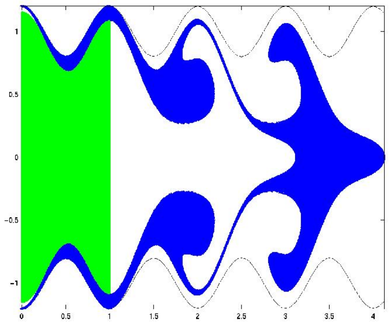
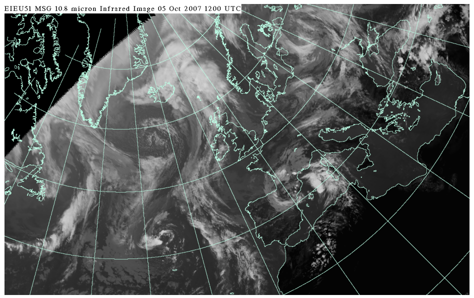
What Maths Can Learn From Industry and What Industry Can Learn From Maths



Chris Budd



Trondheim, Sept 2016

IMI

UNIVERSITY OF BATH

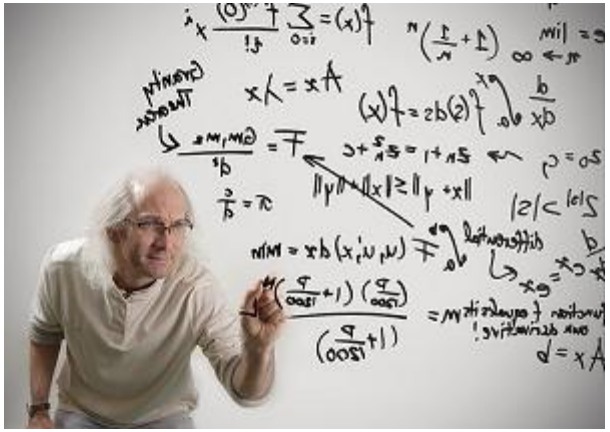
Traditional ‘school/undergraduate/general view’ of maths?

- Maths is useless

- The only jobs for mathematicians are in accountancy and teaching

- Industry is full of people with oily rags

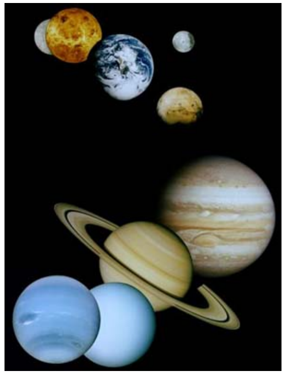
- All mathematicians are mad



My own view:

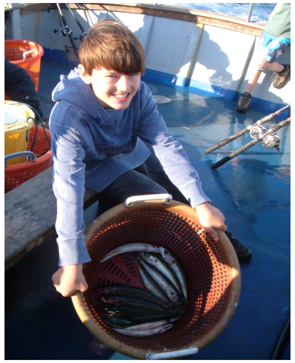
- Almost all maths can be applied to almost all problems

… And this simple fact is truly amazing!!!!



- We can learn lots of new maths from almost all applications: Fourier Series, Calculus, …

- My whole career has been involved in applying really nice maths to messy industrial problems!



Indeed ….

Much of industry has problems which can potentially be formulated, and solved using mathematics

Maths connects with all areas and knows no bounds or constraints!

Too few people recognize that the high technology so celebrated today is essentially a mathematical technology

Edward David, ex-president of Exxon R&D

Parts to the Presentation:

1. Examples of working with industry, the sorts of maths needed and what we can learn
2. How to model a problem using maths
3. Some general thoughts on the whole process
4. How to get your hands dirty

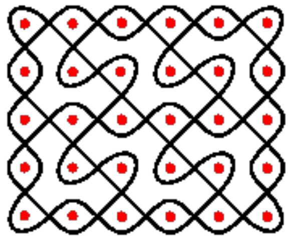
Part 1: Where do good problems come from?

Traditional industrial users of maths are

Telecommunications, aerospace, power generation, iron and steel, mining, oil, weather forecasting, security, finance

But they could equally well be …

Retail, food, zoos, sport, entertainment, graphic design, media, forensic service, hospitals, air-sea-rescue, education, transport, risk, health, biomedical, environmental agencies, art, …



All lead to great and diverse problems. Many/most of which can be tackled or illuminated using math

Good maths

↓

Hard applied problems

↓

More good maths

↓

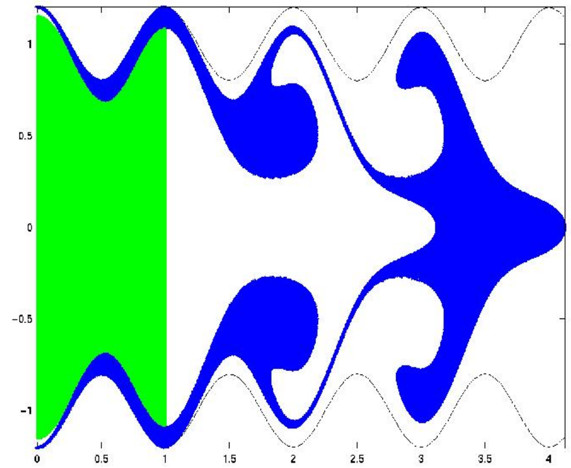
More hard applied problems

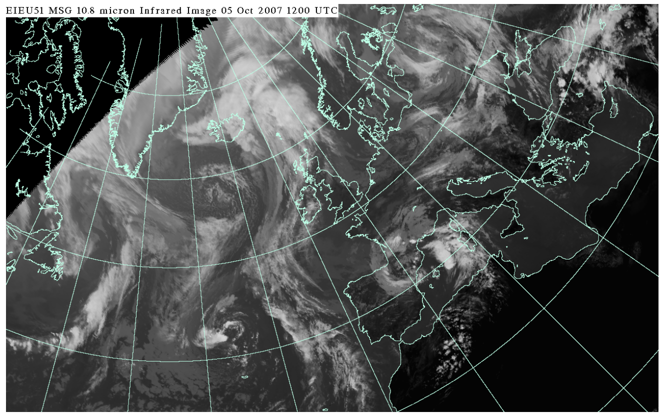
↓

Start with the maths

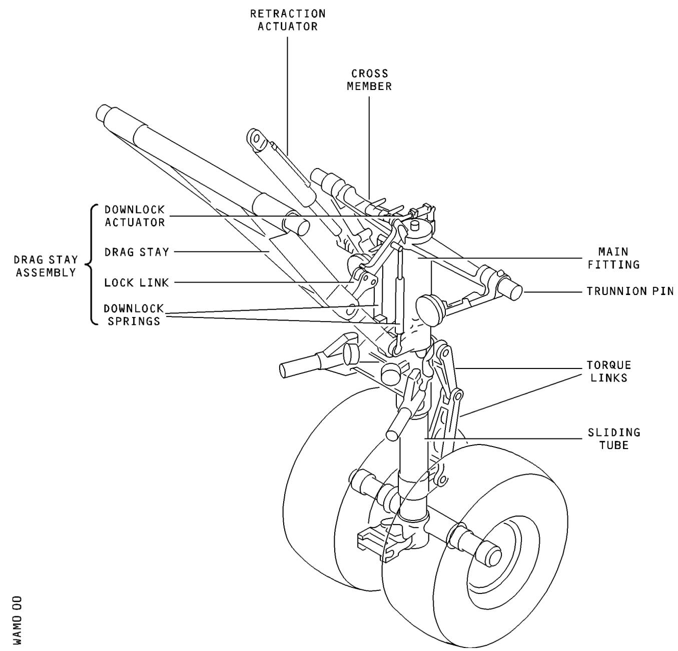
Classical continuum mechanics and statistics is very important many industrial problems eg …

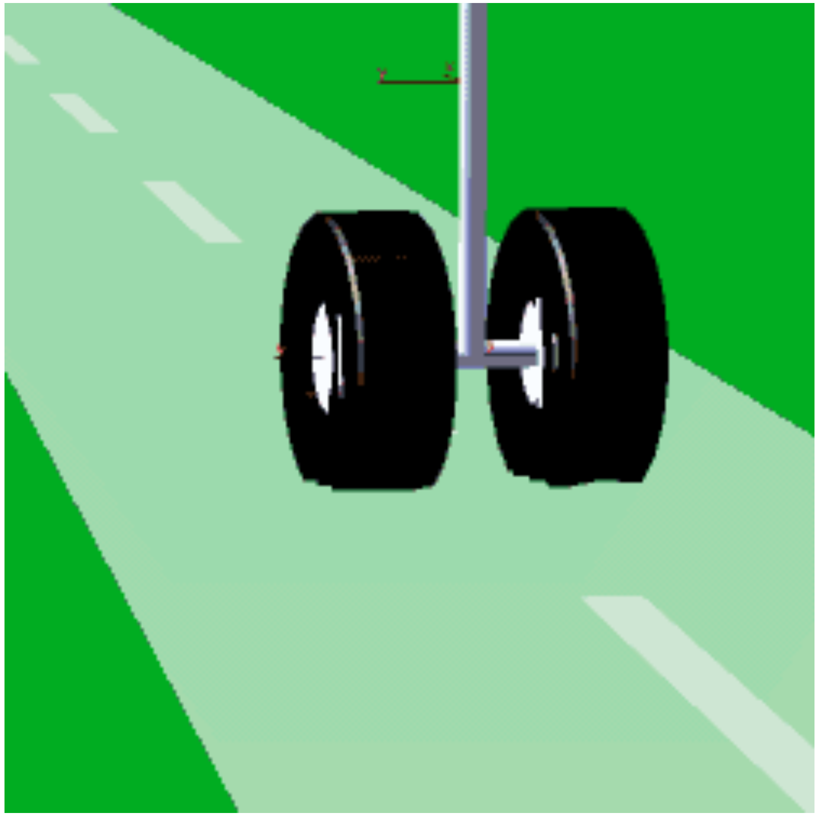
- Fluids + Solids
- Statistics and probability
- Electro-magnetism
- Reaction-diffusion problems
- Asymptotics
- Dynamical systems
- Partial differential equations
- Calculus of variations
- Numerical analysis
- Signal processing
- Inverse problems





Eg. 1: Aircraft undercarriage: Airbus



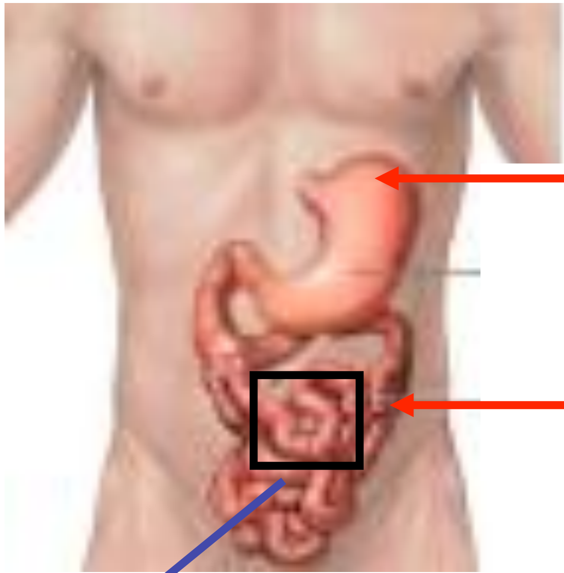


Develop and analyse a piecewise-smooth dynamical model to describe the undercarriage dynamics and the interaction of the tyres with the runway.

