Modelling peat behaviour with an elasto-viscoplastic model for clay

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Introduction

- 2D-ABC model (Den Haan 2009) describes the behaviour of soft soils by means of Maxwell element. It was extended to general stress space. The creep strain rate is time dependent.
- AEP model (Anisotropic Elasto-Plastic) model (based on SClay1 (Sivasithamparam, 2012) is used to predict the behavior of peat. Plastic strain rate uses the plastic multiplier.
- B-AEP model (Bubble Anisotropic Elasto-Plastic) model (based on B-SClay1) is presented.
- SSC model (Plaxis) is used for some comparisons.

A conventional strain-log(stress) relationship describes this:

$$\varepsilon_d = a\ln(\sigma_1' / \sigma_{1o}')$$

and

$$\dot{\varepsilon}_d = a\dot{\sigma}_1' / \sigma_1'$$

with a being the direct compression index.

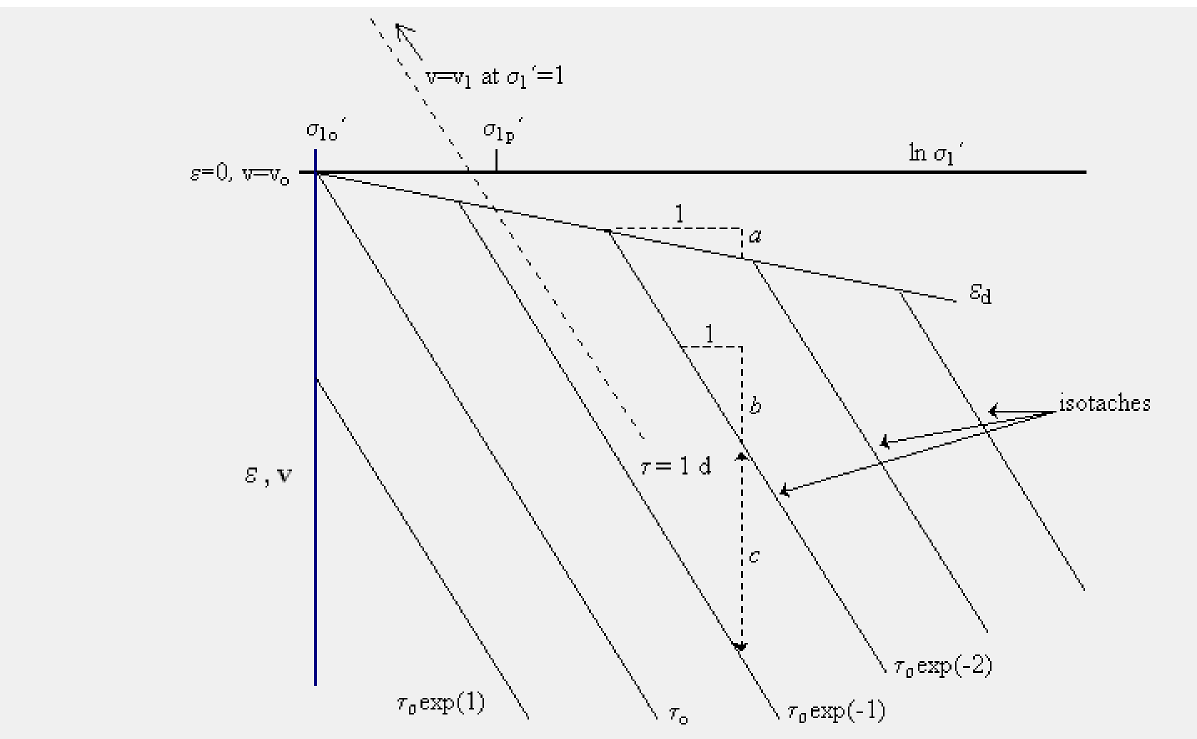
The series of parallel lines in the figure are creep isotaches. They are given by

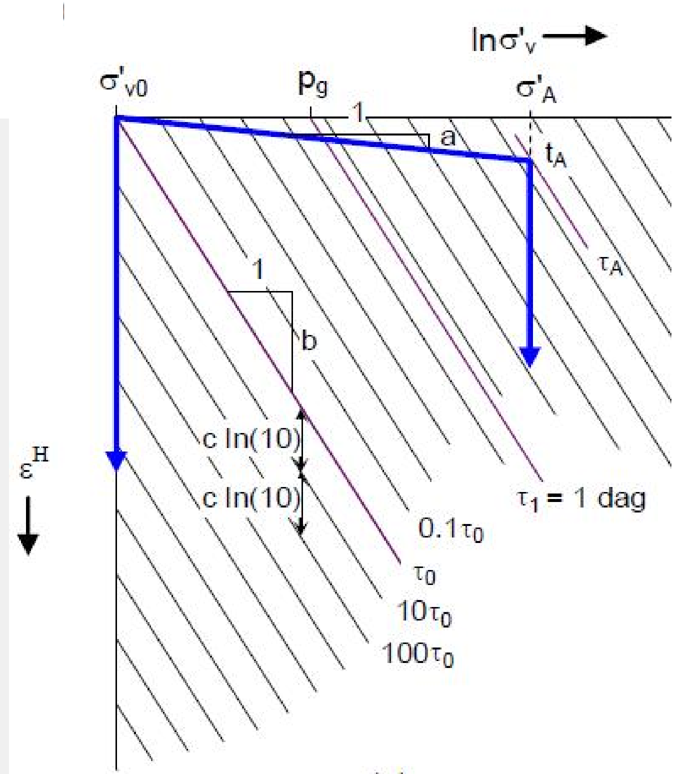
$$\varepsilon = b\ln(\sigma_1' / \sigma_{1o}') + c\ln(\tau / \tau_o)$$

where b is the secular compression index and c is the coefficient of rate of secular compression. The *intrinsic time* τ is related to secular strain rate $\dot{\varepsilon}_s$ by

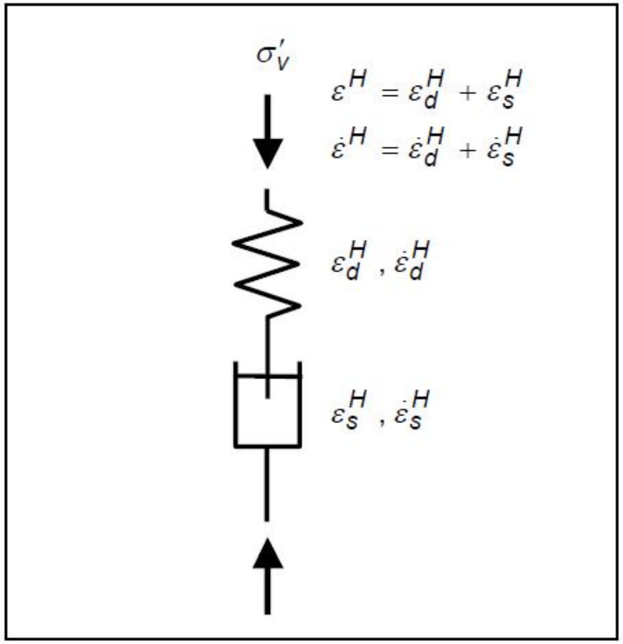
$$\dot{\varepsilon}_s = c / \tau$$

2D-ABC Model (Den Haan, 2009)





(a)



- ABC Isotache model represented as a Maxwell element

$$\varepsilon_d = a\ln(\sigma'_1 / \sigma'_{1o})$$

$$\dot{\varepsilon}_d = a\dot{\sigma}'_1 / \sigma'_1$$

$$\varepsilon = b\ln(\sigma'_1 / \sigma'_{1o}) + c\ln(\tau / \tau_o)$$

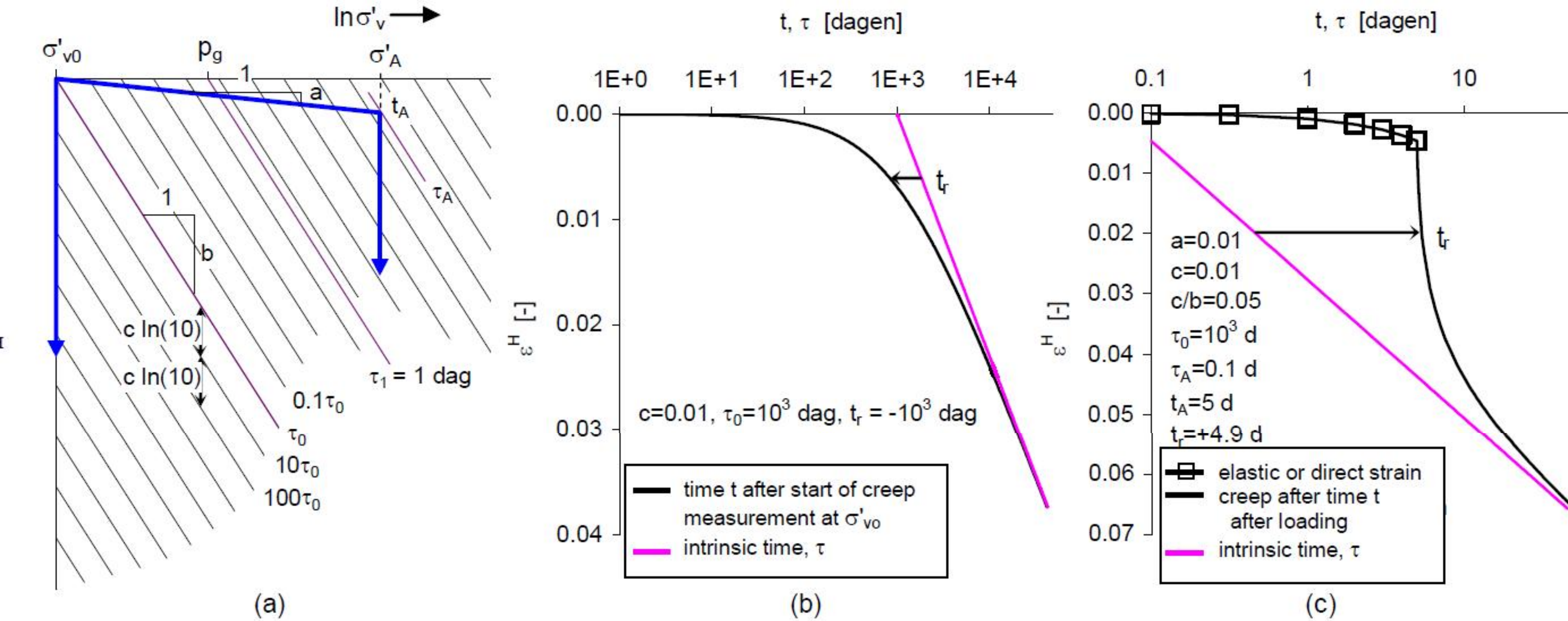
where b is the secular compression index and c is the coefficient of rate of secular compression. The *intrinsic time* τ is related to secular strain rate $\dot{\varepsilon}_s$ by

$$\dot{\varepsilon}_s = c / \tau$$

The expression for $\dot{\varepsilon}_s$ can be elaborated in several ways by expressing τ as a function of various parameters and state variables. For example:

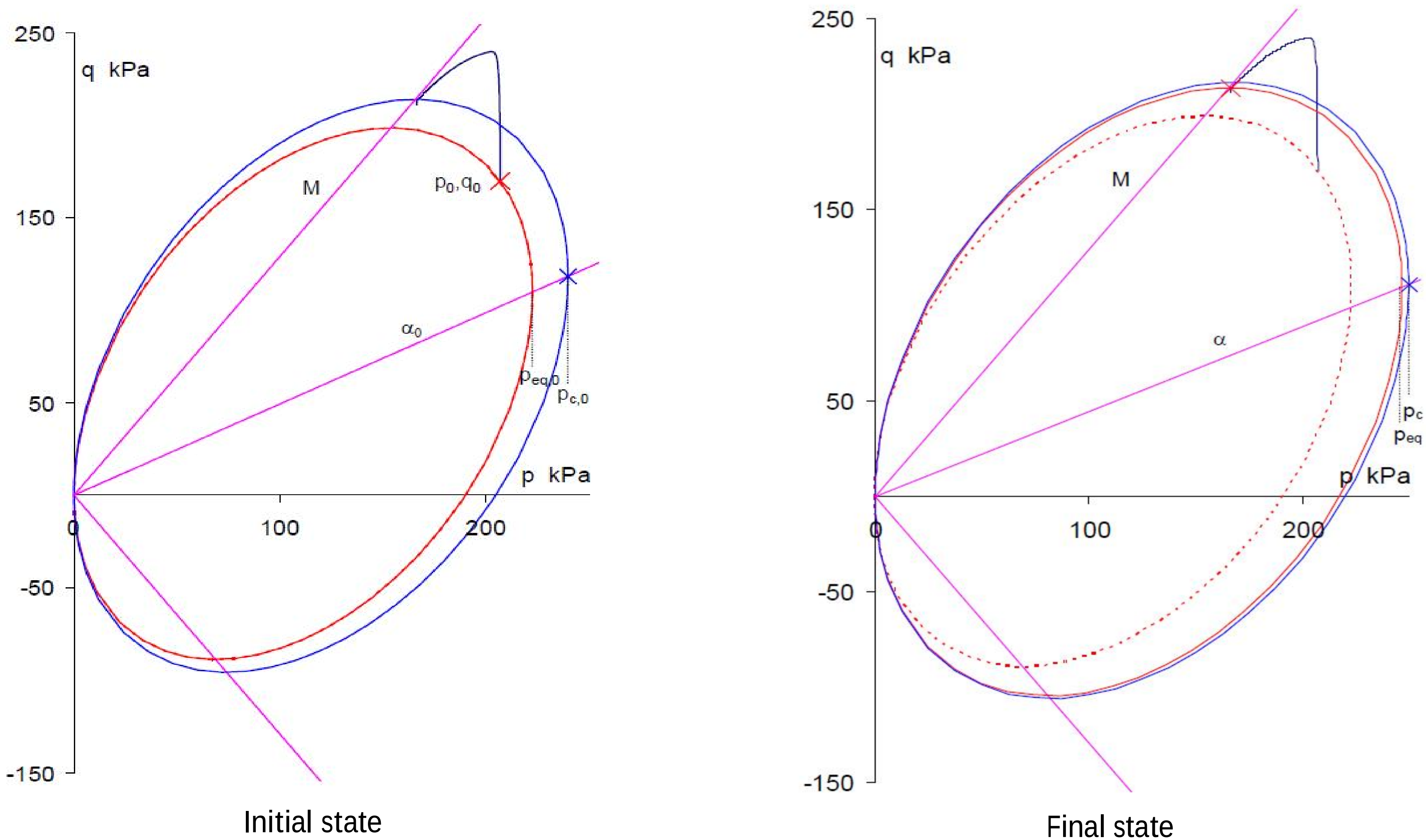
$$\dot{\varepsilon}_s = \frac{c}{\tau_o} e^{-\varepsilon / c} \left(\frac{\sigma'_1}{\sigma'_{1o}} \right)^{b/c}$$

- Isotache model , in ABC model the total rate of strain is obtained by adding the “elastic” rate of strain and “creep” or “viscoplastic” rate of strain



. Isotaches defined by intrinsic time (a), Development of creep if load is not increased (b), Development of creep after application of a relatively large load (c)

- In 2D dimensions, 2D-ABC model



Principle of the 2D-ABC model

Red: ellipse through stress state characterised by p_{eq}. Blue: ellipse with reference rate of volumetric plastic creep strain, characterised by p_c. Purple: M line and ellipse axis. The latter rotates as a function of relative volumetric and shear strains.

- The stress-state ellipse is given by

$$p'^2 + \frac{(q-\alpha p)^2}{M^2-\alpha^2} = p'_{eq}\, p' \qquad p_c = p_{c0}\exp(\varepsilon_p^p/(\lambda-\kappa))$$

- The creep strain rate is given by

$$\dot{\varepsilon}_{ij}^{c} = \frac{\mu^*}{(\partial p'_{eq}/\partial p')\,\tau_1}\left(\frac{p'_{eq}}{p'_c}\right)^m \frac{\partial p'_{eq}}{\partial \sigma'_{ij}} \qquad m = \frac{\lambda^* - \kappa^*}{\mu^*}$$

$$\tau_1 = 1\ \text{day}$$

- The plastic multiplier $d = \partial p'_{eq}/\partial p'$

- Rotational hardening parameter α

$$\dot{\alpha} = \omega\left[\left(\frac{3q}{4p'}-\alpha\right)\dot{\varepsilon}_{\text{vol}}^{c} + \omega_d\left(\frac{q}{3p'}-\alpha\right)\dot{\gamma}^{c}\right]$$

- Parameters of 2D-ABC model

κ^*	λ^*	μ^*	M_C	ν	p_0 [kPa]	q_0 [kPa]	p_{c0} [kPa]	ω	ω_d	α_0, α_{nc}

A.E.P (Anisotropic Elasto-Plastic) model (based on SClay1 (Wheeler 1997, Sivasithamparam 2012)

$$\underline{\sigma}'_d = \begin{bmatrix} \sigma'_x - p' \\ \sigma'_y - p' \\ \sigma'_z - p' \\ \sqrt{2}\tau_{xy} \\ \sqrt{2}\tau_{yz} \\ \sqrt{2}\tau_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{3}(2\sigma'_x - \sigma'_y - \sigma'_z) \\ \frac{1}{3}(-\sigma'_x + 2\sigma'_y - \sigma'_z) \\ \frac{1}{3}(-\sigma'_x - \sigma'_y + 2\sigma'_z) \\ \sqrt{2}\tau_{xy} \\ \sqrt{2}\tau_{yz} \\ \sqrt{2}\tau_{zx} \end{bmatrix}$$

Deviatoric stress tensor

$$p' = \frac{1}{3}(\sigma'_x + \sigma'_y + \sigma'_z)$$

$$\underline{\alpha}_d = \begin{bmatrix} \alpha_x - 1 \\ \alpha_y - 1 \\ \alpha_z - 1 \\ \sqrt{2}\alpha_{xy} \\ \sqrt{2}\alpha_{yz} \\ \sqrt{2}\alpha_{zx} \end{bmatrix}$$

Deviatoric fabric tensor

- The equation of the yield surface is:

$$p'_{eq} = p' + \frac{3}{2p'} \frac{\{\tilde{\sigma}_d - p'\tilde{\alpha}_d\}^T \{\tilde{\sigma}_d - p'\tilde{\alpha}_d\}}{M^2 - \frac{3}{2}\{\tilde{\alpha}_d\}^T \{\tilde{\alpha}_d\}}$$

