Fluid Dependence of Rock Compressibility, post Biot-Gassmann

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Introduction

At seismic frequencies and below, the standard theory for understanding the fluid dependence of seismic velocities is that due originally to Biot (1941) and Gassmann (1951). However, despite its near-universal acceptance for over 65 years, its experimental support is extremely thin. Further, Brown and Korringa (1975) concluded theoretically that this B-G theory is not valid in the case of rocks with poly-mineralic solid grains, but requires an additional parameter (denoted K_M), a property of the rock.

It is also known (*e.g*. Thomsen, 1985) that the classic B-G result is not consistent with effective medium theory for rocks containing cracks. Although the B-G derivation does not make any explicit assumptions about pore geometry, such an assumption is in fact implicit within their derivation. The inconsistency with effective medium theory is resolved by B&K; with the additional parameter K_M empirically accounting for heterogeneity in both mineralogy and pore-space geometry.

Experimental support for B-G theory

It is sobering to reach the conclusion that a theory that has been well accepted for over 65 years is, in fact, not quite correct. In such a case, one should immediately re-examine the experimental support for that well-accepted theoretical prediction. However, most experimental tests of the B-G theory are flawed by a frequency issue: the theory is explicitly restricted to low frequency, but the experiments are usually of high-frequency (*c.f., e.g.* Thomsen (1985), Wang (2000)), and are affected by high-frequency "fluid squirt". Hence, the commonly-observed failures of the B-G theoretical prediction have been attributed to this frequency effect, irrelevant at seismic frequencies.

Hart and Wang (**Geophysics**, 2010) performed undrained, drained, and unjacketed compression experiments (zero frequency) on samples of Berea sandstone, as functions of both external pressure p and fluid pressure p_{Fld}. In Fig. 1, the data of H&W (black squares) are used to test the predictions of B-G (green diamonds). The B-G predictions are biased severely low at low differential pressures, but are biased high at high differential pressures. The crossover point corresponds to a depth of about 1400' (assuming normal pressures), so that the important discrepancy, for geophysicists, is at the high-pressure end of this dataset. (An implication is that the low bias observed at all pressures, in most ultrasonic data, is a result of the frequency issue mentioned above.)

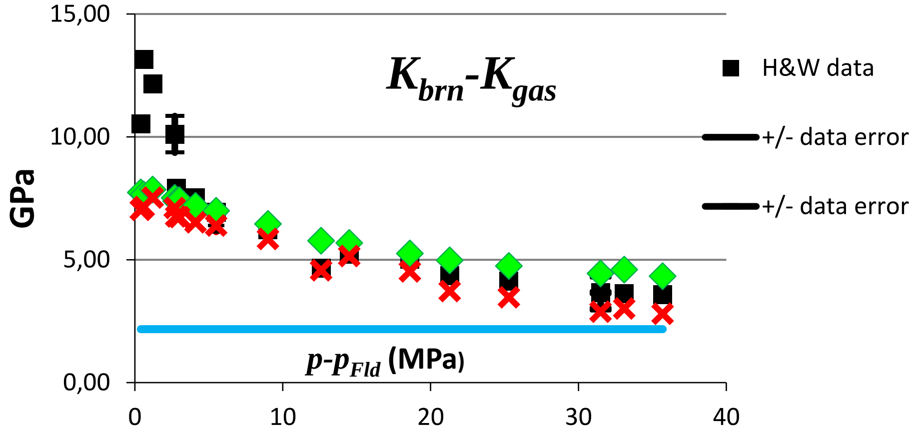

Fig. 1

Brown and Korringa refinement

Over 30 years ago, B&K showed that B-G theory contains an approximation; without that approximation the dependence of rock incompressibility on fluid incompressibility is given by:

$$K_{fld}(K_{Fld}) - K_{gas} = \frac{K_{Fld}(1 - K_{gas}/K_M)^2}{\phi + (K_{Fld}/K_{Sol})(1 - K_{gas}K_{Sol}/K_M^2 - \phi)} \qquad \text{(B\&K) (1)}$$

where K_M was defined precisely by B&K, and the other notation is conventional. The B-G result corresponds to a special case of the B&K result (1) in which $K_M = K_{Sol}$. B&K argued that this was not true for most rocks; Thomsen (2010) argued that it was not true, even for mono-mineralic rocks.

