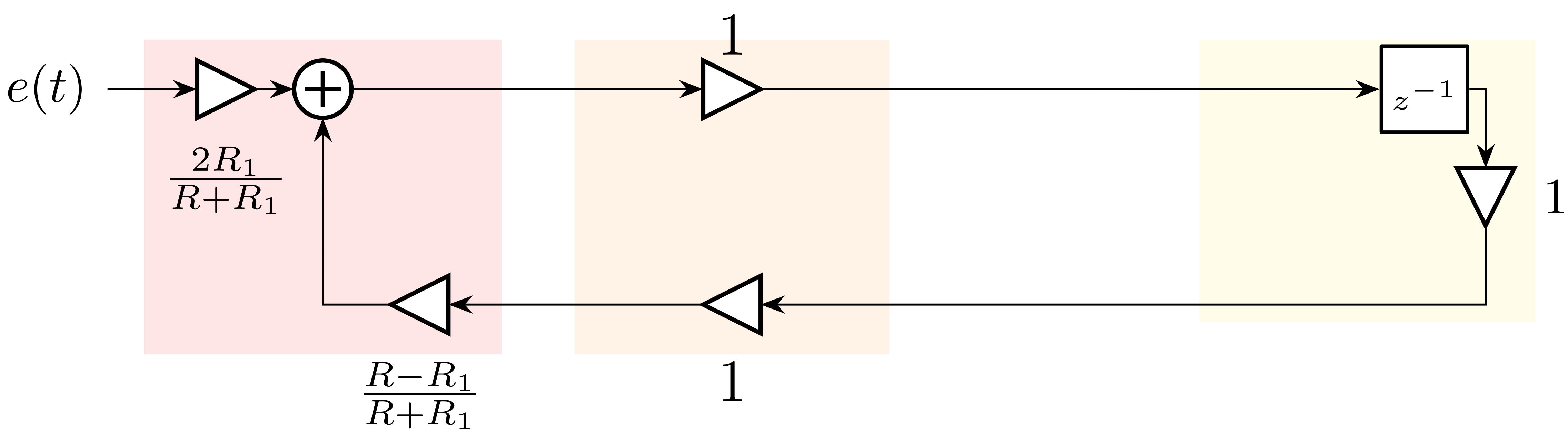


but, this time, we can fix things!

by tuning R

Discretize RC Network (WDF approach)

$$R_2 = \frac{T}{2C} \leftarrow R_3 = R_4 \text{ (by def.)} \leftarrow R_4 = \frac{T}{2C}$$

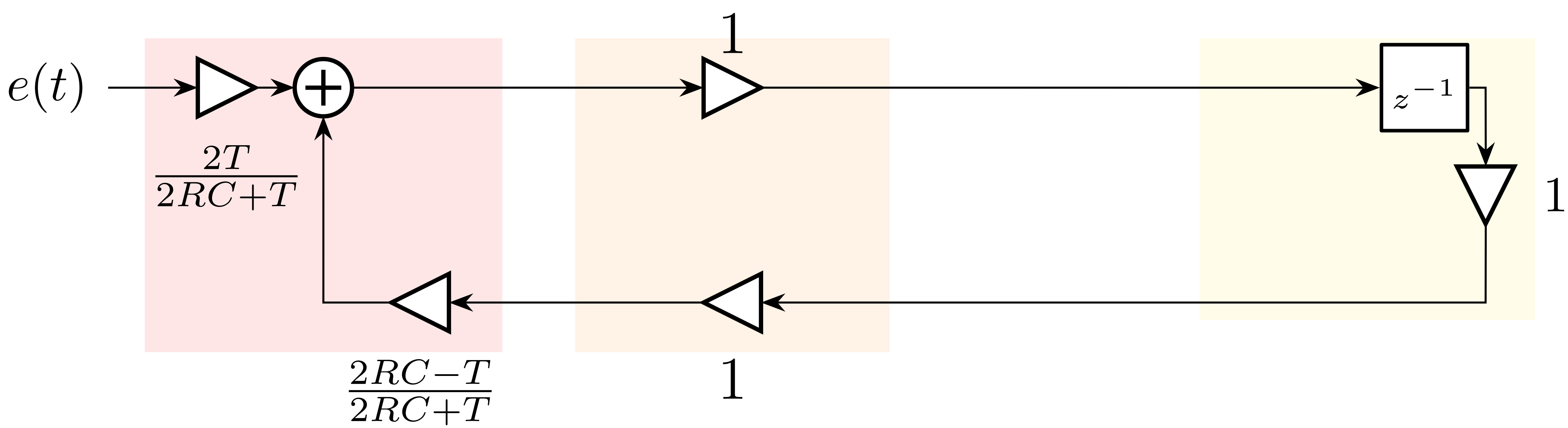


but, this time, we can fix things!

by tuning R

Discretize RC Network (WDF approach)

$R_1 = R_2$ (by def.) ← $R_2 = \frac{T}{2C}$ ← $R_3 = R_4$ (by def.) ← $R_4 = \frac{T}{2C}$



structure is computable!

Wave Digital Filters

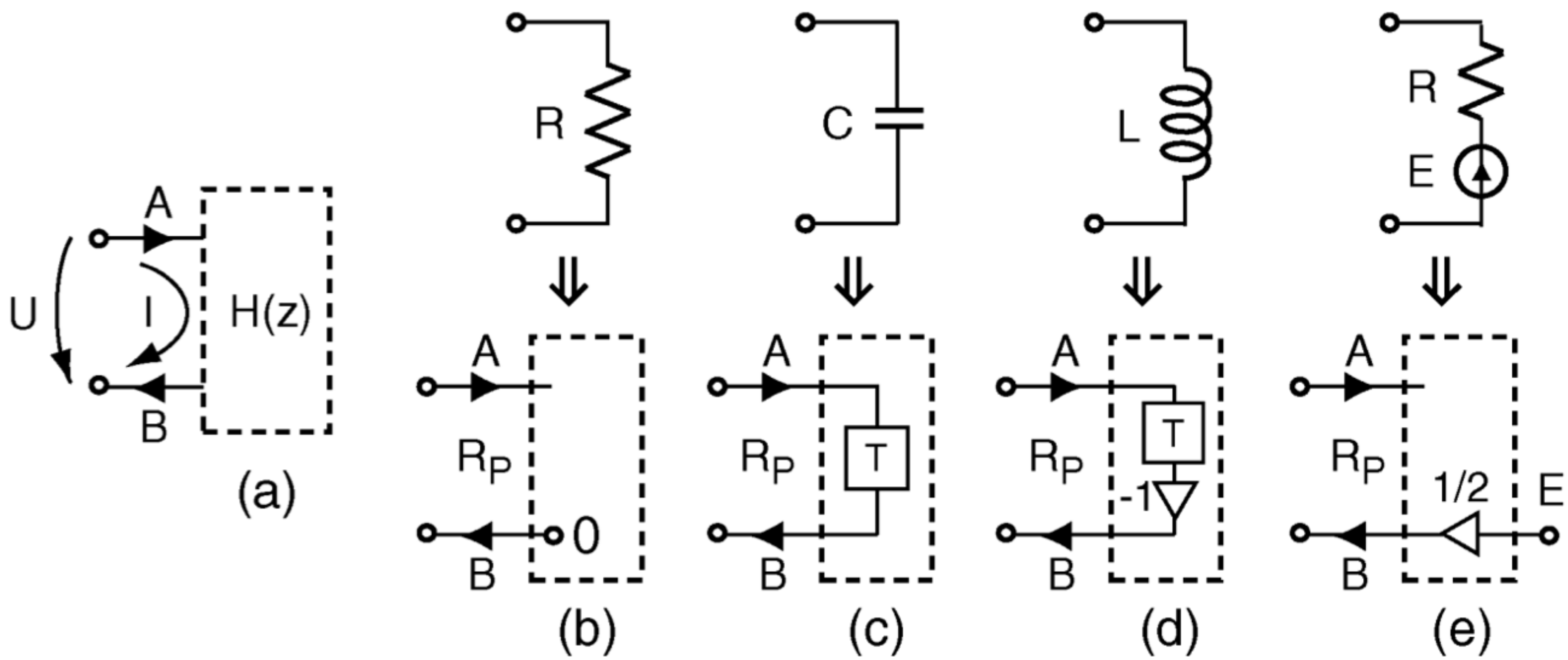
- modular
- no transfer function representation needed
- no factoring into biquads needed

- structure arranged as a "tree"
  - one element as the root, resolve loops upwards

- energetic properties in reference domain used to guarantee stability *by construction* $p_n = \left(a_n^2 - b_n^2\right) / R_n$
- good on quantization/sensitivity (original purpose)

Alfred Fettweis, Pseudopassivity, Sensitivity, and Stability of Wave Digital Filters," IEEE Transactions on Circuit Theory, Vol. CT-19, No. 6, November 1972.

WAVE DIGITAL FILTERS (resolved one ports)



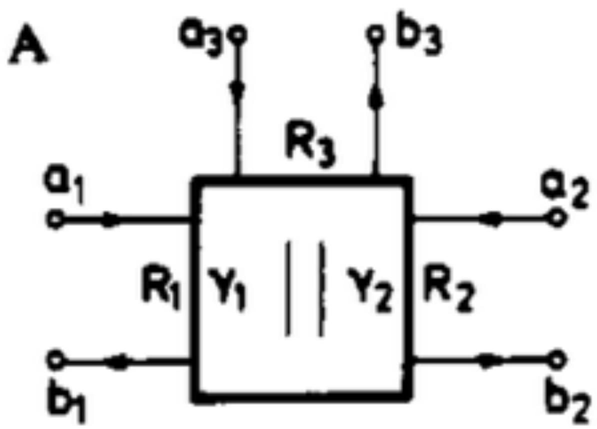
(a) A generic one-port
(b) resistor: $R_p=R$
(c) capacitor: $R_p=T/2C$
(d) inductor: $R_p=2L/T$
(e) voltage source $R_p=R$

C, *L*, R, are physical capacitance, inductance, and resistance, *T* is unit delay

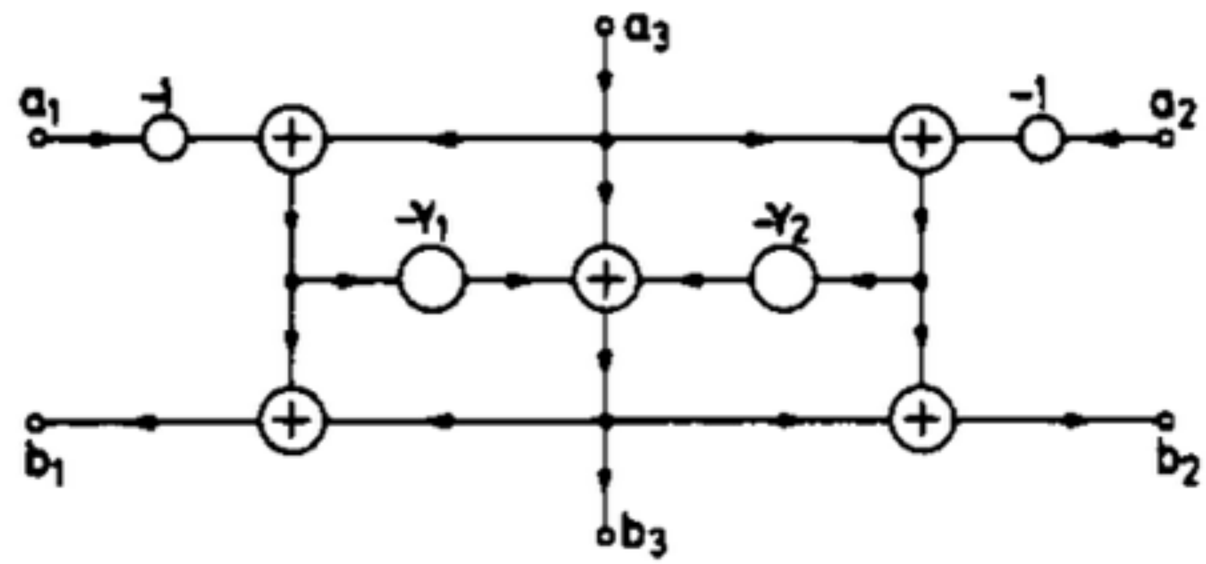
M. Karjalainen, "Efficient Realization of Wave Digital Components for Physical Modeling and Sound Synthesis," IEEE Trans. Audio, Speech, Language Process., July 2008

WAVE DIGITAL FILTERS (adaptors)

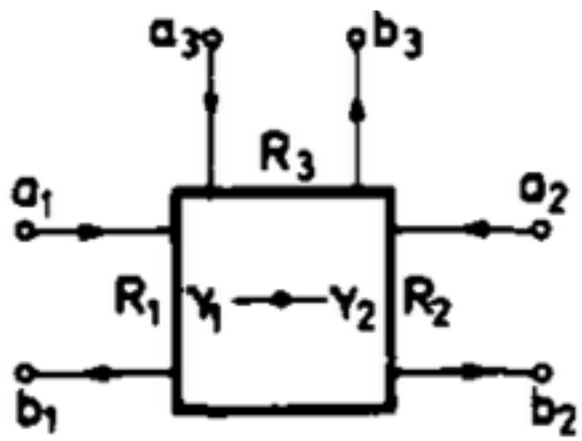
Parallel Adaptor



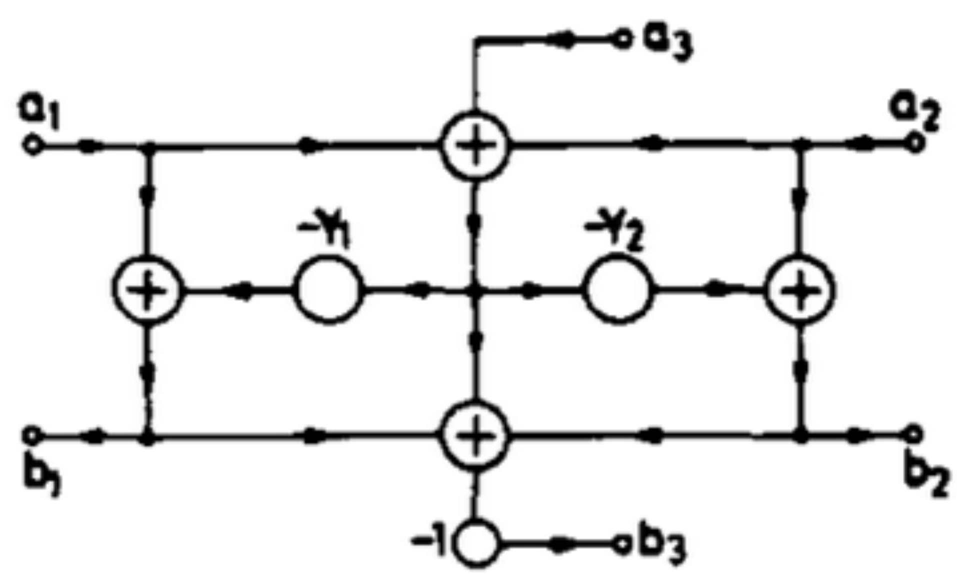
$$\gamma_\nu = \frac{2G_\nu}{G_1 + G_2 + G_3}, \quad \nu = 1,2$$



Series Adaptor



$$\gamma_\nu = \frac{2R_\nu}{R_1 + R_2 + R_3}, \quad \nu = 1,2$$



A. Fettweis, “Wave Digital Filters: Theory and Practice,” Proc. IEEE, 1986.

WAVE DIGITAL FILTERS (binary connection tree)

- binary connection tree (BCT) systematizes WDF with only series and parallel connections
- up to one nonlinearity
- N-port series connections implemented with *(N–2)* 3-port series adaptors
- N-port parallel connections implemented with *(N–2)* 3-port parallel adaptors
- see also: Alfred Fettweis and Klaus Meerkötter, "On adaptors for wave digital filters," 1975.

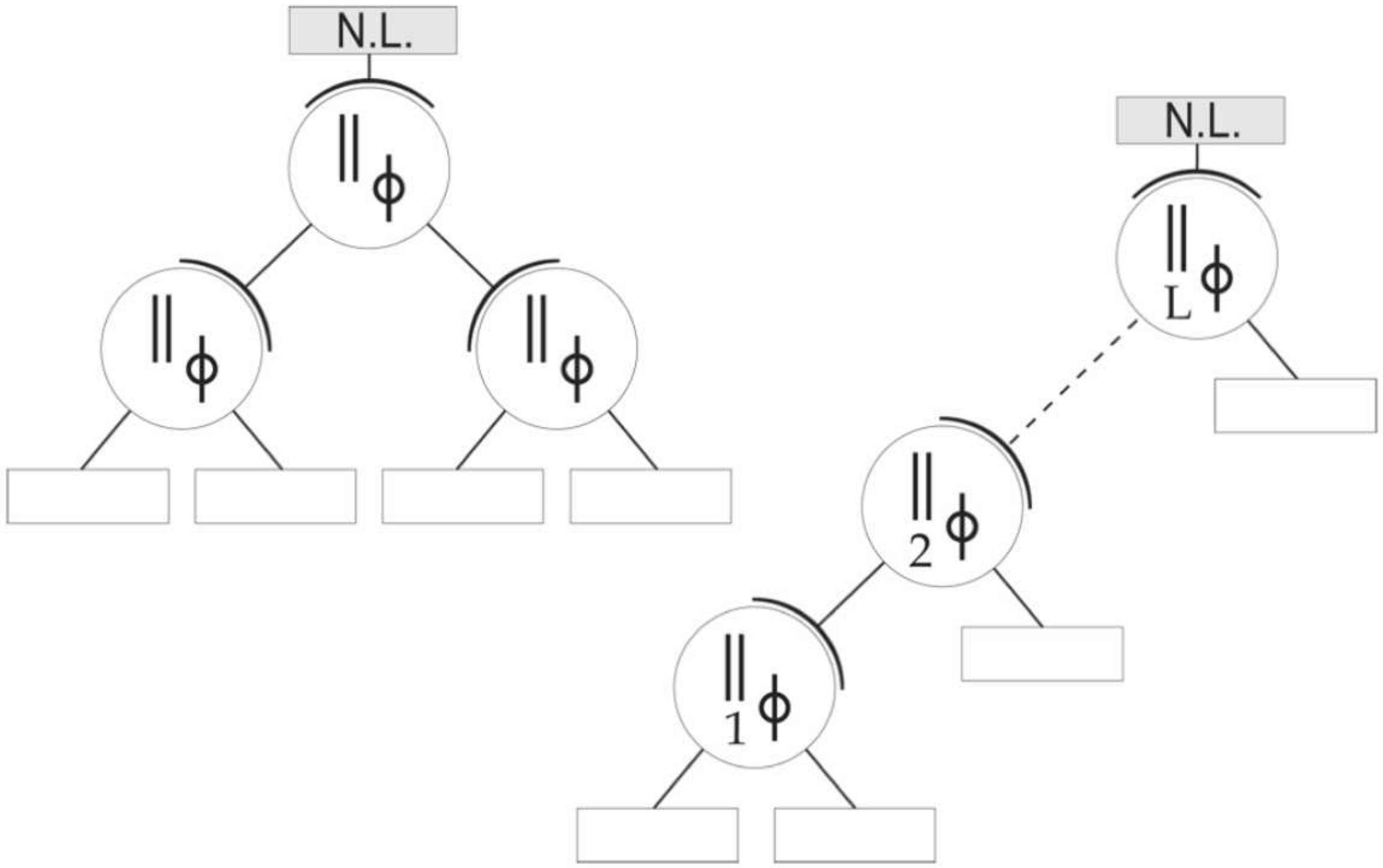


Fig. 5. Two examples of BCTs. (left) Generic one and (right) chainlike circuit. The circular box represents an instantaneous adaptor, in which the adapted port is clearly specified. This particular notational choice simplifies the drawing of connection trees with a great amount of branching.

