

Symbolic Dynamics

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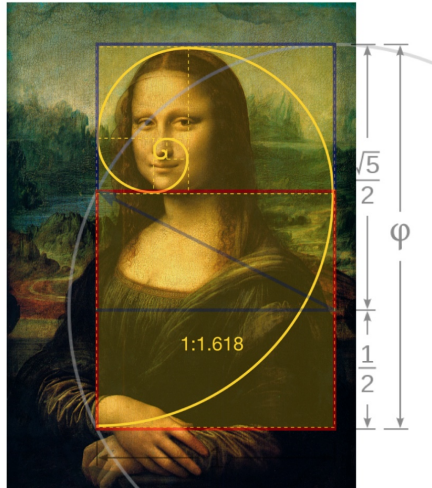
Math Slam!

September 22, 2016



NTNU

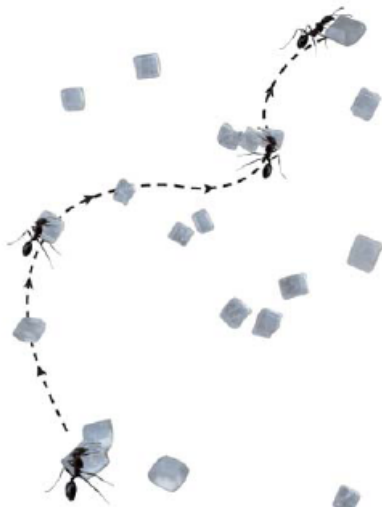
Mathematics is fun and beautiful!



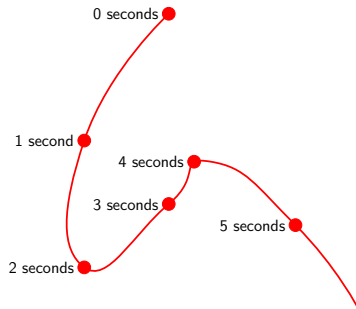
Dynamical Systems



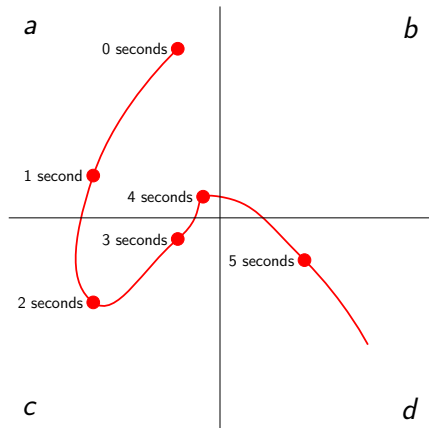
Dynamical Systems



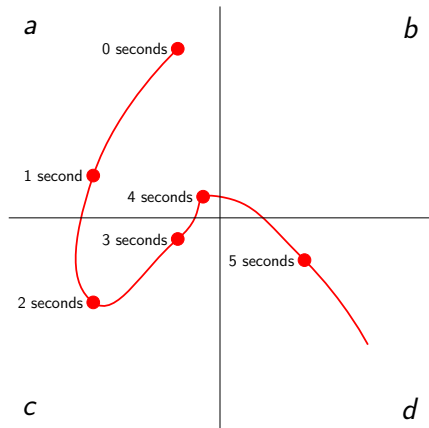
Discretization



Discretization

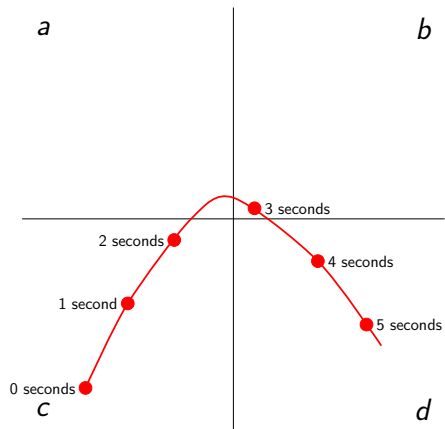


Discretization



- Sequence: *aaccad* ...

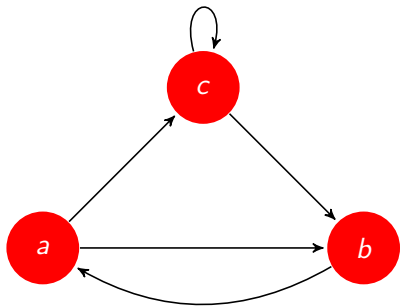
Discretization



- Sequence: *cccbdd* ...