AEP Anisotropic Elasto-Plastic model

The yield function fy is given by

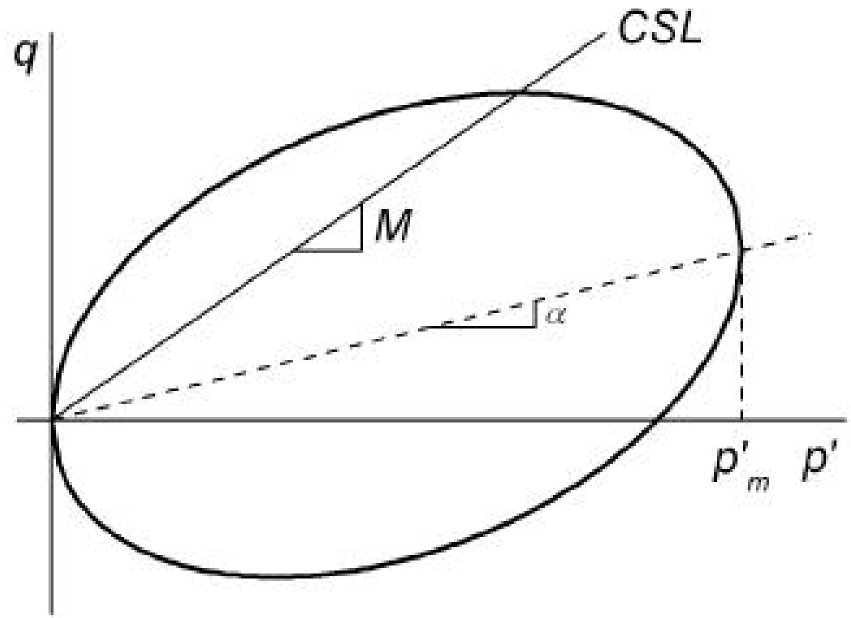
$$f_y = \frac{3}{2}\frac{\{\underline{\sigma}'_d - \underline{\alpha}_d p'\}^T \{\underline{\sigma}'_d - \underline{\alpha}_d p'\}}{M^2 - \alpha^2} + \left(p' - \frac{p'_m}{2}\right)^2 - \left(\frac{p'_m}{2}\right)^2 = 0$$

$$\underline{s} = \underline{\sigma}'_d - \underline{\alpha}_d p'$$

$$f_y = \frac{3}{2}\frac{\{\underline{s}\}^T \{\underline{s}\}}{M^2 - \alpha^2} + \left(p' - \frac{p'_m}{2}\right)^2 - \left(\frac{p'_m}{2}\right)^2 = 0$$

The potential function py is given by

$$p_y = \frac{3}{2}\frac{\{\underline{s}\}^T \{\underline{s}\}}{M^2 - \alpha^2} + \left(p' - \frac{p'_m}{2}\right)^2 - \left(\frac{p'_m}{2}\right)^2 = 0$$



Increment of plastic strain

$$\Delta\underline{\epsilon}^p = \Delta\Lambda \frac{\partial p_y}{\partial \underline{\sigma}'}$$

$\Delta\Lambda$ plastic multiplier

Hardening laws, AEP model

First hardening law, describes the changes in size of the yield surface

$$\Delta p'_m = \frac{(1+e)p'_m}{\lambda - \kappa}\Delta\epsilon_v^p$$

Rotational hardening law, describes the changes in orientation of the yield surface

$$\Delta\underline{\alpha}_d = \mu\left(\left[\frac{3\underline{\sigma}'_d}{4p'} - \underline{\alpha}_d\right]\langle\Delta\epsilon_v^p\rangle + \beta\left[\frac{\underline{\sigma}'_d}{3p'} - \underline{\alpha}_d\right]\Delta\epsilon_d^p\right)$$

μ and β : soil constants

Hardening modulus H = $H_{\boldsymbol{\alpha}}$ + H_0

$$\mathcal{H}_\alpha = \left\{\frac{\partial f_y}{\partial\underline{\alpha}_d}\right\}^T\left[\left\{\frac{\partial\underline{\alpha}_d}{\partial\epsilon_v^p}\right\}\left\langle\frac{\partial p_y}{\partial p'}\right\rangle + \left\{\frac{\partial\underline{\alpha}_d}{\partial\epsilon_d^p}\right\}\sqrt{\frac{3}{2}\left\{\frac{\partial p_y}{\partial\underline{\sigma}'_d}\right\}^T\left\{\frac{\partial p_y}{\partial\underline{\sigma}'_d}\right\}}\right]$$

$$H_0 = -p'.p_m\frac{(1+e)}{(\lambda-\kappa)}\frac{\partial p_y}{\partial p'}$$

Elasto-plastic matrix [Dep] relates increments of strain to increments of stress

$$\{\Delta\underline{\sigma}'\} = [D^{ep}]\,\{\Delta\underline{\epsilon}\}$$

$$[D^{ep}] = [D^e] - \frac{[D^e]\left[\frac{\partial p_y}{\partial \underline{\sigma}'}\right]\left[\frac{\partial f_y}{\partial \underline{\sigma}'}\right]^T [D^e]}{\left[\frac{\partial f_y}{\partial \underline{\sigma}'}\right]^T [D^e]\left[\frac{\partial p_y}{\partial \underline{\sigma}'}\right] + \mathcal{H}_0 + \mathcal{H}_\alpha}$$

When a strain increment produces an elastic stress outside the yield surface, a plastic flow occurs

$$\Delta\Lambda = \frac{\left[\frac{\partial f_y}{\partial \underline{\sigma}'}\right]^T [D^e]\,\{\Delta\underline{\epsilon}\}}{\left[\frac{\partial f_y}{\partial \underline{\sigma}'}\right]^T [D^e]\left[\frac{\partial p_y}{\partial \underline{\sigma}'}\right] + \mathcal{H}_0 + \mathcal{H}_\alpha}$$

Increment of plastic strain

$$\Delta\underline{\epsilon}^p = \Delta\Lambda \frac{\partial p_y}{\partial \underline{\sigma}'}$$

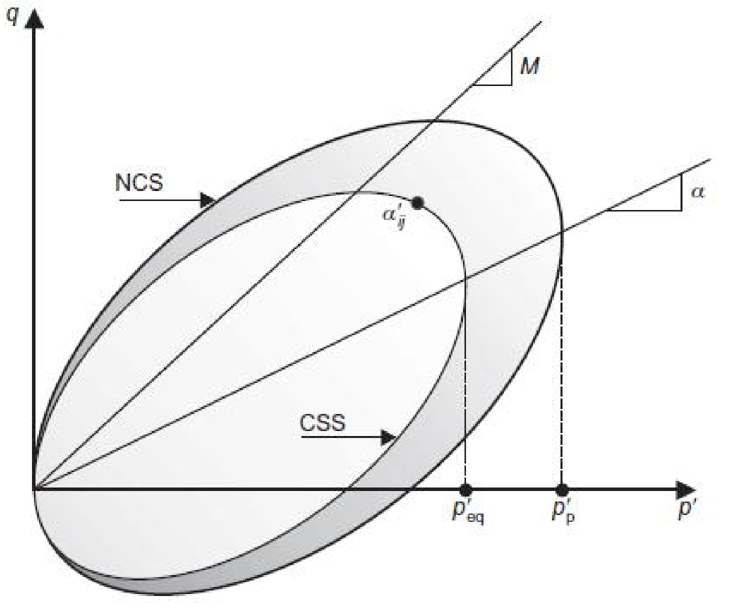
- Parameters of AEP model

$\boldsymbol{\kappa}$ $\boldsymbol{\lambda}$ M_c M_e $\boldsymbol{\nu}$ $p_{\mathbf{m}}$ p_0 q_0 $\boldsymbol{\omega}$ $\boldsymbol{\omega}_d$ $\boldsymbol{\alpha}_0$

SSC (Soft Soil Creep) model (Plaxis)

$$\dot{\varepsilon} = \dot{\varepsilon}^{e} + \dot{\varepsilon}^{cr}$$

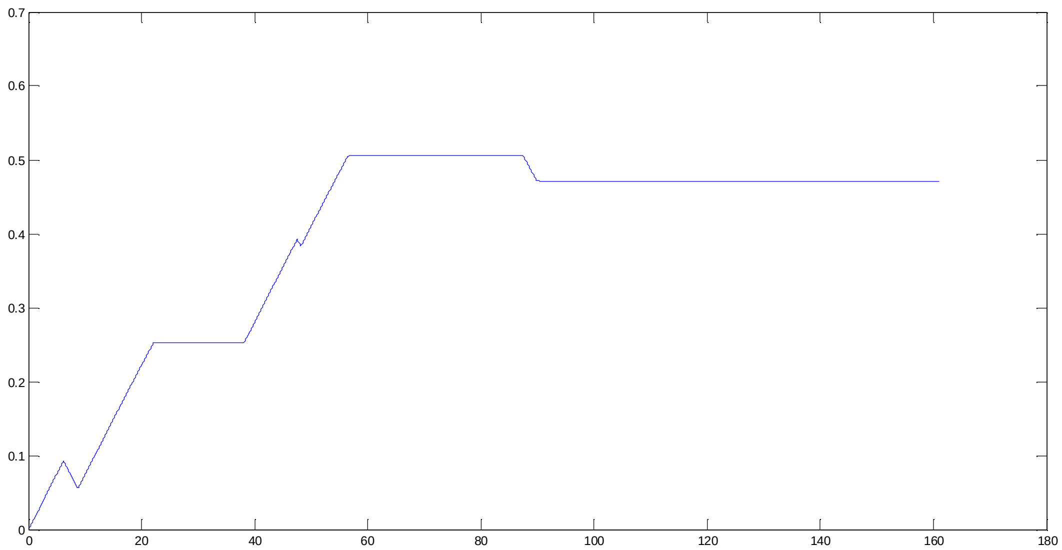
$$\dot{\varepsilon}_v = \kappa^* \frac{\dot{p}'}{p'} + \frac{\mu^*}{T}\left(\frac{p'}{p'_p}\right)^{\frac{\lambda^* - \kappa^*}{\mu^*}}$$



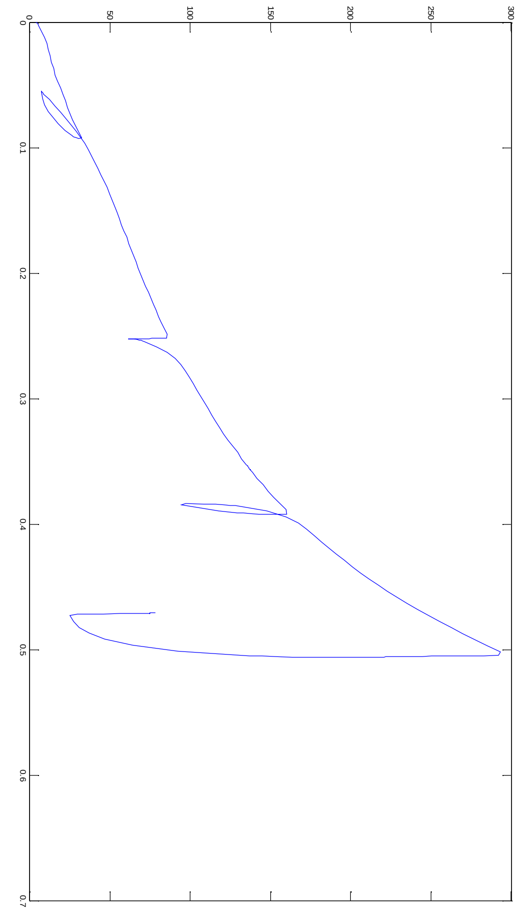
$$\varepsilon = \varepsilon^{e} + \varepsilon^{c} = -A\ln\left(\frac{\sigma'}{\sigma_0}\right) - B\ln\left(\frac{\sigma_{pc}}{\sigma_{p0}}\right) - C\ln\left(\frac{\tau_c + t}{\tau_c}\right)$$

Results, Influence of M

A CRS Kn0 test was performed on peat,
Pm=13.5kPa, ν=0.15, λ = 0,2102 (Cc=0.484) , κ=0,03 (Cs=0.0345) ,
e0=0.5, ϕ_{cv}=41 °

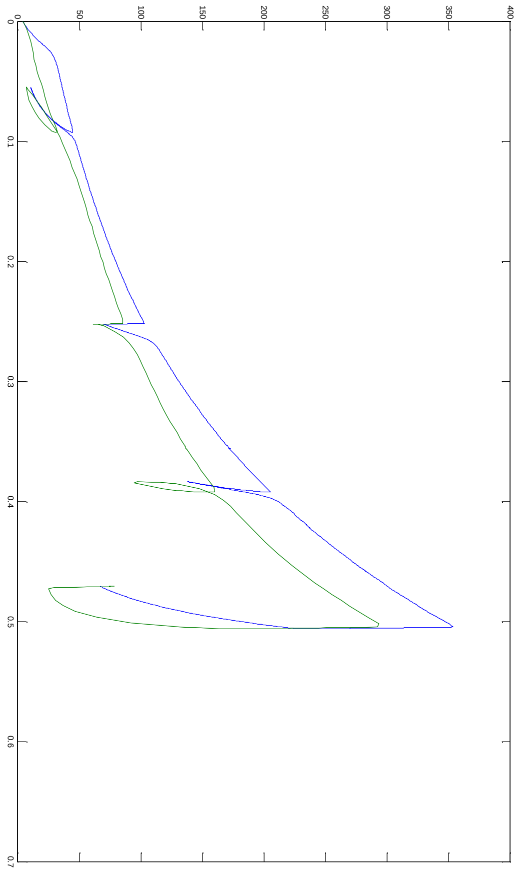


Applied axial strain $\boldsymbol{\varepsilon}_{11}$ vs time

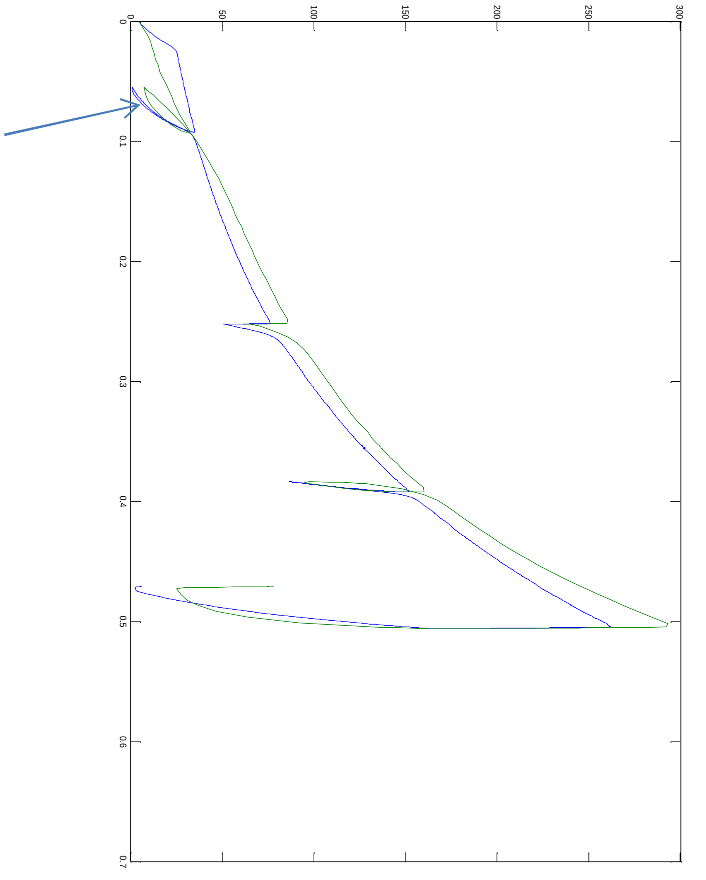


stress strain evolution($\boldsymbol{\varepsilon}_{11}$ vs $\boldsymbol{\sigma}_{11}$)

2D-ABC model

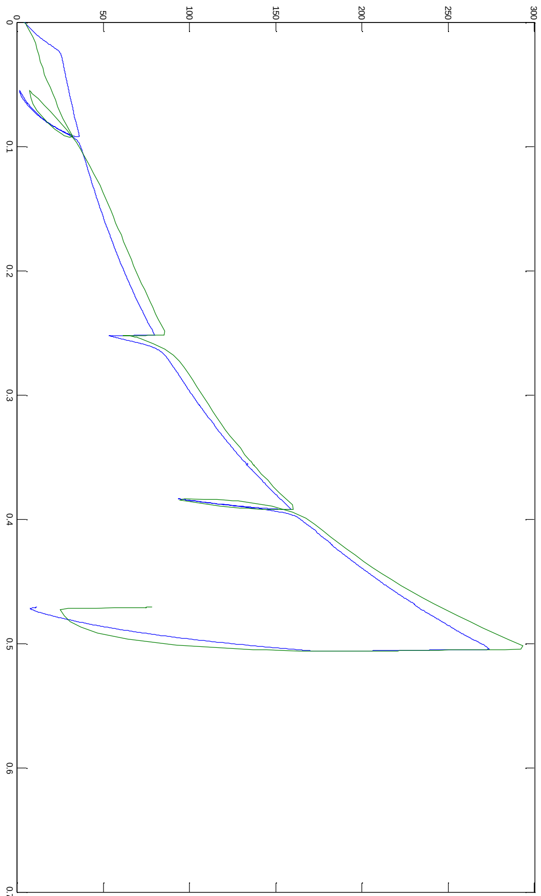


(σ_{11}, ε_{11}) 2D-ABC model, Mc=3.
(in green experimental results)