A. Sarti and G. De Sanctis, "Systematic methods for the implementation of nonlinear wave-digital structures," IEEE Trans. Circuits Syst. I: Reg. Papers, vol. 56, no. 2, pp. 460–472, 2009.

INTRODUCTION

1. tutorial review of WDF principles
2. recent theoretical progress in WDFs
3. WDF software overview and demo

CURRENT RESEARCH at CCRMA

- Wave Digital Filter Adaptors for Arbitrary Topologies and Multiport Linear Elements
@ Int. Conf. Digital Audio Effects (DAFx-15), Trondheim, Norway, Nov. 30 – Dec. 3, 2015
↳Kurt James Werner, Julius O. Smith III, and Jonathan Abel

- Resolving Wave Digital Filters with Multiple/Multiport Nonlinearities
@ Int. Conf. Digital Audio Effects (DAFx-15), Trondheim, Norway, Nov. 30 – Dec. 3, 2015
↳Kurt James Werner, Vaibhav Nangia, Julius O. Smith III, and Jonathan Abel

- A General and Explicit Formulation for Wave Digital Filters with Multiple/Multiport Nonlinearities and Complicated Topologies
@ IEEE Work. Appl. Signal Process. Audio Acoust. (WASPAA), New Paltz, NY, Oct. 18–21, 2015
↳Kurt James Werner, Vaibhav Nangia, Julius O. Smith III, and Jonathan Abel

- An Improved and Generalized Diode Clipper Model for Wave Digital Filters
@ AES 139th Convention, New York, USA, Oct. 29 – Nov. 1, 2015
↳Kurt James Werner, Vaibhav Nangia, Alberto Bernardini, Julius O. Smith III, and Augusto Sarti

- An Energetic Interpretation of Nonlinear Wave Digital Filter Lookup Table Error
@ IEEE Int. Symp. Signals, Circuits, Syst. (ISSCS), Iasi, Romania, July 9–10, 2015
↳Kurt James Werner and Julius O. Smith III

CURRENT RESEARCH at Politecnico di Milano

- Modeling Nonlinear Wave Digital Elements using the Lambert Function
  (*submitted* to IEEE Transactions on Circuits and Systems I: Regular Papers)
  ↳Alberto Bernardini, Kurt James Werner, Augusto Sarti, and Julius O. Smith III

- Modeling a Class of Multi-Port NonLinearities in Wave Digital Structures
  @ European Signal Process. Conf. (EUSIPCO), Nice, France, August 31, 2015
  ↳Alberto Bernardini, Kurt James Werner, Augusto Sarti, and Julius O. Smith III

- Multi-Port NonLinearities in Wave Digital Structures
  @ IEEE Int Symp. Signals, Circuits, Syst. (ISSCS), Iasi, Romania, July 9–10, 2015
  ↳Alberto Bernardini, Kurt James Werner, Augusto Sarti, and Julius O. Smith III

- Modeling NonLinear Circuits with Multi-port Elements in the Wave Digital Domain
  Master's thesis, Politecnico di Milano, Italy, April 2015
  ↳Alberto Bernardini

NONLINEARITIES IN WDFS

1. single nonlinearity
2. consolidated one-port combination
3. cross-controlled multiport
4. simplified multiports
5. linearized multiport
6. piecewise linear models
7. iterative schemes

NONLINEARITIES IN WDFS : single nonlinearity

- accommodate **one** one-port NL element, e.g.:
  - ideal rectifier (ideal diode)
  - piecewise linear resistance
- can view as lookup table with interpolation or piece-wise linear segments
- must solve $b = f(a)$ at root

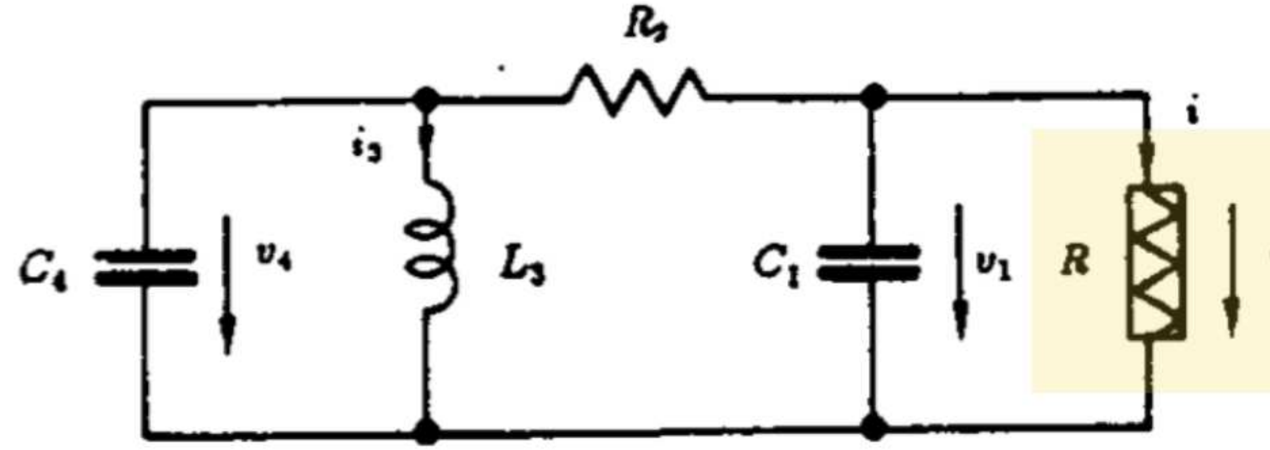


Fig. 2. Nonlinear circuit according to Ref. 2.

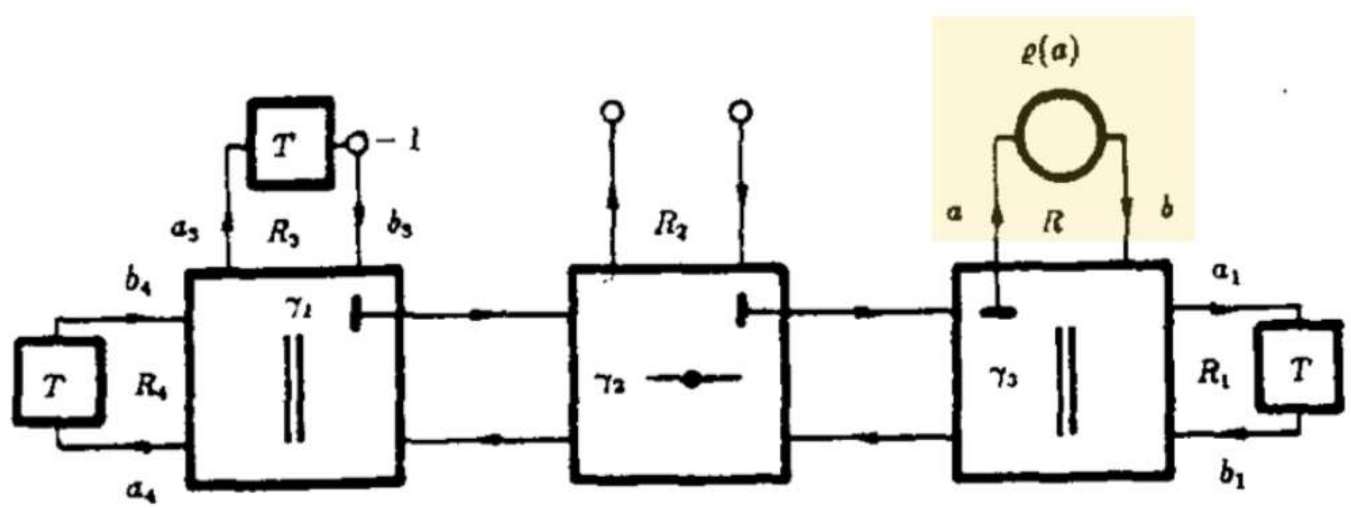


Fig. 4. Wave digital model of the circuit of Fig. 2.

Klaus Meerkötter and Reinhard Sholz, “Digital simulation of nonlinear circuits by wave digital filter principles.” in proc. IEEE Int. Symp. Circuits Syst., June 1989.

NONLINEARITIES IN WDFS : single nonlinearity

wave (*a*–*b*) domain

Kirchhoff (*i*–*v*) domain

voltage wave transformation

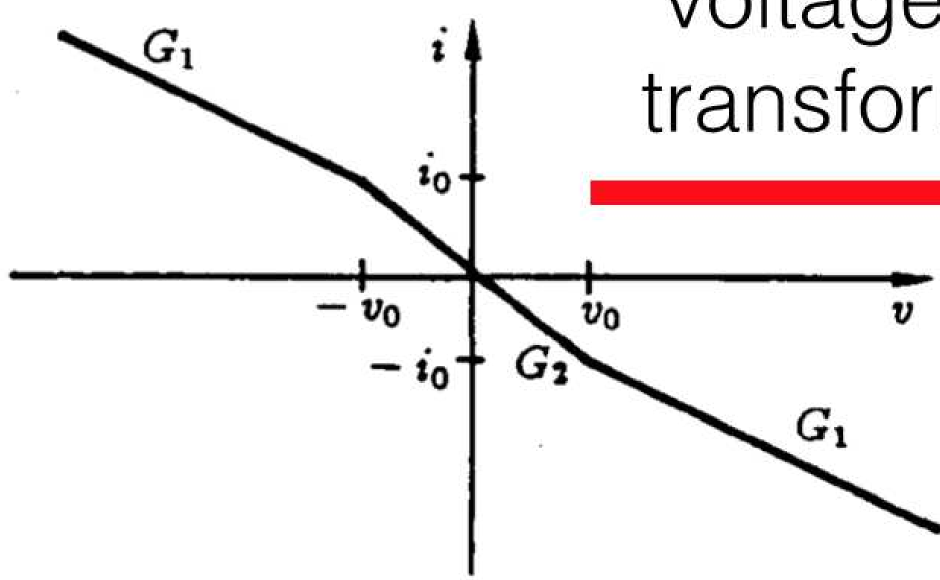


Fig. 3. Characteristic of the nonlinear resistance defined by (16).

$$i = G_1 v + \frac{1}{2}(G_2 - G_1)\left(|v + v_0| - |v - v_0|\right), \qquad (16)$$

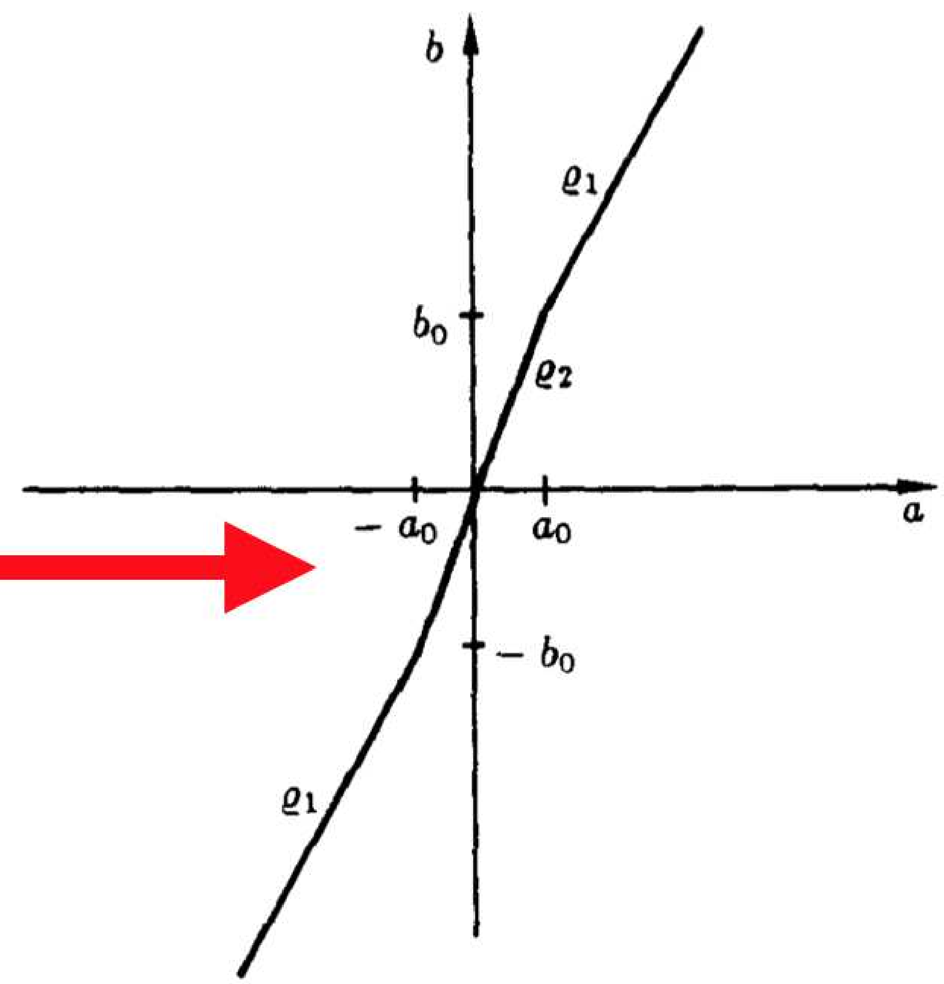


Fig. 5. Plot of the characteristic defined by (17).

$$b = \varrho(a) = \varrho_1 a + \frac{1}{2}(\varrho_2 - \varrho_1)\left(|a + a_0| - |a - a_0|\right), \quad (17)$$

Klaus Meerkötter and Reinhard Sholz, “Digital simulation of nonlinear circuits by wave digital filter principles.” in proc. IEEE Int. Symp. Circuits Syst., June 1989.

NONLINEARITIES IN WDFs : single nonlinearity

- use "mutators" from classical network theory to enable, e.g., nonlinear *q–v* relationships
- these are needed for nonlinear elements "with memory"
- for example, nonlinear capacitors and inductors where flux or charge can saturate

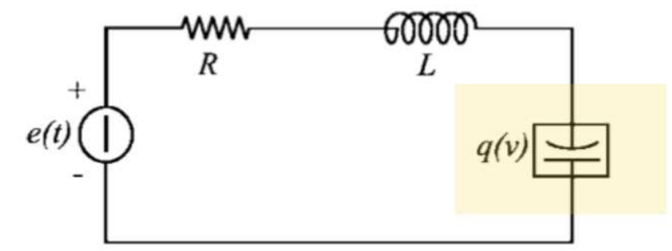


Fig. 6. Electrical circuit of the anharmonic oscillator.

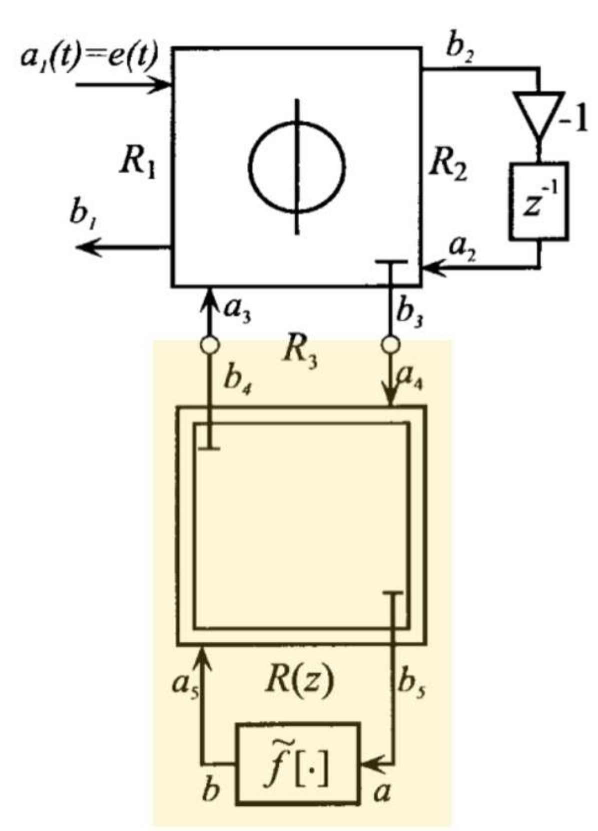


Fig. 9. Wave implementation of the anharmonic oscillator based on instantaneous adaptation. The double-bordered box represents an R-C mutator, and the presence of two "stubs" in its outputs denotes the absence of local instantaneous reflections.

Augusto Sarti and Giovanni De Poli, "Toward nonlinaer wave digital filters," IEEE Trans. Signal Process., vol. 47, no. 6, pp 1654–1668, 1999.

NONLINEARITIES IN WDFS : consolidated one-ports

- multiple nonlinearities handled by consolidating into a single one-port
- implicit nonlinear function solved as *b=f(a)* with numerical methods

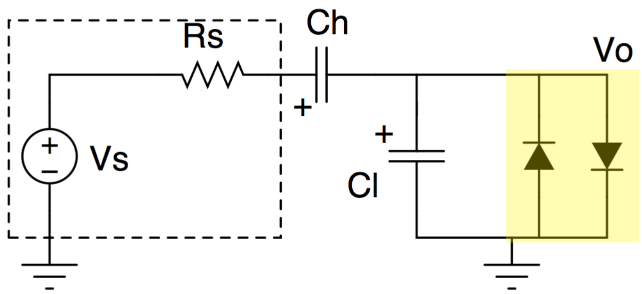


Figure 6: Schematic of the diode clipper with high-pass and low-pass capacitors.

