Optimal bounds for attenuation of P and S waves in porous fluid-saturated media

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Introduction

It is well known that moduli of an isotropic mixture of isotropic elastic constituents must lie within Hashin and Shtrikman (1963) (HS) bounds. Moreover HS bounds are optimal (realisable): for each value K_0 of the bulk modulus that lies between the HS bounds, there exists a geometrical configuration, for which the bulk modulus is exactly K_0. The same is true for the shear modulus.

Real rocks exhibit attenuation and are therefore viscoelastic rather than elastic. In frequency domain viscoelastic media can be described by Hooke's law with moduli that are complex-valued and frequency dependent (Hashin, 1970). For body waves, the real parts of the moduli define the phase velocities while the imaginary parts define the (phase) attenuation factors. Hence bounds for moduli of viscoelastic media are regions in the complex plane. Such rigorous bounds have been derived in 1990s (Gibiansky and Milton, 1993, Milton and Berryman, 1997) but it is not known whether these bounds are realisable.

Poroelastic rocks are viscoelastic mixtures of an elastic porous solids and a Newtonian fluid. Gurevich and Makarynska (2012) derived the bounds for the bulk modulus as a special case of the general viscoelastic bounds of Gibiansky and Milton (1993) and showed that these bulk modulus bounds are realisable. However whether the viscoelastic shear modulus bounds are also optimal is unknown. The aim of this paper is two-fold: (1) derive rigorous bounds for both bulk and shear moduli of a mixture of an elastic solid and a Newtonian fluid and (2) prove that these bounds are realisable.

Construction of the bounds

General bounds for the bulk modulus of a viscoelastic mixture are formed by circular arcs in the complex plane. The arcs are defined by six special points corresponding to (1) moduli of the constituents, (2) formal Reuss and Voigt bounds and (3) formal lower and upper HS bounds (note that neither Reuss-Voigt nor HS bounds are bounds for viscoelastic media because they are just points on a complex plane, not curves). For a mixture of an ideally elastic solid and a Newtonian fluid all of these moduli are real-valued except the lower HS bound, whose real value is very close to the Reuss bound while its imaginary part is very small (but not zero!). It then follows that all the bounding arcs, except one, are segments of the real axis while one arc is a semi-circle connecting the Reuss bound and the upper HS bound in the upper half of the complex plane. This means that the bounding region is a half disc bounded by this semi-circle and the real axis. A similar technique is used to construct the shear modulus bounds.

Optimality of the bounds

The optimality of the bounds is shown by using a solid squirt model of Glubokovskikh *et al.* (2016), which is based on the analysis of the problem of deformation of an intergranular contact modelled as a flat slab filled with an elastic solid, and using elastic/viscoelastic correspondence principle. We assume that the rock is a Hashin sphere assemblage with a small density of penny shaped cracks. These cracks make negligible contribution to overall porosity but substantial effect on its moduli. As illustrated in Figure 1, by varying the crack density we can span the entire region countered by the bounds (see).

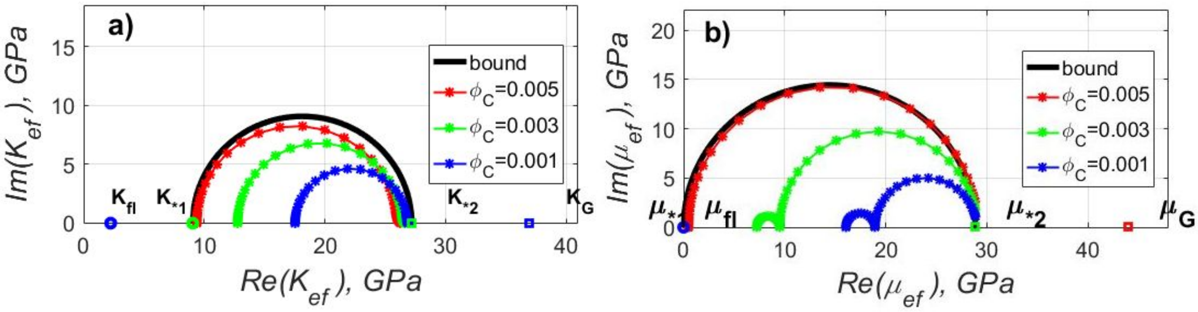

Figure 1. Bounds for complex bulk (a) and shear (b) moduli (black solid lines) and predictions of the solid squirt model (colour dotted lines) of a Hashin sphere assemblage penetrated by a small number of thin randomly oriented cracks, and saturated with a viscous fluid. Different curved colour lines correspond to different crack porosities while different points on the same curve correspond to different frequencies (increasing clockwise).

Conclusions

The bounds for complex moduli of fluid-saturated rocks are semi-circular regions in the complex plane. These bounds are independent of frequency and fluid viscosity, and are optimal (realisable). The bounds can be used to test the validity of experimental measurements and theoretical models of attenuation and dispersion in rocks.

References

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