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Scattering versus intrinsic attenuation in periodically layered media

Alexey Stovas$^1$ and Yuriy Roganov$^2$

1 NTNU, Trondheim, Norway
2 USGPI, Kiev, Ukraine

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Abstract
The propagation of acoustic waves in a layered medium results in the energy loss due to scattering effect. The intrinsic attenuation is an additional effect which plays a very important role in seismic data inversion and reservoir characterization. In this paper we provide the theoretical and numerical study to compare both effects for a periodically layered medium. We also investigate the complex frequency roots (the organ modes) of the reflection/transmission responses.

Keywords: scattering, attenuation, periodically layered medium, organ modes

Introduction
Many authors are considering stratigraphic filtering in their studies (O’Doherty and Anstey 1971, Shapiro and Hubral 1999, Stovas and Arntsen 2006). The approach can be deterministic or stochastic, but the model is based on an elastic medium. To introduce the layering effect within the deterministic approach, a periodically layered elastic medium is often used.

Physical experiments for periodically layered media were performed by Marion and Coudin (1992) and analysed by Hovem (1995), and Stovas and Ursin (2007). The standard eigenvalue decomposition method used for analysis of reflection and transmission responses in periodically layered media does not take into account intrinsic attenuation.

In this paper we extend this method for an anelastic medium and compute the reflection–transmission responses as well as the energy loss for the waves propagating in periodically layered structures. To compute the reflection–transmission responses, we use the matrix propagator method developed by Stovas and Ursin (2007) for a periodically layered medium. We consider the effect of layering and different contrasts between the layers. The low frequency limit for the phase velocity is exactly the same as for the elastic case while the high frequency limit for the anelastic case is different from the one computed for the elastic case.

The complex frequency roots of reflection/transmission responses are related to the self-induced oscillation in a periodical medium. We show that for a large number of cycles, the position of the roots can be obtained by a simple scaling of the real part of the frequency scale.

Intrinsic attenuation model
In order to introduce the intrinsic attenuation we choose the standard linear solid model (Carcione 2007). The velocity dispersion is given by

$$v(\omega) = v_0 \sqrt{\frac{1+i\omega\tau_e}{1+i\omega\tau_\sigma}},$$

where $\tau_e$ and $\tau_\sigma$ are the relaxation times, and $v_0 = v(\omega = 0)$. The high frequency limit is $\text{Limit } \omega \rightarrow \infty v = v_0 \sqrt{\tau_e/\tau_\sigma}$. The quality factor inverse is defined by

$$Q^{-1}(\omega) = \frac{\omega(\tau_e - \tau_\sigma)}{1 + \omega^2\tau_e\tau_\sigma}.$$
incidence reflection coefficient is given by
\[ r(\omega) = \frac{\xi v_2^2 - |v_1|^2 - 2i\xi v_2 \text{Im} v_1(\omega)}{\xi v_2^2 - |v_1|^2 + 2\xi v_2 \text{Re} v_1(\omega)}, \] (3)
where the density ratio \( \xi = \rho_2/\rho_1 \). Equation (3) is derived from a simple normal incidence reflection coefficient by substituting the complex velocity \( v_1(\omega) = \text{Re} v_1(\omega) + i\text{Im} v_1(\omega) \). If \( \text{Im} v_1(\omega) = 0 \) and \( \text{Re} v_1(\omega) = v_1 \), equation (3) is reduced to a well-known equation for the normal incidence reflection coefficient.

The propagator matrix

Hovem (1995) and Stovas and Ursin (2007) show that for the propagator matrix for one period (two layers of thicknesses \( d_1 \) and \( d_2 \) and velocities \( v_1 \) and \( v_2 \) is given by
\[ S = \begin{pmatrix} a_1 & b_1 \\ b_2 & a_2 \end{pmatrix}. \] (4)
For the elastic case (Stovas and Ursin 2007), \( a_2 = a_1^* \) and \( b_2 = b_1^* \), where the star denotes a complex conjugate, and functions
\[ a_{1,2} = e^{i\theta_1} e^{i\text{Re}(\theta_1 + \theta_2)} \frac{1 - r^2 e^{2i\theta_2}}{1 - r^2}, \]
\[ b_{1,2} = \pm e^{i\theta_1} e^{i\text{Re}\theta_2} \frac{2ir \sin \theta_2}{1 - r^2}, \] (5)
with \( r \) being the reflection coefficient for the interface between the layers, and \( \theta_i = \omega d_i/v_k \) are the corresponding phase factors. For the anelastic media, both \( r = r(\omega) \) and \( v_k(\omega) \) are the complex functions.

The real part of the function \( \text{Tr} S/2 \) is given by
\[ \frac{1}{2} \text{Re}(\text{Tr} S) = \frac{1}{2} \text{Re}(a_1 + a_2) = \frac{e^{-i\theta_1}}{1 - 2|r|^2 \cos \theta_1 + |r|^4} \left[ \cos(\text{Re} \theta_1 + \theta_2) - 2|r|^2 \cos(\text{Re} \theta_1) \cos \theta_2 \cos 2\varphi + |r|^2 \cos(\text{Re} \theta_1 - \theta_2) \right], \] (6)

The total thickness of \( d_1 \) and \( d_2 \) is given by
\[ d = \sum_{i=1}^{30} d_i \] for periods, and \( d \) is the total thickness of the medium can be computed as
\[ E = 1 - |t_D|^2 - |r_D|^2. \] (9)

For an elastic model we have \( E = 0 \), while for an anelastic model \( 0 \leq E \leq 1 \).