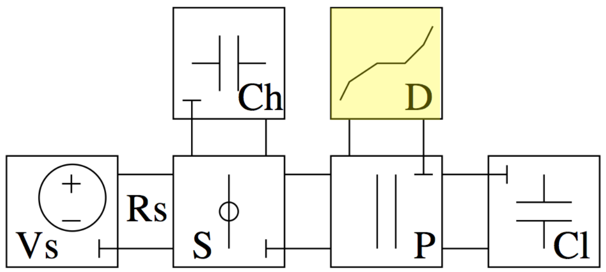


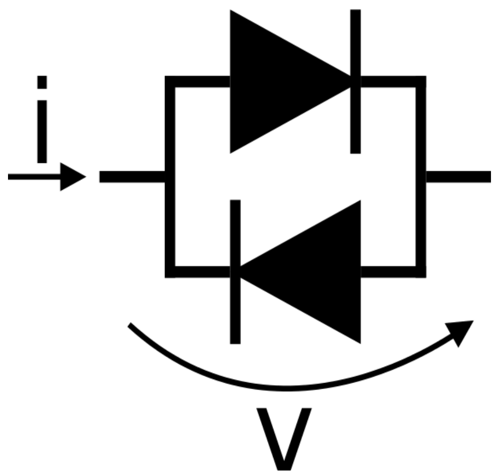
Figure 7: WDF tree of the two-capacitor diode clipper. Diode D is root.

$$\frac{b-a}{2R} = I_s\left(e^{\frac{a+b}{2V_T}} - 1\right) - I_s\left(e^{-\frac{a+b}{2V_T}} - 1\right)$$

David T. Yeh and Julius O. Smith III, "Simulating guitar distortion circuits using wave digital and nonlinear state-space formulations," in proc. Int. Conf. Digital Audio Effects (DAFx-08), pp. 19–26, 2008.

NONLINEARITIES IN WDFS : consolidated one-ports

b=f(a) for diode pair solved using Lambert *W* function, assuming one diode dominates: (ignoring saturation current)

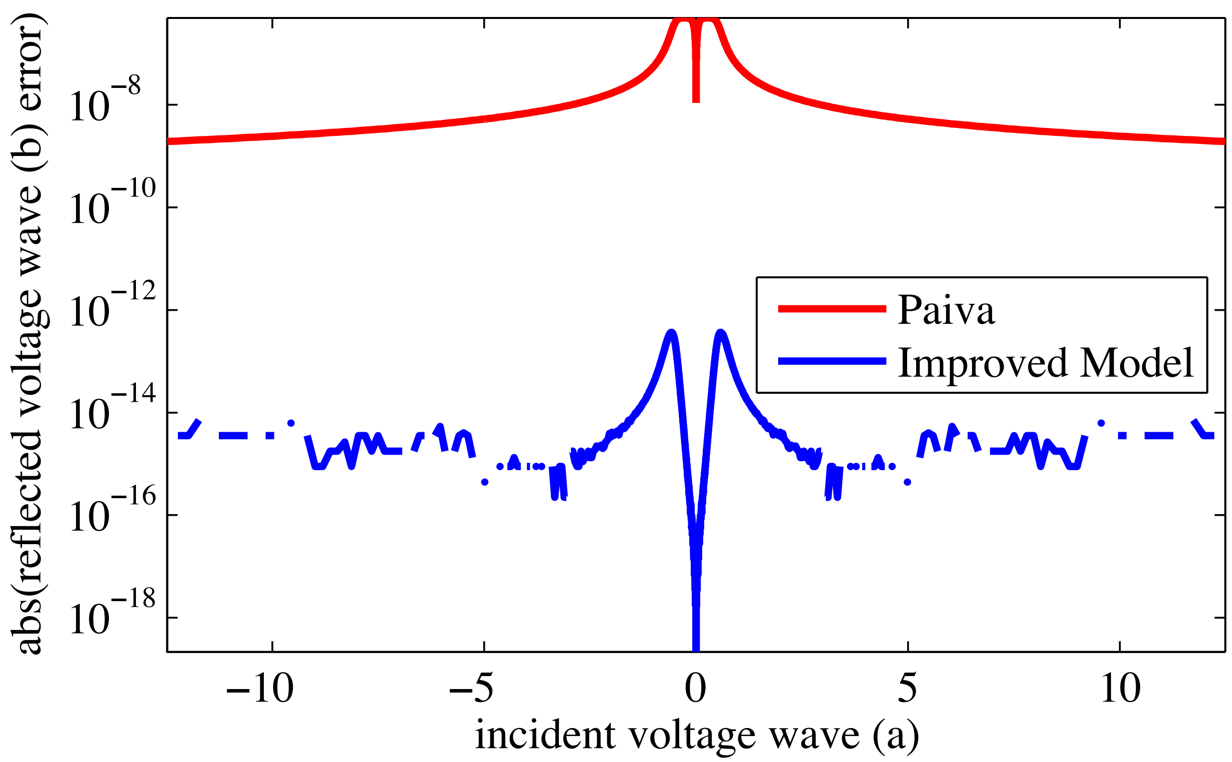


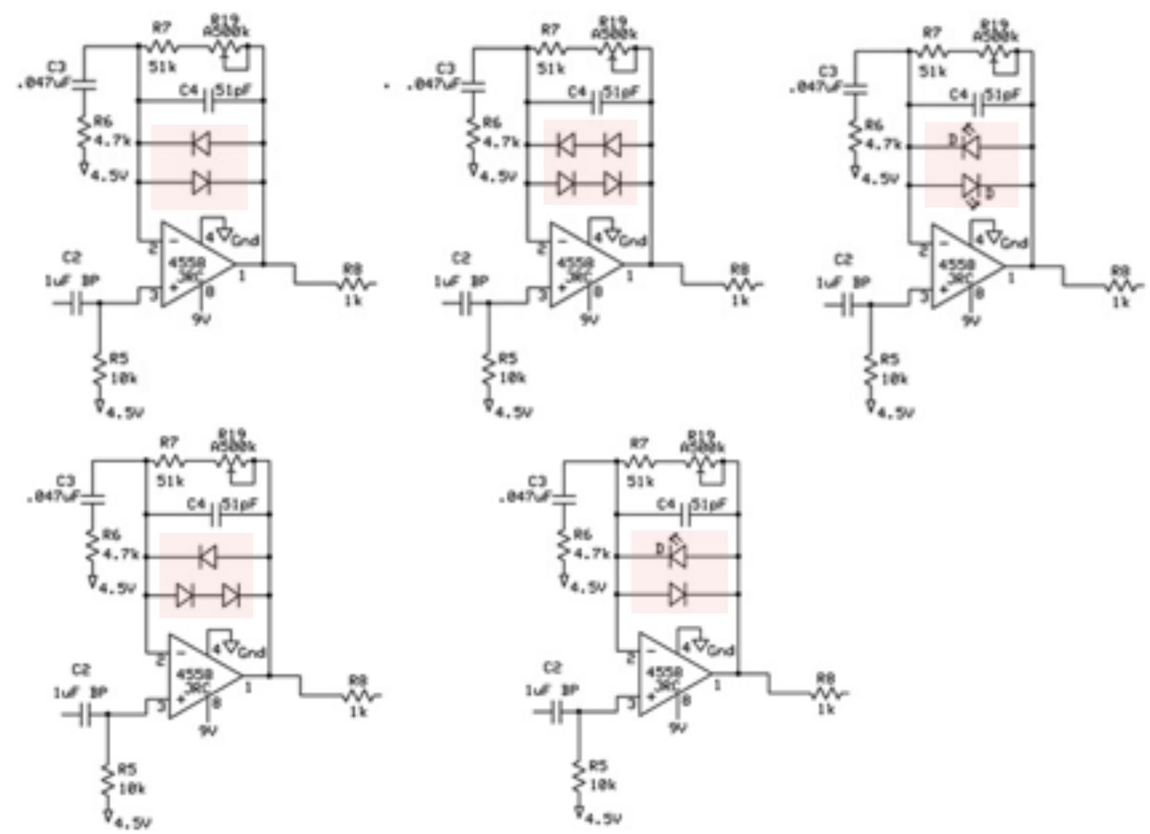
- One diode (Kirchhoff): $i = I_s\left(e^{\frac{v}{V_T}} - 1\right)$
- One diode (wave, implicit): $\dfrac{a-b}{2R} = I_s\left(e^{\frac{a+b}{2V_T}} - e^{-\frac{a+b}{2V_T}}\right)$
- One diode (wave, explicit): $b = f(a) = a + 2RI_s - 2V_T\mathcal{W}\left(\dfrac{RI_s}{V_T}e^{\frac{RI_s+a}{V_T}}\right)$
- Diode pair (approximate): $b = \operatorname{sgn}(a)\cdot f(|a|)$

Rafael C. D. de Paiva, Stefano D'Angelo, Jyri Pakarinen, and Vesa Välimäki, "Emulation of operational amplifiers and diodes in audio distortion circuits," IEEE Trans. Circuits Syst. II: Expr. Briefs, Vol. 59, No. 10, Oct. 2012.

NONLINEARITIES IN WDFS : consolidated one-ports

- explicit model improved by canceling some approximation error of Paiva *et al.* (2013) model with an additional Lambert *W* term
- generalized to any number of diodes in each direction (stock and hacked guitar distortion pedals)



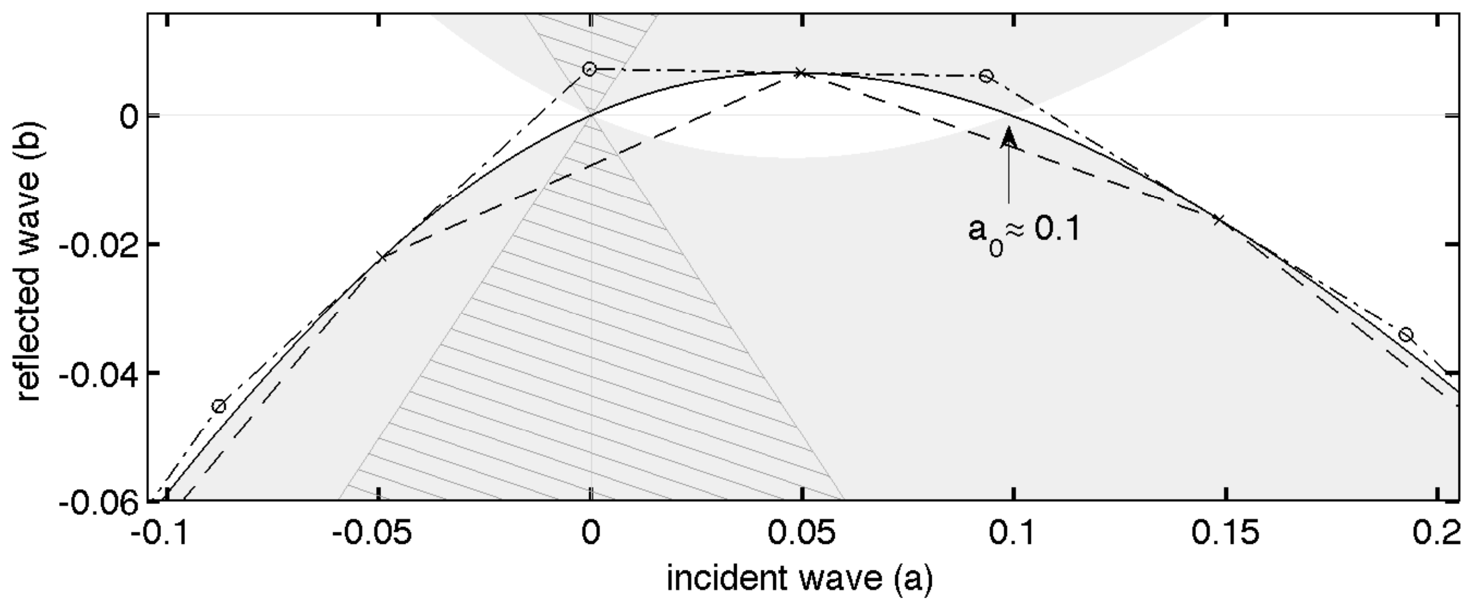


Kurt James Werner, Vaibhav Nangia, Alberto Bernardini, Julius O. Smith III, and Augusto Sarti, "An improved and generalized diode clipper model for wave digital filters," in proc. Audio Eng. Soc. (AES), New York, NY, 2015.

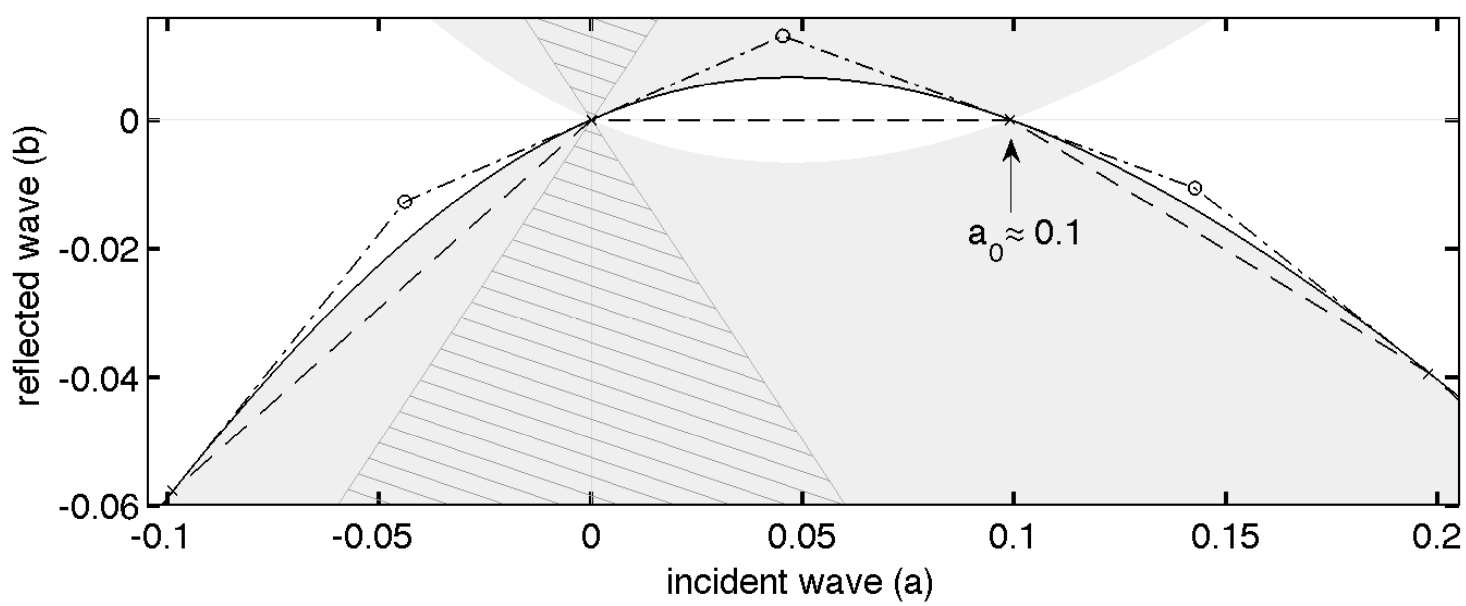
B. Wampler, "5 DIY Mods to Perfect Your Ibanez TS9 and Boss SD-1," Sept. 2012, Online: http://www.premierguitar.com/articles/5_DIY_Mods_to_Perfect_Your_Ibanez_TS9_and_Boss_SD-1?page=3

NONLINEARITIES IN WDFS : consolidated one-ports

Linear secant interpolation and tangent extrapolation can be incrementally (gray) or globally (thatched) non-passive



Choosing table points and secant/tangent properly (considering sgn(*a*) and *a*”) yields interpolation methods that respect passivity



Kurt James Werner and Julius O. Smith III, and Augusto Sarti, “An energetic interpretation of nonlinear wave digital filter lookup table error,” in proc. Int. Symp. Signals, Circuits, Syst. (ISSCS), Iași, Romania, July 2015.

NONLINEARITIES IN WDFS : cross-controls

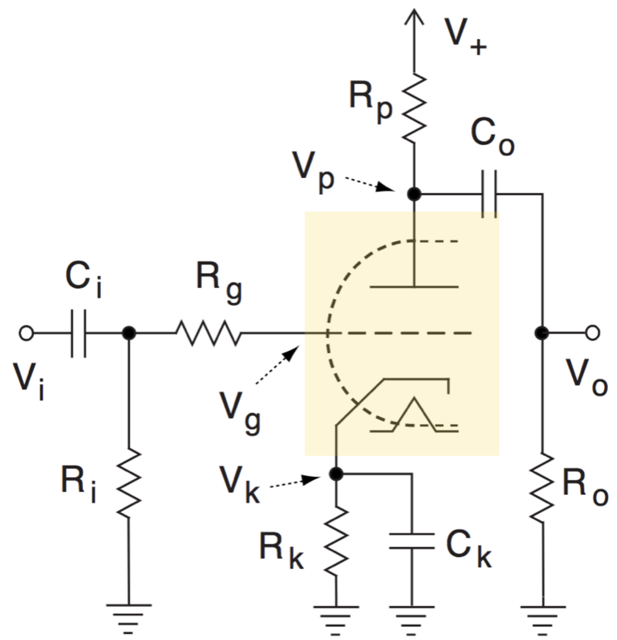


Fig. 1. A typical triode amplifier stage.

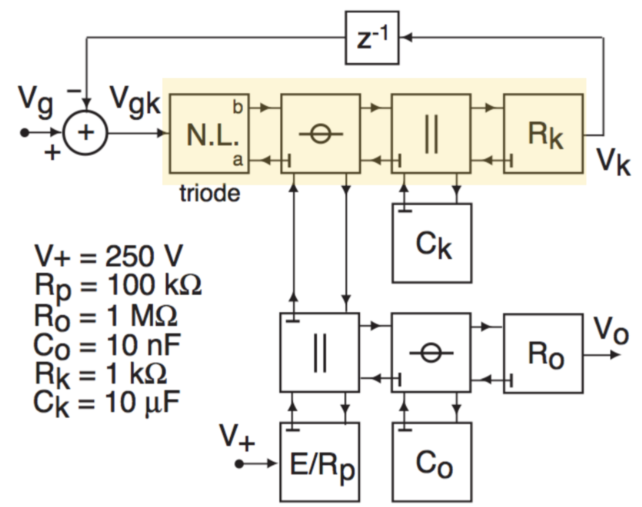
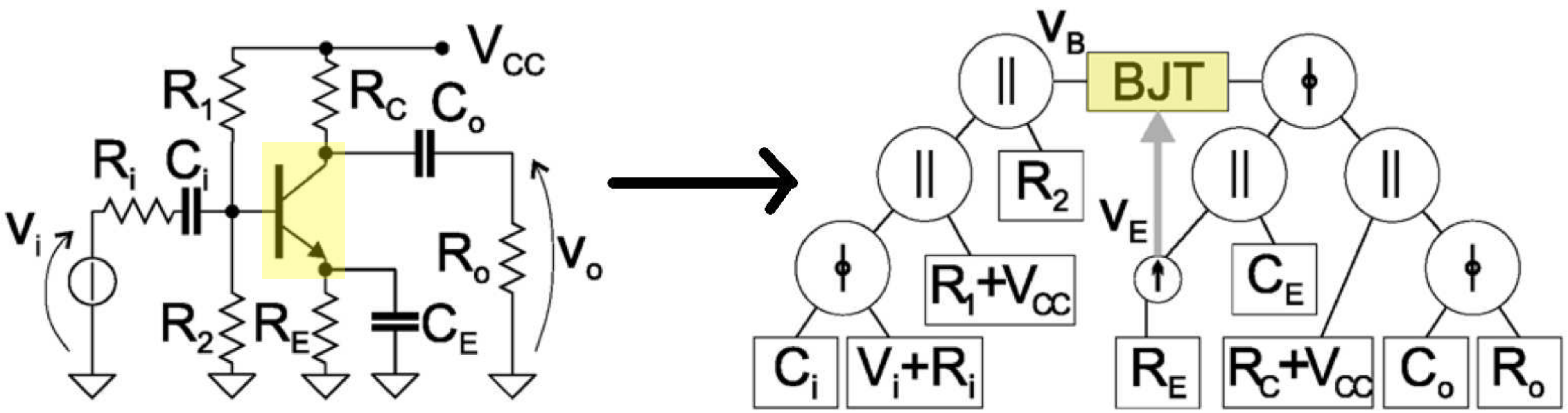


Fig. 4. WDF binary tree for simulation of the triode stage in Fig. 1. The input circuit (C_i, R_i, R_g) is omitted and the cathode voltage V_k is used throught a unit delay to get the grid-to-cathode voltage V_{gk}.