Piecewise smooth dynamics is a rapidly evolving area of maths stimulated by industry!

Eg. 2: The digestive system: Non-Newtonian flow in a moving geometry Unilever

Need to develop a PDE model

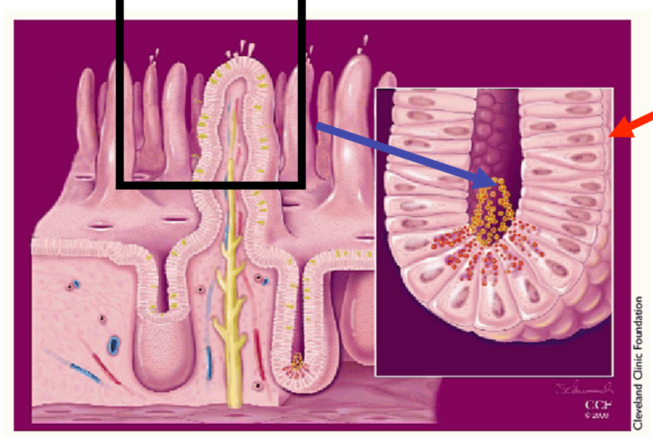


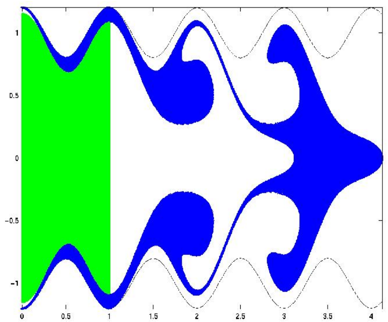
Stomach

Intestine

Intestinal wall:

Villi and Microvilli



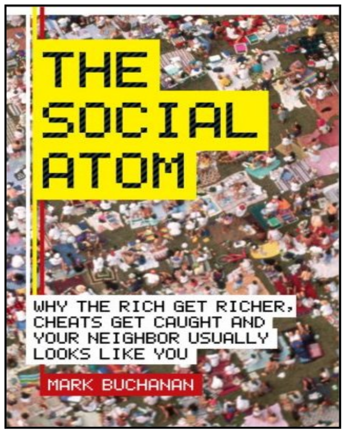


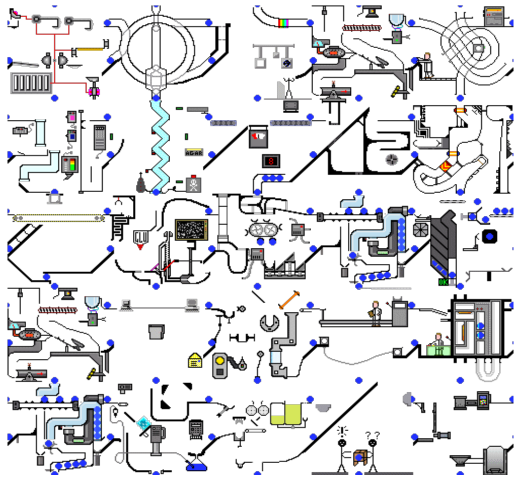
Flow patterns

[A Leger]

But much of the more recent interactions between industry and math is with a different sort of industry with different types of problem

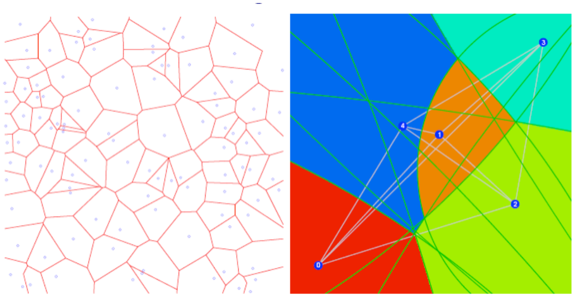
- Information/Bio-informatics/Genetics
- Digital economy
- Big Data
- Commerce/retail sector
- Complexity
- Traffic
- People based activity



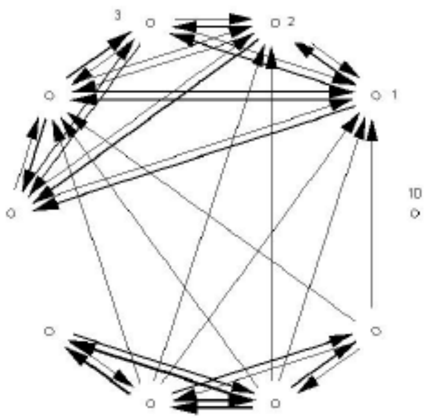


What sort of (new) maths do we also use?

- Discrete maths eg. Graph theory, Combinatorics, Colouring, Cohomology, Tropical math



- Geometry
- Topology
- Data and data assimilation
- Multi-scale methods
- Stochastic methods
- Very large scale computations
- Complex systems .. Agent based models
- Networks eg. Small-world systems
- Optimisation (discrete and continuous)



Eg 3: Crowd (and traffic) Dynamics: Home Office, Japan/New Zealand Tsunami warning

Develop and analyse social force agent based model of Helbing and others describing how crowds interact and respond to extreme events … another rapidly evolving area of research

Part 2: The mathematical modelling process

What is a mathematical model?

Simulation: Detailed mathematical description of a problem with as many effects as possible. Often millions of lines of computer code (in C++ or Fortran) taking weeks to run

Gives numbers but not much insight

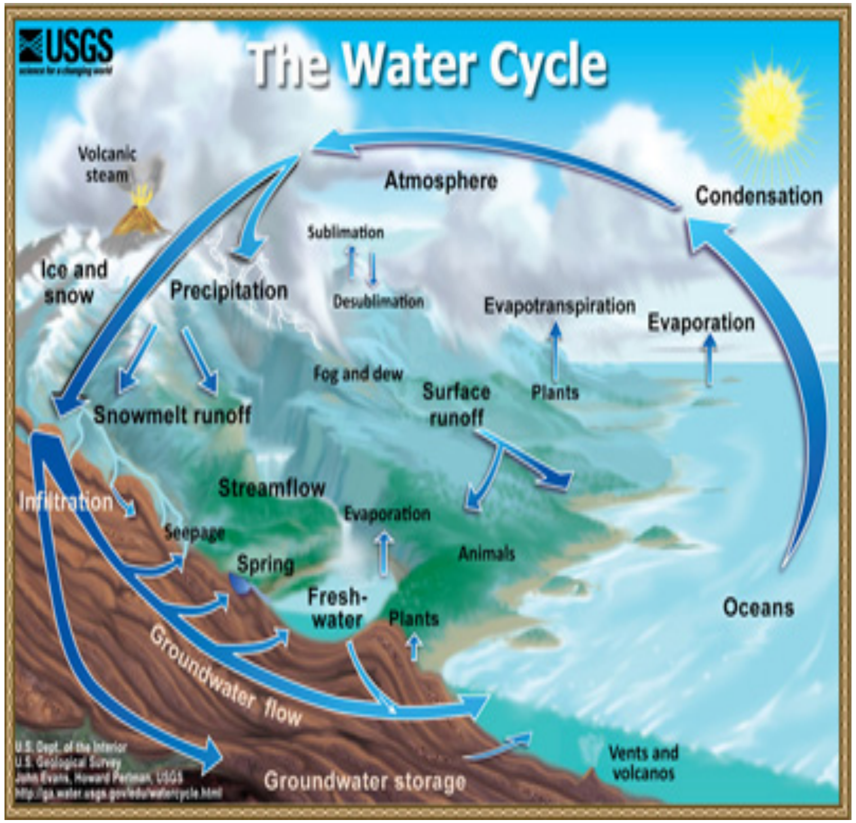
Model: Simplification containing the essential processes which can be analysed and computed on quickly (in Matlab/Python)

Gives insight and sometimes numbers

A model should be as simple as possible, and no simpler
A. Einstein

Example: Climate

Simulation:



$$\frac{Du}{Dt} + 2f \times u + \frac{1}{\rho}\nabla p + g = \nu \nabla^2 u,$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0,$$

$$C\frac{DT}{Dt} - \frac{RT}{\rho}\frac{D\rho}{Dt} = \kappa_h \nabla^2 T + S_h + LP,$$