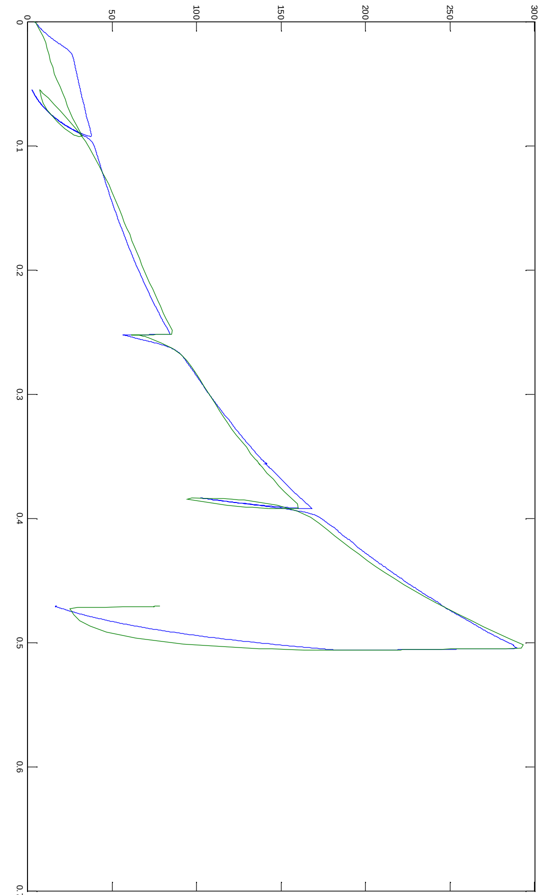


(σ_{11}, ε_{11}) Mc=1.85

2D-ABC model



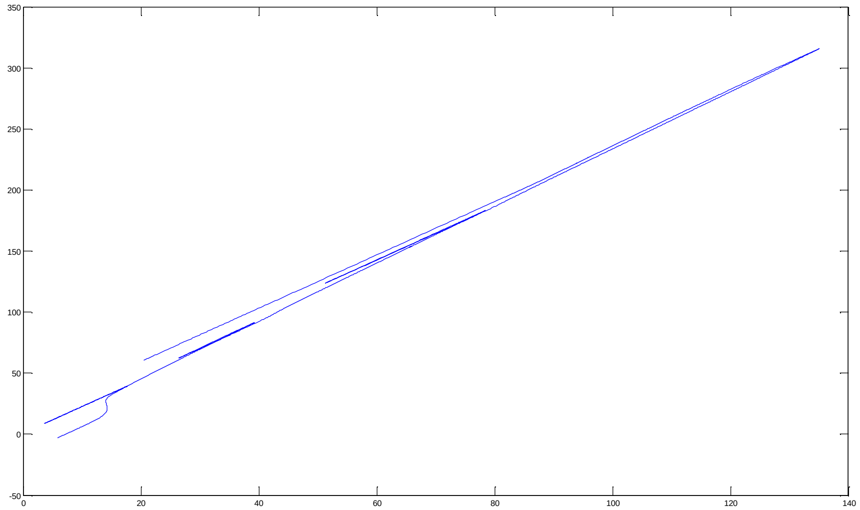
(σ_{11}, ε_{11}) 2D-ABC model Mc=2,



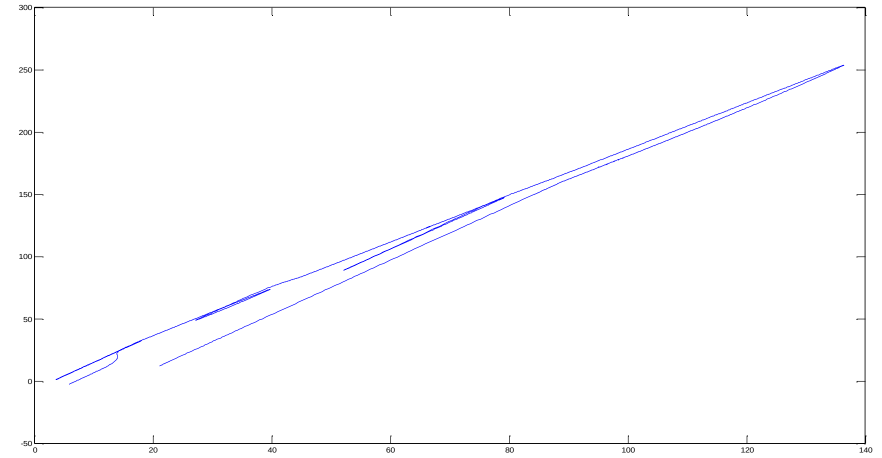
(σ_{11}, ε_{11}) 2D-ABC model Mc=2.2,

The results from the model fit experimental data for Mc=2,2

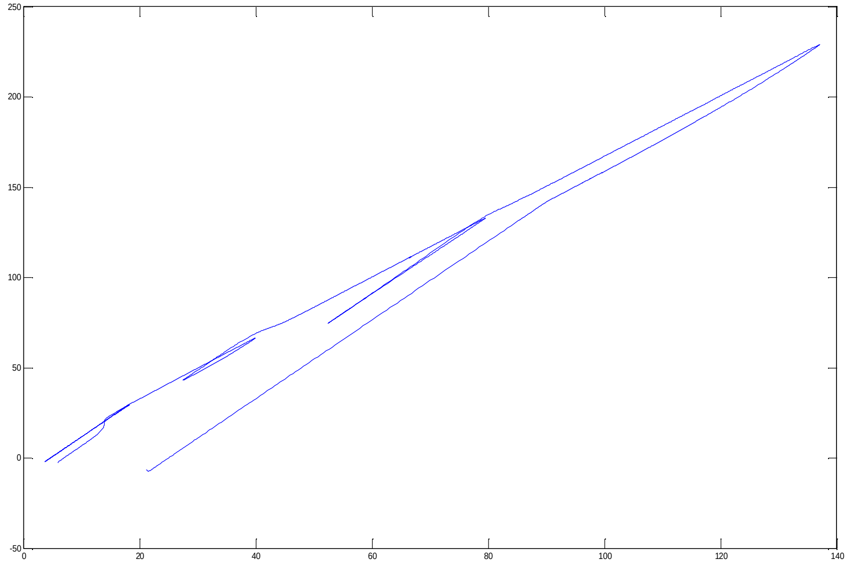
2D-ABC model



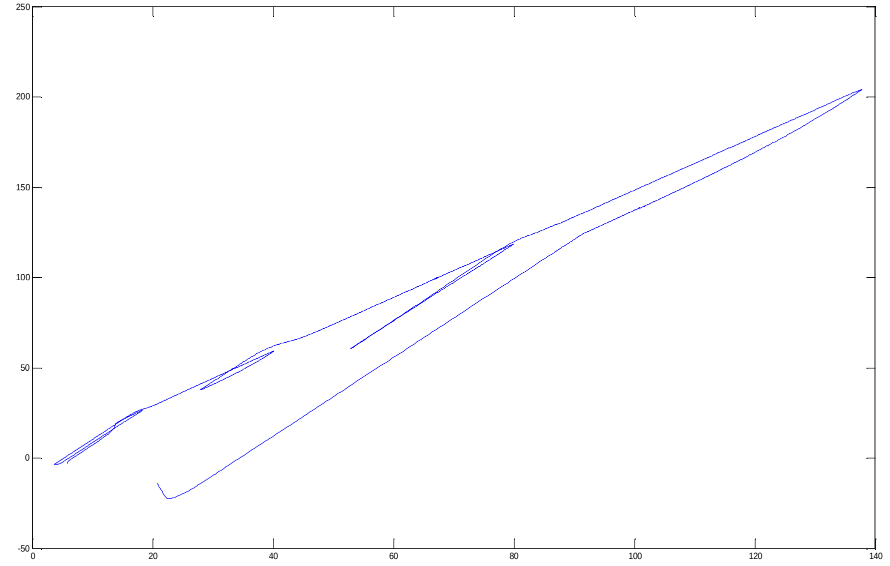
(p,q) 2D-ABC model Mc=2.9



(p,q) 2D-ABC model Mc=2.4

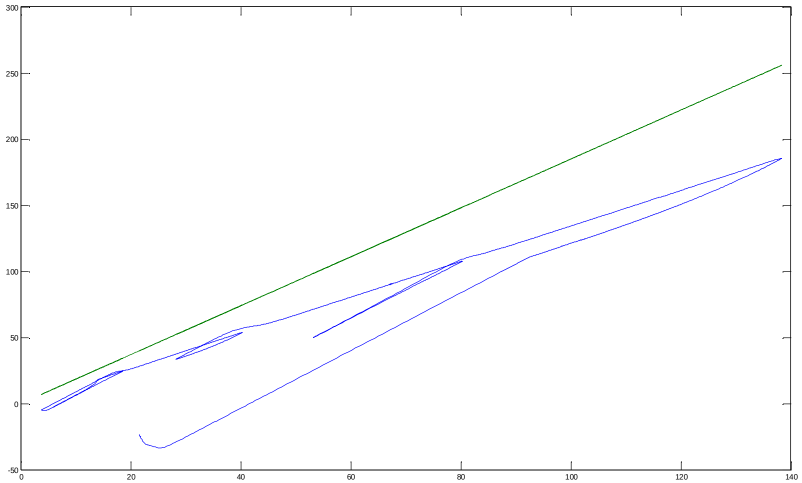


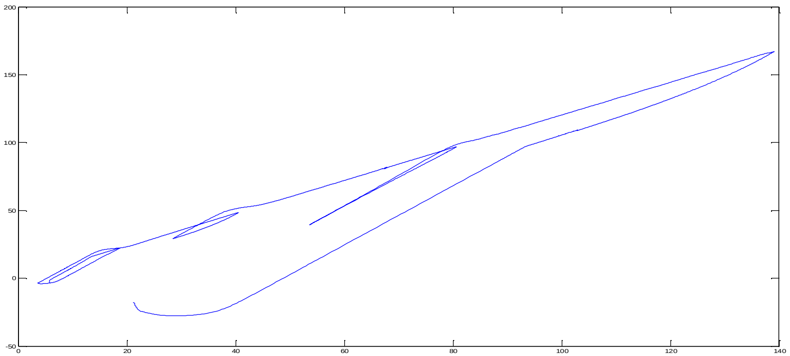
(p,q) 2D-ABC model Mc=2.2



(p,q) 2D-ABC model Mc=2

2D-ABC model



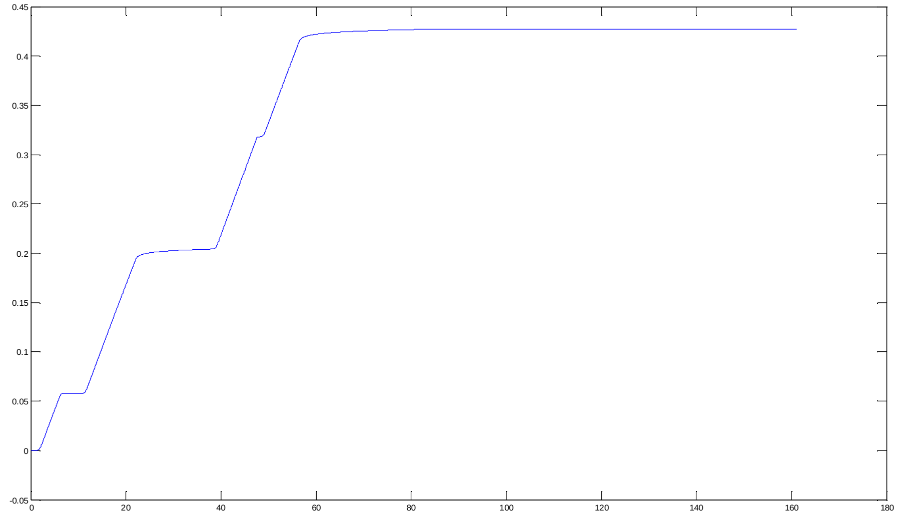


(p,q) 2D-ABC model, Mc=1.85 ,

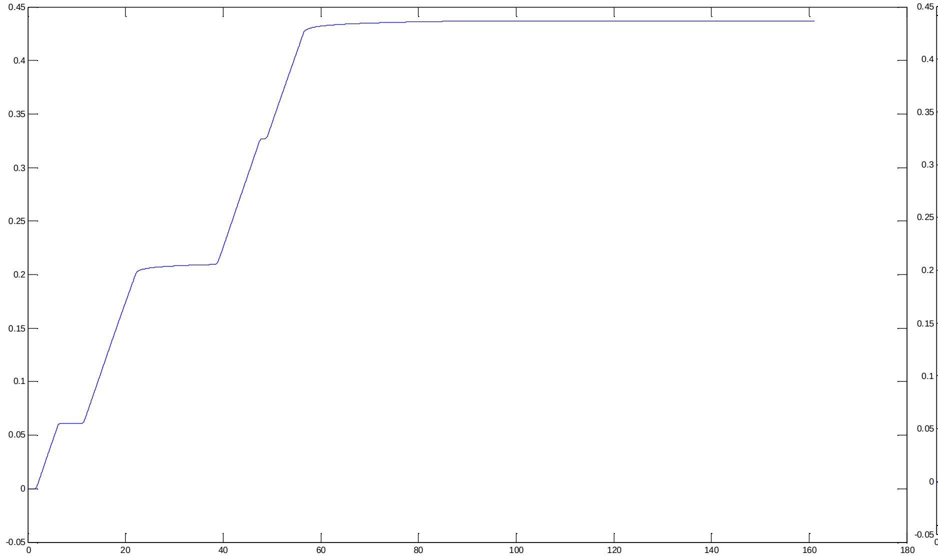
(p,q) 2D-ABC model Mc=1.7

- When Mc decreases stresses decrease and (p,q) curves become wider and more deformations are expected

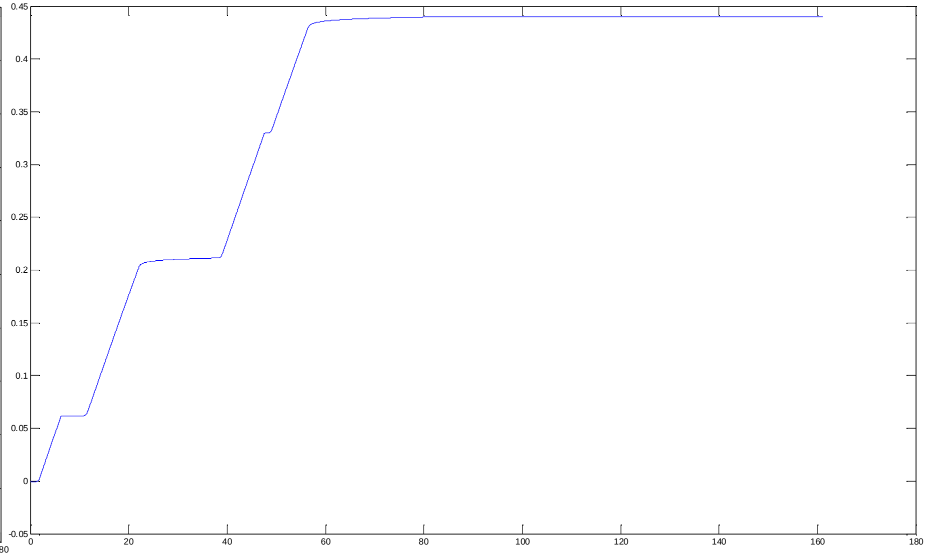
Creep deformation $\boldsymbol{\varepsilon}_{c11}$ 2D-ABC model



(t,$\boldsymbol{\varepsilon}_{c11}$) 2D-ABC model Mc=2.9

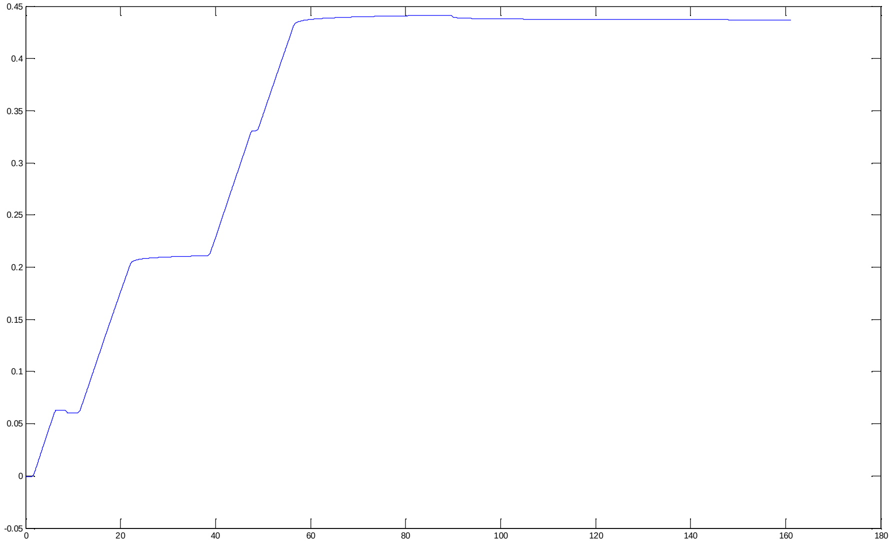


(t,$\boldsymbol{\varepsilon}_{c11}$) 2D-ABC model Mc=2.4

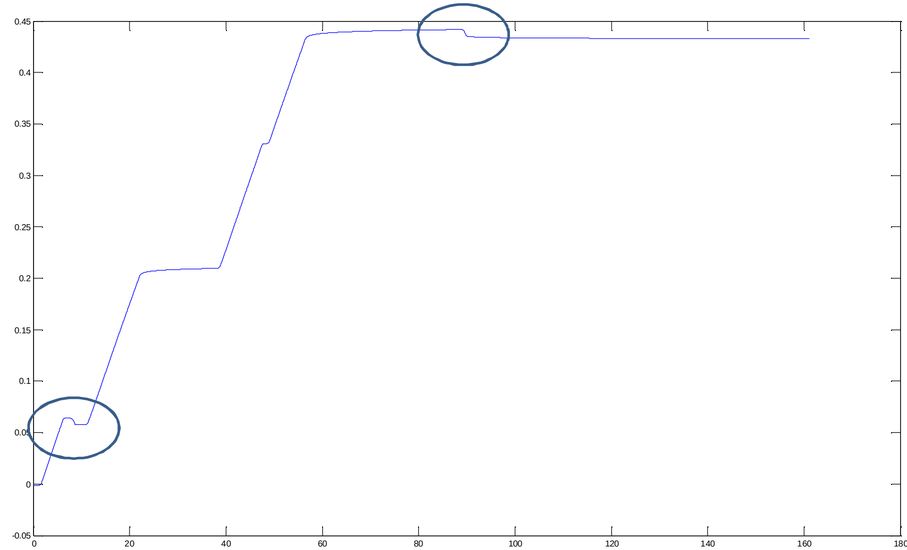


(t,$\boldsymbol{\varepsilon}_{c11}$) 2D-ABC model Mc=2.2

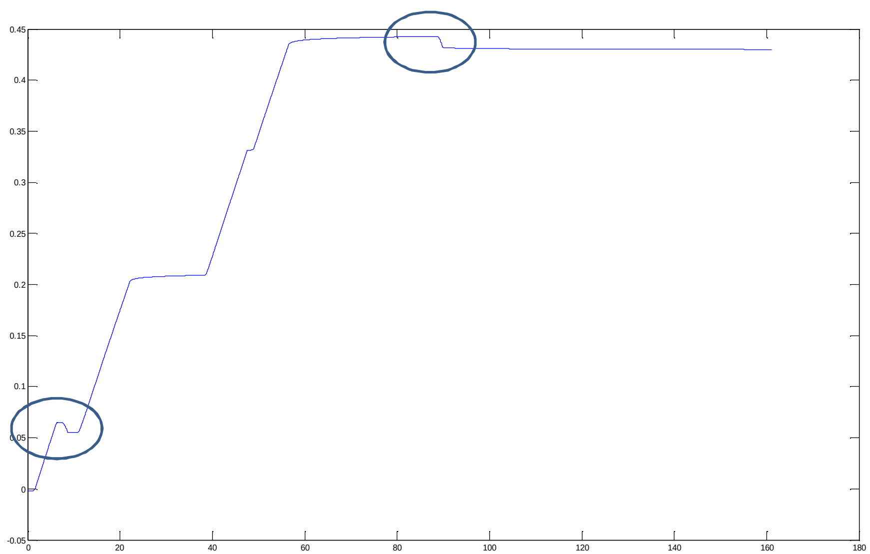
2D-ABC model



$(t,\boldsymbol{\varepsilon}_{c11})$ 2D-ABC model Mc=2.