- grid–cathode voltage modeled as "cross-control" of plate–cathode nonlinearity, with ad-hoc delay to aid realizability
- Refined in Jyri Pakarinen and Matti Karjalainen, "Enhanced wave digital triode model for real-time tube amplifier emulation," IEEE Trans. Audio, Speech, Language Process., vol. 18, no. 4, pp. 738–746, May 2010.

Matti Karjalainen and Jyri Pakarinen, "Wave digital simulation of a vacuum-tube amplifier," in proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP), Toulouse, France, May 14–19. 2006.

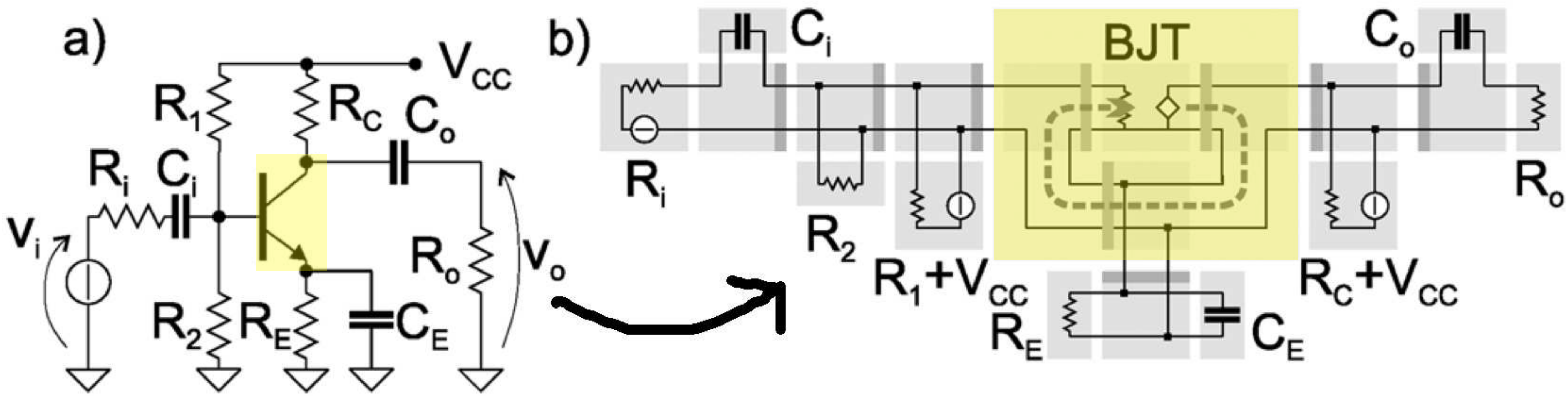
NONLINEARITIES IN WDFs : cross-controls



- frame BJT as two-port nonlinear element
- two linear WDF subtrees
- Emitter voltage V_E proposed as "cross-control" on BJT

Giovanni De Sanctis and Augusto Sarti, "Virtual analog modeling in the wave-digital domain," IEEE Trans. Audio, Speech, Language Process., vol. 18, no. 4, pp. 715–727, 2010.

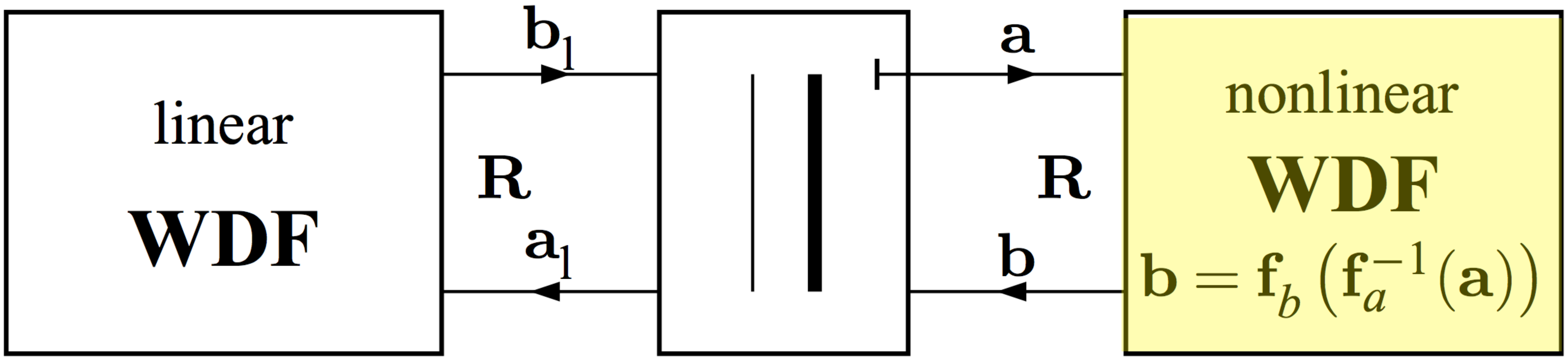
NONLINEARITIES IN WDFS : linearized multiport



- proposes using linearized Hybrid-π model of BJT
- three linear WDF subtrees

Giovanni De Sanctis and Augusto Sarti, "Virtual analog modeling in the wave-digital domain," IEEE Trans. Audio, Speech, Language Process., vol. 18, no. 4, pp. 715–727, 2010.

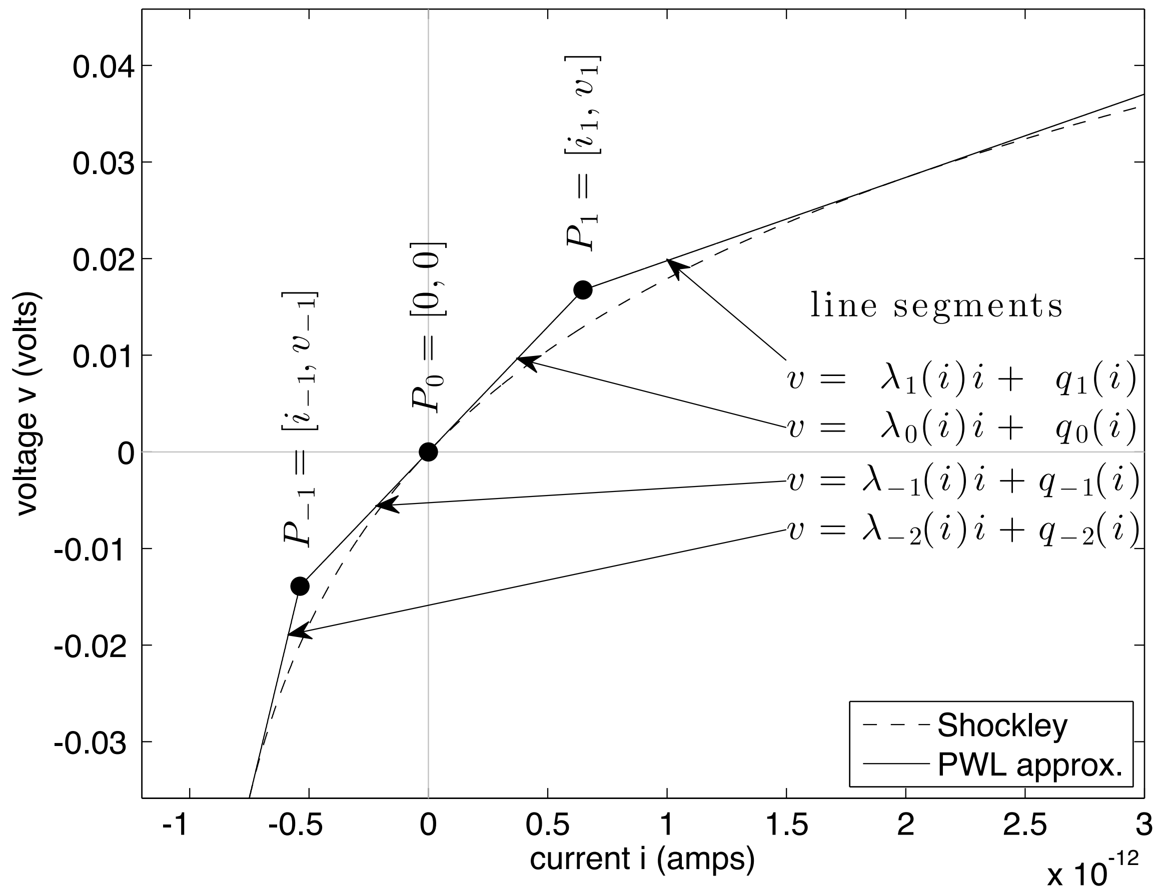
NONLINEARITIES IN WDFS : piecewise linear models



- represent vector of nonlinear root elements with piecewise linear approximation
- limited to vector parallel relationship between "internal" (***a*** and ***b***) and "external" ($\boldsymbol{a}_l$ and $\boldsymbol{b}_l$) root ports

Stefan Petrausch and Rudolf Rabenstein, "Wave digital filters with multiple nonlinearities," in proc. European Signal Process. Conf. (EUSIPCO), vol. 12, Vienna, Austria, Sept. 2004.

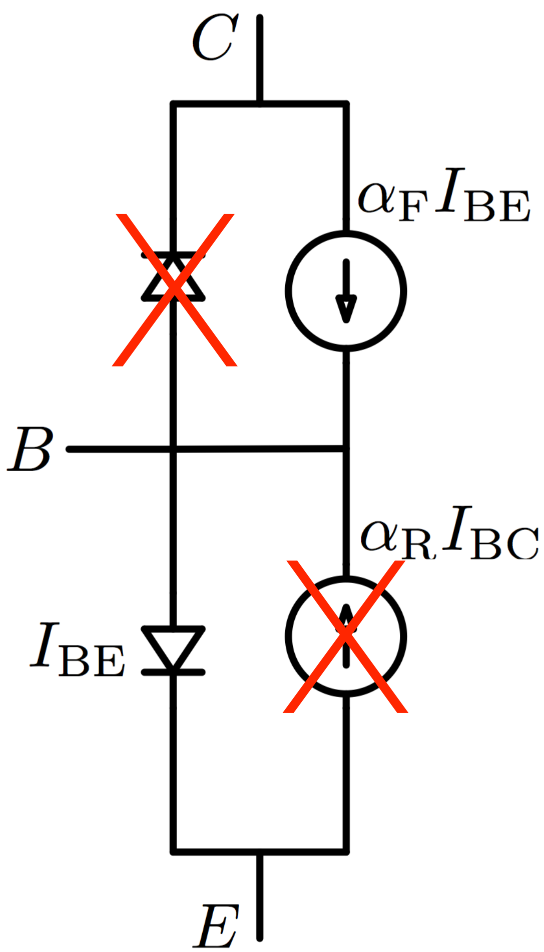
NONLINEARITIES IN WDFS : piecewise linear models

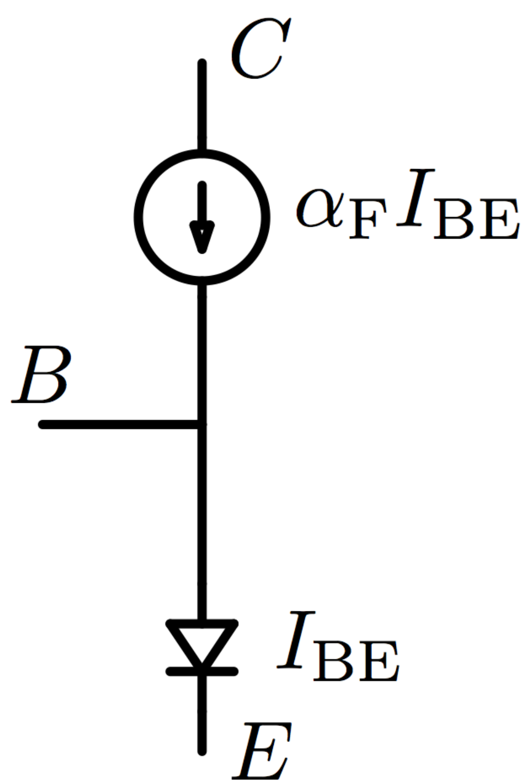


- addresses Petrausch & Rabenstein (2004) limit to vector parallel case for multiple one-port nonlinearities

Alberto Bernardini, Kurt James Werner, Augusto Sarti, and Julius O. Smith III, "Multi-port nonlinearities in wave digital structures," in proc. IEEE Int. Symp. Signals, Circuits, Syst. (ISSCS), Iasi Romania, July 9–10, 2015.

NONLINEARITIES IN WDFS : simplified multiports





- depending on operating point, transport across particular *p–n* junctions in a BJT can be reasonably neglected
- introduces some approximation error, but renders equations tractable using the Lambert W, as in Paiva *et al.* (2013)
- Opportunities for treating cases with feedback

Alberto Bernardini, “Modeling nonlinear circuits with multi-port elements in the wave digital domain,” M.S. thesis, Politecnico di Milano, 2015.

(Alberto Bernardini, Kurt James Werner, Augusto Sarti, and Julius O. Smith III, “Modeling Nonlinear Wave Digital Elements using the Lambert Function,” *recently submitted* to IEEE TCAS I.)

NONLINEARITIES IN WDFS : iterative schemes

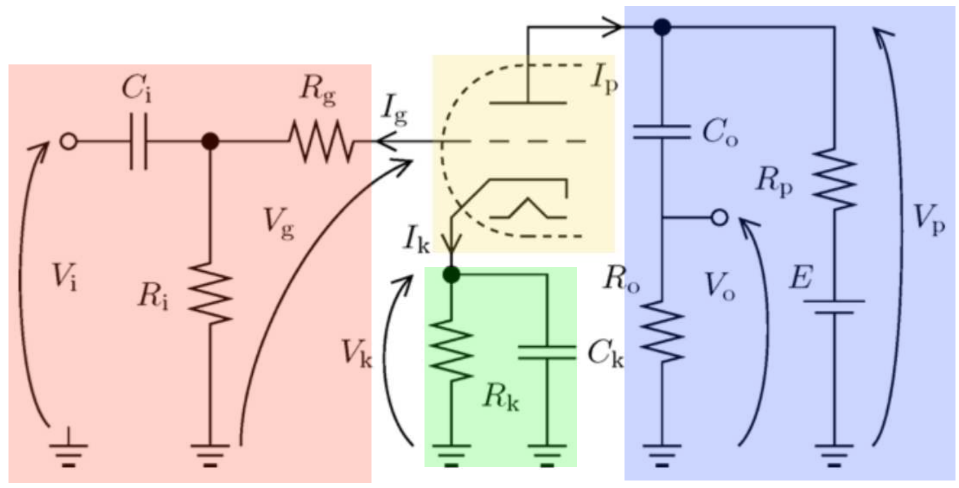


Fig. 2. The common-cathode triode gain stage, typically found in tube amplifiers.

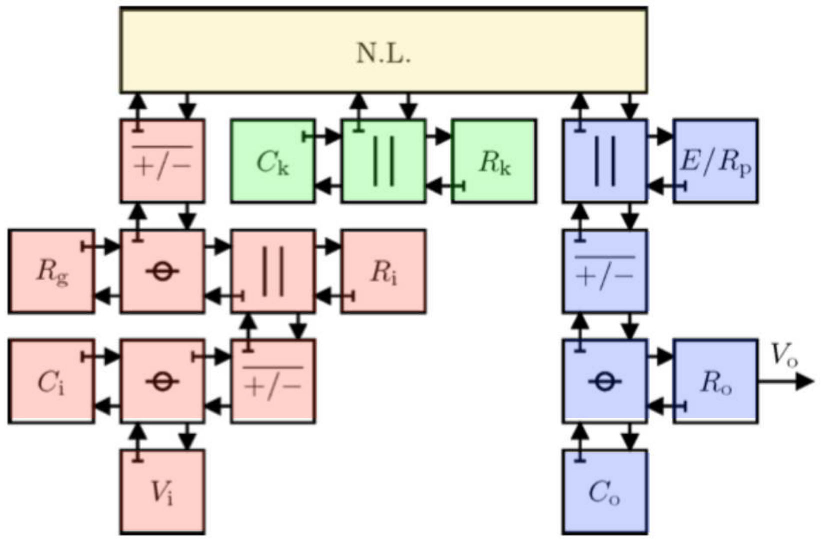


Fig. 4. Implementation of new WDF simulators. The same structure is used in both cases (w/o and w/ grid current).

- entire triode nonlinearity contained in root element
- three linear WDF subtrees (1 2 3)
- root solved with customized secant method (specific to triode model)

Stefano D'Angelo, Jyri Pakarinen, and Vesa Välimäkii, "New family of wave-digital triode models," IEEE Trans. Audio, Speech, Language Process., vol. 21, no. 2, pp. 313–321, 2013.