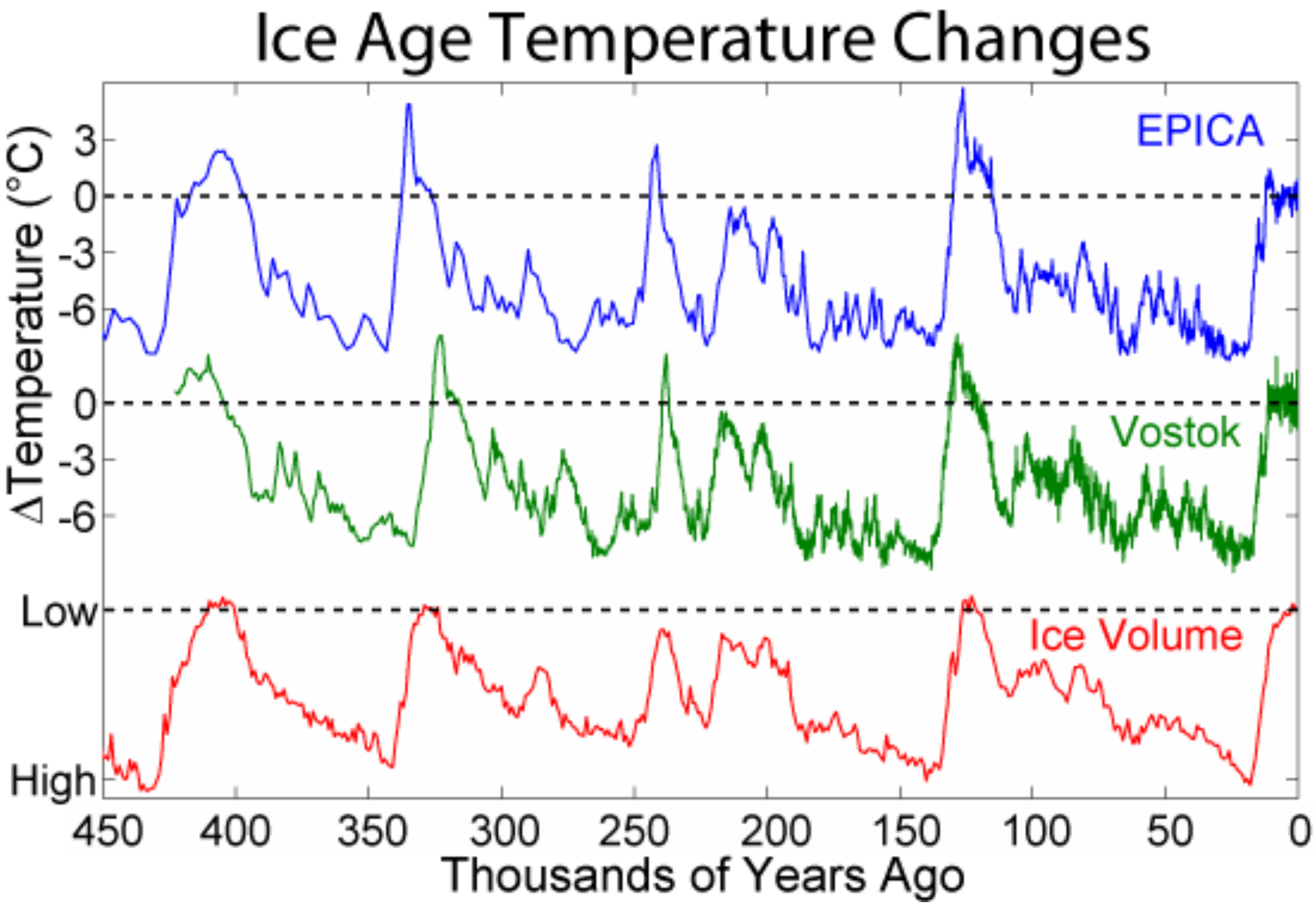
$$\frac{Dq}{Dt} = \kappa_q \nabla^2 q + S_q - P,$$

$$p = \rho RT.$$

For climate add in ice, CO2, ocean currents, vegetation, volcanos, solar variation, ….

Very complex. Hard to solve, hard to test, hard to interpret!

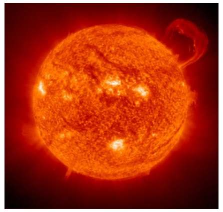
No real use in helping us to understand past climate



Looks regular .. Can a model predict this?

A simple model

Heat absorbed → $(1-a)S$



a Albedo: How well the earth reflects the Sun's rays

Heat radiated away → $e\sigma T^4$

e emissivity: How much energy is radiated into space

Balance these

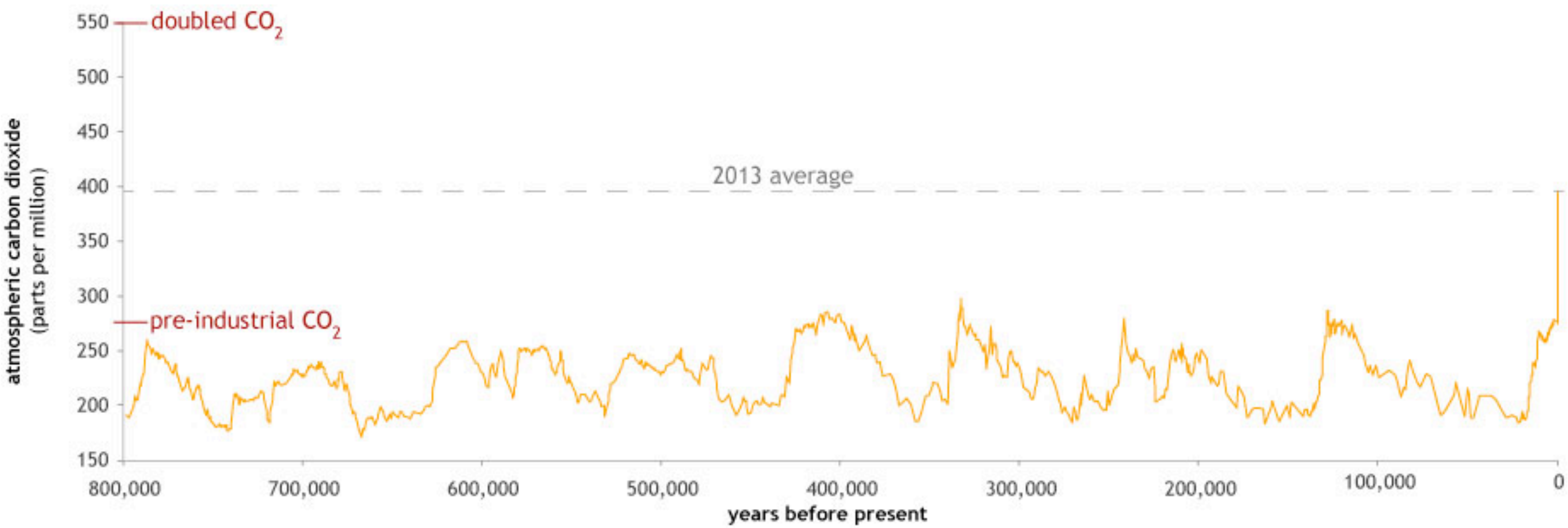
$$e\sigma T^4 = (1-a)S$$

Solve to find the temperature T

a depends upon Ice cover

e depends on Carbon Dioxide

Level of Carbon Dioxide (ppm)	Emissivity e_{CO2}
200	0.194
400	0.14
600	0.108
800	0.085

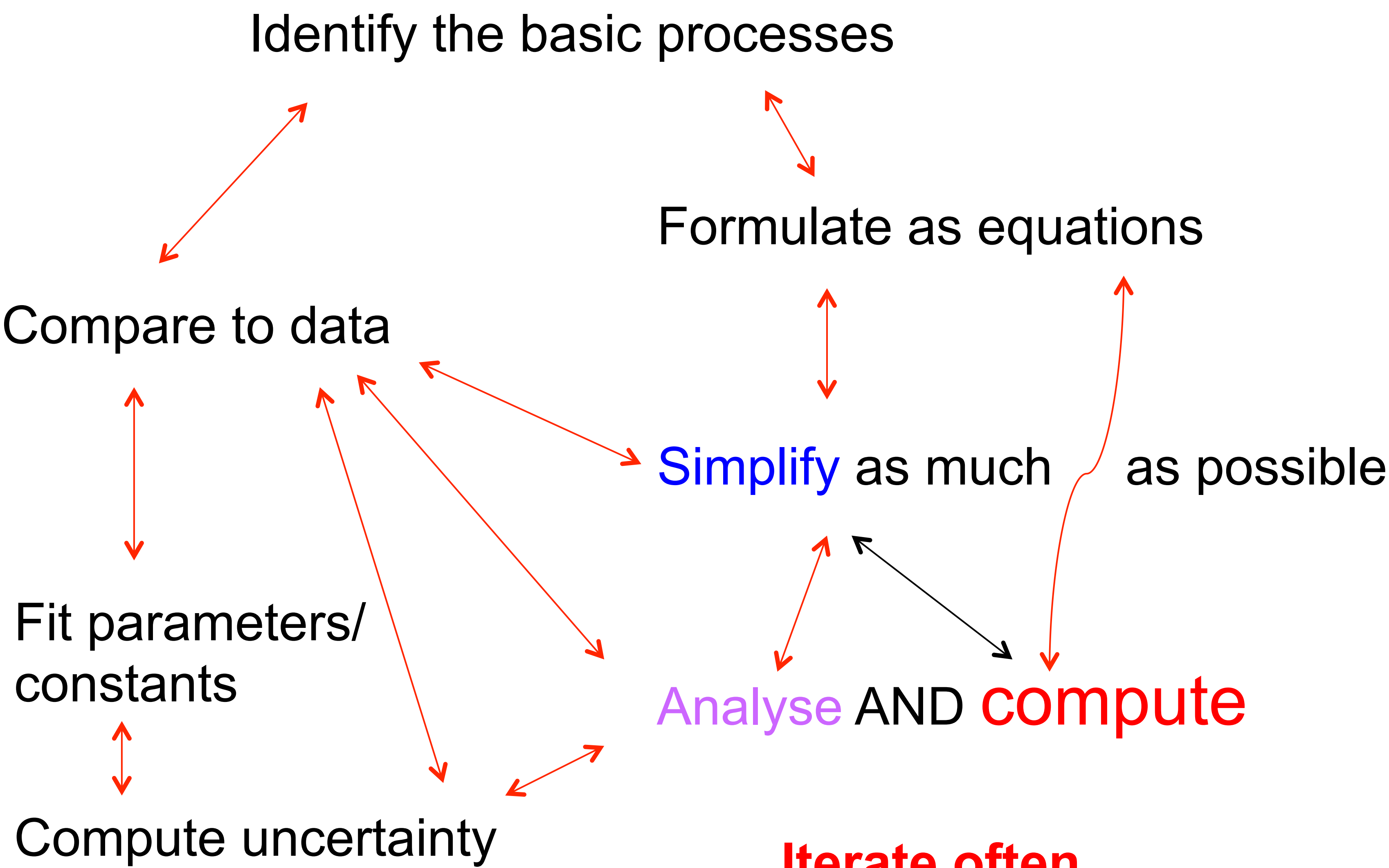


Model helps predict global warming but not much else

Need a hierarchy of models to do the job!