Numerical examples

To test the intrinsic attenuation versus scattering for the periodically layered medium we consider a similar model as in Marion and Coudin (1992) and with different reflection coefficients: the original \( r(0) = 0.87 \), \( r(0) = 0.48 \) and \( r(0) = 0.16 \). Change in reflection coefficient is obtained by changing the densities in the layers (Stovas and Ursin 2007). By doing that, we preserve the travel time in the high frequency limit. The layering is represented by the index of the model, \( M_j, j = 1, 2, \ldots, 64 \), where \( j \) denotes the number of periods. The total thickness \( D = d_1 + d_2 = 51 m \) is the same for all the models. Schematically these models are illustrated in figure 2. The model parameters are taken from Stovas and Ursin (2007).
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In figure 3 we show the reflection and transmission responses for anelastic and elastic cases from equation (8) as well as the energy loss for the anelastic case from equation (9) for the models $M_1 - M_{32}$ with the reflection coefficient $r(0) = 0.48$. One can see that, regardless of the model, both reflection and transmission responses are smoother for the anelastic case, and their magnitudes decrease with increasing frequency. The model $M_8$ captures the transition zone (or the stop-band) which is clearly pronounced for both elastic and anelastic cases. The graph for energy loss has a distinctive anomaly coinciding with the stop-band position. When the medium is blocked for propagation, we do not have too much energy loss, since there is almost no propagation through the medium. One can also see the stretch effect of the reflection–transmission responses.
Figure 4. The phase factor computed for one cycle (model M1) for elastic and visco-elastic cases.

Figure 5. Reflection (top) and transmission (bottom) responses in the time domain computed for models M1–M64 and elastic/elastic, elastic/anelastic and anelastic/anelastic cases. The zero-frequency reflection coefficient is 0.48.

Figure 6. The phase velocities computed for elastic and visco-elastic models (M1–M64) and zero-frequency reflection coefficients of 0.16 (top), 0.48 (middle) and 0.87 (bottom). For notation see figure 3.

due to the presence of attenuation. This stretch is illustrated in figure 4. The model M_{32} represents the effective medium. The reflection–transmission responses and the energy loss curve are very similar to those from the model M_{1}. The overall magnitude level for the reflection response is smaller for the effective medium.

The reflection and transmission responses in the time domain are shown in figure 5 for all the models M_j and reflection coefficient r(0) = 0.48 (the Gaussian-type wavelet with central frequency of 350 Hz was used (Stovas and Ursin (2007))). Here we considered three cases: elastic–
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The phase velocities for models with $r(0) = 0.87$, $r(0) = 0.48$ and $r(0) = 0.16$ are shown in figure 6. Note that the high-frequency limits are different for elastic and anelastic cases. The larger the reflection coefficient the larger the difference between low and high frequency velocity limits. One can also see that the phase velocities are smoother in the case of intrinsic attenuation presence.

As the reflection coefficient increases, the stop-bands become wider and deeper (figure 7). This effect is very similar to the one observed for an elastic medium (Stovas and Ursin 2007).

To illustrate the effect of intrinsic anisotropy we consider three cases: elastic–elastic, anelastic–elastic and anelastic–anelastic ones. In figure 8 one can see the reflection and transmission responses; the energy loss and the phase velocities for model $M_8$ and reflection coefficient $r(0) = 0.48$. The effect of intrinsic attenuation represented by the anelastic–anelastic model is mostly pronounced for the frequencies larger than the position for the stop-band.

The overall conclusion from our study is that the presence of intrinsic attenuation is the second-order effect with respect to the stratigraphic filtering. The anelasticity results in the small shift of the general picture for reflection–transmission responses and does not very much affect the position of the effective medium zone. The major effect is observed for the stop-bands (transition zones) where we have the resonance for the reflection response. In this respect, the applicability of the multi-layered model with intrinsic attenuation depends on the frequency range of seismic data.

A very important characteristic of the layered medium is the position of the complex roots of the self-induced oscillations (or organ modes). The method for computation of the complex roots is shown in the appendix. The roots can be computed from equation $C^2 + 1 = 0$ (setting denominators in equation (8) to zero). In figures 9(a)–(c) we show the location of the roots for the models $M_j$, $j = 1, 2, 4, 8, 16, 32, 64$, and different values of the reflection coefficient $r = 0.16, 0.48, 0.87$, respectively. One can see that from the value of $M = 8$, the roots are very close to the scaled curve given by equation $\beta = \chi(\alpha)/64$.