NONLINEARITIES IN WDFS : iterative schemes

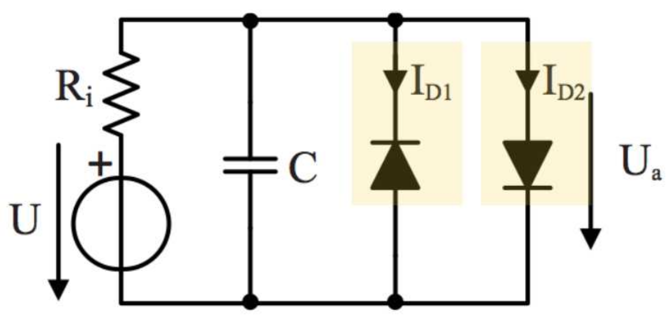
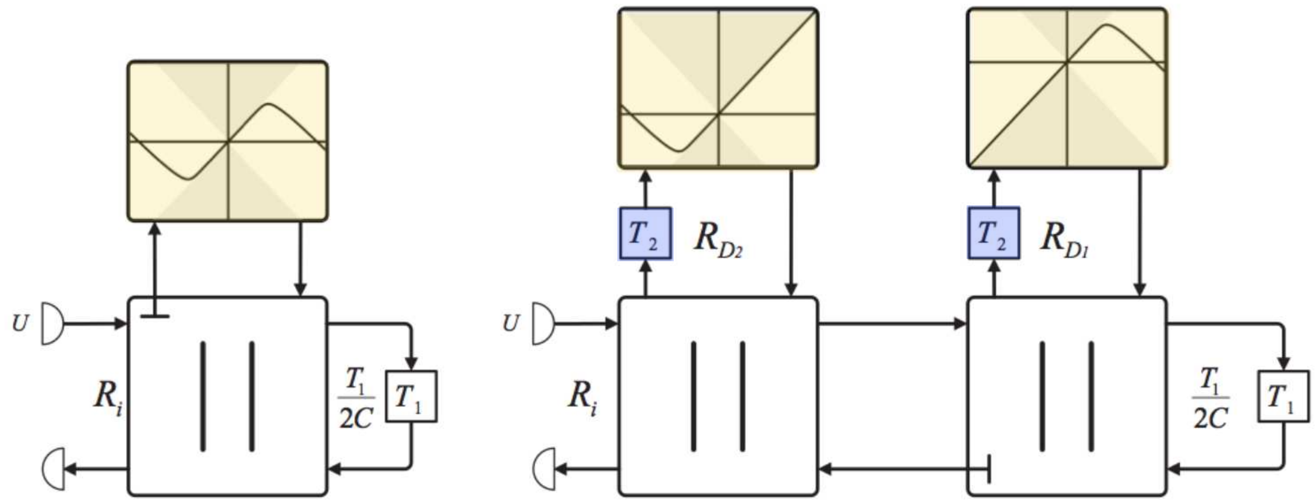


Fig. 1. Prototype circuit



(a) Reference WDF (b) Lumped multidimensional WDF

- multiple nonlinearities create delay-free loops
- resolved by inserting extra delay elements as second time dimensions (T_2)
- framed as extension to multidimensional case
- T_2's solved by iteration, convergence guaranteed by contractivity of WDF properties energy metric

Tim Schwerdtfeger and Anton Kummert, "A multidimensional approach to wave digital filters with multiple nonlinearities," in proc. European Signal Process. Conf. (EUSIPCO), Lisbon, Portugal, Sept. 1–5, 2014.

NONLINEARITIES IN WDFS : iterative schemes

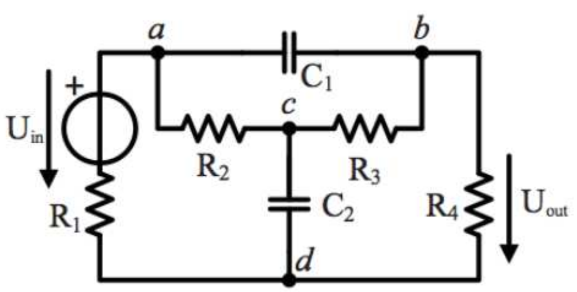
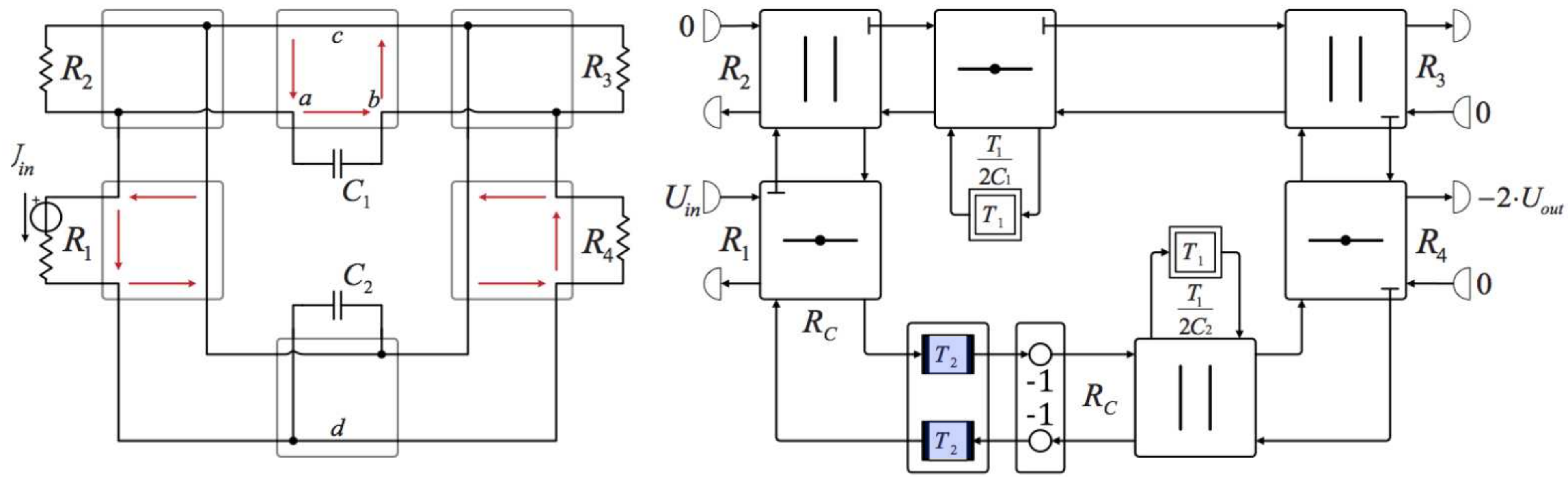


Fig. 1. Prototype circuit with bridged-T topology



(a) Rewritten prototype circuit

(b) WDF realization

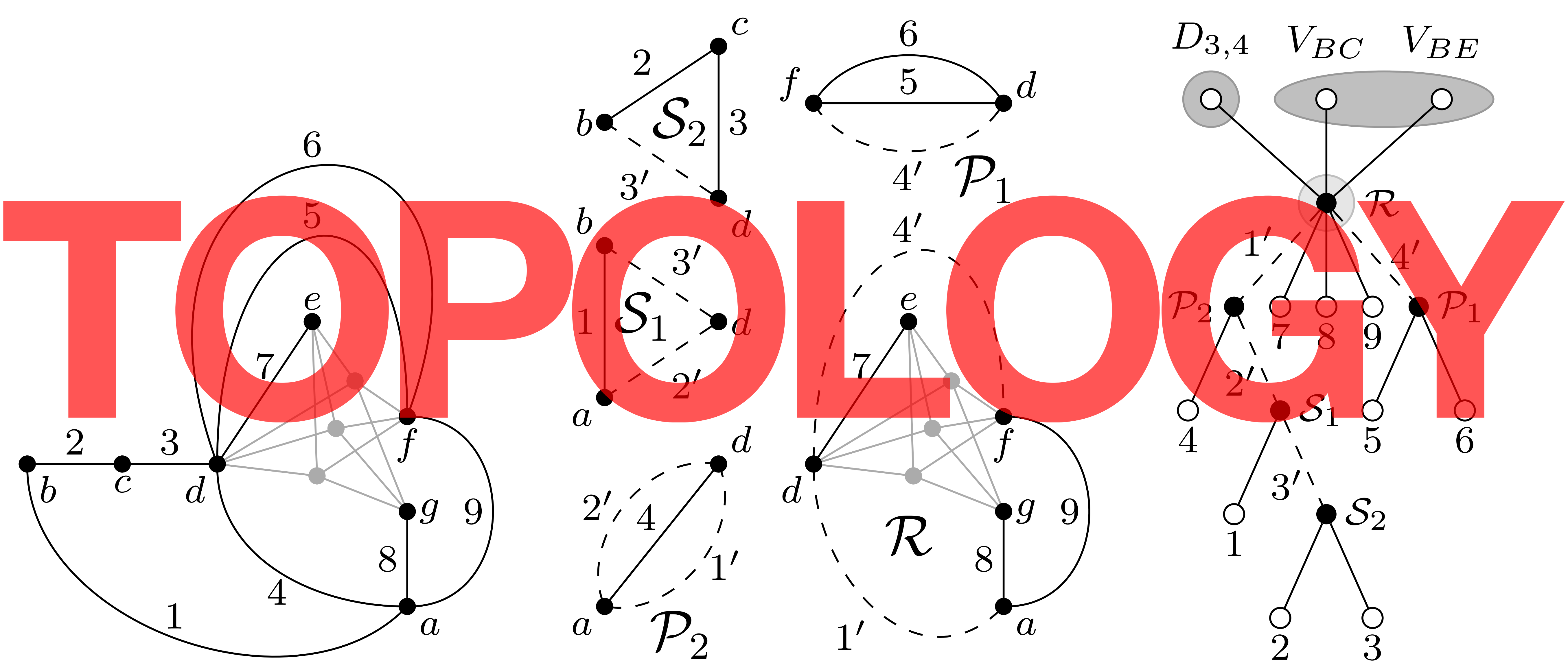
- same technique applied to topological problems in linear circuits (e.g., bridged-T topology)

Tim Schwerdtfeger and Anton Kummert, “A multidimensional approach to wave digital filters with topology-related delay-free loops,” in proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP), Florence, Italy, May 4–9, 2014.

Nonlinearities in WDFs

- recent research focuses on multiple nonlinearities
- since 1989, we've known that >1 requires especial treatment
- solutions with a single multiport at the root seem promising
- what about MULTIPLE multiports?
- ad hoc solutions sometimes work, but no general solutions

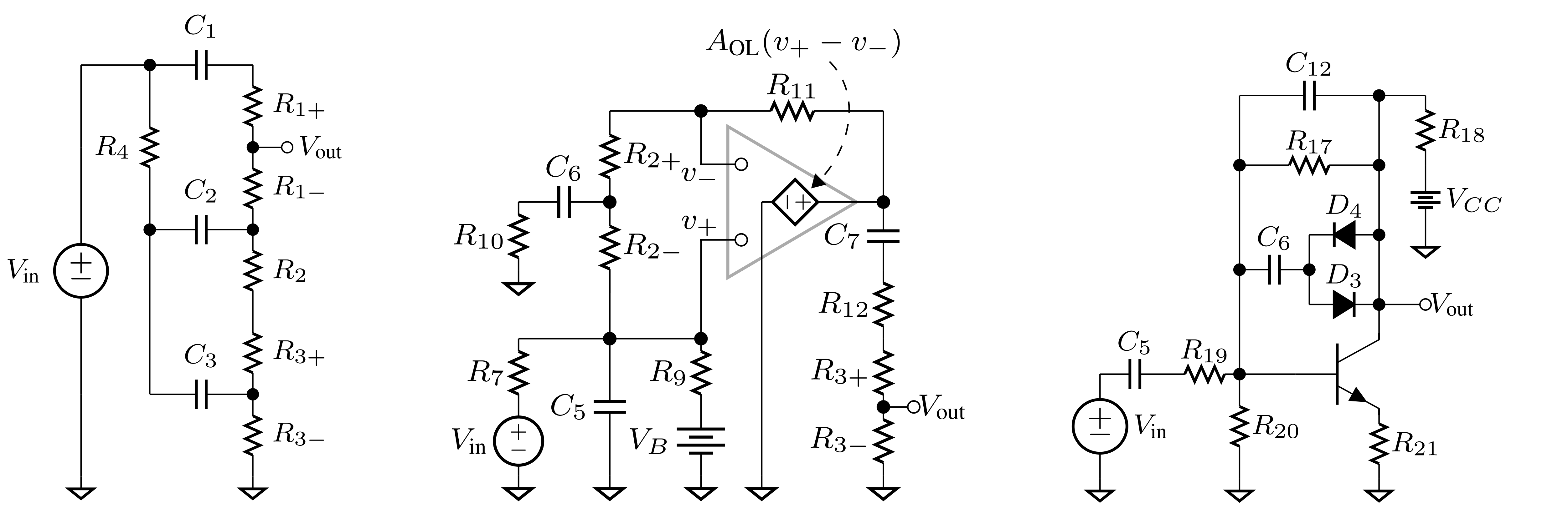
Topological Aspects of WDFs



Topological Aspects of WDFs

WDFs originally intended for *ladder* and *lattice* circuits
(topology comprised of **only** series and parallel)

in general, circuits have ANY topology
(**infinite** class of other types of connections)



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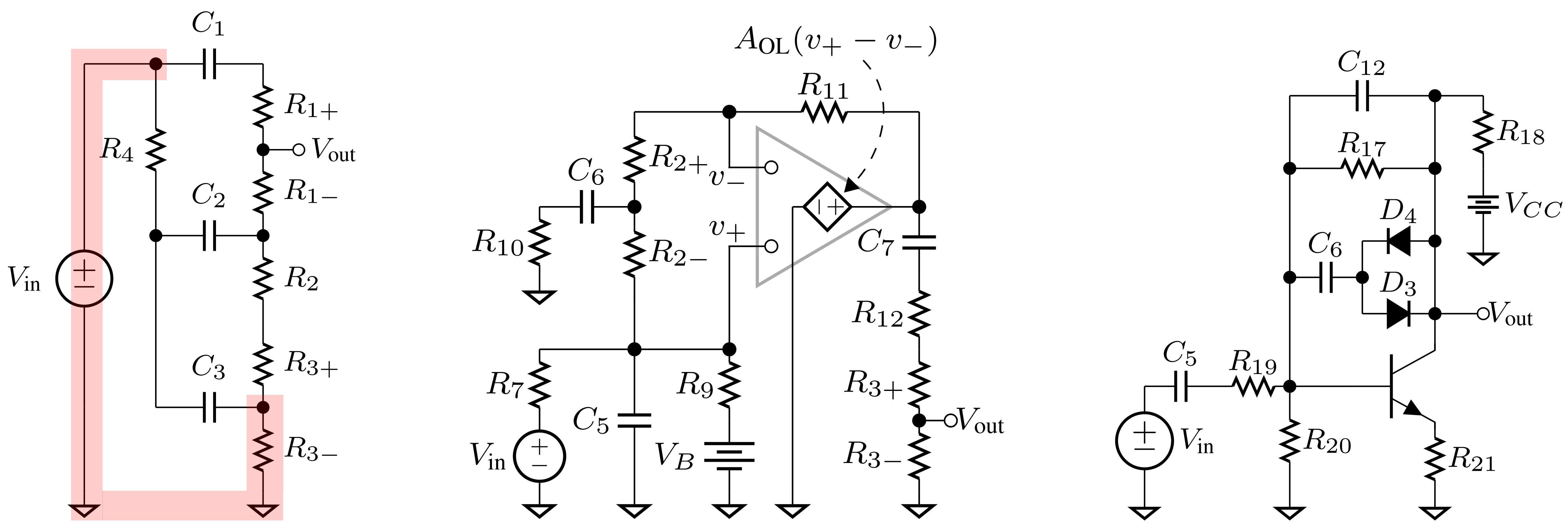

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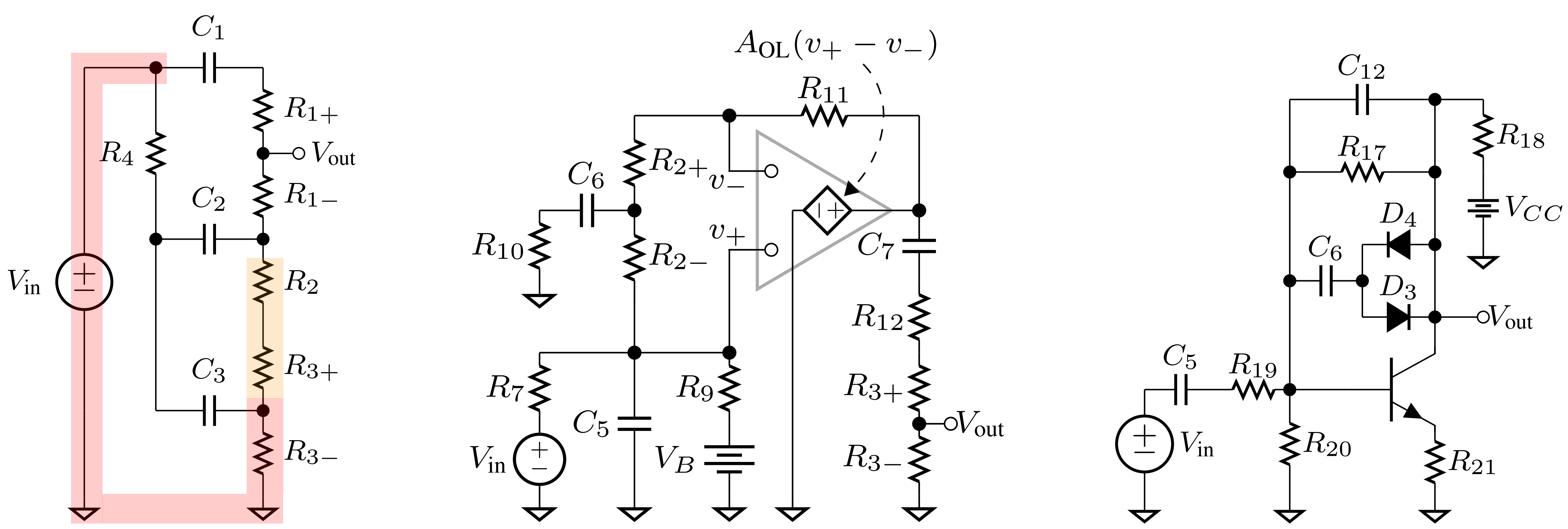

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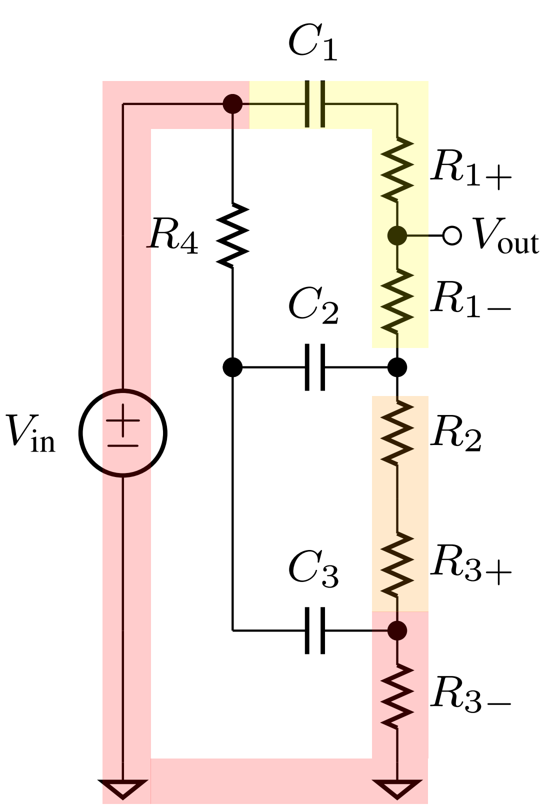