How do we create and develop a mathematical model?

My own experience!



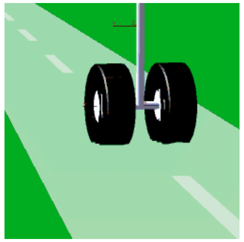
Iterate often

Then …..

- Extend the model if needed .. Have you REALLY considered all of the issues?
- Do lots of what if experiments
- Discover things you never knew before (Neptune)
- Optimise your systems
- Change the world! (eg. Newton, Maxwell, Google, ..)
- BUT always take a good hard look at what you have done and

DON'T Eat the menu (often done!!)

Physical/Engineering/??biology problems



- Basic processes often quite well understood (ODEs/PDEs)

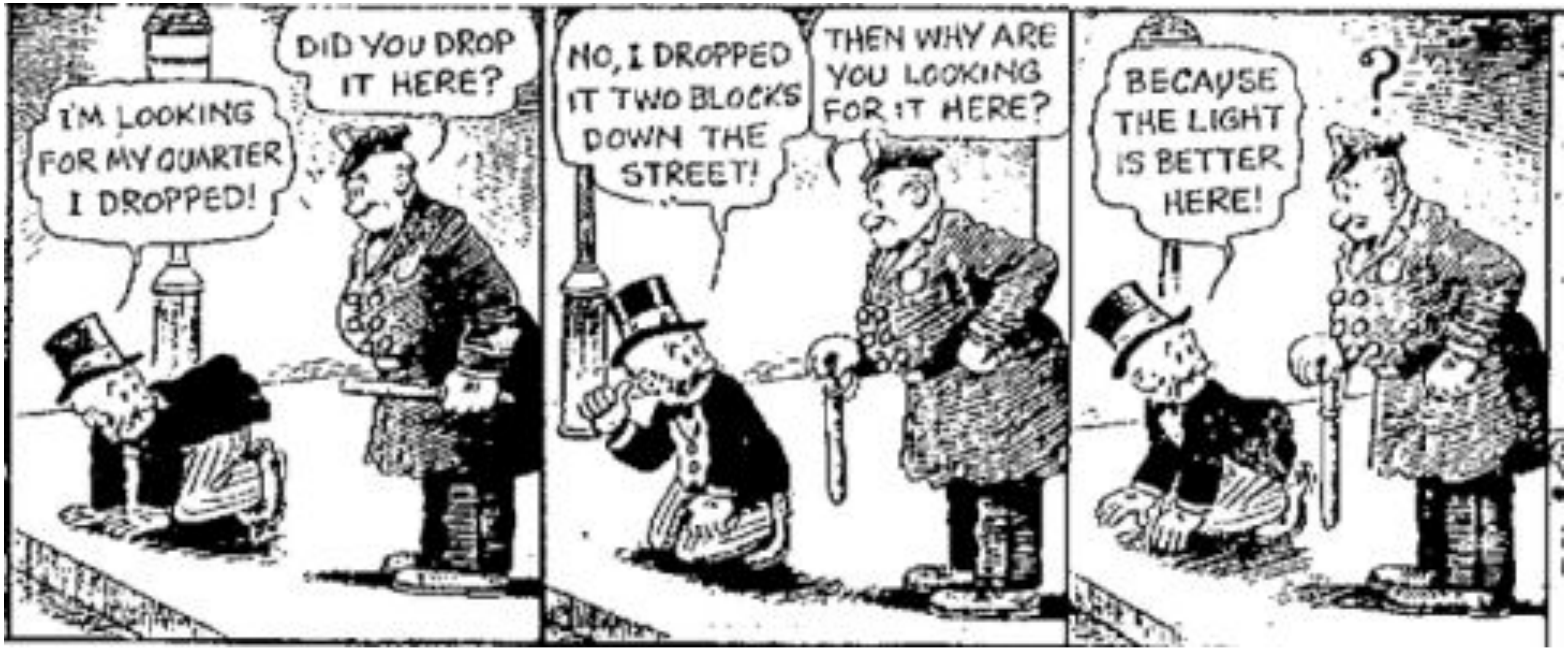
- Rescale/non-dimensionalise equations to identify small parameters to identify important processes

- Analyse these processes where possible:

  perturbation methods, homogenisation, dimension reduction and exact solutions can be very helpful

- Never be afraid to compute as early as possible!

But ….try not to be a mathematical drunkard



Social science problems (including finance and much of industry)

- Common and important
- MUCH less well understood
- Have to be creative in identifying the key processes involved
- Don't aim for too much sophistication
- Get as much data as possible
- Always include stochastic effects
- COMPUTE

Two big guiding principles

1. Make sure that you are solving the right problem (this may not be quite the problem originally asked)

2. Make sure that you solve the problem right

> **We thought that we had the answers, it was the questions we had wrong.** - Bono

Industrial Case Study: Microwave cooking

How safe is microwave cooking?

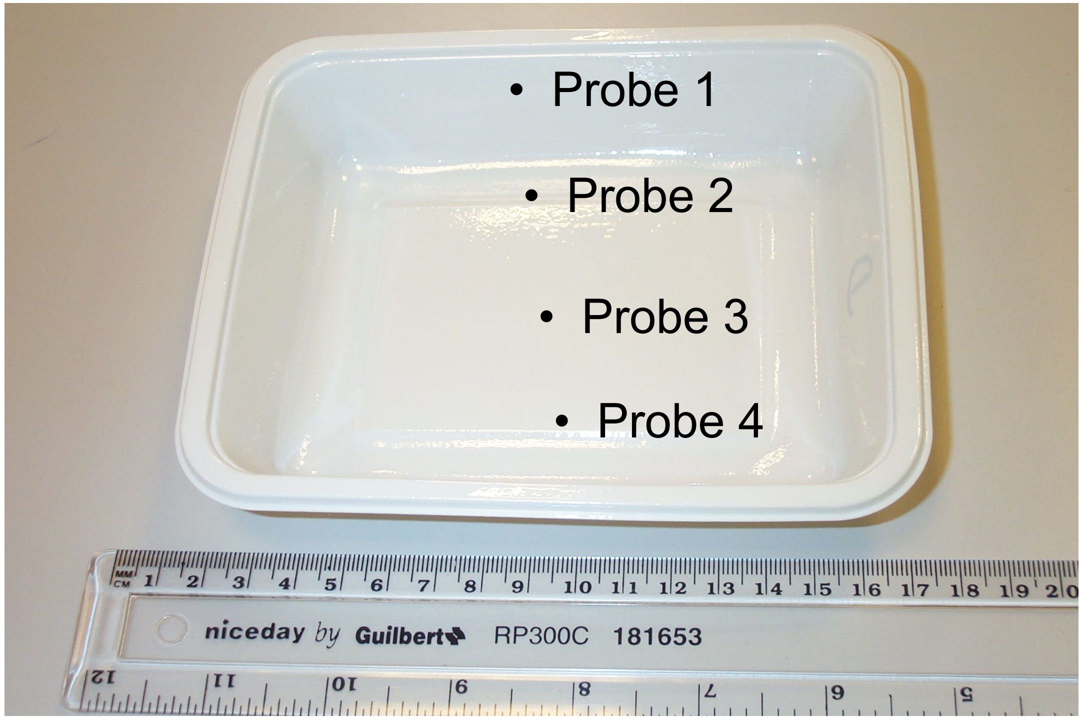
If you cook a potato in a microwave cooker what gets hotter?

- The outside
- The middle
- Somewhere else?

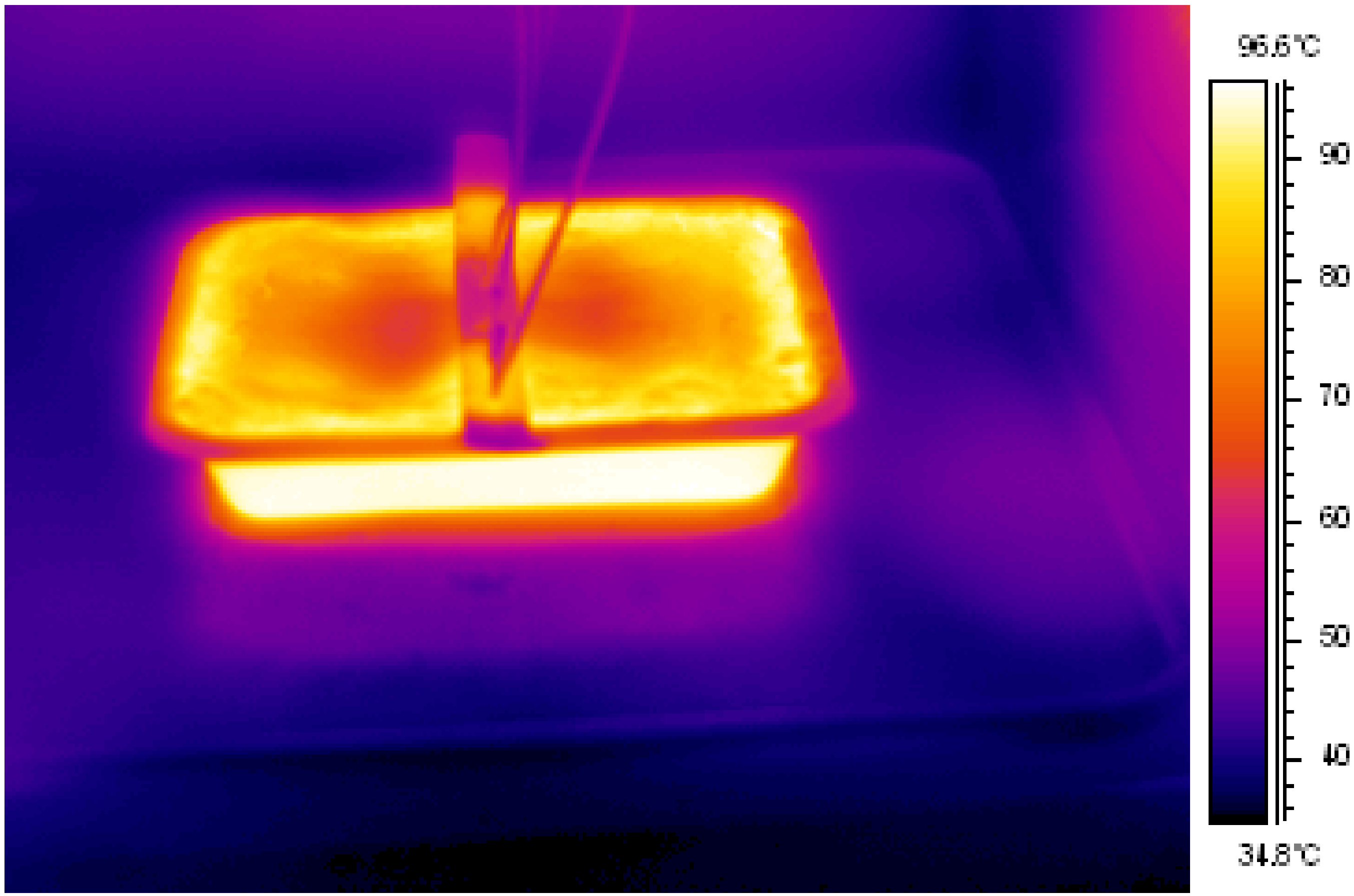
What is the best design of a microwave cooker?