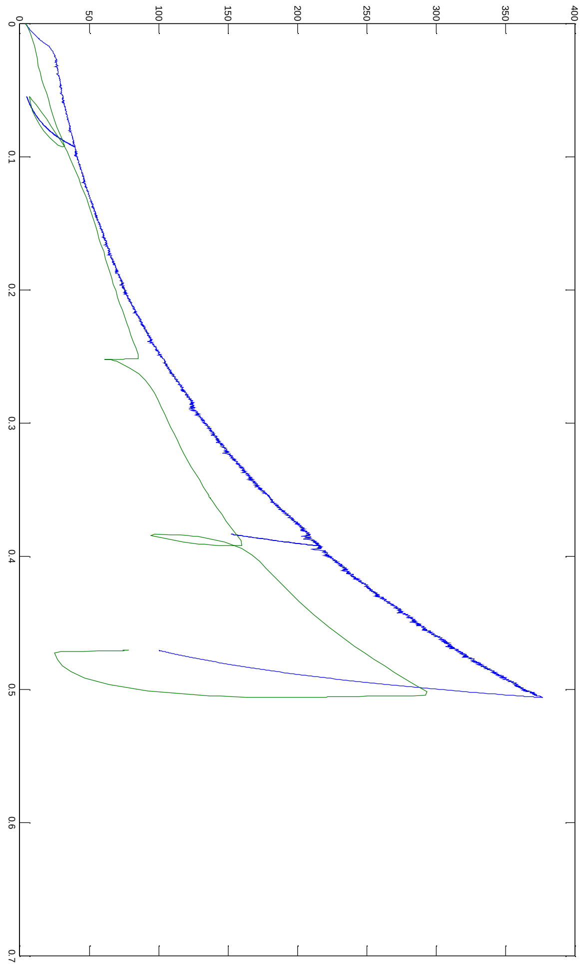
$(t,\boldsymbol{\varepsilon}_{c11})$ 2D-ABC model Mc=1.8



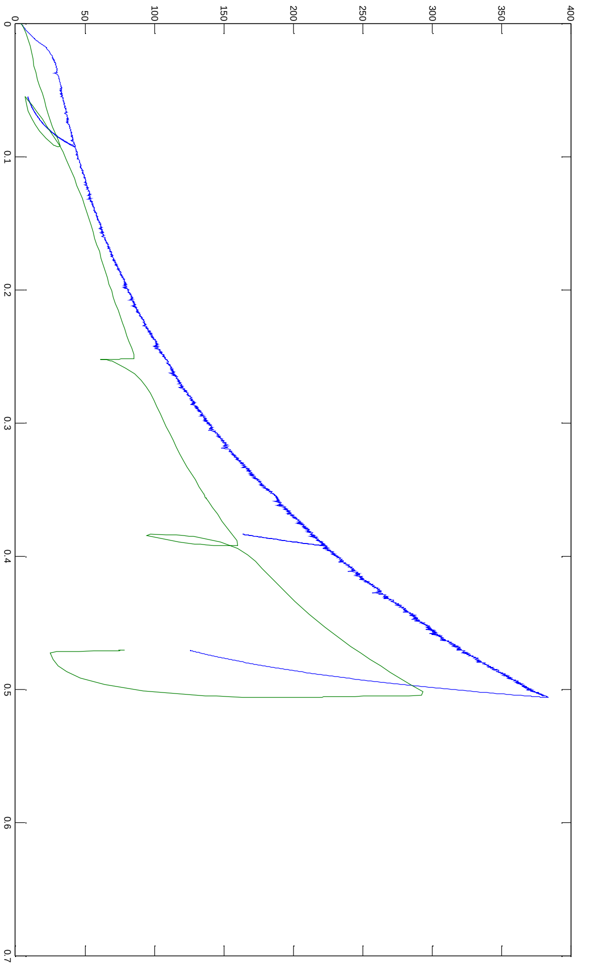
$(t,\boldsymbol{\varepsilon}_{c11})$ 2D-ABC model Mc=1.6,

- Plastic deformation increases when Mc takes lower values

AEP model

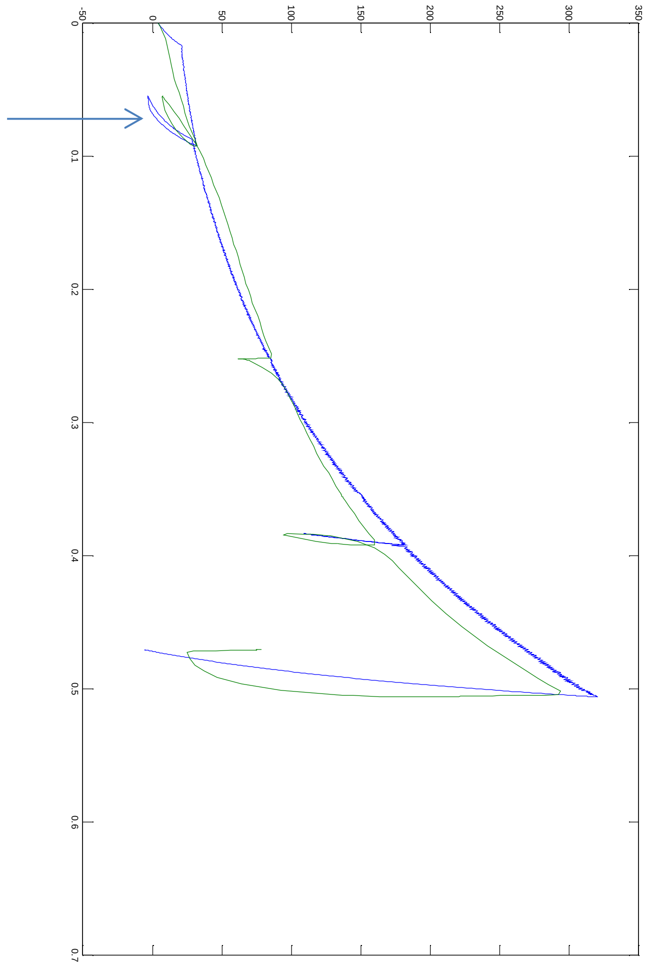


(σ_{11}, ε_{11}) AEP model Mc=2.6

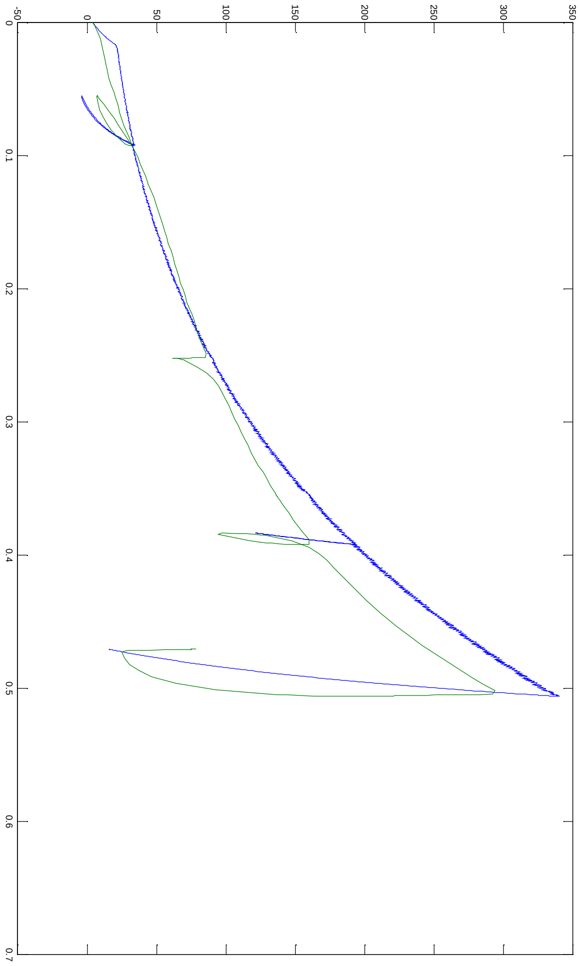


(σ_{11}, ε_{11}) AEP model Mc=3.

AEP model

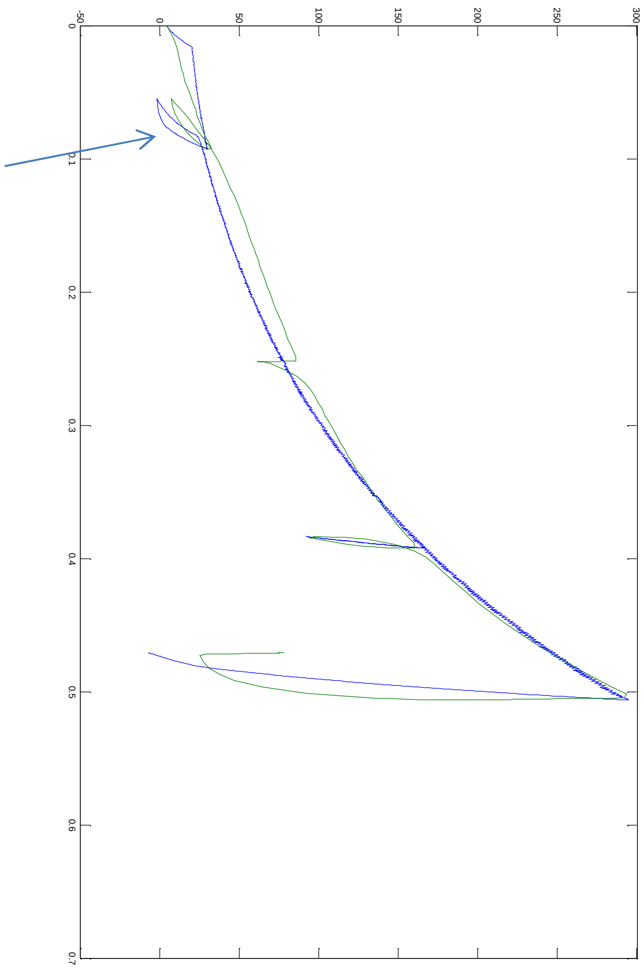


(σ_{11}, ε_{11}) AEP model Mc=1.4

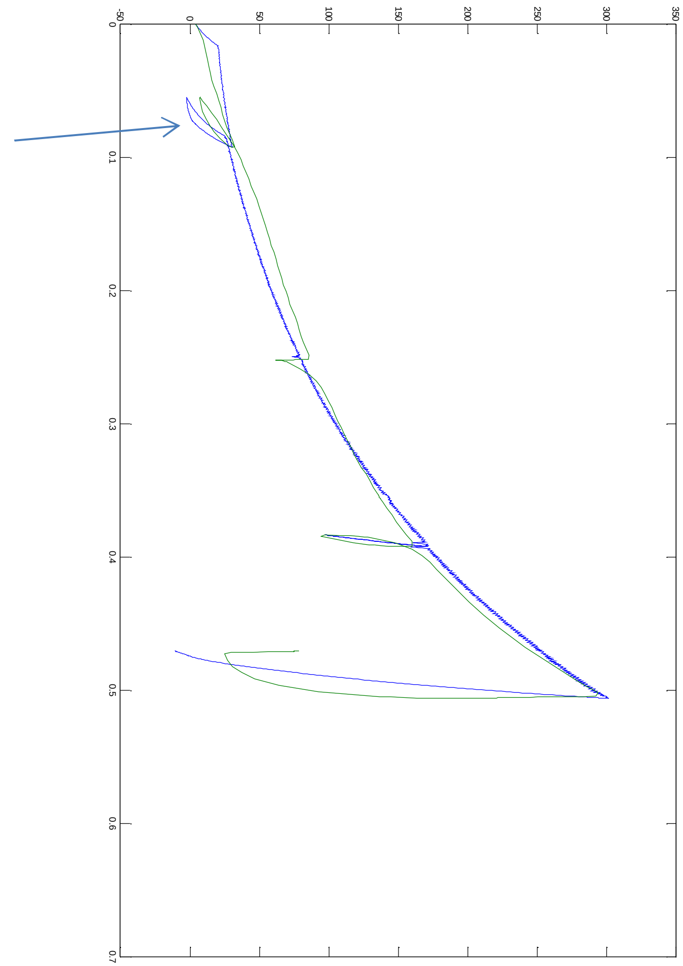


(σ_{11}, ε_{11}) AEP model Mc=1.6

AEP model



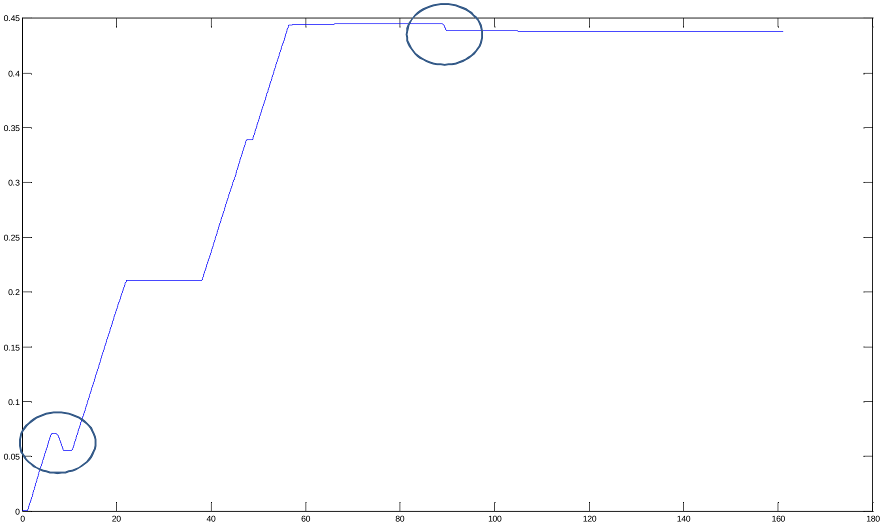
($\boldsymbol{\sigma}_{11}$, $\boldsymbol{\varepsilon}_{11}$) AEP model Mc=1.1



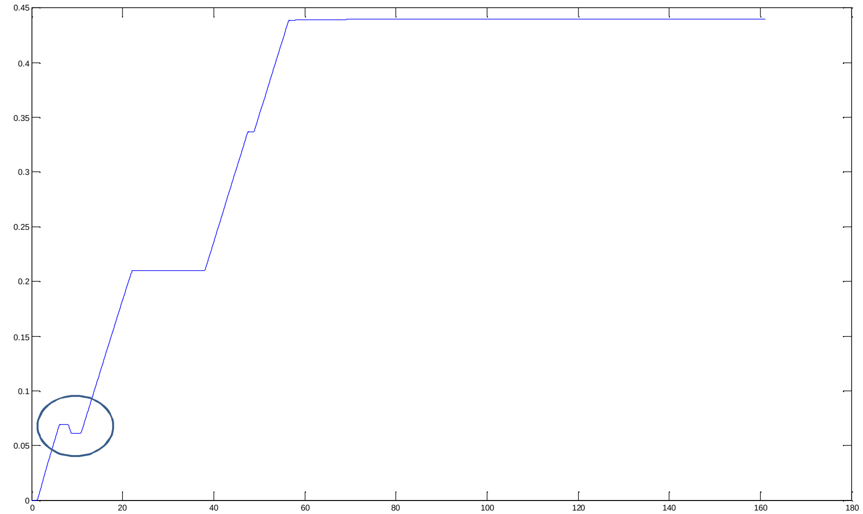
($\boldsymbol{\sigma}_{11}$, $\boldsymbol{\varepsilon}_{11}$) AEP model Mc=1.2

- Mc lower, more plasticity

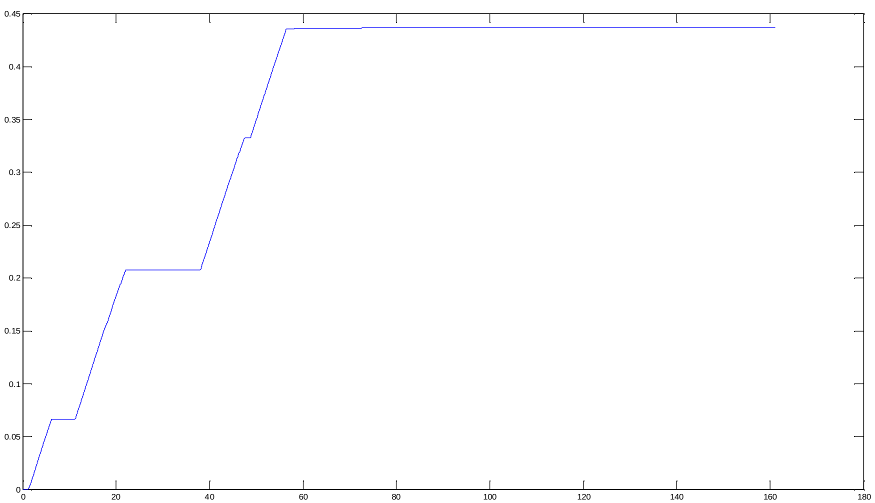
Plastic deformation $\boldsymbol{\varepsilon}_{p11}$ AEP model



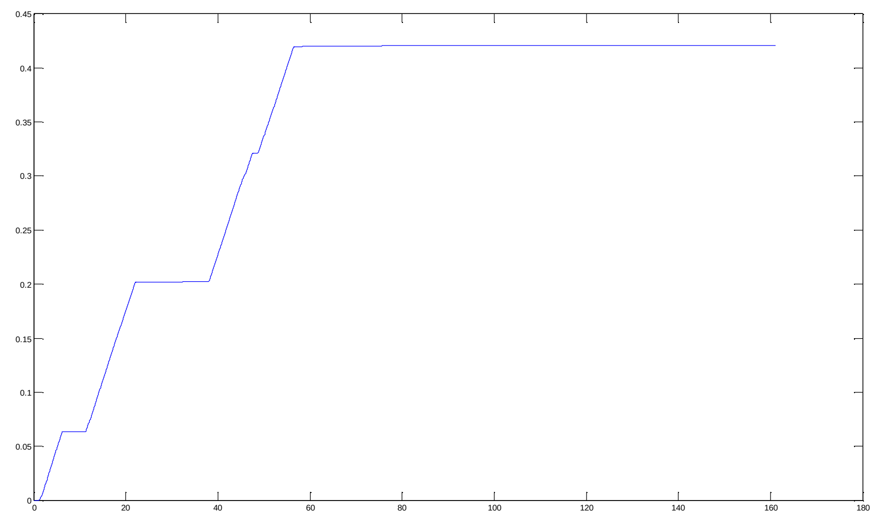
(t, $\boldsymbol{\varepsilon}_{p11}$) AEP model Mc=1.1



(t, $\boldsymbol{\varepsilon}_{p11}$) AEP model Mc=1.4

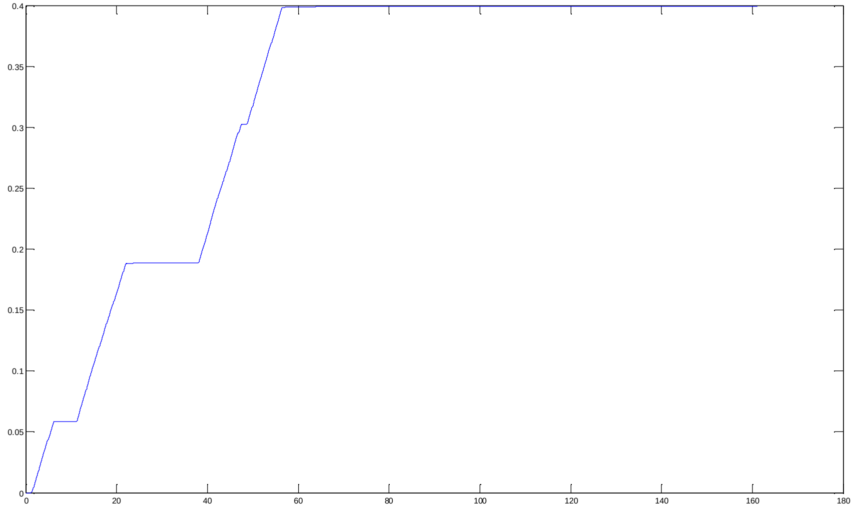


(t, $\boldsymbol{\varepsilon}_{p11}$) AEP model Mc=1.9

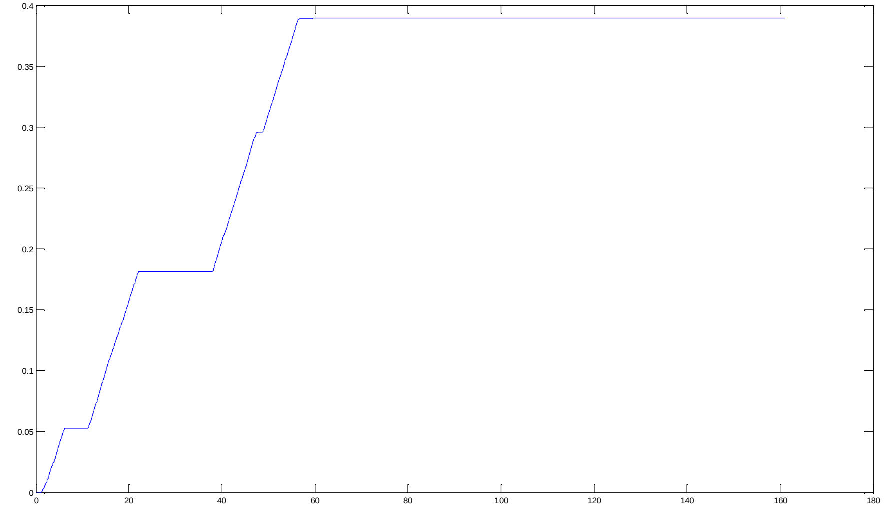


(t, $\boldsymbol{\varepsilon}_{p11}$) AEP model Mc=2.2

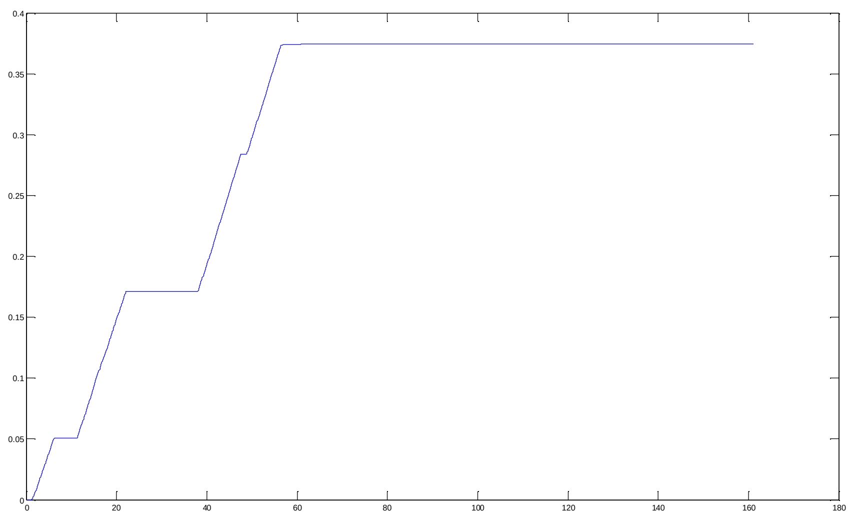
Plastic deformation $\boldsymbol{\varepsilon}_{p11}$ AEP model