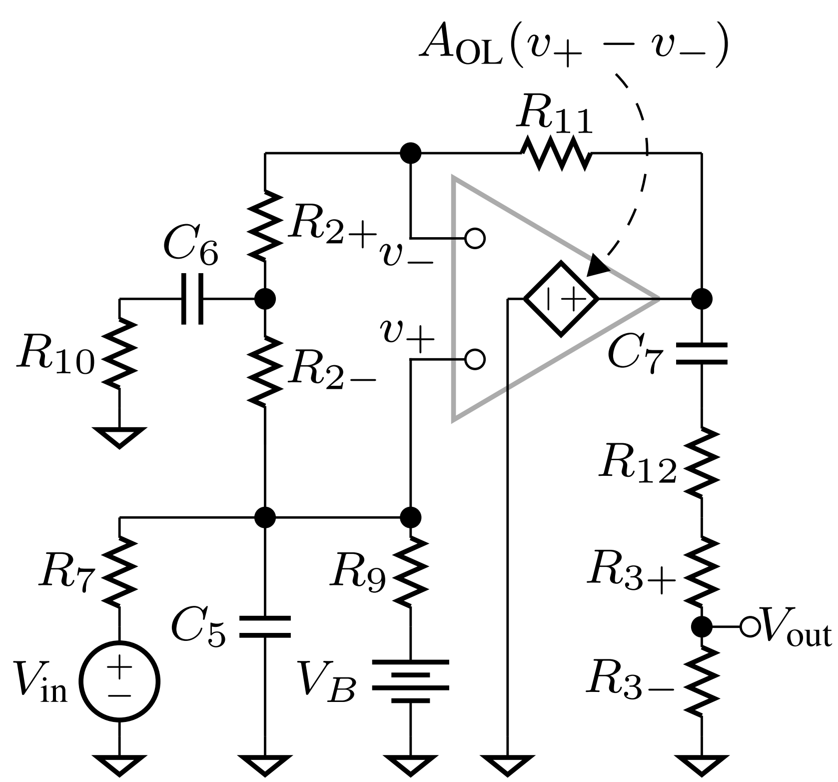

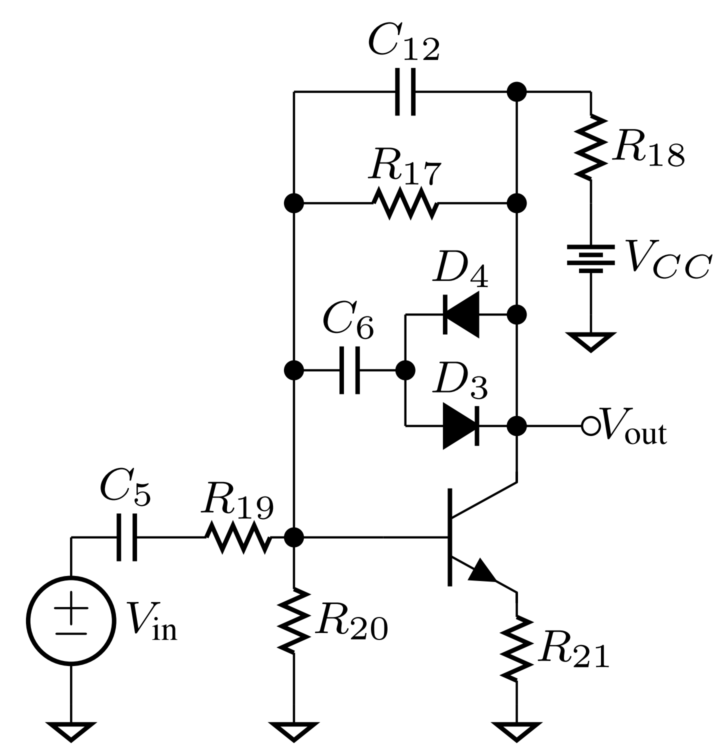

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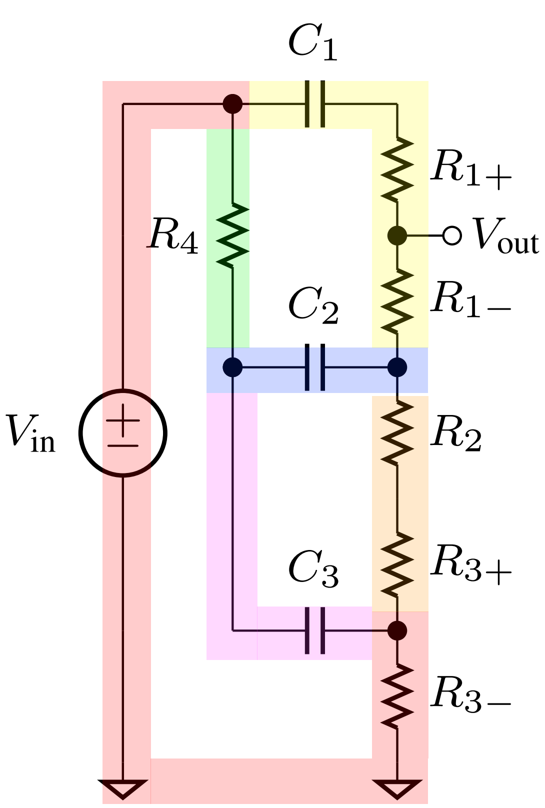

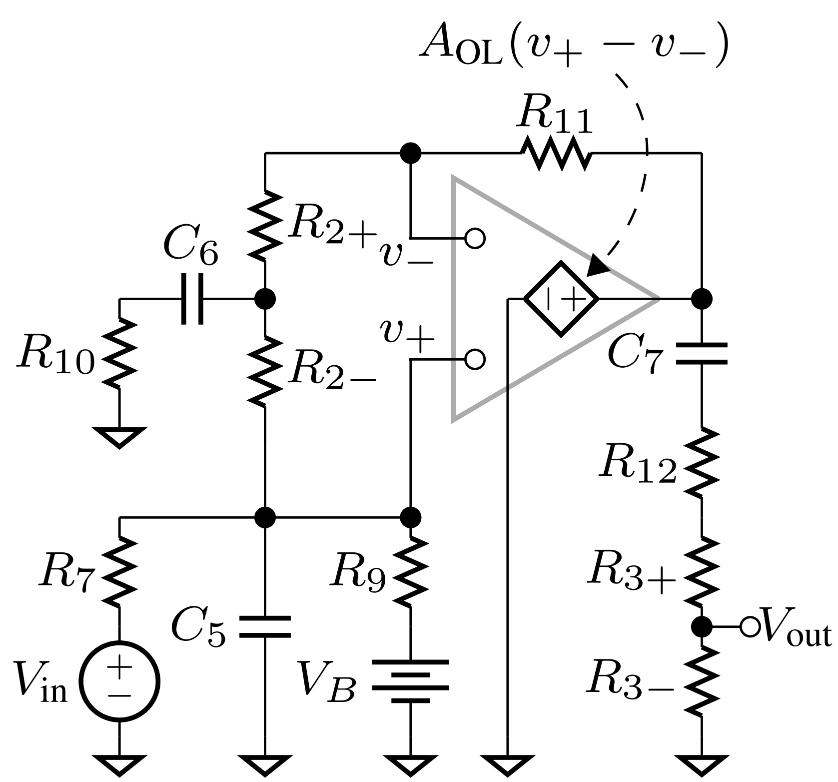

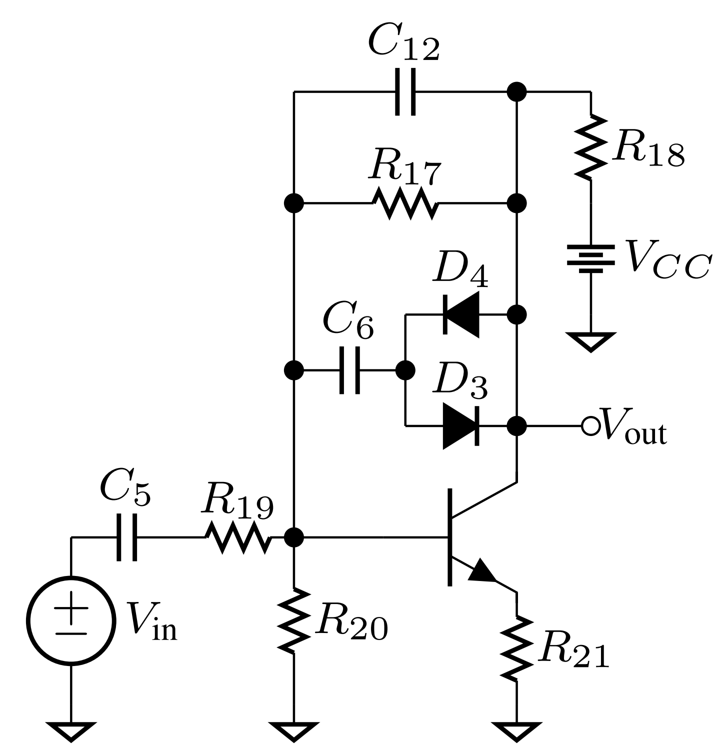

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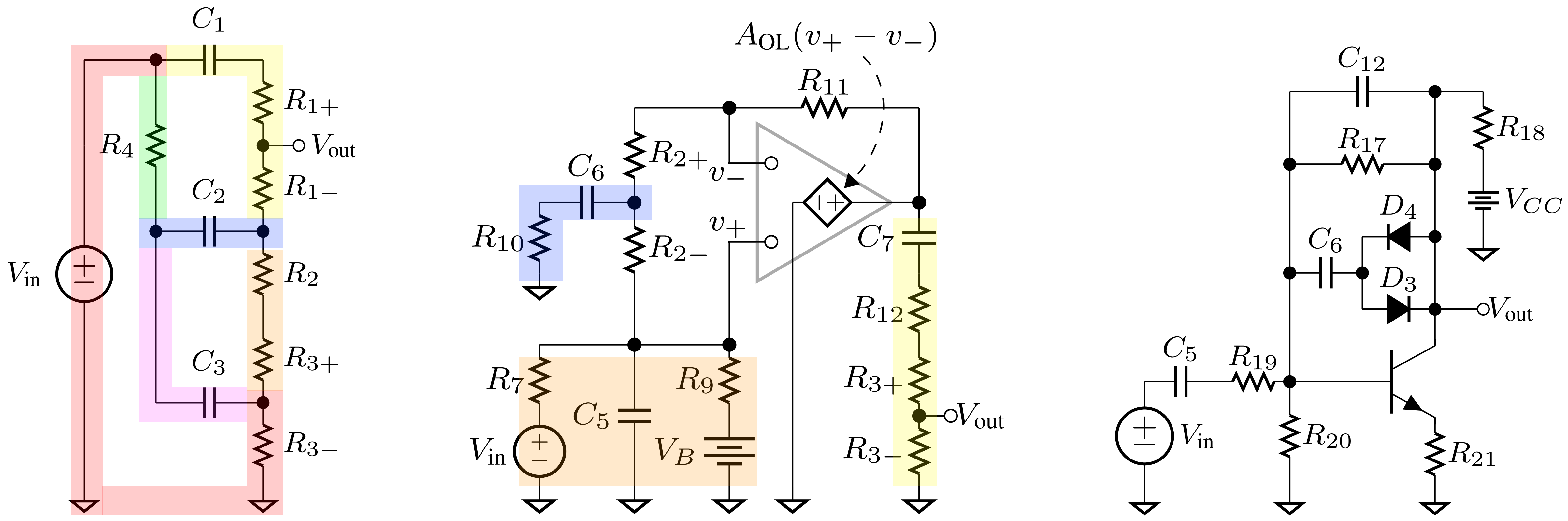

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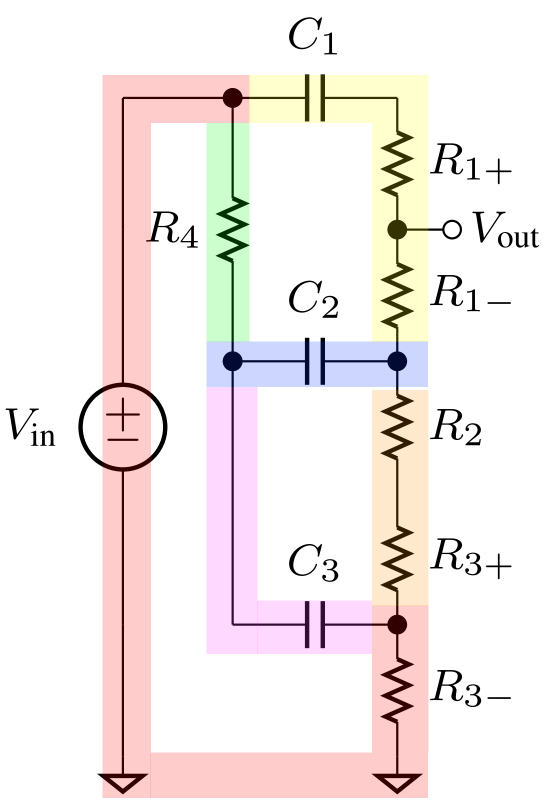

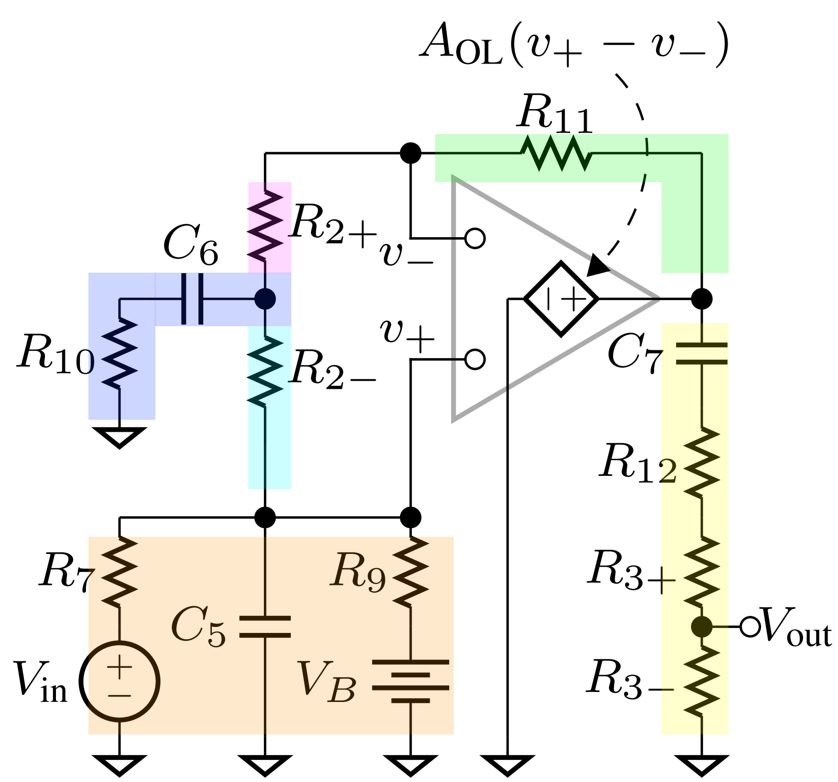

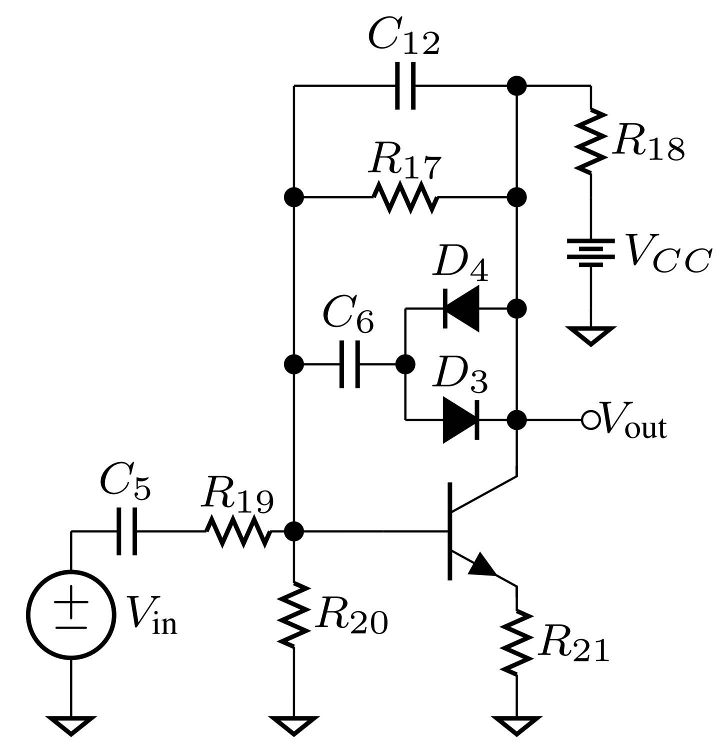

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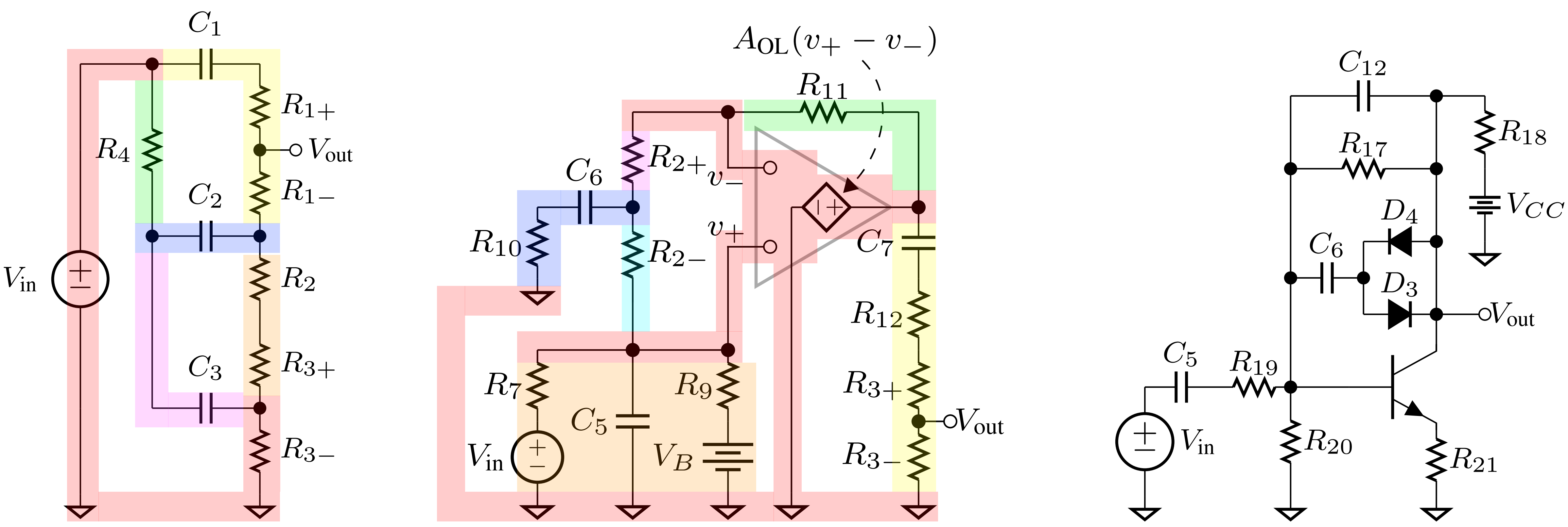