- Mode stirred
- Turntable

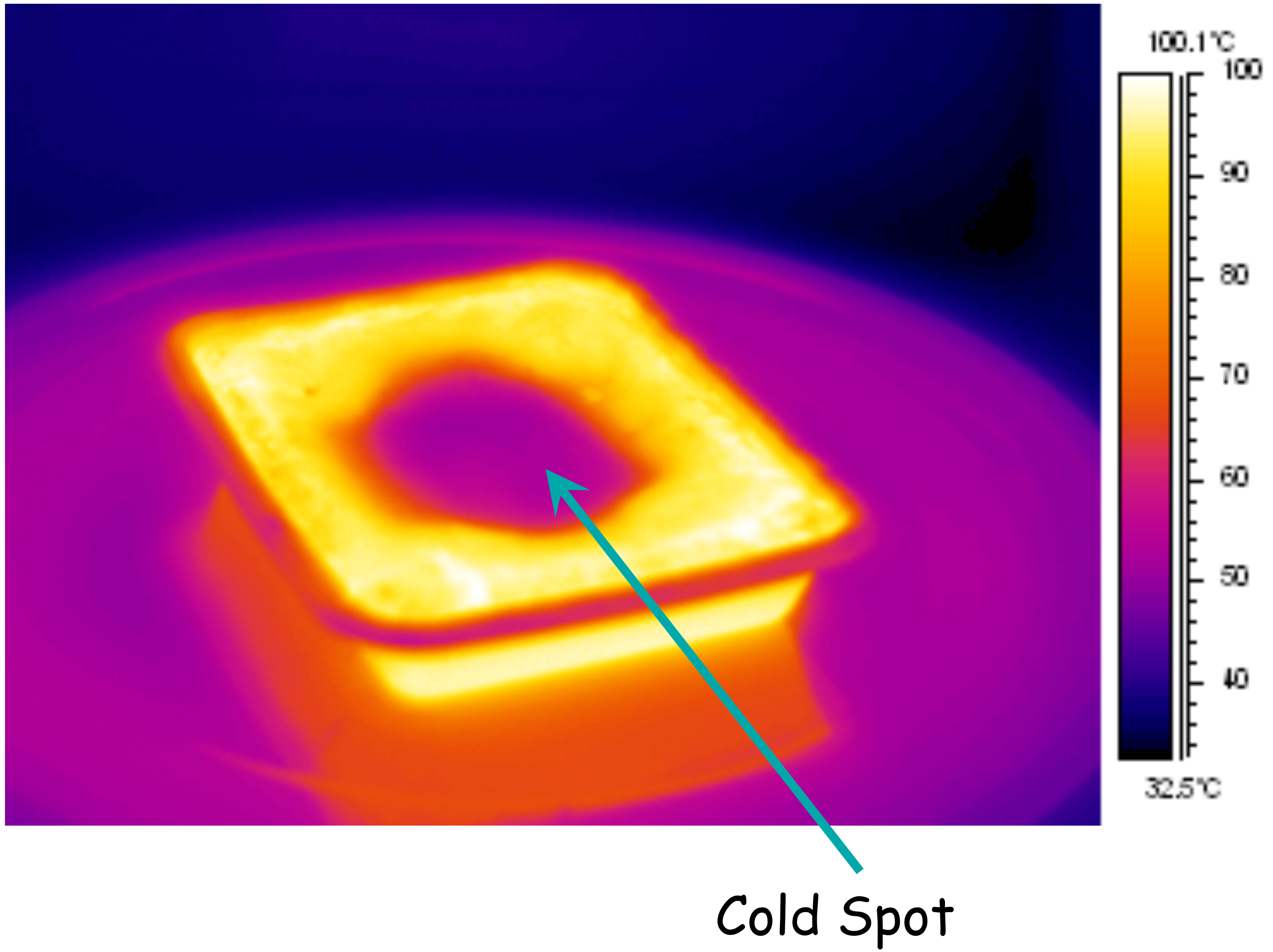
1. Gather data



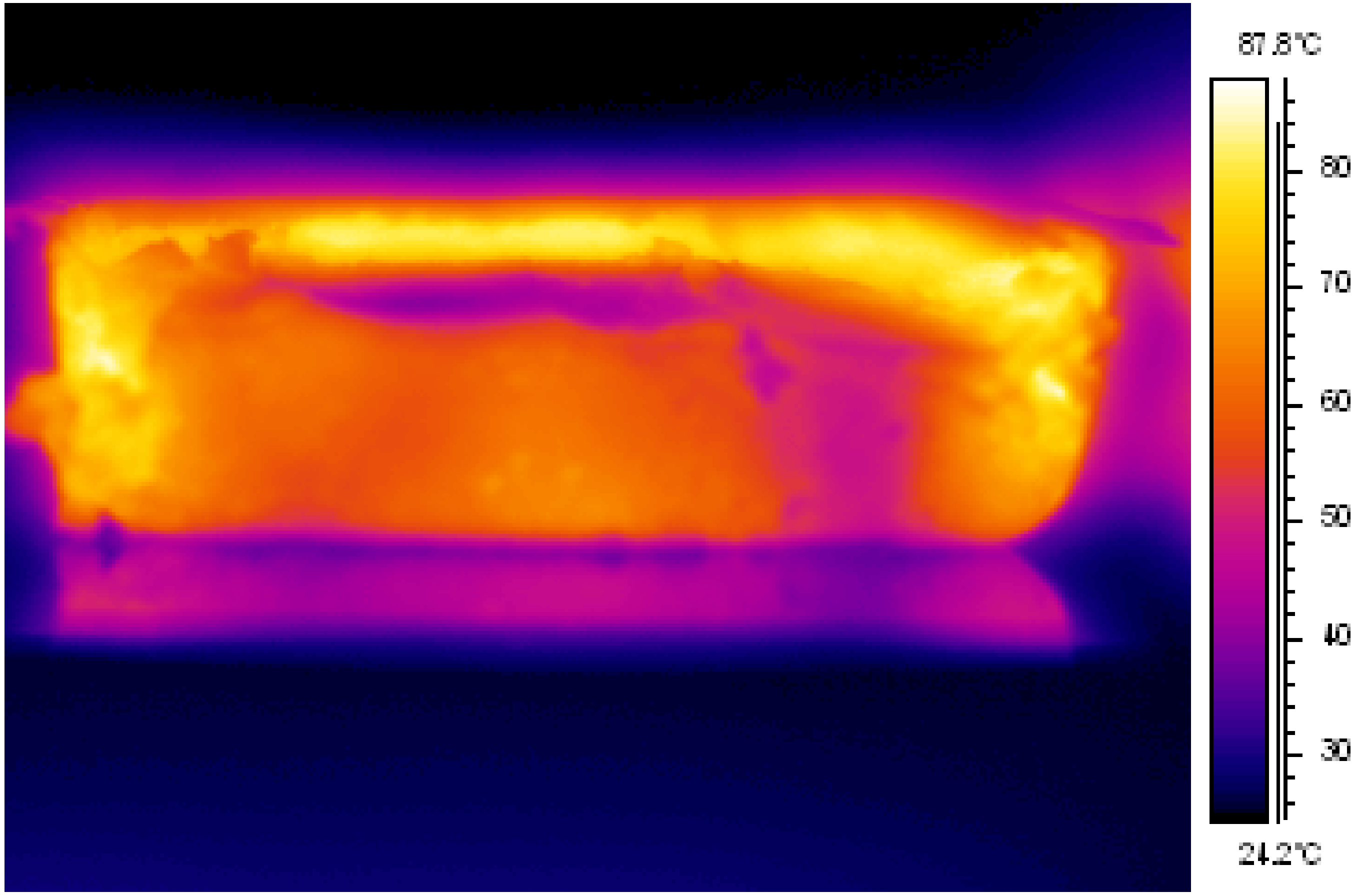
Thermal image of the food surface after 5 minutes heating (stirred oven)



