PhD

Multiphase flow modelling: Numerical methods

Tore Flåtten

Objective
Arrive at robust and efficient methods for transient multiphase simulations based on two fluid and/or drift flux formulations.

Background
A two fluid model is suitable for separated flow modeling, whereas a drift flux model, together with an algebraic slip relation, may be preferred for mixed flows. Flow in pipelines include both regimes, meaning that both a separated and a mixed formulation may be considered solved for all regimes.

Model based control and optimization need robust and simplified models and an objective of the work is to arrive it a suitable scheme for this purpose. The activity is a part of the Petronics project.

Activities
Activities have included include:

- Development of a direct numerical scheme with non-staggered grid
- Extending methods derived for single phase to two phase flows
- Testing of the scheme on benchmark problems with gas-liquid fronts
- Reporting in the form of 4 publications and a thesis

The defense of the PhD thesis took place on March 5th 2003.

Thesis Summary

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- *Thesis title:* Hybrid Flux-Splitting Schemes for Numerical Resolution of Two-Phase Flows.

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- *Papers to be included in thesis:*

  1. *Hybrid Flux-Splitting Schemes for a Common Two-Fluid Model*, Steinar Evje and Tore Flåtten. *Journal of Computational Physics*, to appear. **Enclosed.**

  2. *A Mixture Flux Approach for Accurate and Robust Resolution of Two-Phase Flows*, Tore Flåtten and Steinar Evje. Submitted to *Journal of Computational Physics*. **Enclosed.**

3. *Weakly Implicit Numerical Schemes for the Two-Fluid Model*, Steinar Evje and Tore Flåtten. Submitted to *SIAM Journal of Scientific Computing*. **Enclosed.**

4. *CFL-Free Numerical Schemes for the Two-Fluid Model*, Steinar Evje and Tore Flåtten. Submitted to *Journal of Computational Physics*. **Enclosed.**

5. *MF-AUSM$^+$: A Refinement of AUSM$^+$ for Two-Phase Flows*, Tore Flåtten and Steinar Evje. In preparation. **Not enclosed.**

Abstract

The work deals with the construction of numerical schemes for a compressible two-fluid model, consisting of separate conservation equations for mass and momentum for each fluid. More spesifically, the model can be written on the following vector form

$$\partial_t \begin{pmatrix} \rho_{\rm g}\alpha_{\rm g} \\ \rho_{\rm l}\alpha_{\rm l} \\ \rho_{\rm g}\alpha_{\rm g}v_{\rm g} \\ \rho_{\rm l}\alpha_{\rm l}v_{\rm l} \end{pmatrix} + \partial_x \begin{pmatrix} \rho_{\rm g}\alpha_{\rm g}v_{\rm g} \\ \rho_{\rm l}\alpha_{\rm l}v_{\rm l} \\ \rho_{\rm g}\alpha_{\rm g}v_{\rm g}^2 + \alpha_{\rm g}p \\ \rho_{\rm l}\alpha_{\rm l}v_{\rm l}^2 + \alpha_{\rm l}p \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ (p-\Delta p)\partial_x\alpha_{\rm g} \\ (p-\Delta p)\partial_x\alpha_{\rm l} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ Q_{\rm g} + M_{\rm g}^{\rm D} \\ Q_{\rm l} + M_{\rm l}^{\rm D} \end{pmatrix}. \tag{1}$$

Here α_k is the volume fraction of phase k with $\alpha_{\rm l} + \alpha_{\rm g} = 1$, ρ_k and v_k denote the density and fluid velocities of phase k, and p is the pressure common to both phases. Moreover, Δp represents the pressure difference on the interface between the two fluids. $M_k^{\rm D}$ represents interfacial drag force with $M_{\rm g}^{\rm D} + M_{\rm l}^{\rm D} = 0$ whereas Q_k represent source terms due to gravity, friction, etc.

The model is structurally similar to the Euler (one-phase) equations, with added complications due to the introduction of non-conservative products $p\partial_x\alpha$ as well as the existence of strong couplings between the various variables.

The aim of this work is to obtain efficient, robust and accurate numerical schemes for the two-phase flow model. Our starting point is the observation that numerical schemes for hyperbolic conservation laws may be divided into two main classes:

- *Flux Vector Splitting Schemes* (FVS), where the numerical flux $\mathbf{F}_{j+1/2}(\mathbf{U}_j, \mathbf{U}_{j+1})$ is split as follows

$$\mathbf{F}_{j+1/2}(\mathbf{U}_j, \mathbf{U}_{j+1}) = \mathbf{F}^{+}(\mathbf{U}_j) + \mathbf{F}^{-}(\mathbf{U}_{j+1}). \tag{2}$$