Graphs & Matrices

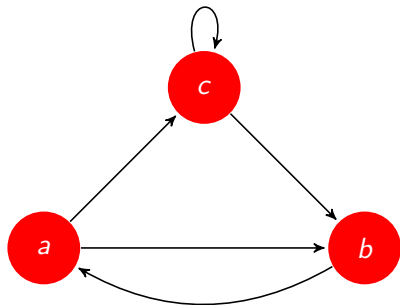
State transitions:



- Example path: *abaccb...*

Graphs & Matrices

State transitions:

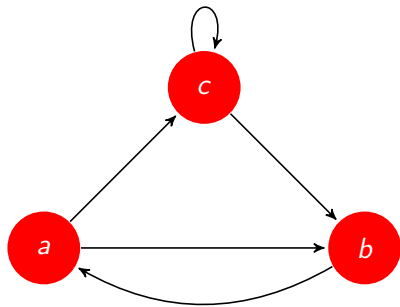


- Quantum Mechanics
- Position of an electron
- Markov Chain
- CD storage (0,1)

- Example path: *abaccb...*

Graphs & Matrices

State transitions:

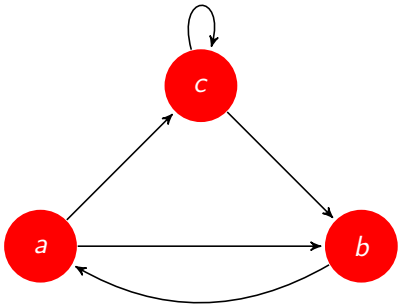


- Example path: *abaccb...*

Adjacency matrix:

$$A = \begin{matrix} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

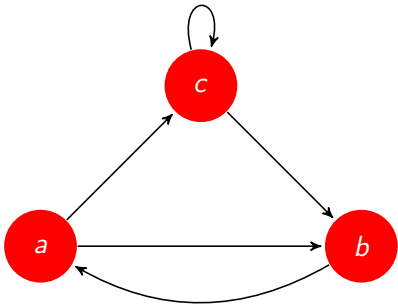
Invariants



From $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ we can compute:

- Entropy: $0.694241\dots (= \log_2 \left(\begin{array}{c} \text{Mona Lisa} \\ \text{1.198} \\ \text{1.078} \end{array} \right))$

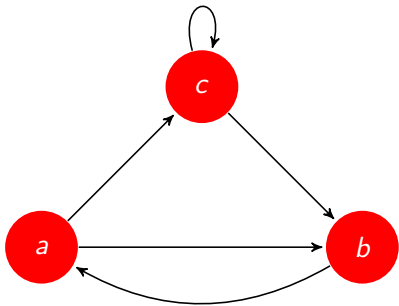
Invariants



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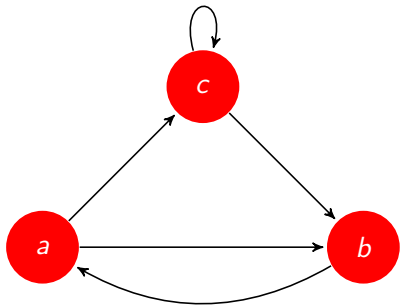
Invariants



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- Determinant Sign: $+1$

Invariants

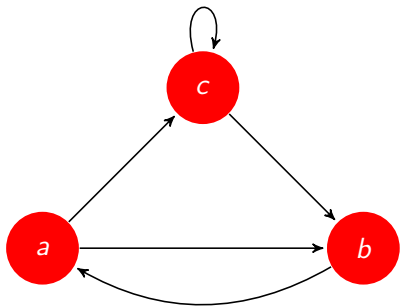


Analytic Objects:

- Zeta function:

$$\zeta(s) = \frac{1}{-s^3 - 2s^2 + 1}$$

Invariants



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- Zeta function:

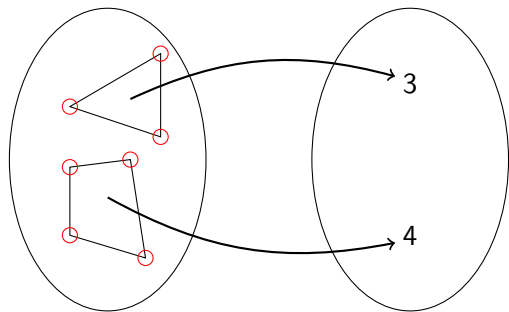
$$\zeta(s) = \frac{1}{-s^3 - 2s^2 + 1}$$

- C^* -algebra: \mathcal{O}_2

The philosophy of invariants

Convex Polygons

Integers

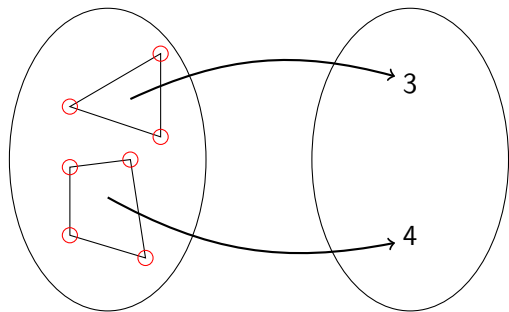


Different!

The philosophy of invariants

Convex Polygons

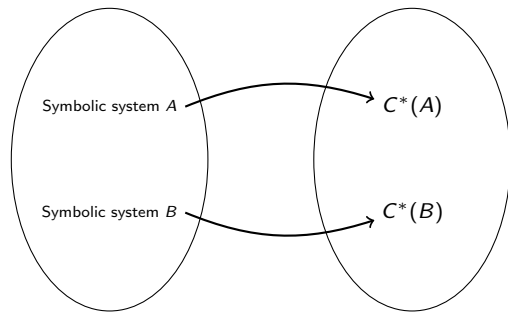
Integers



Different!

Symbolic Systems

C^* -algebras



Different???

Thank you!