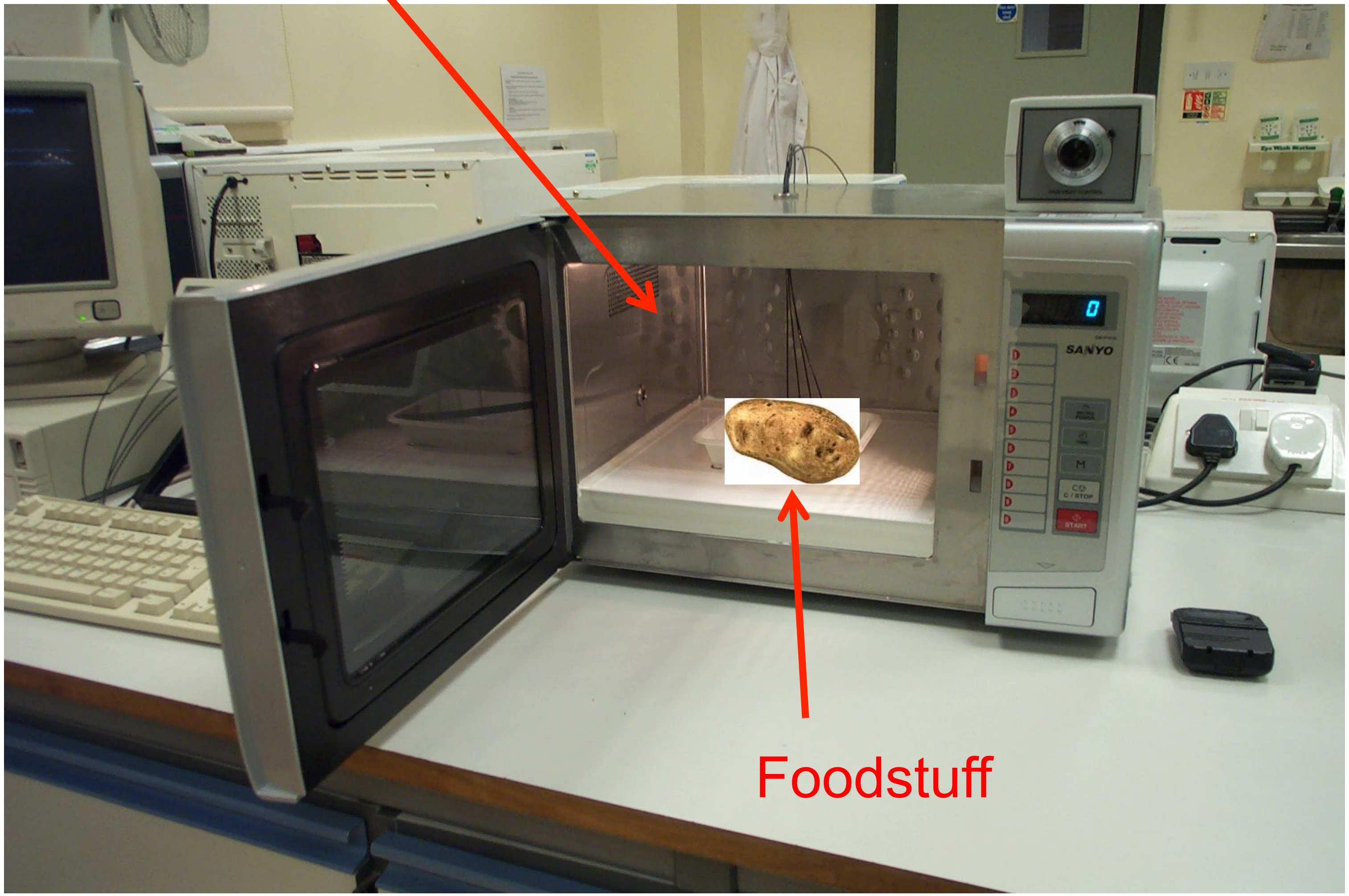
Turntable oven, thermal image taken after 5 minutes heating



Thermal image of cross section after 3 minutes heating

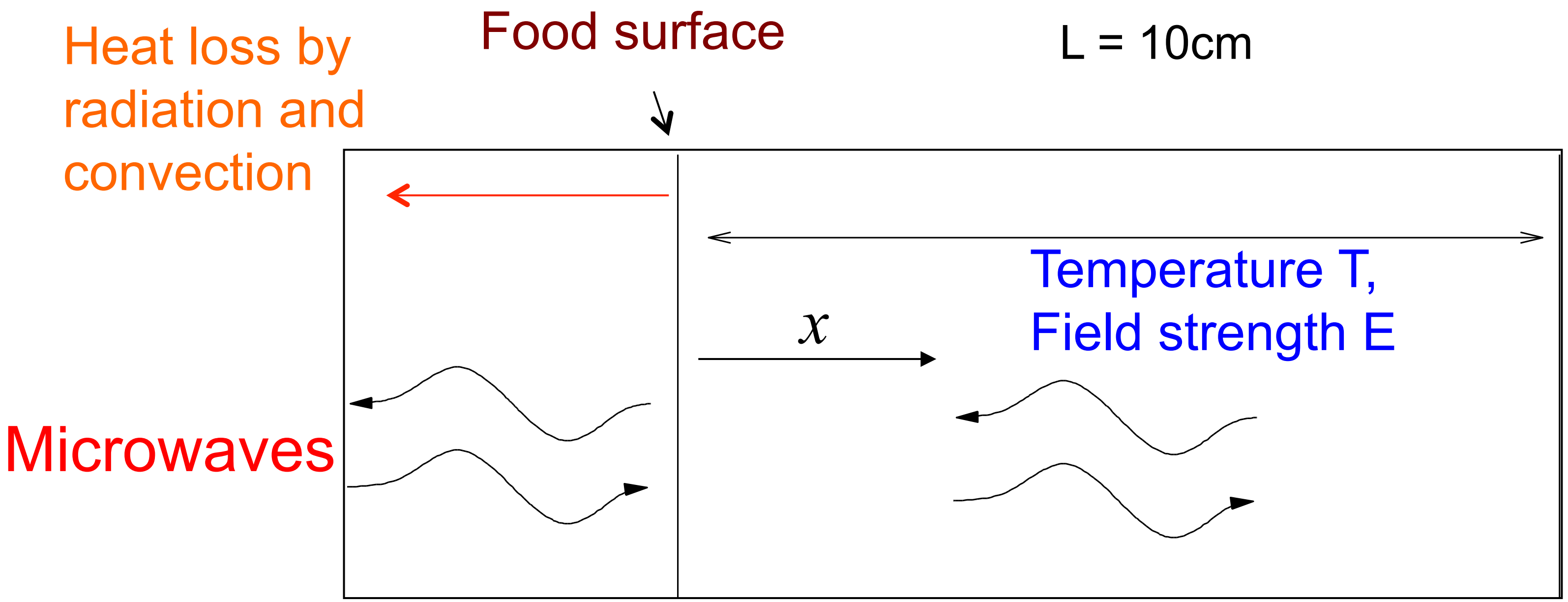


2. Identify geometry and quantities of interest



Close to foodstuff

Starchy food containing moisture



L: Length: 2-14cm

d: Penetration depth: 8mm

Microwave Wavelength 1-2cm

Only really interested in what is happening in the food. Microwaves absorbed and heat food

3. Identify the basic physics

1. Maxwells equations in the cavity

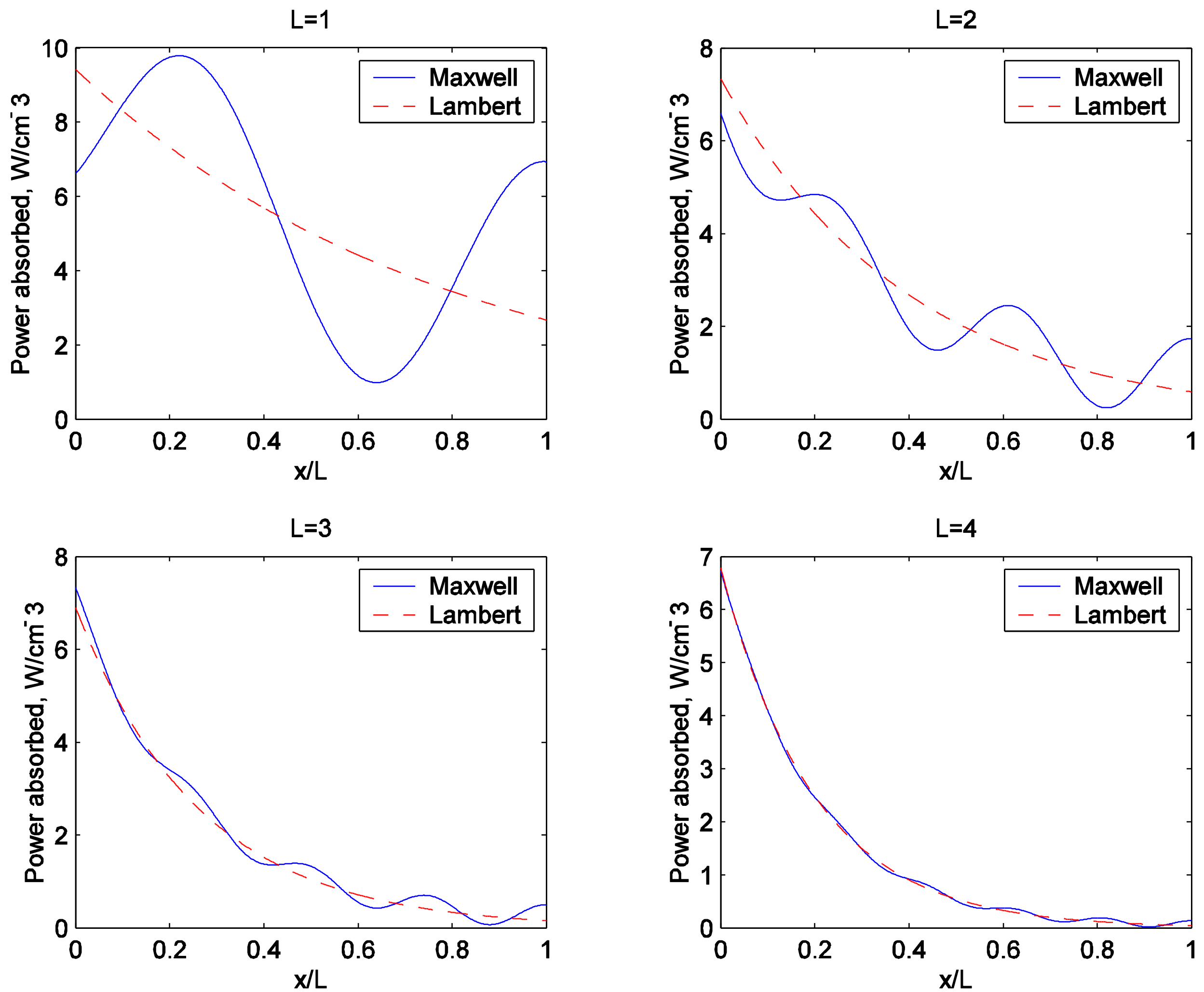
2. Maxwells equations in the food

3. Food acts as a two phase porous medium

4. Surface of the food allows microwaves to penetrate, reflect and reradiate

5. Inside the food, microwaves cause power dissipation (by friction) and heat the food

4. Do a first computation of the Electric Field (Maxwell)



Microwaves decrease in strength as they penetrate the food and appear to take on a simple exponential form as the penetrate .. Vital clue!!