- *Flux Difference Splitting Schemes* (FDS), where the numerical flux difference $\mathbf{F}_{j+1/2}(\mathbf{U}_j, \mathbf{U}_{j+1}) - \mathbf{F}_{j-1/2}(\mathbf{U}_{j-1}, \mathbf{U}_j)$ is split as follows

$$\begin{aligned} & \mathbf{F}_{j+1/2}(\mathbf{U}_j, \mathbf{U}_{j+1}) - \mathbf{F}_{j-1/2}(\mathbf{U}_{j-1}, \mathbf{U}_j) \\ = \; & \mathbf{A}^{+}_{j-1/2}(\mathbf{U}_j - \mathbf{U}_{j-1}) + \mathbf{A}^{-}_{j+1/2}(\mathbf{U}_{j+1} - \mathbf{U}_j). \end{aligned} \tag{3}$$

FVS is based on scalar calculations and is consequently quite efficient. Classical central schemes like Lax-Friedrichs and Lax-Wendroff are categorized in the FVS class. Another classical FVS scheme is the van Leer FVS for the Euler equations, where the convective fluxes are obtained from the sonic splitting formulas

$$V^{\pm}(v, c) = \pm\frac{1}{4c}(v \pm c)^2. \tag{4}$$

However, FVS is quite diffusive as the scalar computation does not allow for accurate resolution of all elementary waves. In practice, the numerical viscosity is determined by the fastest moving waves.

On the other hand, classical FDS schemes like the Roe scheme decompose the system into its full eigenstructure and an accurate resolution of all waves may be obtained. This comes at a price of being less efficient than FVS schemes, as the scalar computations are replaced by matrix computations.

This situation suggests attempting to hybridize the FVS and FDS approaches, such that we largely keep the simplicity of FVS while exploiting the accuracy of FDS. For the Euler equations, a strategy termed AUSM (Advection Upstream Splitting Methods) has been highly successfull at this.

In this doctoral work hybrid FVS/FDS schemes are constructed for the two-fluid model. In particular, a novel strategy termed *mixture flux* (MF) methods is developed, which takes advantage of the mathematical structure of the model to derive highly efficient schemes which allow for robust and accurate resolution of all waves. The strategy is derived essentially from imposing two principles:

1. A robust resolution of pressure,

2. An accurate resolution of a (moving or stationary) contact discontinuity.

The idea is to hybridize a simple flux that satisfies 1) with a simple flux that satisfies 2) such that both principles are satisfied simultaneously.

The numerical investigations demonstrate that the resulting approach is indeed very fruitful, comparing very well with an approximate Riemann solver in accuracy and robustness while allowing for highly improved efficiency. It is also believed that the ideas presented in this work may have some general applicability for other systems of conservations laws.

The main results of the enclosed papers are summarized below:

Paper 1

Hybrid Flux-Splitting Schemes for a Common Two-Fluid Model. A strategy termed AUSMD/AUSMV, originally suggested for the Euler equations, is adapted to the two-phase flow model. It is demonstrated that the approach allows for accurate and efficient solutions to several benchmark problems. However, tendencies for spurious oscillations and overshoots are observed. The results of the paper are consistent with results previously obtained by other authors, using a related strategy termed AUSM^+.

The numerical results indicate that there is room for improvement. The direct generalization of the AUSM strategies, which essentially treat the system as two sets of one-phase equations, may be a bit naive for such a *mixture* model.

Paper 2

A Mixture Flux Approach for Accurate and Robust Resolution of Two-Phase Flows. Here the MF approach is introduced and elaborated. Building upon Paper 1, we modify the AUSMD scheme to obtain a better scheme termed MF-AUSMD. We formally prove that, like the AUSMD, the MF-AUSMD recovers the upwind flux for a general contact discontinuity. Numerical simulations are presented that demonstrate the following points:

- MF-AUSMD represents major improvements compared to AUSMD in robustness.
- MF-AUSMD compares very well with the Roe scheme in terms of accuracy and robustness, while being much more efficient.

Paper 3

Weakly Implicit Numerical Schemes for the Two-Fluid Model. We here formulate the MF approach more generally in a semi-discrete setting. We demonstrate that the MF approach naturally allows for a *weakly implicit* implementation, denoted as WIMF-AUSMD, where only the fluxes corresponding to fast waves are treated implicitly. This allows for a violation of the CFL criterion for the fast waves, allowing for larger integration timesteps and more efficient and accurate resolution of the slow (volume fraction) waves.

Paper 4

CFL-Free Numerical Schemes for the Two-Fluid Model. We here investigate implicit versions of the MF schemes further. We introduce a *strongly implicit* MF scheme termed SIMF-AUSM that is unconditionally stable for a general contact discontinuity. Furthermore, we formally prove that the weakly implicit (WIMF) class of schemes allow for an *exact* resolution of such a general (moving or stationary) contact discontinuity. Explicit or strongly implicit schemes can not possess this property.

Numerical simulations are performed shedding light on the differences between the WIMF and SIMF class of schemes. We conclude that while the SIMF schemes possess excellent stability properties, the WIMF schemes are better suited for practical applications (relevant for the petroleum industry) due to their accuracy on slow transient phenomena.