As B&K point out, Eqn. (1) is much more difficult to apply, in practice, than is the B-G result, because it contains K_M, which is a property of the saturated rock, rather than of the solid, and cannot be looked up in a handbook, like K_{Sol}. This obviously complicates the task of understanding the effects of fluid substitution, *e.g*. in a 4D context. For this reason of practicality, it has remained common practice (post B&K) to use the B-G result to estimate the fluid dependence of $K_{fld}(K_{Fld})$.

Determining K_M and application to field data

Thomsen (submitted to **Geophysics**, 2017) shows that K_M may be determined experimentally through the measurement of the Skempton (1954) coefficient B (defined as the ratio of pore pressure to confining pressure, in an undrained experiment). Figure 2 shows the values of $\kappa_M = 1/K_M$ thus determined for Berea sandstone, from the data for B taken by H&W (red X's). It shows that for this sample, contrary to the assumption of B-G, K_M differs from K_{Sol}, outside of experimental uncertainty.

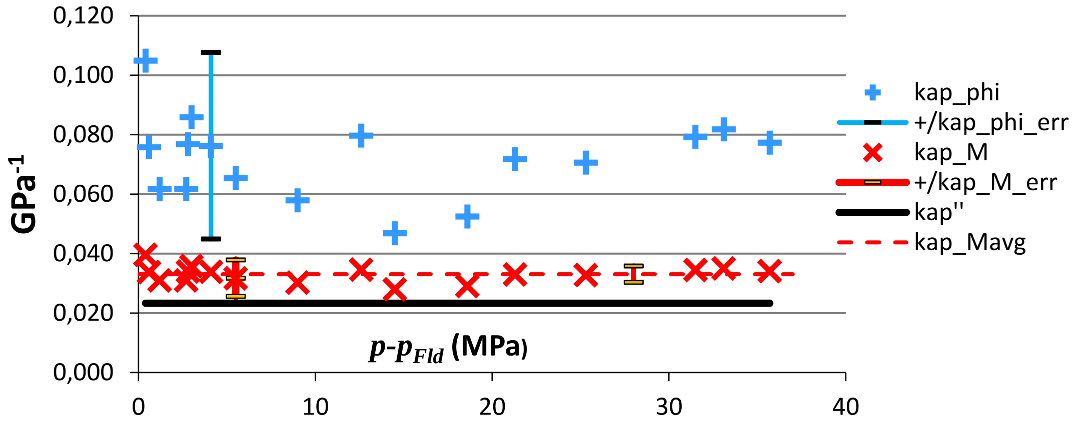

Fig. 2

The variation of the compressibility-difference in (1), to first order in the difference (K_M - K_{Sol}), is:

$$\left(\frac{1}{K_{gas}} - \frac{1}{K_{fld}}\right) = \left(\frac{1}{K_{gas}} - \frac{1}{K_{fld}}\right)_{K_M = K_{Sol}} - 2B_0(1 - B_0)\left(\frac{1}{K_M} - \frac{1}{K_{Sol}}\right) \qquad (2)$$

Of course, the Skempton coefficient B_0 is not known, for most *in-situ* rocks. But, it probably lies between 0.7 at 5 MPa and 0.4 at 40 MPa (as in the H&W data). The corresponding limits on the Taylor coefficient above are: $-0.4 > -2B_0 (1 - B_0) > -0.5$. This range is tight enough that a useful approximation for the coefficient above is simply: -1/2. Although the experimental evidence is slim, a maximum plausible value for $1/K_M$ is probably no more than two times $1/K_{Sol}$. Then Eqn. (2) implies that, in field data, the required correction above to the B-G result is negative, and smaller than $1/2K_{Sol}$.

This argument shows that the B-G result should always be replaced by the B&K result, for understanding the fluid dependency of isotropic rock velocity. And it provides a rational estimate for the size of the correction. This fluids correction can then help to understand the effect of 4D changes in pore pressure. The importance of this correction, newly feasible, should be assessed in each case.