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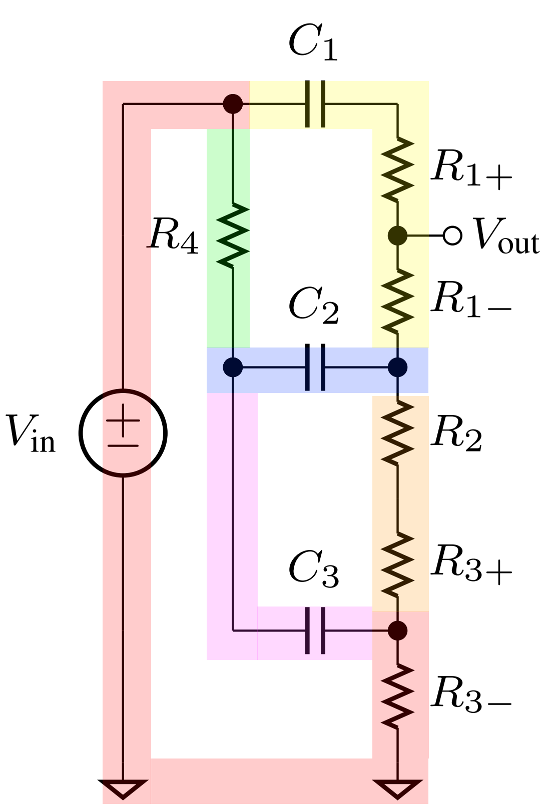

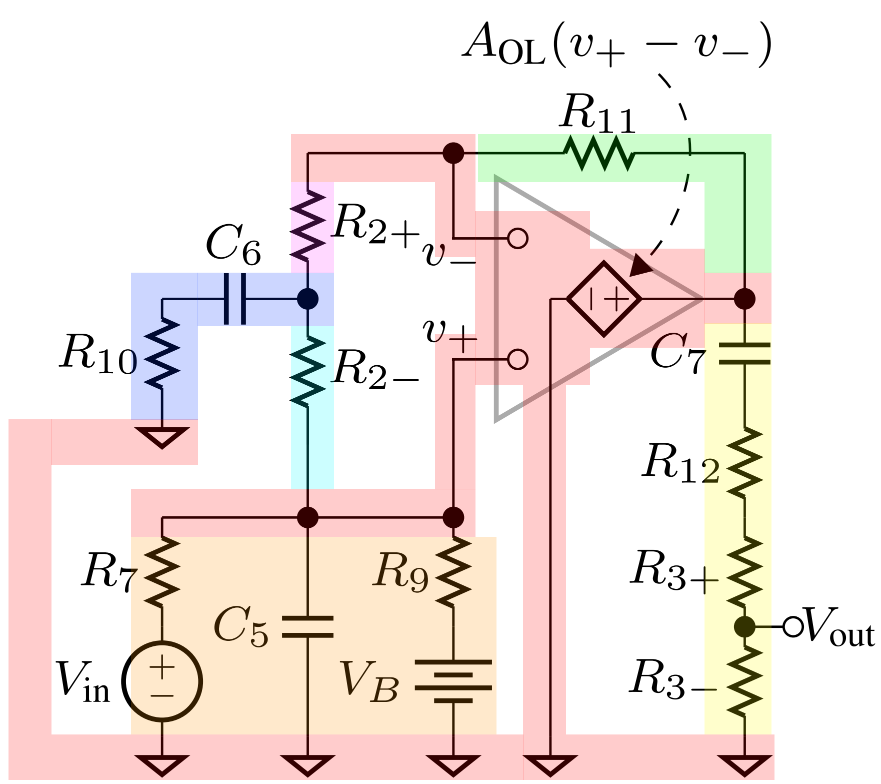

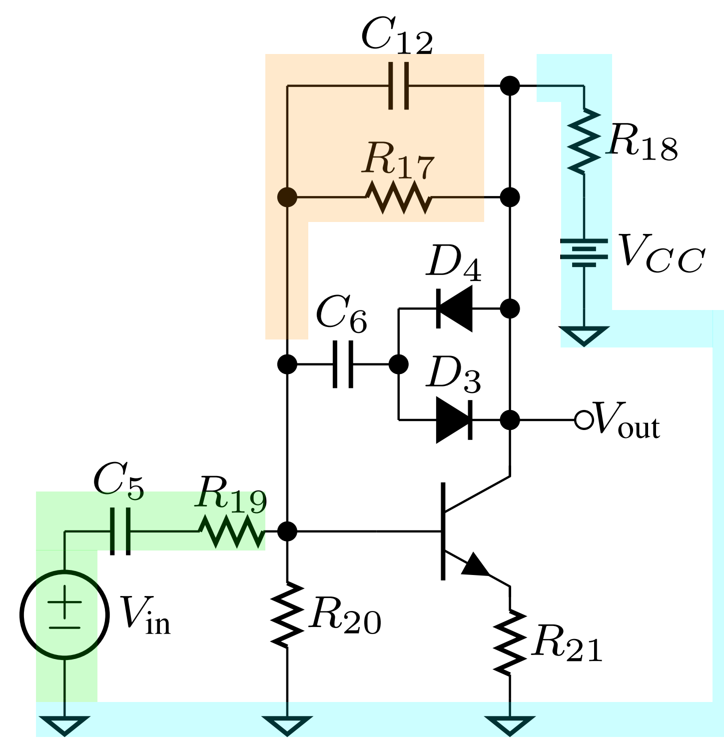

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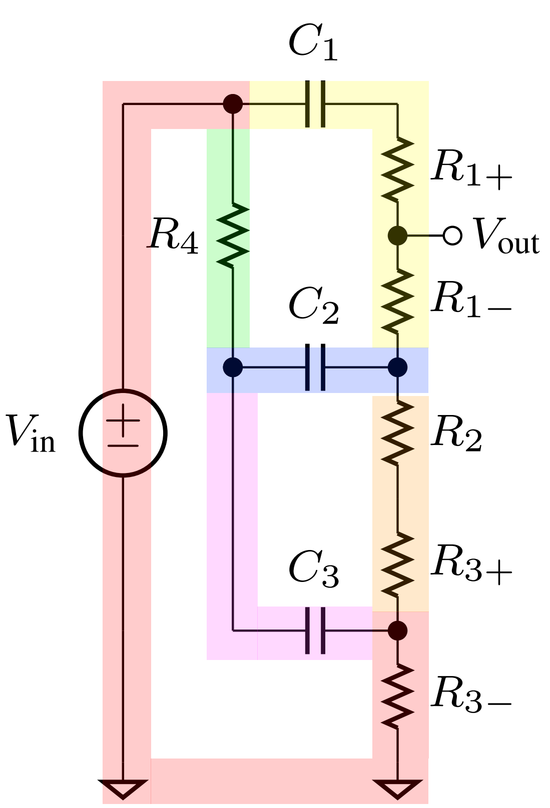

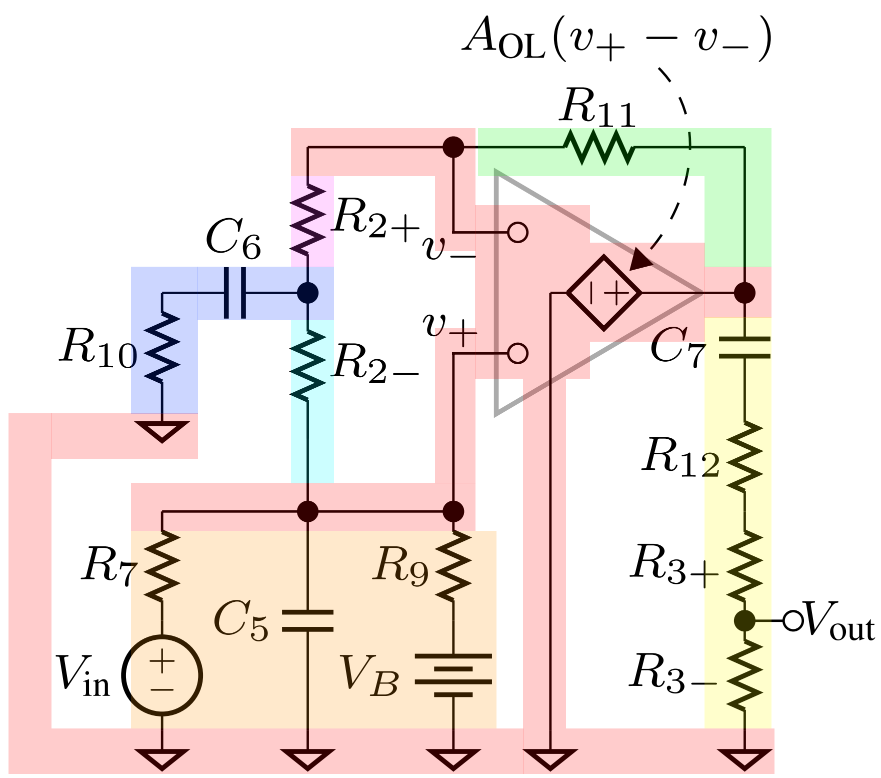

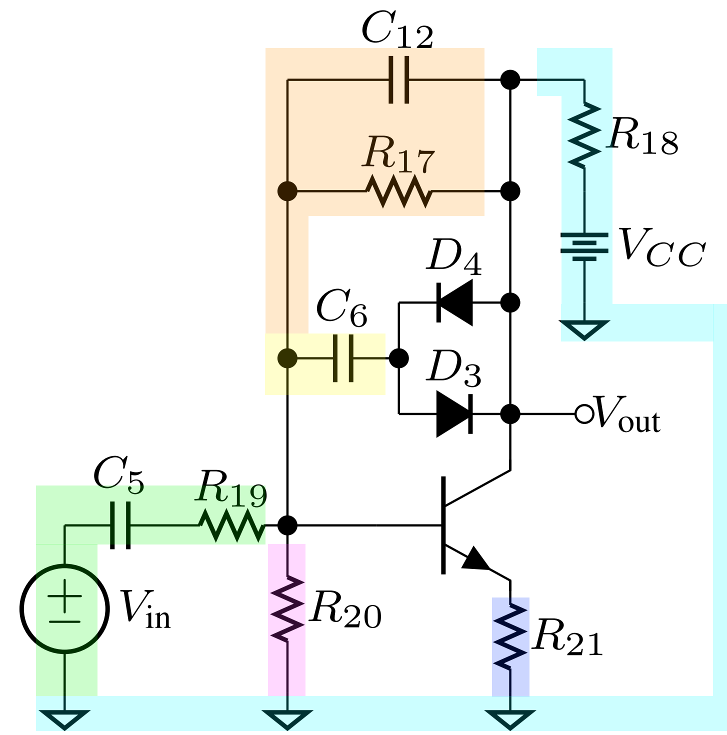

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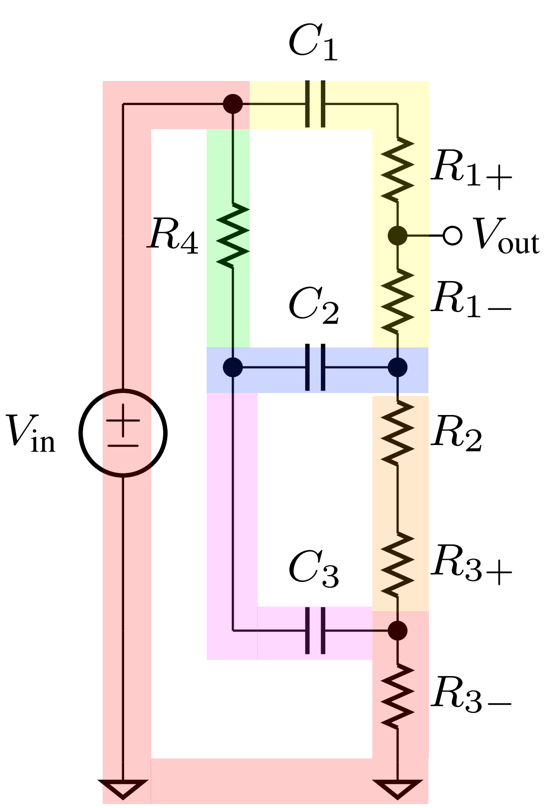
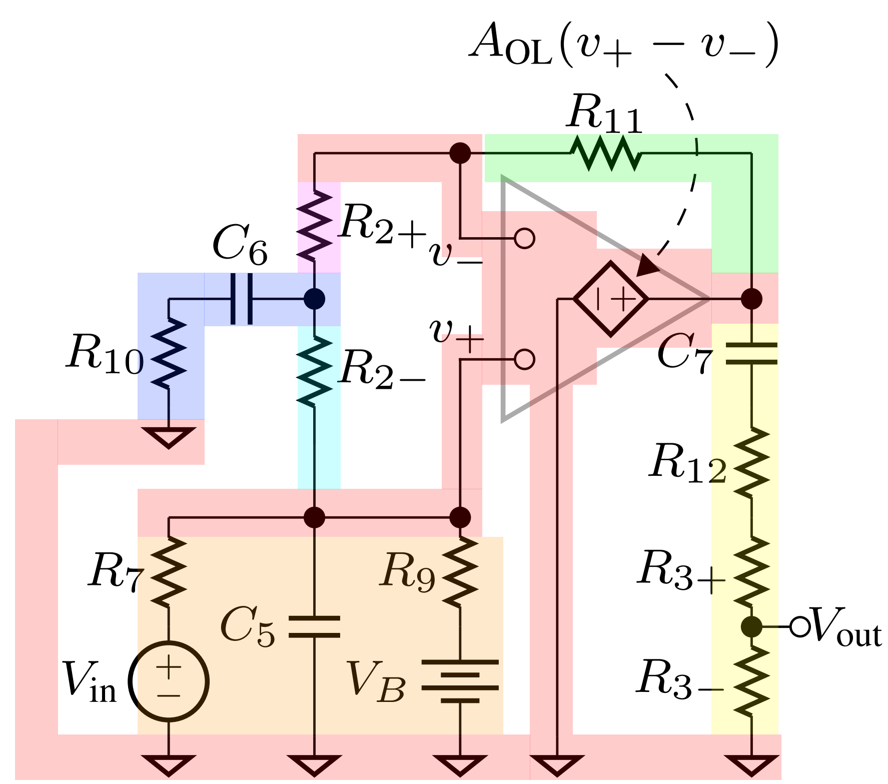
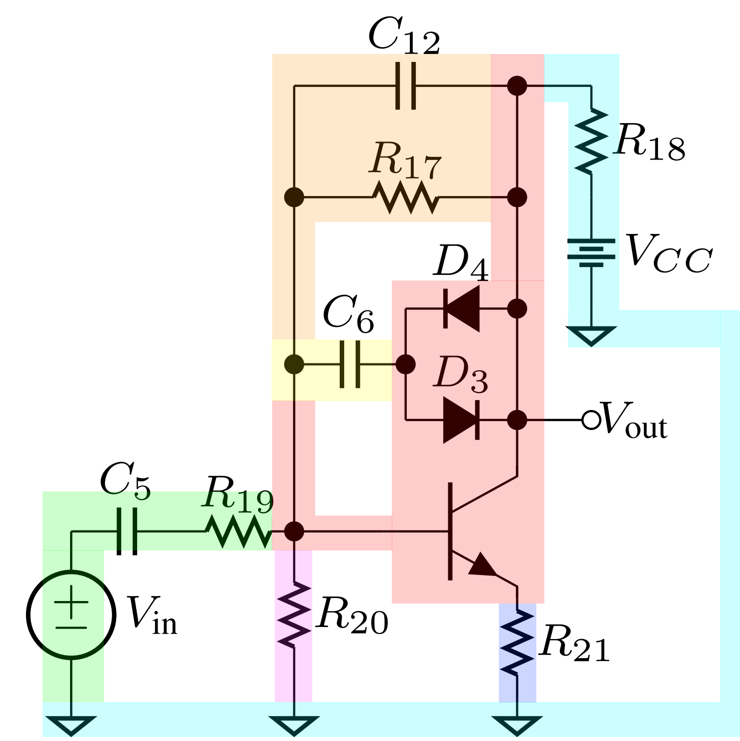
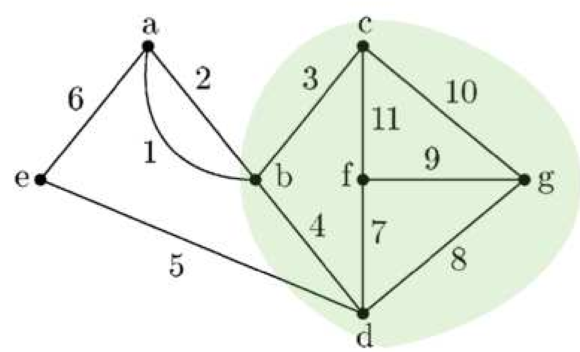


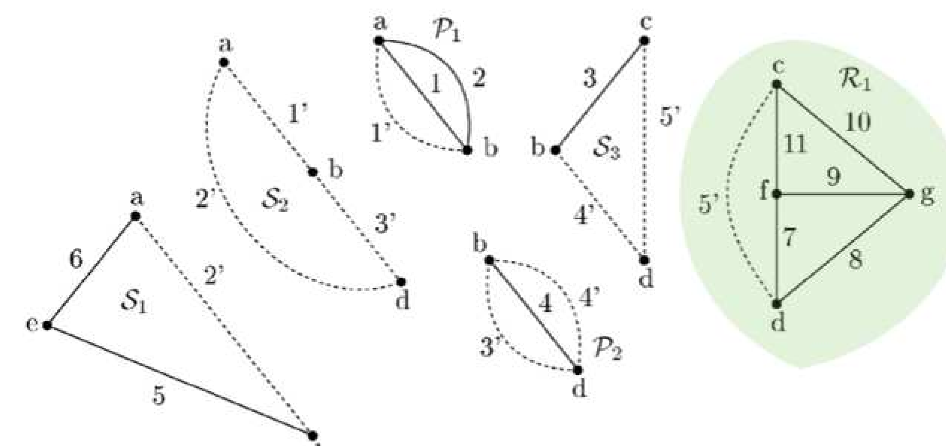




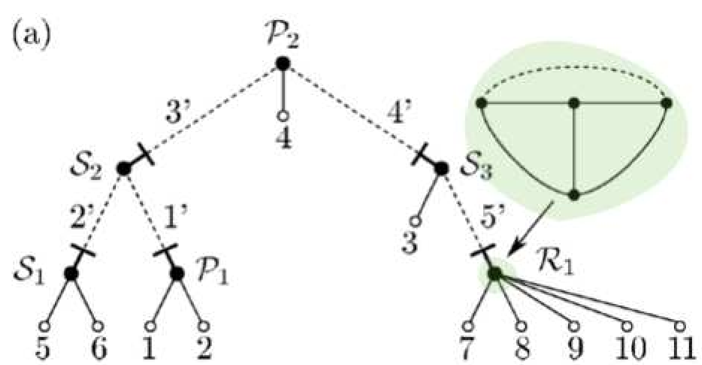
TOPOLOGICAL ISSUES IN WDFS: SPQR tree



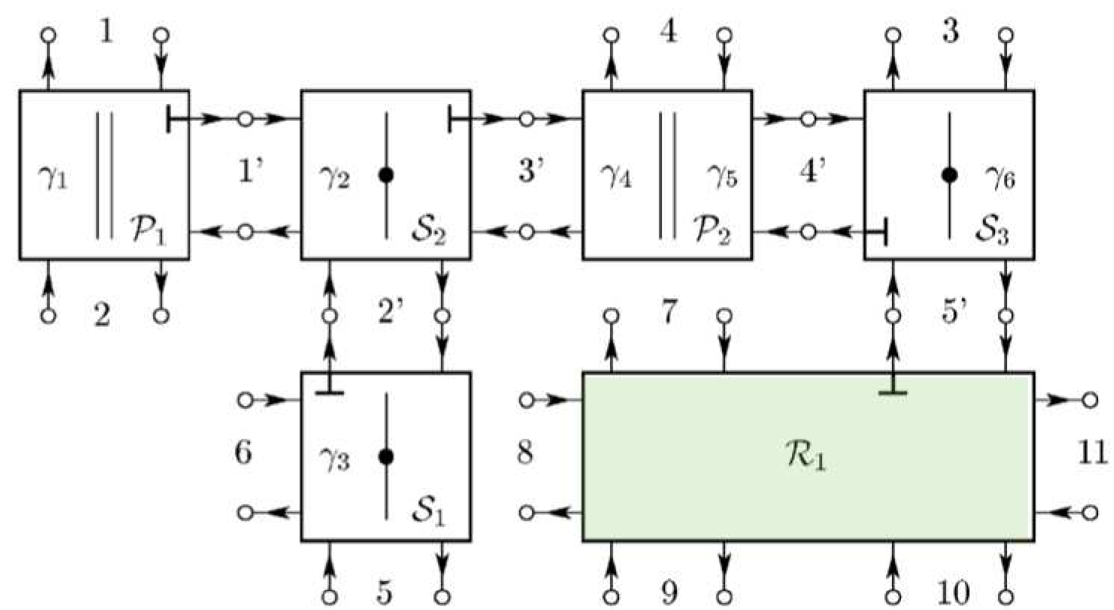
biconnected circuit graph



finding "split components"



SPQR tree



WDF adaptor structure

Dietrich Fränken, Jorg Ochs, Karlheinz Ochs, "Generation of wave digital structures for connection networks containing multiport elements," IEEE Trans. Circuits Syst. I: Reg. Papers, vol. 52, no. 3, pp. 586–596, 2005.