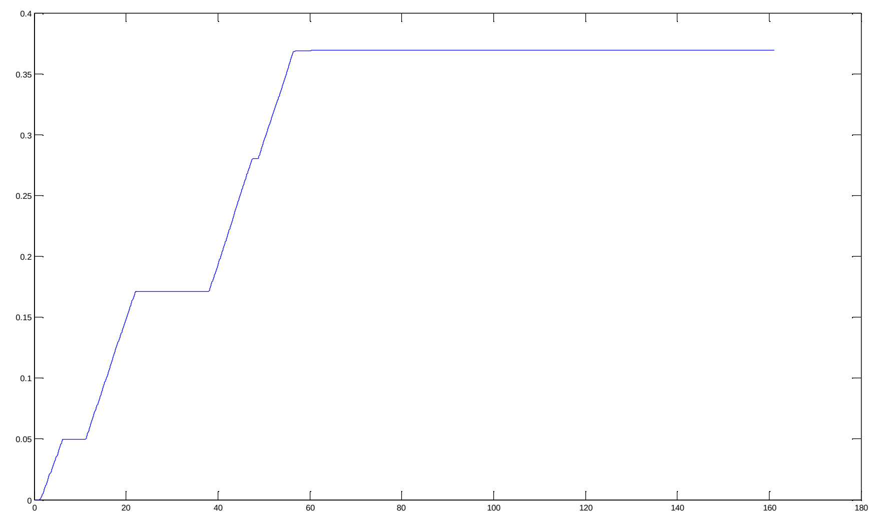
(t, $\boldsymbol{\varepsilon}_{p11}$) AEP model Mc=2.4



(t, $\boldsymbol{\varepsilon}_{p11}$) AEP model Mc=2.6



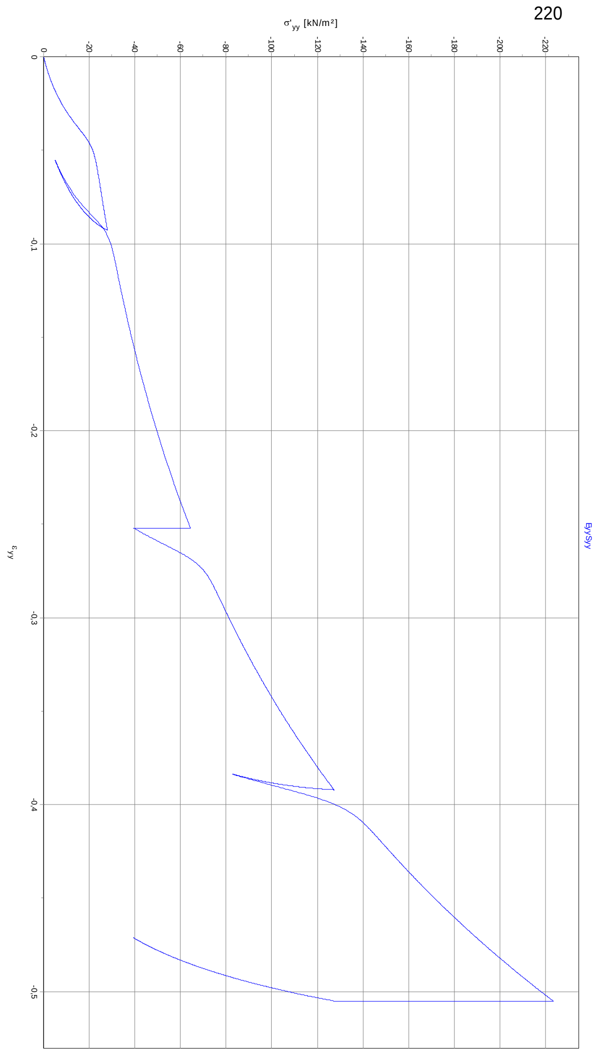
(t, $\boldsymbol{\varepsilon}_{p11}$) AEP model Mc=2.8



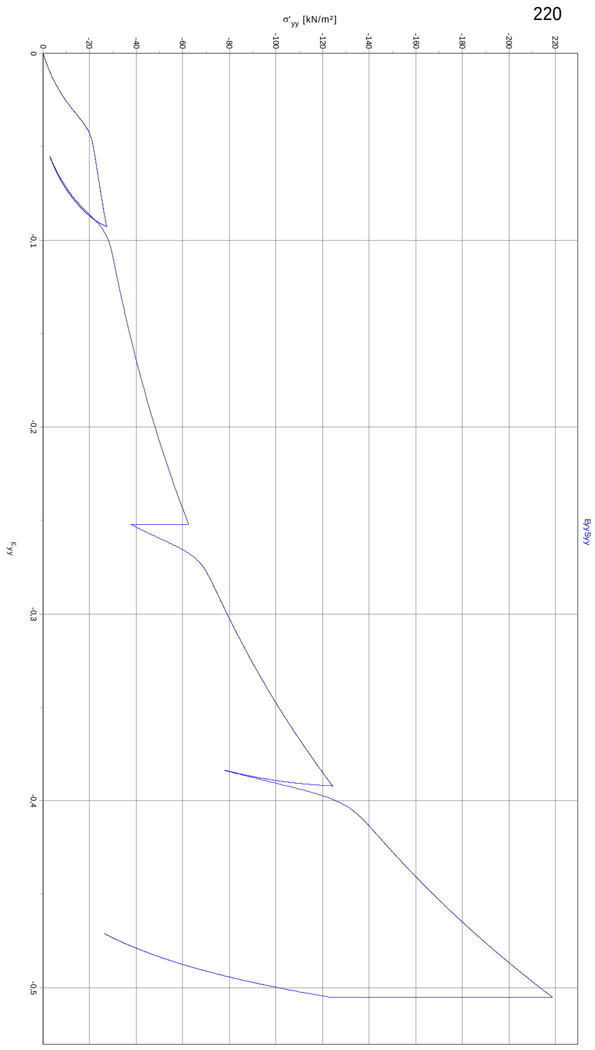
(t, $\boldsymbol{\varepsilon}_{p11}$) AEP model Mc=3