Guided by the clue, do some analysis to simplify the problem:

1. Field penetrates the foodstuff and is a function only of the depth. Locally Maxwell reduces to the Helmholtz equation:

$$E_{xx} + \lambda^2 E = 0, \qquad \lambda = \alpha + i\beta$$

2. From basic physics

$$\beta = \omega\sqrt{\mu\epsilon_0}\sqrt{\frac{\kappa'(\sqrt{1+\tan^2(\delta)} - 1}{2}}, \qquad \tan(\delta) = \frac{\kappa''}{\kappa'}$$

$\omega, \kappa', \kappa'', \mu, \epsilon_0$ All known quantities

2. Calculate $\beta = 60m^{-1}$

$$2\beta = 120m^{-1} = 1/(8mm) \equiv 1/d$$

3. Estimate field

$$|E|^2 = E_0^2 \, e^{-2\beta x}$$

Plus other reflected terms from the boundaries

4. Non-dimensionalise. Length scale L = 10cm

y = x/L $$|E|^2 = E_0^2 \, e^{-12y}$$

Rapid exponential decay. Other terms (due to reflections) decay too rapidly to matter

Other useful simplifications

- External field in a stirred oven is uniform
- External field in a turntable oven is a standing wave
- Internal field is close to one-dimensional
- Starch is a ‘leaky’ two phase material
- Electrical constants do not depend on temperature and depend only weakly on moisture content
- Cooking times are short compared to conduction

The resulting model.

Let N = Enthalpy of the food (thermal energy plus latent heat)

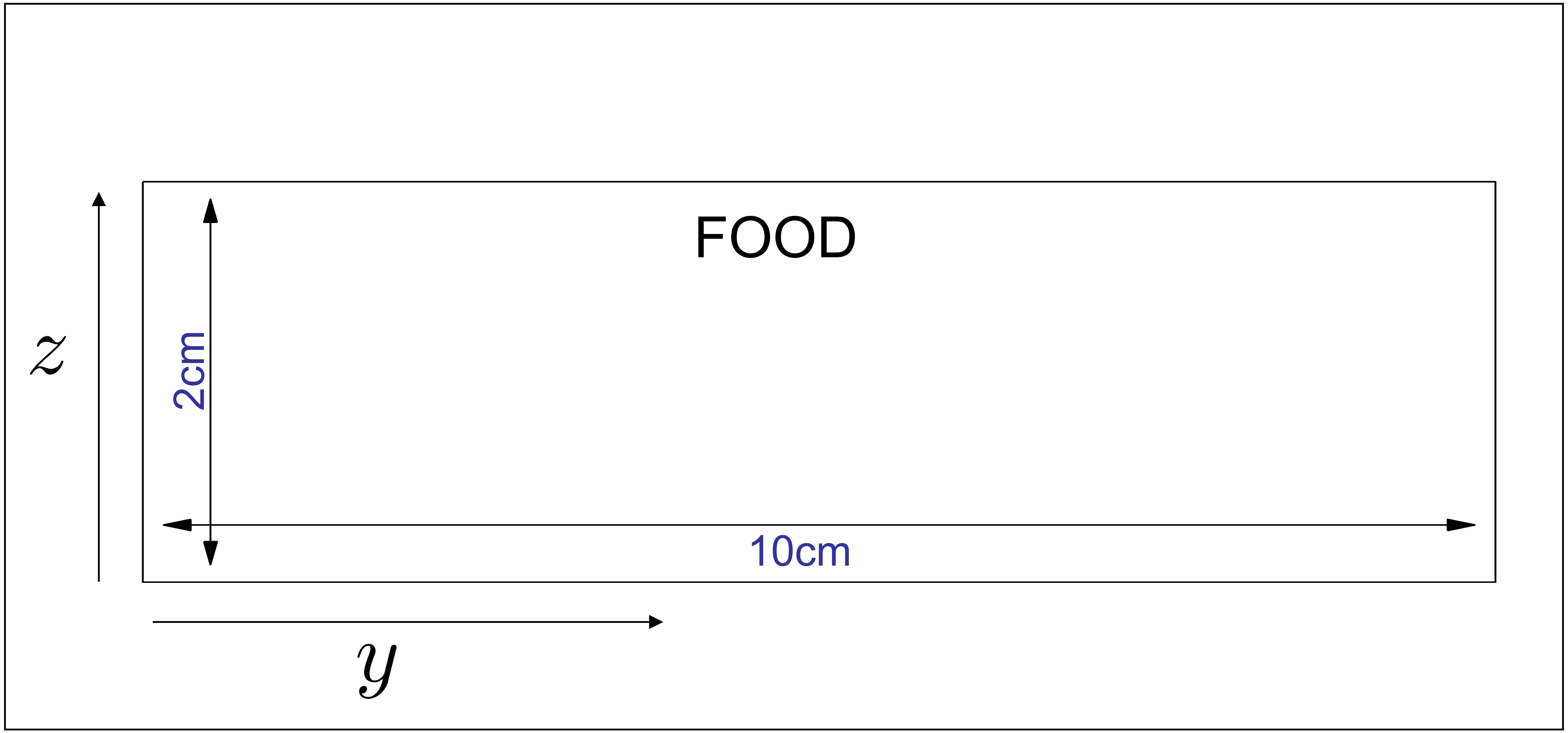
$$N_t = k\nabla^2 T + P, \qquad P = \frac{1}{2}\omega\epsilon_0\kappa''|E|^2$$

$$|E|^2 = E_0^2 \, e^{-12y}$$

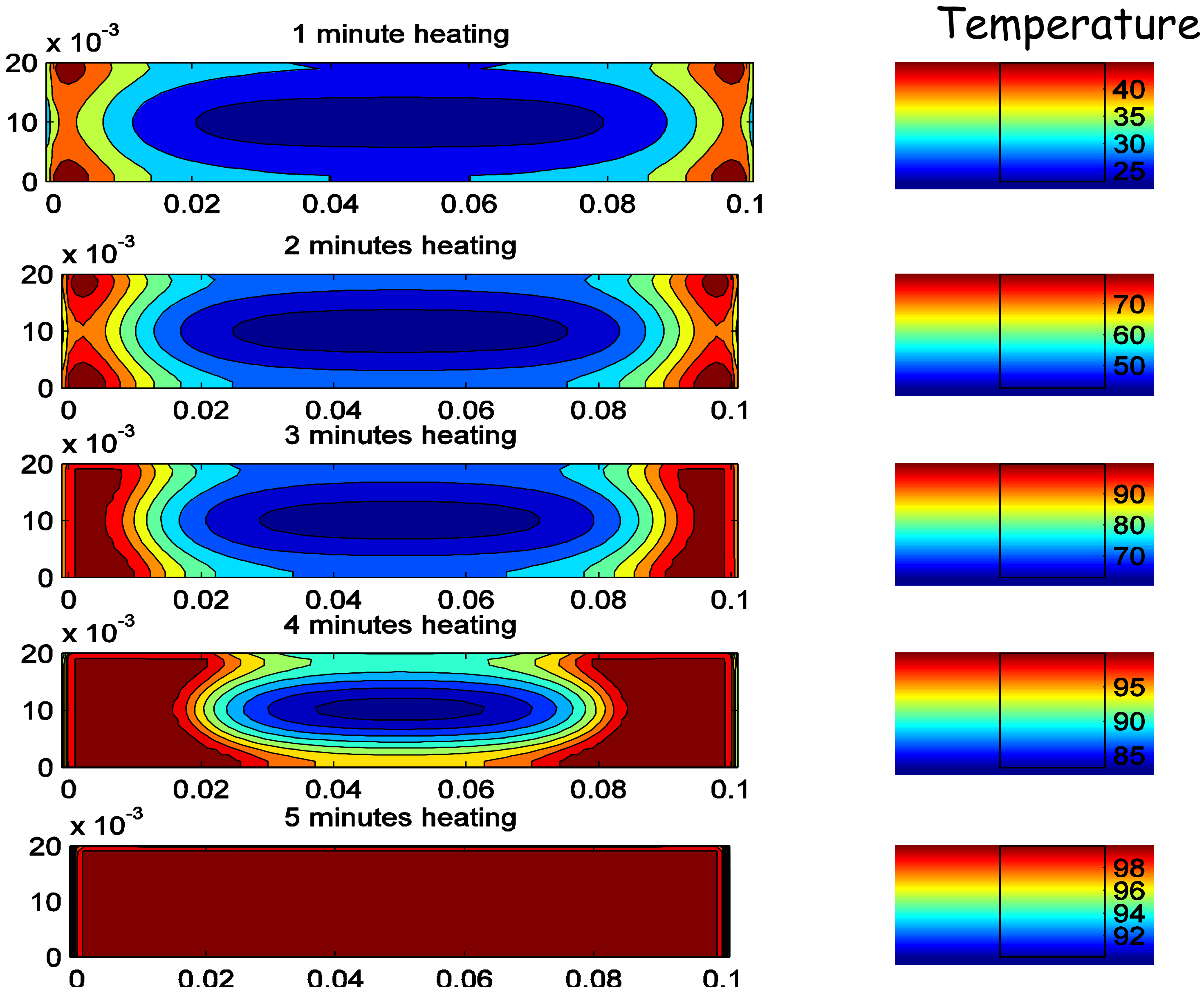
$$kT_y = h\left(T - T_a\right) + \gamma\sigma T^4$$ On the boundary