Problems with Wave Digital Filters

1. No general method for deriving topology
2. No general method for handling complicated topologies
3. No general method for handling multiple nonlinearities

- Wave Digital Filter Adaptors for Arbitrary Topologies and Multiport Linear Elements
  @ Int. Conf. Digital Audio Effects (DAFx-15), Trondheim, Norway, Nov. 30 – Dec. 3, 2015
  ↳Kurt James Werner, Julius O. Smith III, and Jonathan Abel

- Resolving Wave Digital Filters with Multiple/Multiport Nonlinearities
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- A General and Explicit Formulation for Wave Digital Filters with Multiple/Multiport Nonlinearities and Complicated Topologies
  @ IEEE Work. Appl. Signal Process. Audio Acoust. (WASPAA), New Paltz, NY, Oct. 18–21, 2015
  ↳Kurt James Werner, Vaibhav Nangia, Julius O. Smith III, and Jonathan Abel

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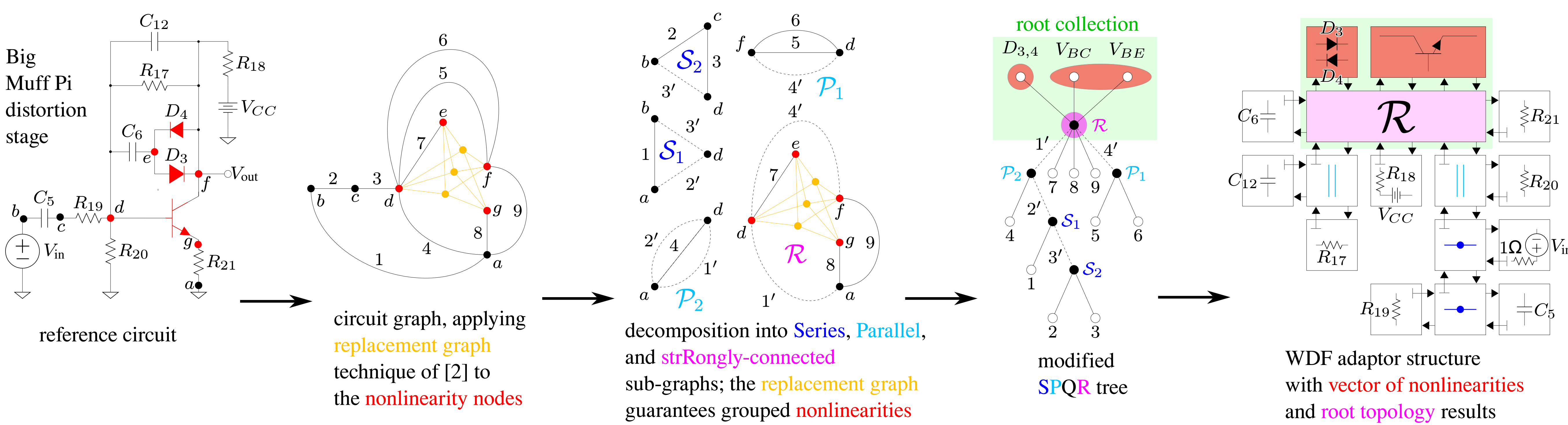

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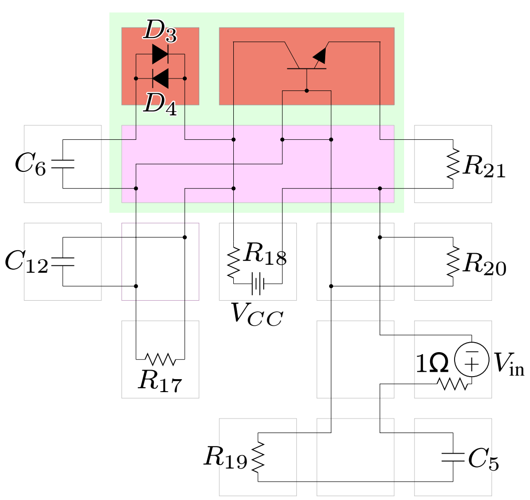
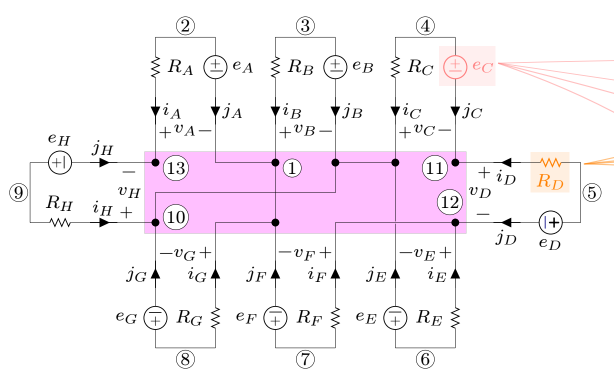
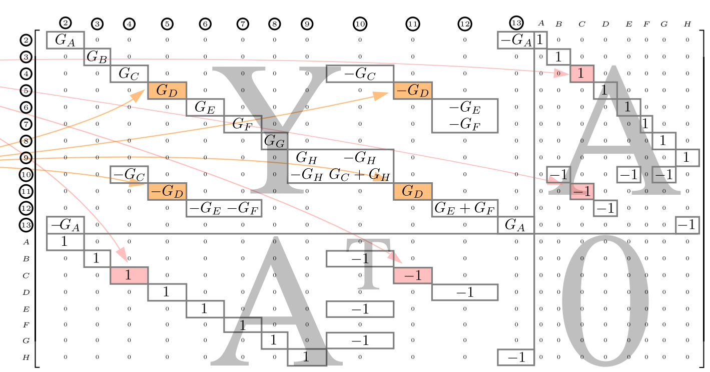


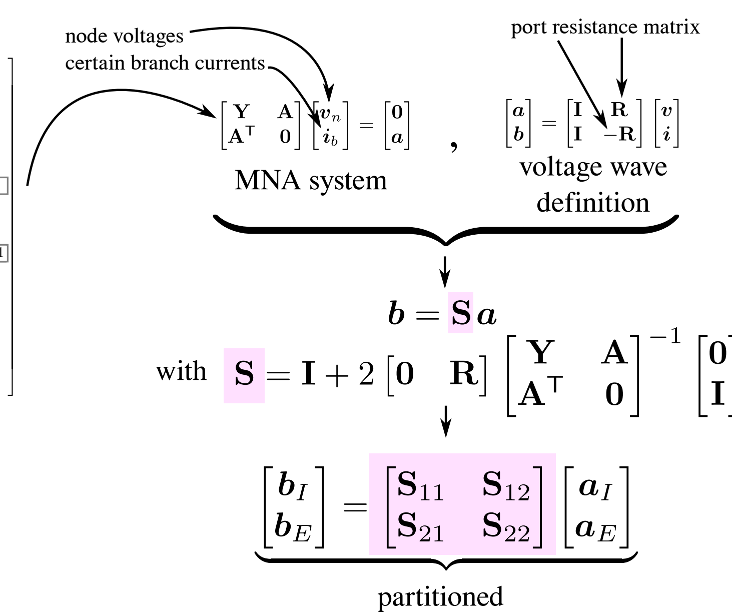
reference circuit rearranged to highlight adaptor structure



— Attach a Thévenin equivalent at each port
— Source values equal to incident waves
— Resistors equal to port resistances



populate system matrix using MNA "element stamps" [3]



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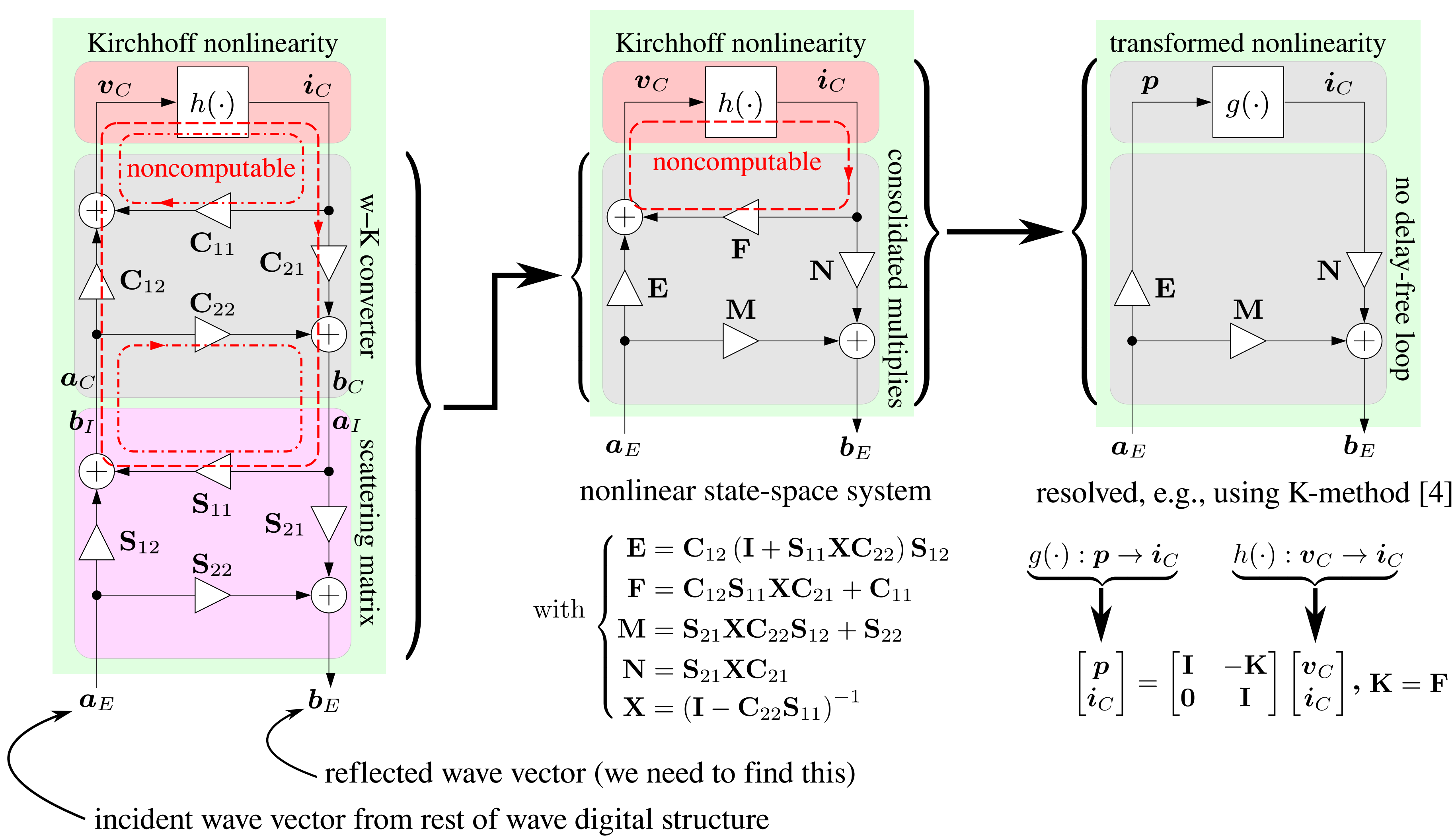

$$\text{with}\begin{cases}\mathbf{E} = \mathbf{C}_{12}(\mathbf{I}+\mathbf{S}_{11}\mathbf{X}\mathbf{C}_{22})\mathbf{S}_{12}\\ \mathbf{F} = \mathbf{C}_{12}\mathbf{S}_{11}\mathbf{X}\mathbf{C}_{21}+\mathbf{C}_{11}\\ \mathbf{M} = \mathbf{S}_{21}\mathbf{X}\mathbf{C}_{22}\mathbf{S}_{12}+\mathbf{S}_{22}\\ \mathbf{N} = \mathbf{S}_{21}\mathbf{X}\mathbf{C}_{21}\\ \mathbf{X} = (\mathbf{I}-\mathbf{C}_{22}\mathbf{S}_{11})^{-1}\end{cases}$$

$$g(\cdot): \boldsymbol{p} \to \boldsymbol{i}_C \qquad h(\cdot): \boldsymbol{v}_C \to \boldsymbol{i}_C$$

$$\begin{bmatrix}\boldsymbol{p}\\ \boldsymbol{i}_C\end{bmatrix} = \begin{bmatrix}\mathbf{I} & -\mathbf{K}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\boldsymbol{v}_C\\ \boldsymbol{i}_C\end{bmatrix}, \mathbf{K} = \mathbf{F}$$

Kurt James Werner, Vaibhav Nangia, Julius O. Smith III, and Jonathan Abel, "Resolving Wave Digital Filters with Multiple/Multiport Nonlinearities," at Int. Conf. Digital Audio Effects (DAFx-15), Trondheim, Norway, Nov. 30 – Dec. 3, 2015.

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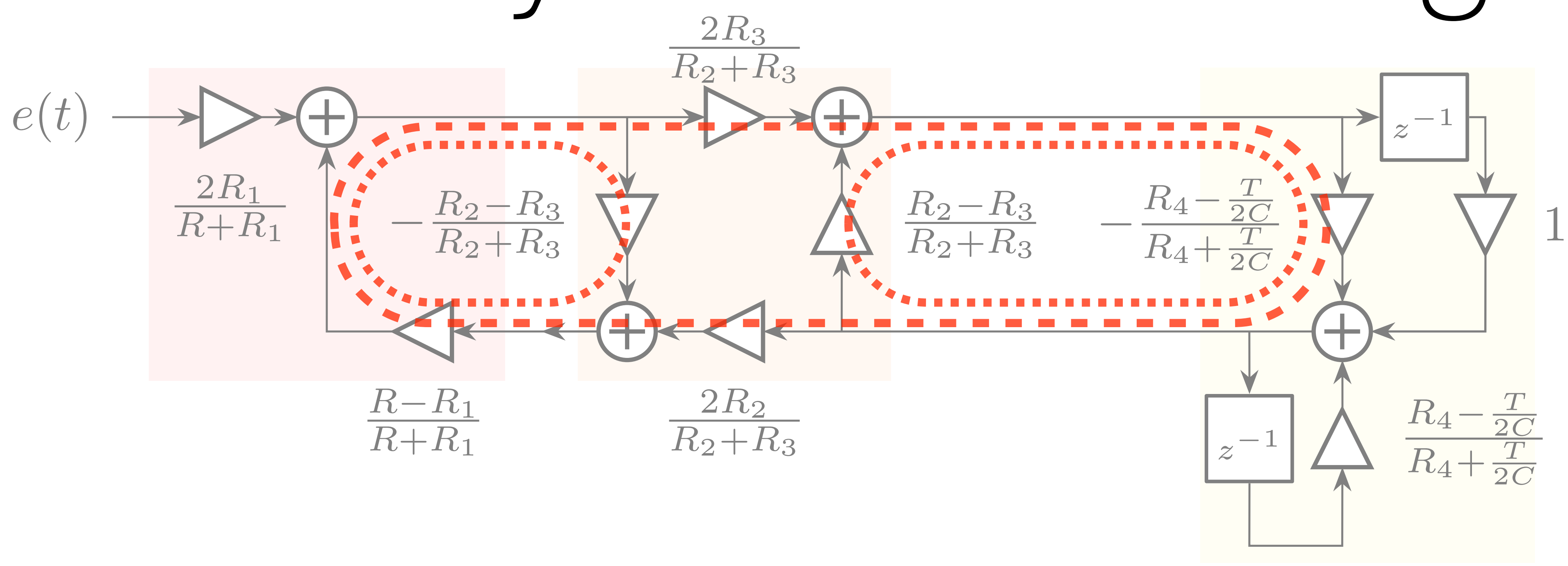
INTRODUCTION

1. tutorial review of WDF principles
2. recent theoretical progress in WDFs
3. WDF software overview and demo

SUMMARY

- WDFs are an elegant solution for circuit modeling
- Frustratingly applicable to only a tiny class of circuits
- Outside that class, ad hoc solutions (focused on nonlinearities) dominate
- New research addressing topological issues (details at talks tomorrow!) **vastly expands** the range of suitable reference circuits

Thank you for listening!



...now go build some WDFs!

Julius O. Smith III & **Kurt James Werner**
[jos,kwerner]@ccrma.stanford.edu