- The smaller Mc , bigger plastic deformations

SSC Plaxis

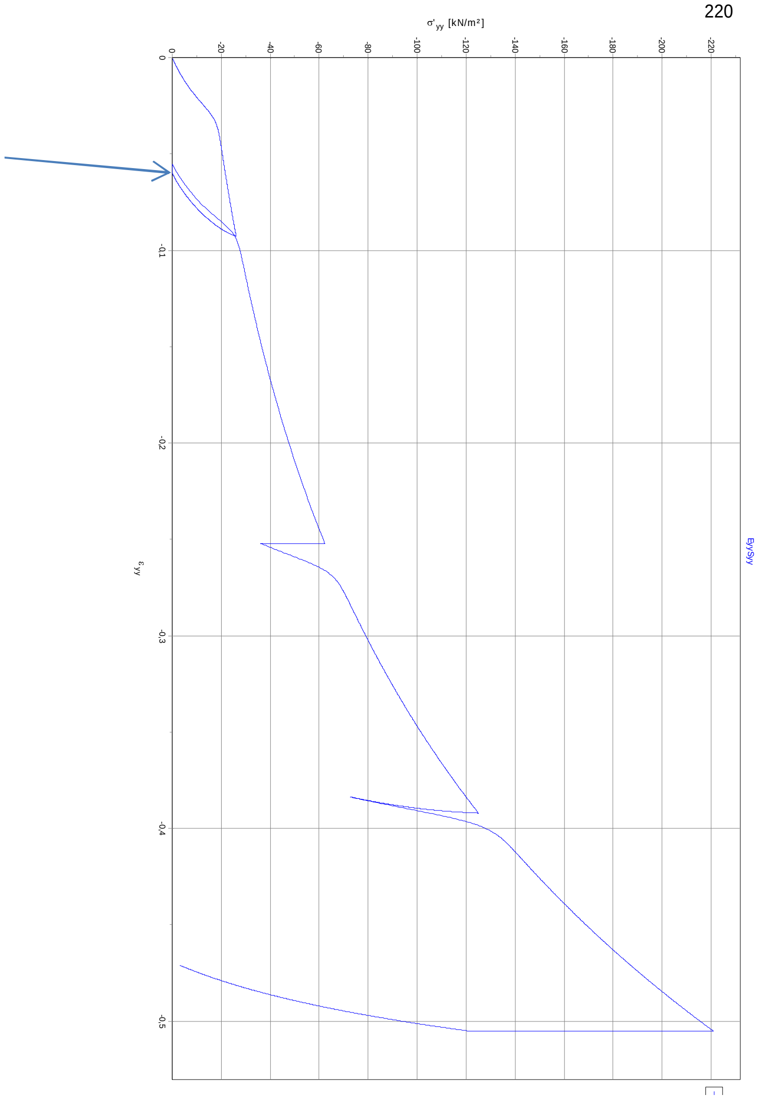


$(\boldsymbol{\sigma}_{11}, \boldsymbol{\varepsilon}_{11})$ SSC Plaxis M=2.972

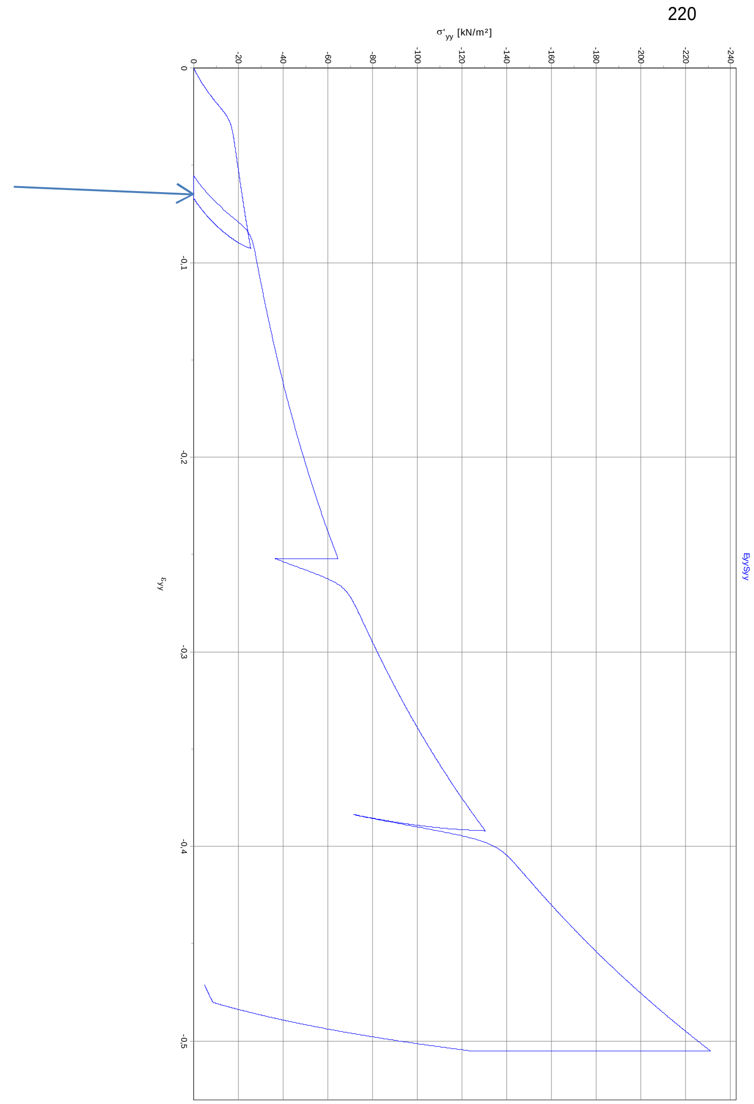


$(\boldsymbol{\sigma}_{11}, \boldsymbol{\varepsilon}_{11})$ SSC Plaxis M=2.61

SSC Plaxis

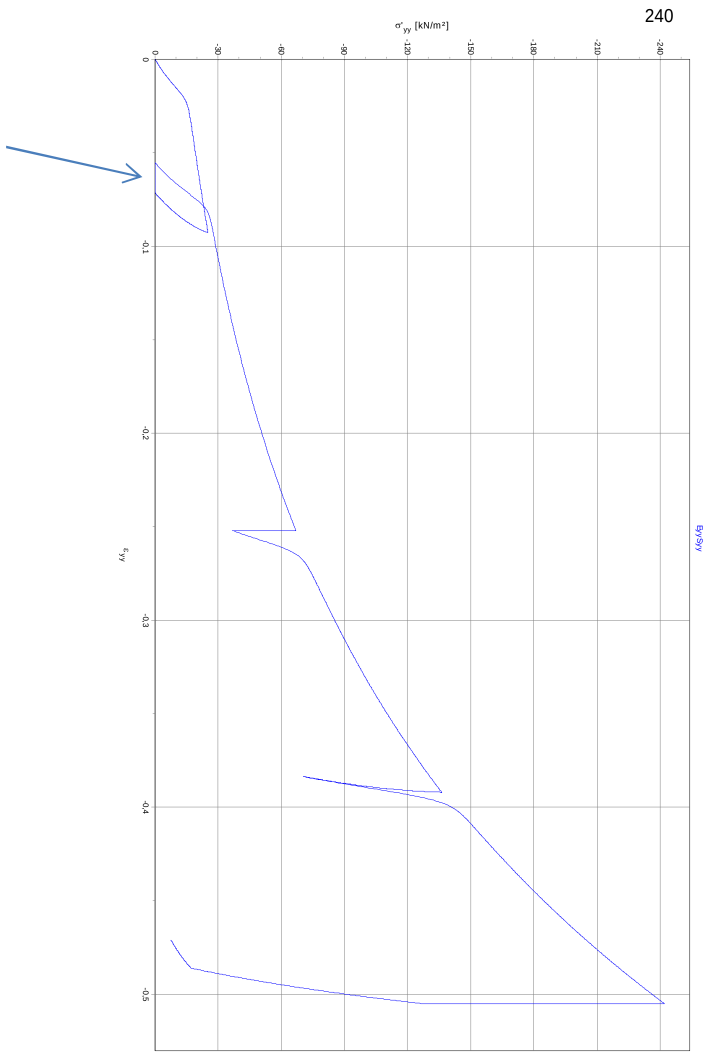


(σ_{11}, ε_{11}) SSC Plaxis M=2.159

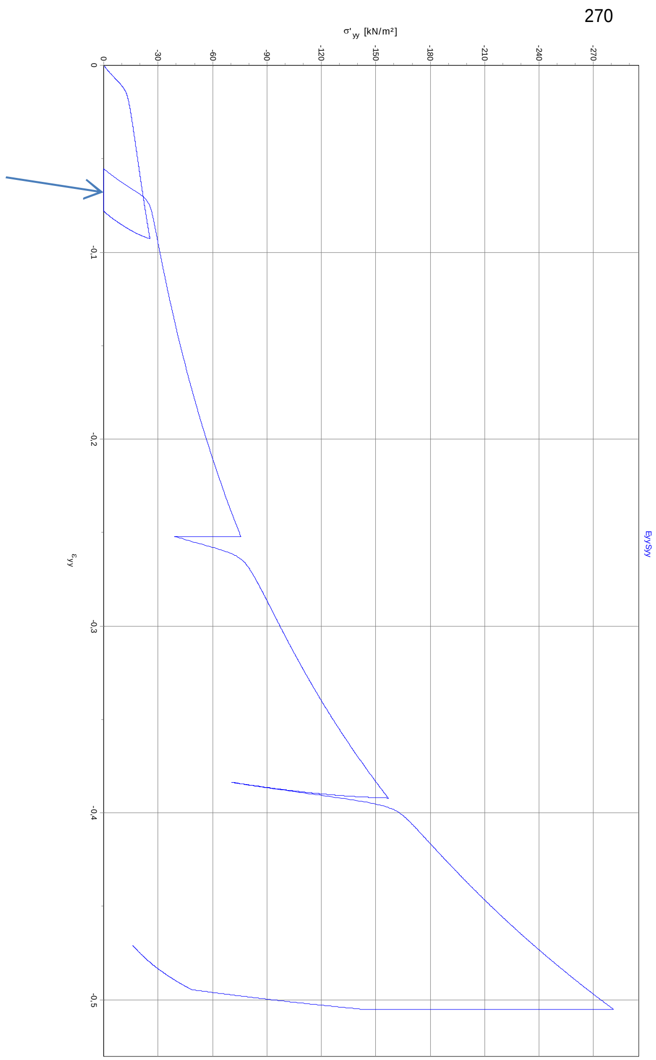


(σ_{11}, ε_{11}) SSC Plaxis M=1.85

SSC Plaxis



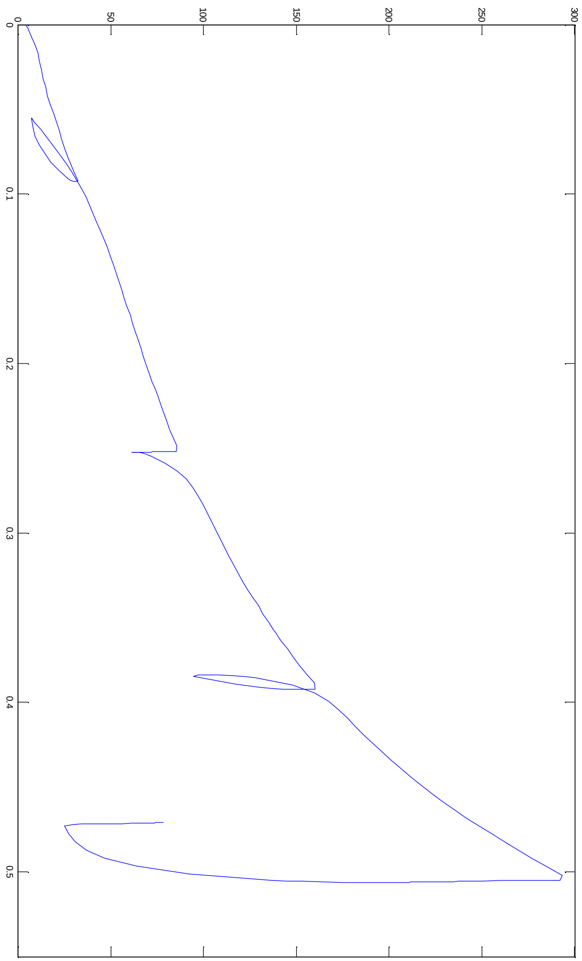
(σ_{11}, ε_{11}) SSC Plaxis M=1.61



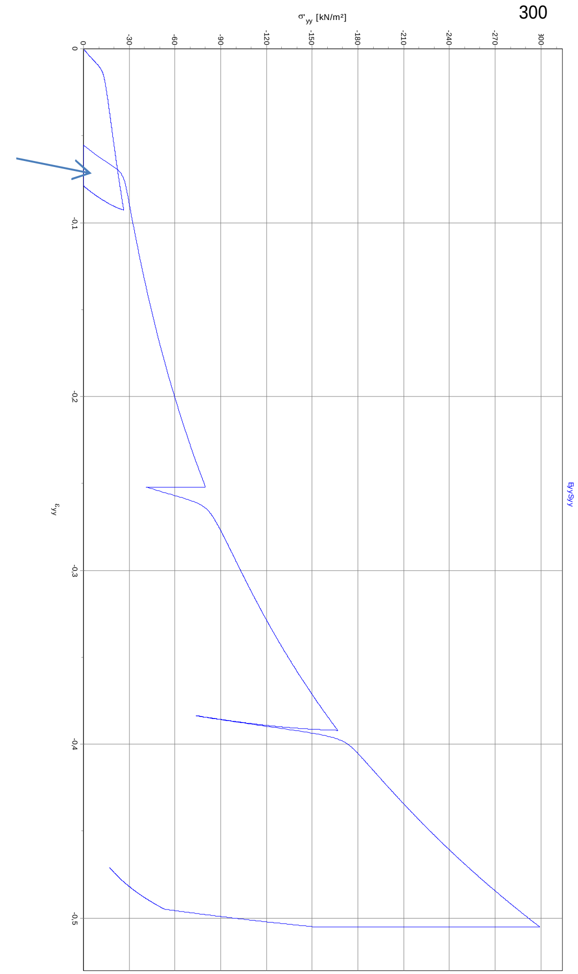
(σ_{11}, ε_{11}) SSC Plaxis M=1.173

- Lower values of Mc bigger plastic deformations

SSC Plaxis

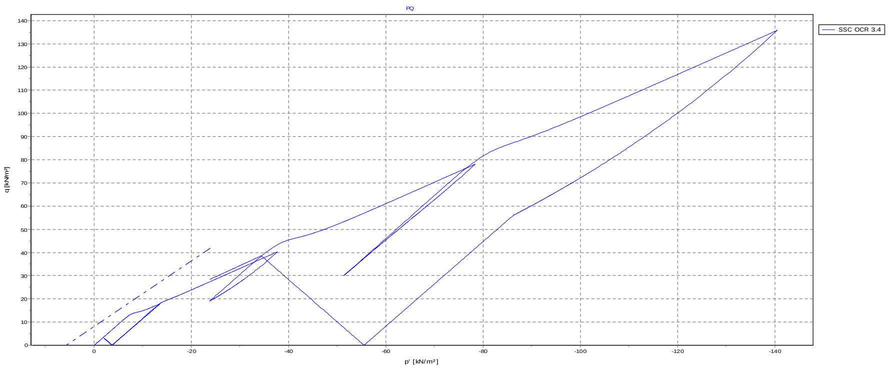


(σ_{11}, ε_{11}) Experimental

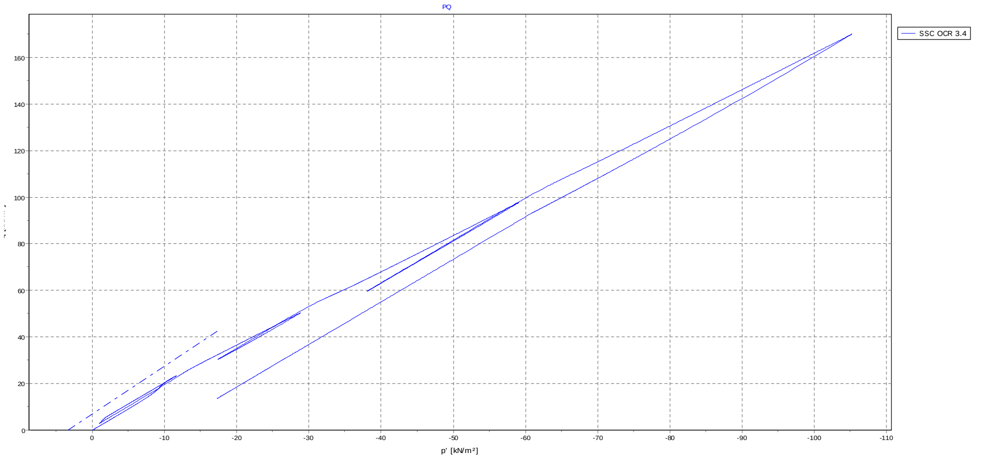


(σ_{11}, ε_{11}) SSC Plaxis M=1.152

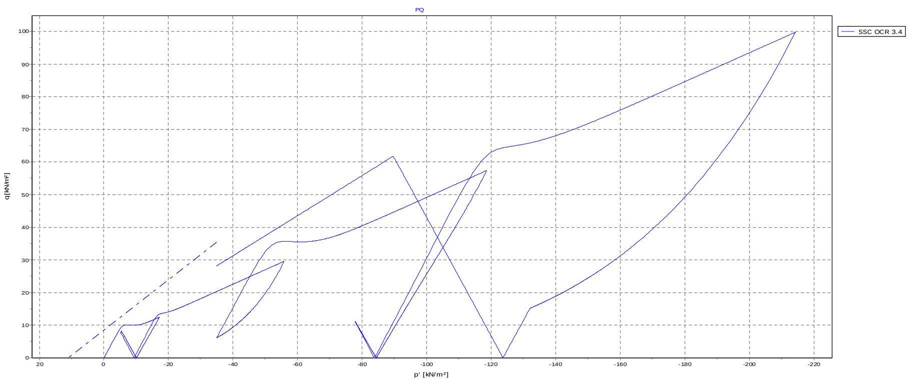
M=1.152, SSC results fit experimental data



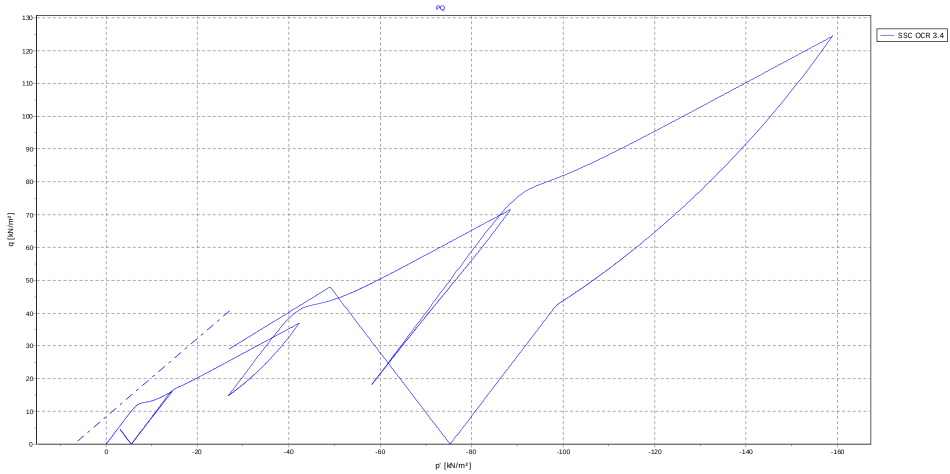
(p,q) SSC plaxis M=1.85



(p,q) SSC plaxis M=2.61



(p,q) SSC plaxis M=1.173



(p,q)SSC plaxis M=1.61

M=1,1 AEP model
M=2,2 2D-ABC model
M= 1,152 (SSC plaxis)

Example II

Axial loading unloading, K_{0nc}

C_c=0.484 ; C_s=0.0745 ;
C_α =0.0414 ;

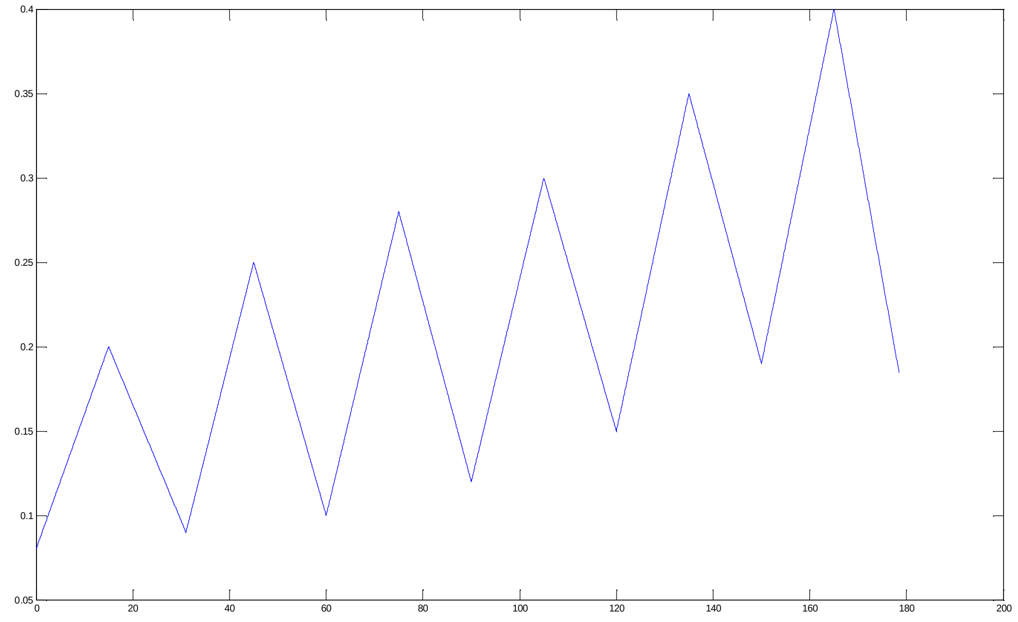
$\mu = C_\alpha / \log(10)$;

ν =0.15 ;

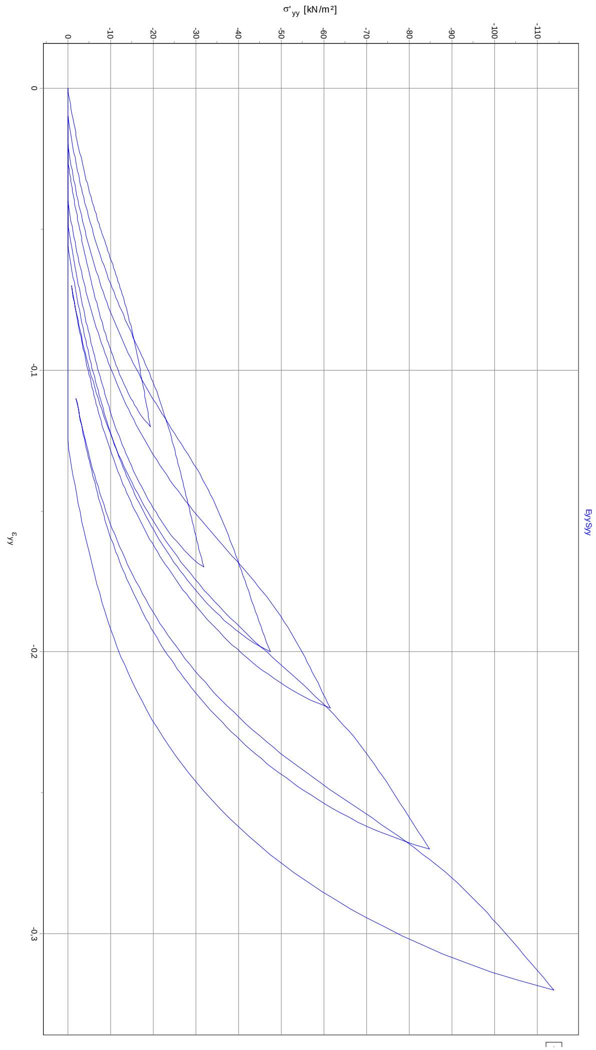
$\lambda = C_c/\log(10) = 0.2102$

$\kappa = 2*C_s/\log(10) = 0.0647$

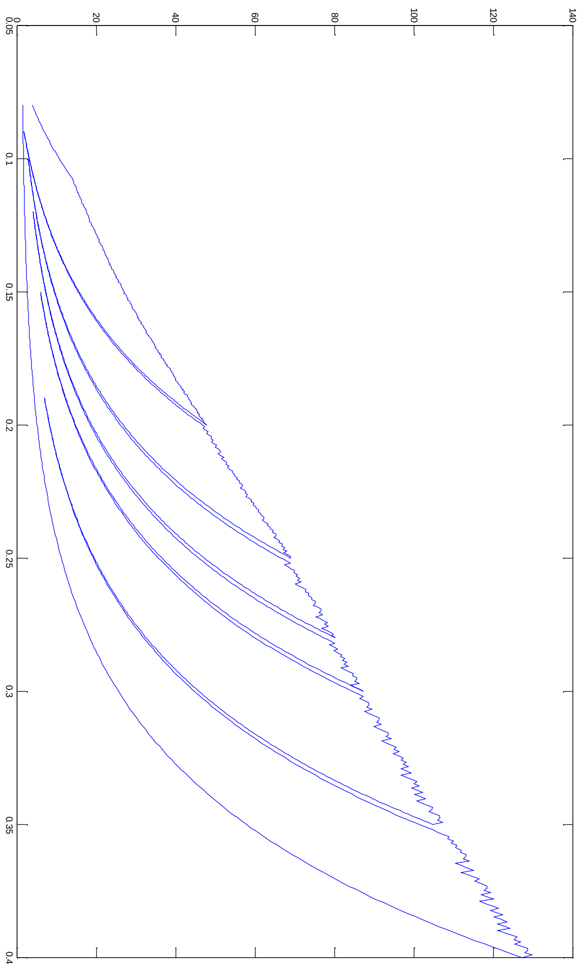
e_0 =0.5;



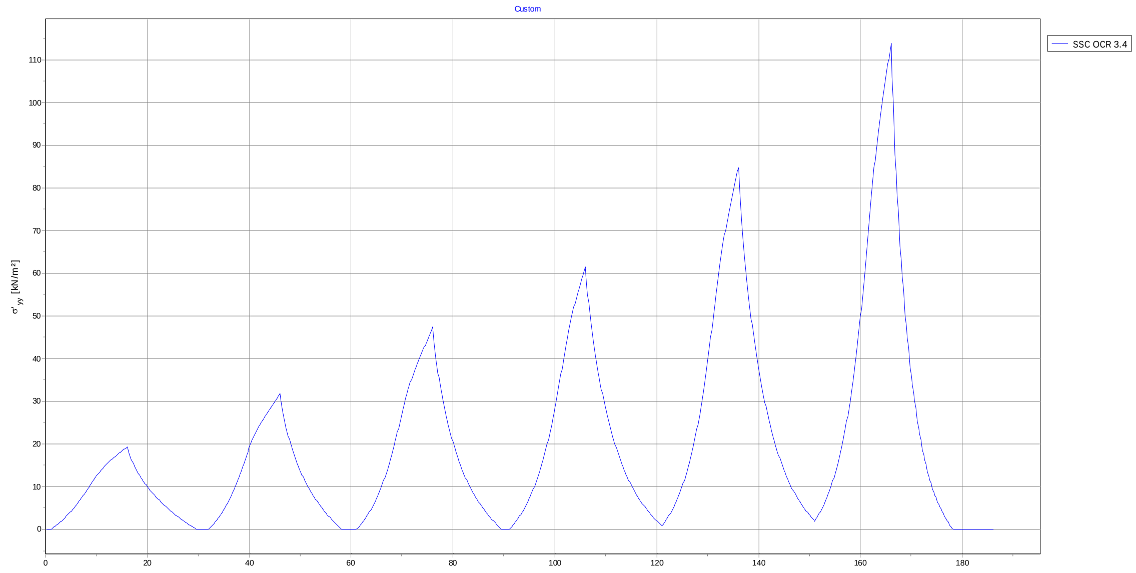
Applied axial strain ($\boldsymbol{\varepsilon}_{11}$, t)