E_0 Uniform (stirrer), sine-wave (turntable)

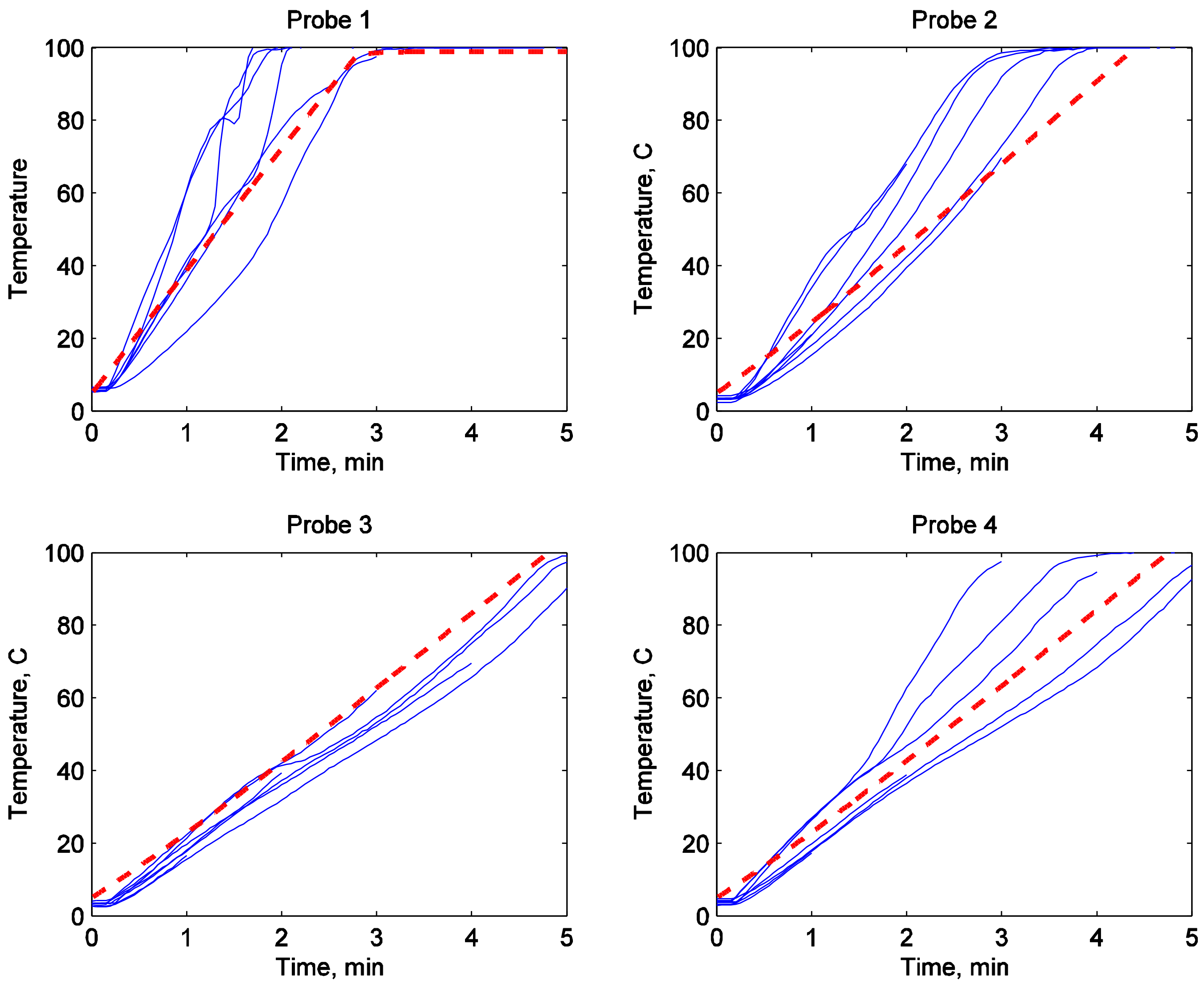
Compute in the following geometry to find the temperature T



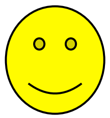
650W Oven, Mode stirrer



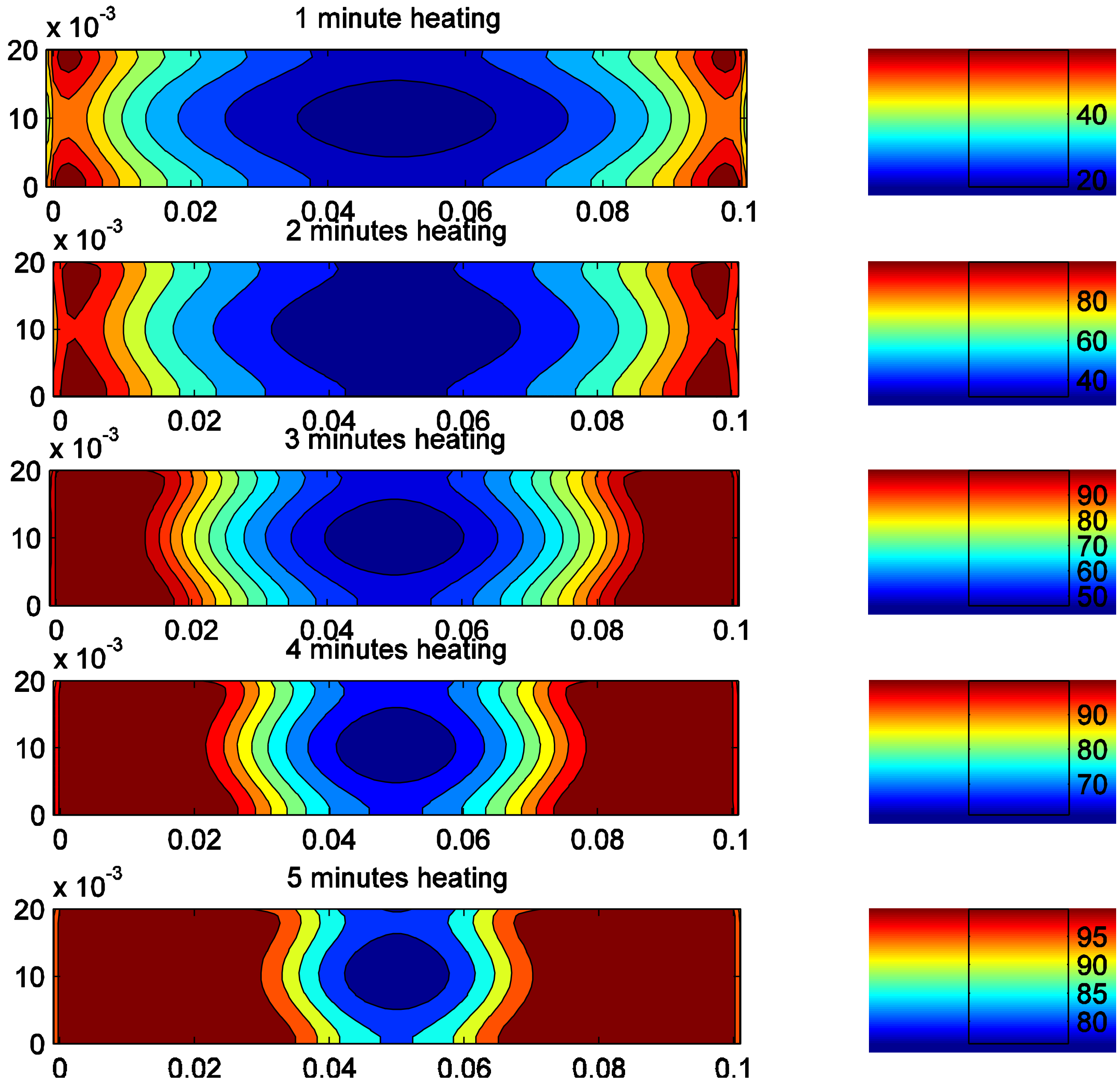
Compare to data : 650W Point Temperatures



Very good agreement



750W turntable oven



Conclusions:

- Simple model works well both qualitatively and quantitatively
- Takes seconds to run. Commercial codes take days
- Now extending (with H Huang) as a model for purification of Silicon Wafers for solar cells

Part 3. The process/mind set of working with industry

- Be prepared to work outside of your comfort zone
- Work as part of a team
- Manage expectations
- Phone a friend
- Industrial maths knows no limits. Use all the math that you know and make more up
- Have clear objectives
- Answer the question set
- … but only once you are clear of the question!
- Put in the hours

Mechanisms to make this interaction work

There are certainly problems in working with industry!

What industry wants

- Short term solutions
- Confidentiality
- Money

What universities want

- Long term and deep research
- Open publication
- Training of young people

Seem irreconcilable .. But there are ways to make it work!

European Study groups: ESGI/ECMI

Study Group Model (in use all over the world)

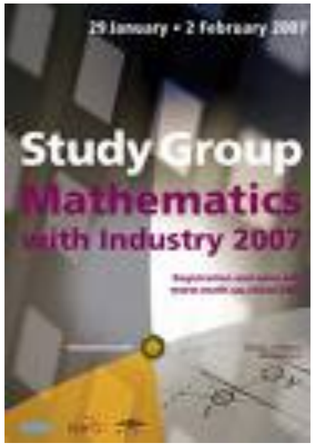
- Bring academics and industrialists together
- Pose industrial problems on the first day
- Work on the problems for a week in teams
- Present a paper at the end
- Follow up with longer term projects
- Including PhD and MSc projects

Major success .. Lots of new maths!!!

A wonderful way to

- Make new contacts
- Find really good (long term) research problems
- Train students and staff
- Get great examples for undergraduate teaching

ESGI, ECMI, MITACS, PIMS, Australia …



Some resources