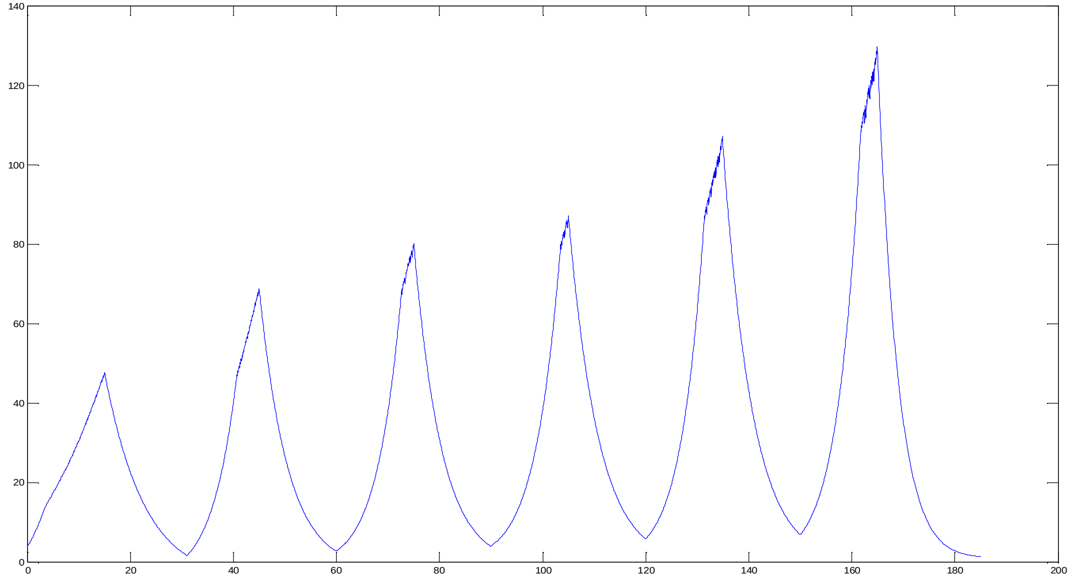
SSC PLAXIS M=2.94



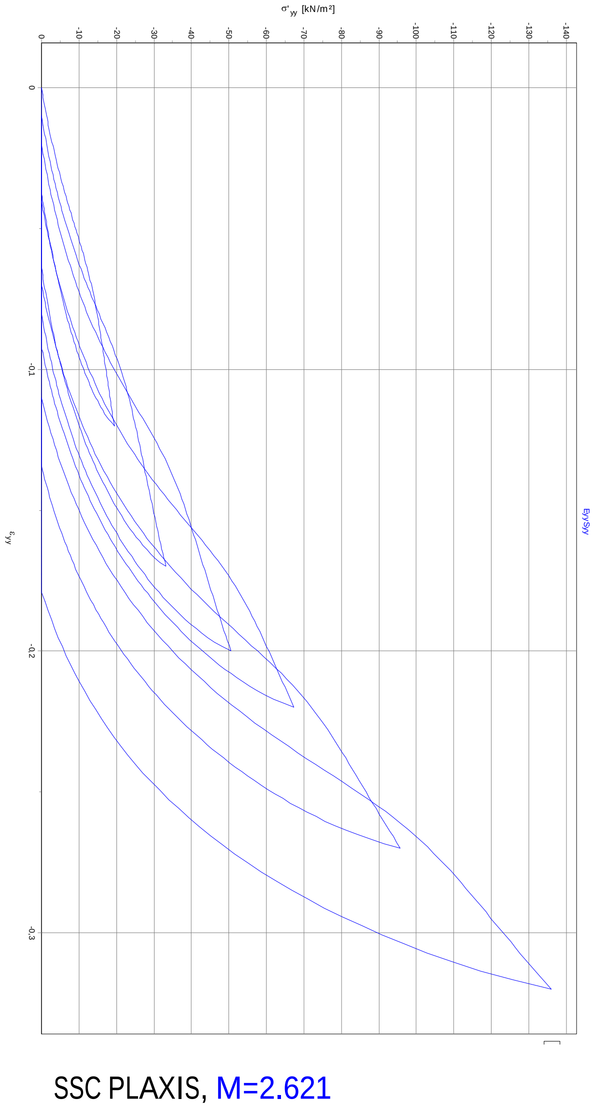
AEP model Mc=2.94



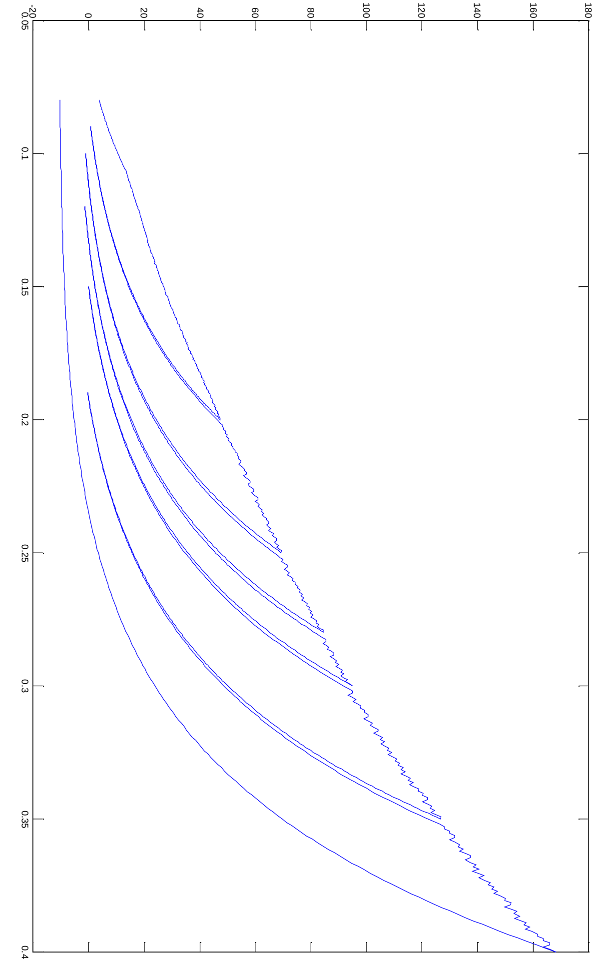
SSC PLAXIS, M=2.94



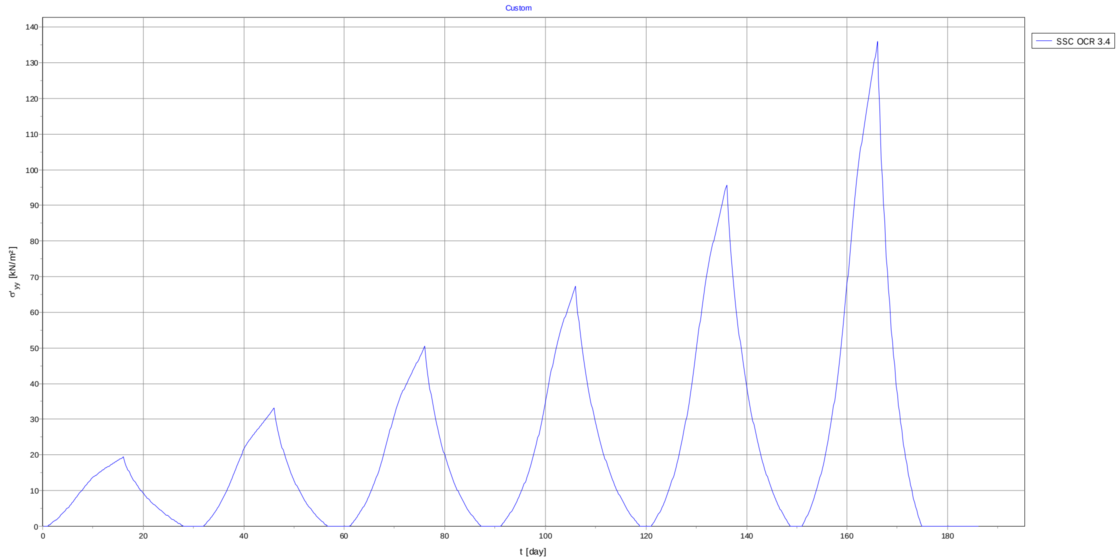
AEP model Mc=2.94



SSC PLAXIS, M=2.621



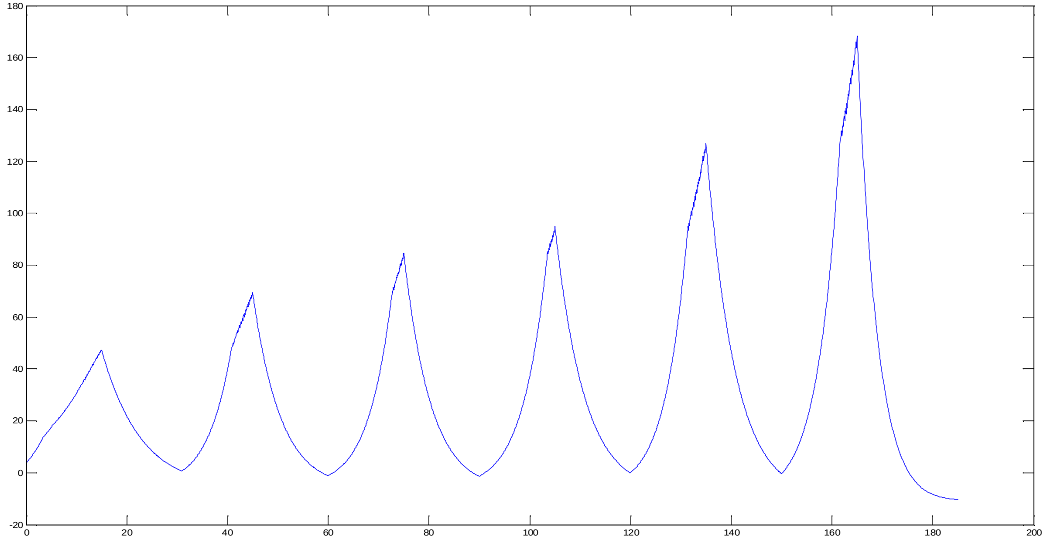
AEP model Mc=2.6



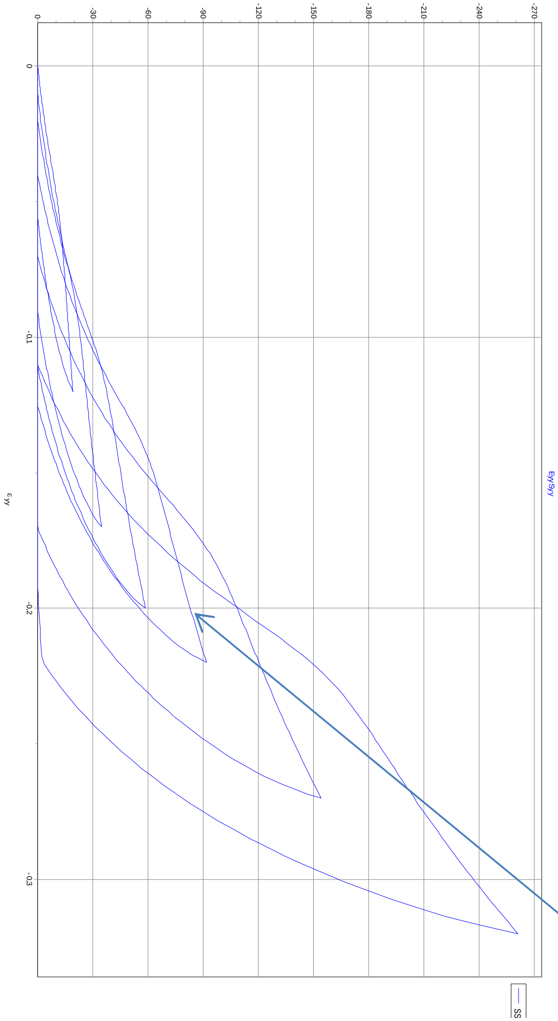
SSC PLAXIS, M=2.621

Same shape, evolution of σ_{11}

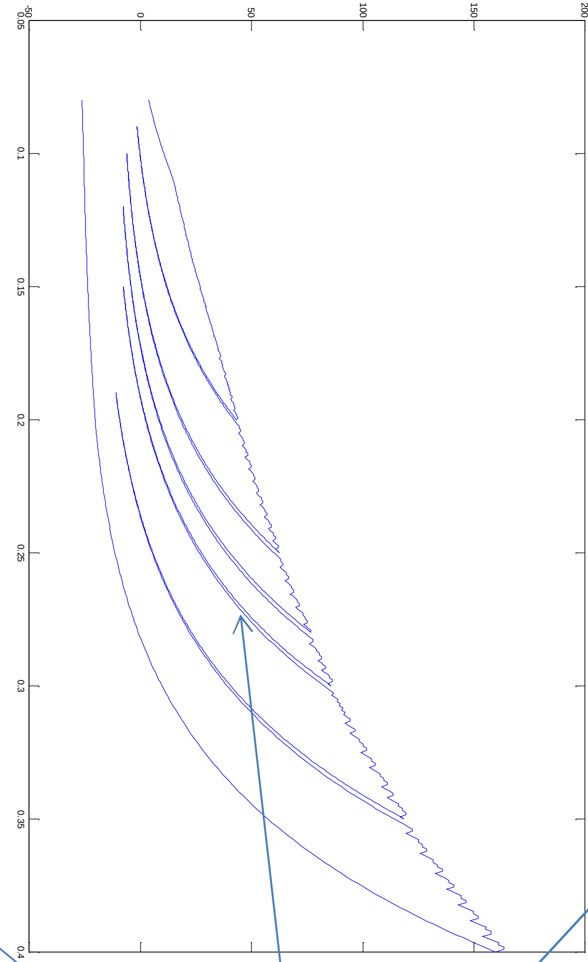
Values of σ_{11} slightly bigger in AEP model



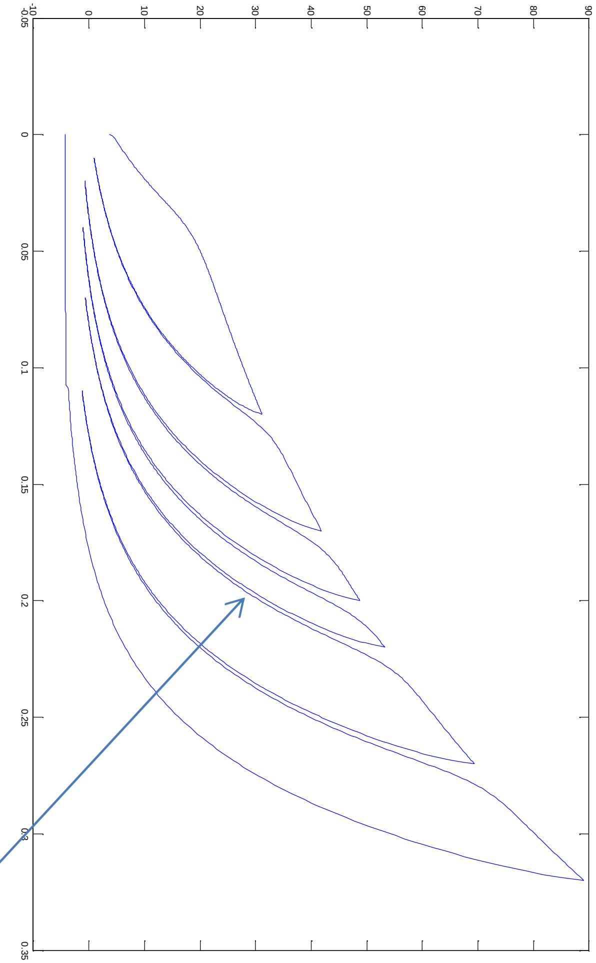
AEP model Mc=2.6



SSC PLAXIS M=2.203, Konc=0.3138 , ϕ=43°



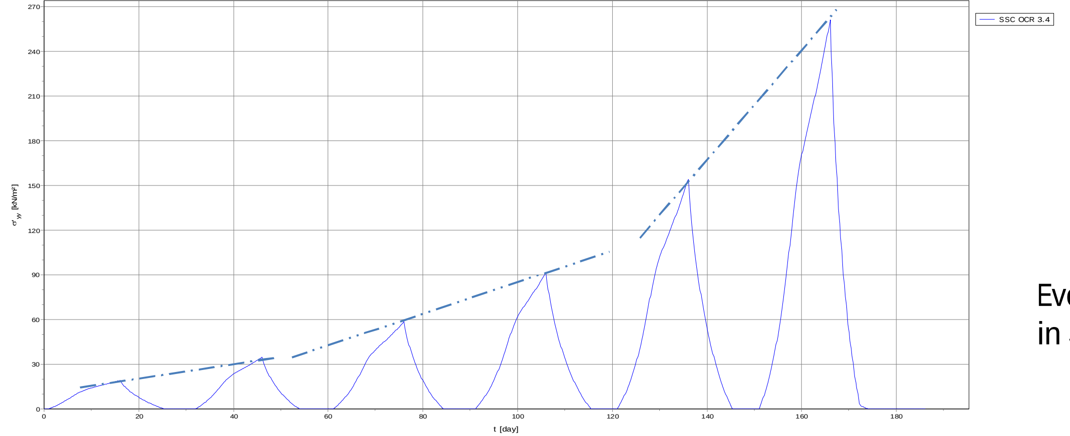
AEP model Mc=2.2



2D-ABC model Mc=2.2

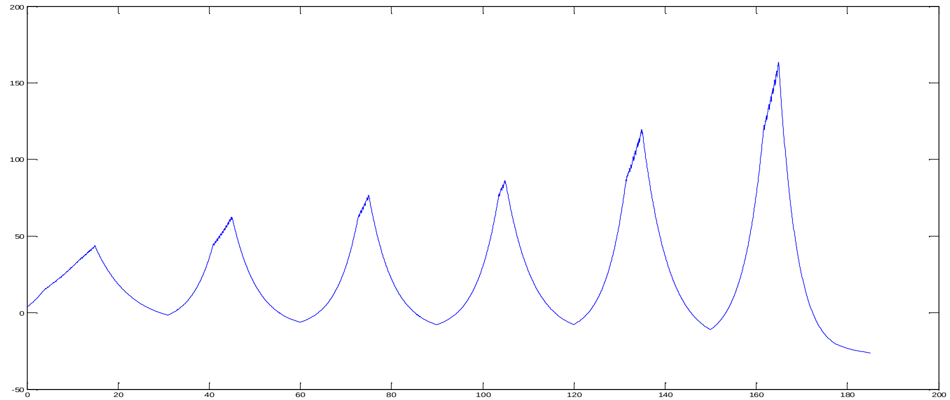
Response to Loading unloading cycles is different between SSC and AEP model

SSC PLAXIS, M=2.203

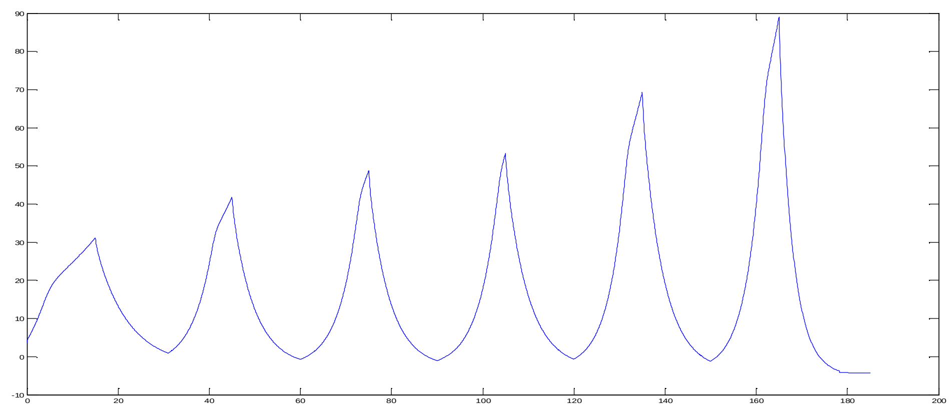


Evolution of σ_{11} is quicker in SSC

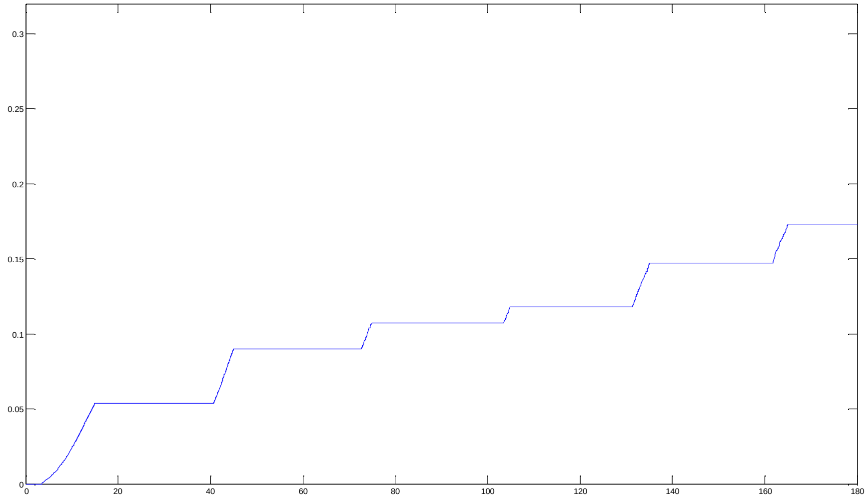
AEP model Mc=2.2



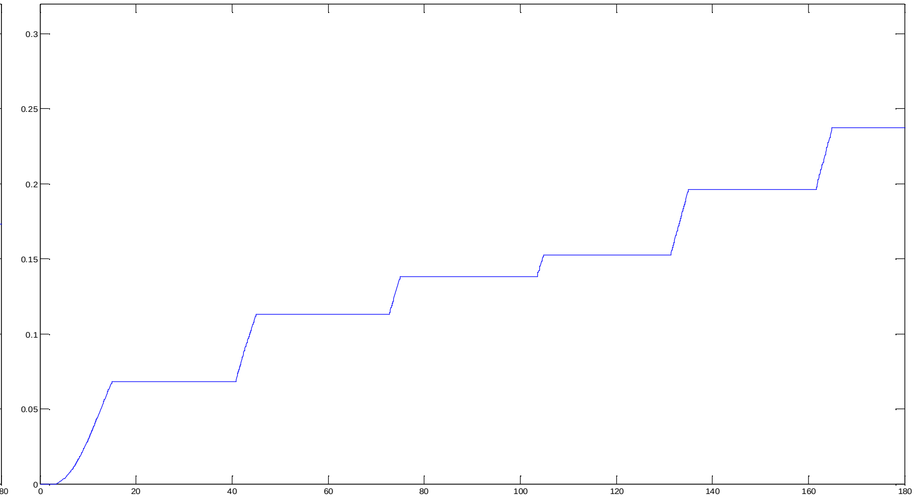
2D-ABC model Mc=2.2



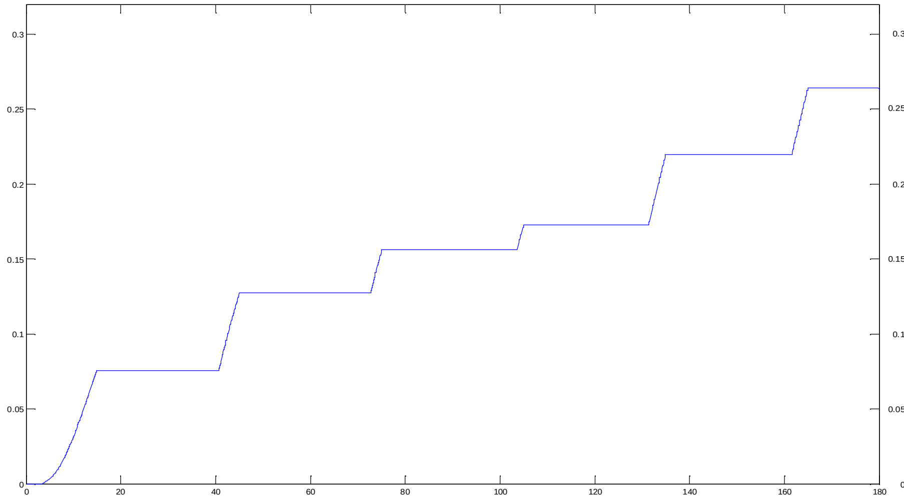
Values of σ_{11} are smaller in 2D-ABC model than those from SSC and AEP



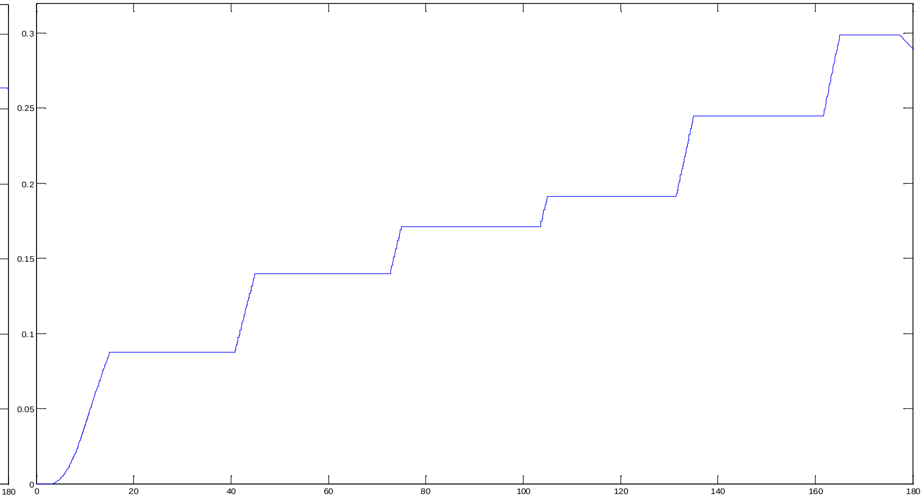
(ε_{p11} ,t) AEP model Mc=2.94



(ε_{p11} ,t) AEP model Mc=2.6



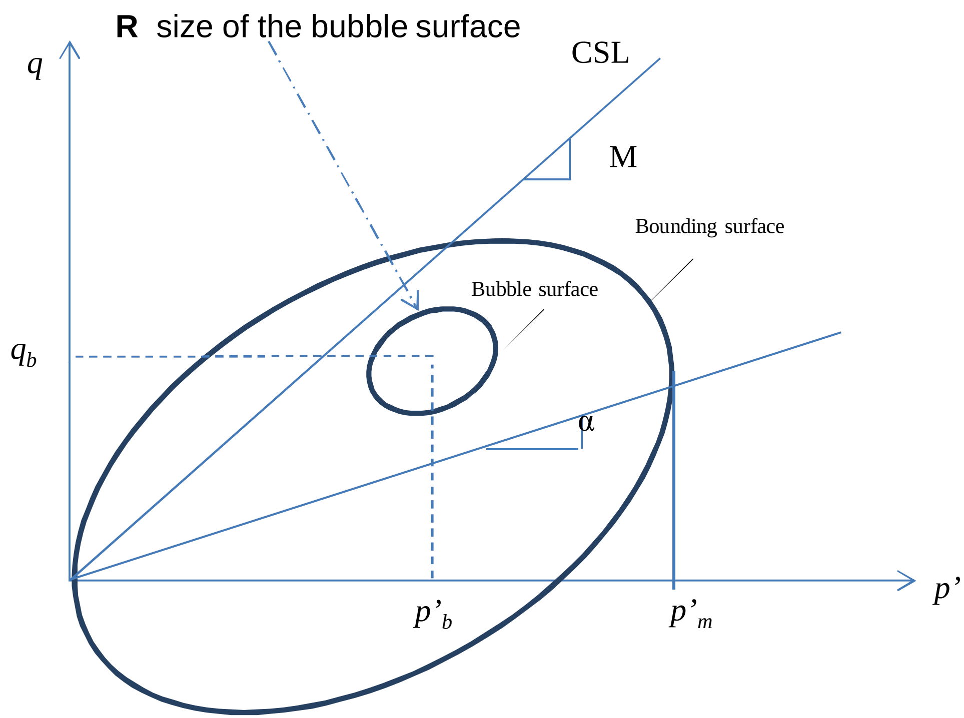
(ε_{p11} ,t) AEP model Mc=2.4



(ε_{p11} ,t) AEP model Mc=2.2

(ε_{p11} ,t) AEP model, more plasticity when Mc decreases from 2,94 to 2,2

B-AEP (Bubble Anisotropic Elasto-Plastic model (based on B-SClay1S, Sivasithamparam 2012)



Bounding surface and bubble surface in the model

Equation of bounding surface

$$f_y = \frac{(q - \alpha p')^2}{M^2 - \alpha^2} + \left(p' - \frac{p'_m}{2}\right)^2 - \left(\frac{p'_m}{2}\right)^2 = 0$$

Equation of the bubble

$$f_b = \frac{[(q - p'\alpha) - (q_b - p'_b\alpha)]^2}{M^2 - \alpha^2} + (p' - p'_b)^2 - R^2\left(\frac{p'_m}{2}\right)^2 = 0$$

Hardening laws, B-AEP model

The flow rule: $$\Delta\epsilon_{ij}^{p} = \Delta\Phi\frac{\partial p_b}{\partial\sigma_{ij}} = \Delta\Phi\frac{\partial f_b}{\partial\sigma_{ij}}$$

Isotropic hardening rule

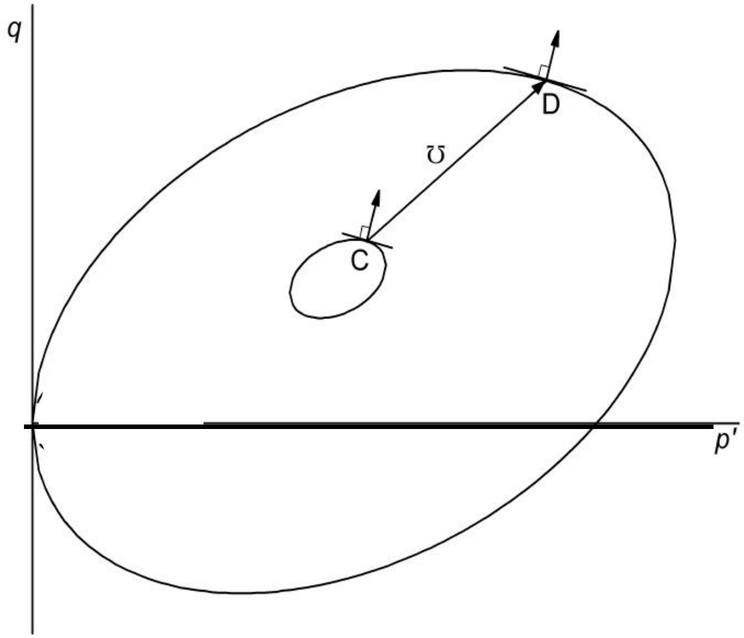
$$\Delta p'_m = \frac{(1+e)p'_m}{\lambda-\kappa}\Delta\epsilon_v^p$$

The rotational hardening law, describes the changes in orientation of the yield surface

$$\Delta\underline{\alpha}_d = \mu\left(\left[\frac{3\underline{\sigma}'_d}{4p'} - \underline{\alpha}_d\right]\langle\Delta\epsilon_v^p\rangle + \beta\left[\frac{\underline{\sigma}'_d}{3p'} - \underline{\alpha}_d\right]\Delta\epsilon_d^p\right)$$

μ and β : soil constants

Translation rule B-AEP model



Translation of the bubble along the vector Ʊ

The translation rule describes the bubble surface mouvement within the bounding surface

$$\mho = \left\{ \begin{array}{c} \dfrac{p' - p'_b}{R} - (p' - p'_m) \\ \\ \dfrac{(q - p'\alpha) - (q_b - p'_b\alpha)}{R} - (q - p'\alpha) \end{array} \right\}$$

The translation of the bubble is defined as :

$$\left\{ \begin{array}{c} dp'_b \\ dq_b \end{array} \right\} = \frac{dp'_m}{p'_m} \left\{ \begin{array}{c} p'_b \\ q_b \end{array} \right\} + S \left\{ \begin{array}{c} \frac{p' - p'_b}{R} - (p' - p'_m) \\ \frac{(q - p'\alpha) - (q_b - p'_b\alpha)}{R} - (q - p'\alpha) \end{array} \right\}$$

S scalar quantity

R size of the bubble surface

Hardening modulus $\mathbf{H_{0b}}$ $\mathbf{H_\alpha}$ $\mathbf{H_b}$

ψ parameter, real positve (>0)

$$\mathcal{H}_\alpha = \left\{ \frac{\partial f_b}{\partial \alpha} \right\} \left[\left\{ \frac{\partial \alpha}{\partial \epsilon_v^p} \right\} \left\langle \frac{\partial f_b}{\partial p'} \right\rangle + \left\{ \frac{\partial \alpha}{\partial \epsilon_d^p} \right\} \frac{\partial f_b}{\partial q} \right]$$

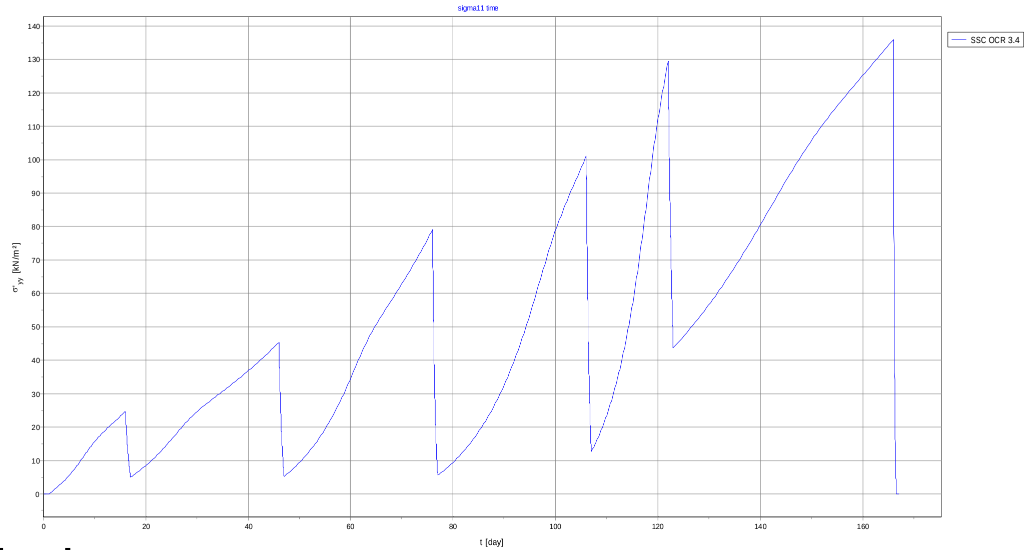
$$\mathcal{H}_{0b} = \frac{4(1+e)}{\lambda_i - \kappa} \left[(p' - p'_b) - \frac{(q - \alpha p') - (q_b - \alpha p'_b)}{M^2 - \alpha^2} (\alpha) \right] \left[p'(p' - p'_b) + \frac{(q - \alpha p')((q - \alpha p') - (q_b - \alpha p'_b))}{M^2 - \alpha^2} \right]$$

$$\mathcal{H}_b = \frac{4(1+e)}{\lambda_i - \kappa} \left(\frac{\ell}{\ell_{max}} \right)^\psi \left(\frac{p'_m}{2} \right)^3$$

EXAMPLE III

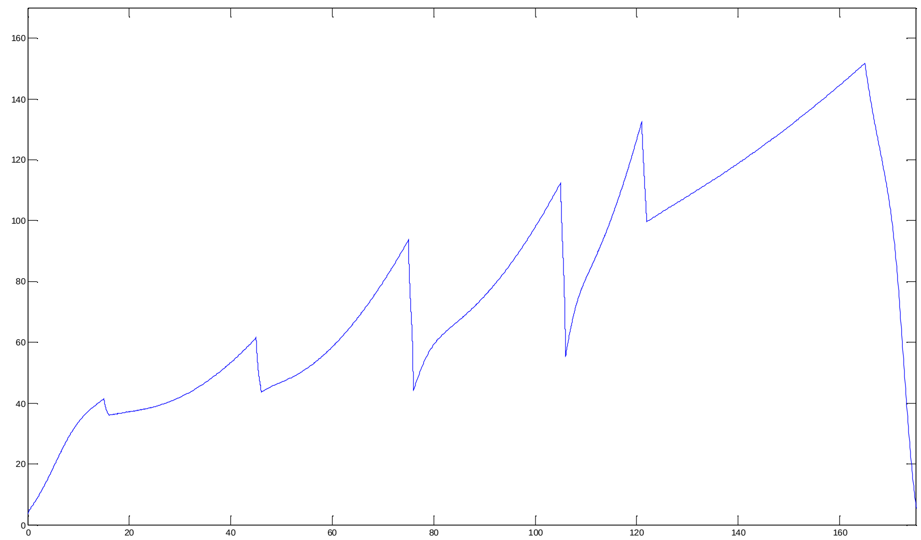
Axially loading unloading

pm=10.5kPa
κ=0.1129
λ=0.2897
ν=0.15
e0= 0,5

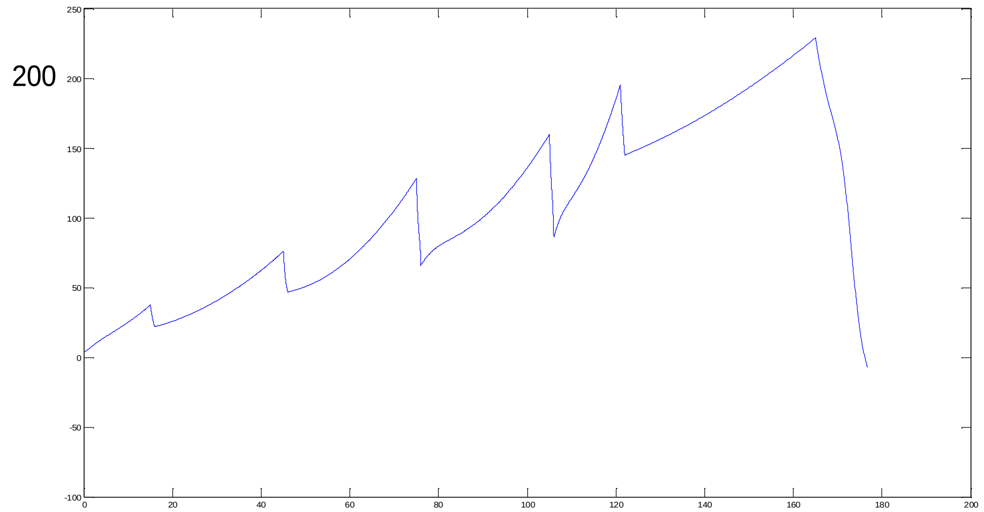


SSC Plaxis(σ_{11}, t) , **κ**=0.1129, **λ**=0.2897

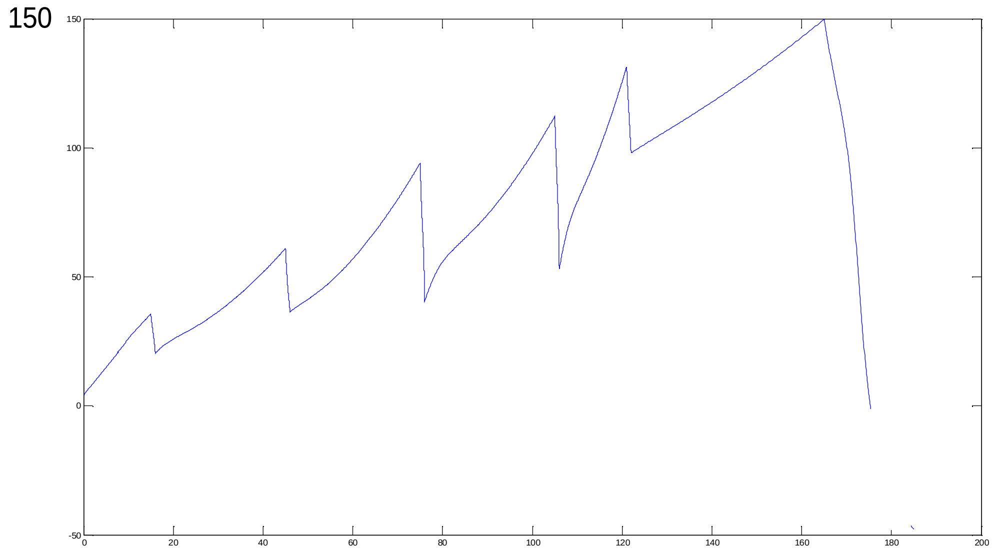
λ=Cc/log(10)
κ=2*Cs/log(10)



B-AEP model,(σ_{11}, t), Cs=0.13(κ=0.1129), **Cc=0.667**(λ=0.2897);**R=0.15**, ψ=0.25



B-AEP model, (σ_{11}, t), **Cs=0.13** (κ=0.1129), **Cc=0.567** (λ=0.2462); **R=0.15**, **ψ=0.25**



B-AEP model, (σ_{11}, t), **Cs=0.13** (κ=0.1129), **Cc=0.7** (λ=0.30); **R=0.15**, **ψ=0.25**

conclusion

- 2D-ABC model for soft soils was extended to general stress space, and showed from simulations the possibility to reproduce correctly peat behavior . The AEP model uses elasto-plastic deformations. The parametric study on M showed more plastic deformations for small values of M.

Thank you