Conclusions

The intrinsic attenuation results in smooth and stretched reflection–transmission responses from a periodically layered medium. The anomalies in the energy loss follow the stop-bands which become wider with increasing reflection coefficient. Both reflection and transmission responses have been damping with increasing frequency. The high-frequency limit for the phase velocity is bigger for the anelastic case. The frequencies of self-induced oscillations from the periodically layered medium are related to the solutions of the secular equation. It is shown that they can be recursively computed from the reference one by using the scaling factor on the axis for real frequency.
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Figure 8. The transmission (top right) and reflection (top left) responses computed for the model M8 for elastic/elastic, anelastic/elastic and anelastic/anelastic cases. The energy loss and the phase velocities are shown in the bottom left and in the bottom right, respectively.

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Appendix. Self-induced oscillations of a periodically layered model (organ modes)

Equation (8), which defines the transmission and reflection responses $t_D$ and $r_D$, has no sense at $C = \pm i$ because in this case the denominator tends to zero. The equation

$$C^2 + 1 = 0$$  \hspace{1cm} (A.1)

is called the secular equation. It has no real roots for $\omega$, while the complex roots $\omega = \alpha + i\beta \in \mathbb{C}$ define the frequencies and attenuation of the self-induced oscillations of a periodically layered medium. Gilbert (1964) and Cochran et al (1970) studied the dependence of these complex roots on the horizontal slowness (or horizontal wave number) in a horizontally layered medium. This dependence is given by the dispersion equation and is algebraically very complicated even for the four-layer medium. For non-zero values of horizontal slowness, the secular function is a multi-valued function and has the branch points. Therefore, this function has to be considered on the Riemann surface.

In this appendix, we study the dependence of the secular equation roots for vertically propagating plane waves on the number of periods $M$ in a periodically layered elastic medium. We use the following notation for the elements of the matrix $\mathbf{S}$: $a_1 = a$, $a_2 = a^*$, $b_1 = b$ and $b_2 = b^*$ (the same as in Stovas and Ursin (2007)). We show that for large values of $M$, the real part of the roots $\alpha$ yields the equation

$$U_{M-1}(x(\alpha)) = 0,$$  \hspace{1cm} (A.2)

where $x(\alpha) = \text{Re} a(\alpha)$, $a(\alpha) = S_{11}$ are the elements of a propagator matrix, and $U_{M-1}(x)$ is the Chebyshev function of second order. The imaginary part of the roots can be found by the equation

$$\beta = \frac{1}{M} \chi(\alpha),$$  \hspace{1cm} (A.3)

where $\chi(z)$ is an analytical function which does not depend on $M$. Consequently, the roots of the secular equation are located in the areas corresponding to the propagating regime, i.e. when $|x(\alpha)| \leq 1$. 

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For a periodically layered medium, Re \( a(\alpha) \) and \( b \) are given by

\[
x(\omega) = \frac{\cos \omega(t_1 + t_2) - r^2 \cos \omega(t_1 - t_2)}{1 - r^2},
\]

\[
b = \frac{2r \sin \omega t_2}{1 - r^2},
\]

where \( t_j = d_j / v_j, j = 1, 2. \)

The Chebyshev polynomials can be computed by the recurrent equation \( U_0(x) = 1, U_1(x) = 2x, U_k(x) = 2xU_{k-1}(x) - U_{k-2}(x), k = 2, 3, \ldots. \)

The initial location of the roots can be given by the minimum of the function \( |\varepsilon(\omega)| \), and the specification is done by the method proposed by Muller (1956).

Let us investigate the roots of the secular equation (A.1) at \( M \rightarrow \infty \). We will use the following notations:

\[
\varphi(\omega) = \varphi(\alpha + i\beta) = p + iq \in \mathbb{C}. \tag{A.7}
\]

In this case, we have the equation

\[
\sin M\varphi = \pm \frac{i \sin \varphi}{b}, \tag{A.8}
\]

where the right-hand part is \( M \) independent. From the other side, we have

\[
\sin M\varphi = \sin Mp \cosh Mq + i \cos Mp \sinh Mq. \tag{A.9}
\]

Therefore, the functions \( \cosh Mq \) and \( \sin Mq \) should be bounded at \( M \rightarrow \infty \). It means that \( q \rightarrow 0 \) and \( \beta \rightarrow 0 \) at \( M \rightarrow \infty \). The value of \( \sin \varphi/b \) tends to the real number, and the value of \( \sin M\varphi \) tends to the imaginary number defined in \( i\mathbb{R} \). Therefore, \( \sin Mp \rightarrow 0, \cos Mp \rightarrow \pm 1 \) and for large \( M \), the following equation is valid:

\[
\sinh Mq = \frac{(1 - r^2)\sqrt{1 - x^2(\alpha)}}{2r \sin \alpha t_2} = g. \tag{A.10}
\]

From equation (A.10) we can find \( q \) and \( \beta \):

\[
q(\alpha) = \frac{1}{M} \ln(g + \sqrt{1 + g^2}),
\]

\[
\beta(\alpha) = -\frac{q(\alpha)}{x'(\alpha)} \sqrt{1 - x^2(\alpha)}, \tag{A.11}
\]

with

\[
x(\alpha) = \frac{\cos \alpha(t_1 + t_2) - r^2 \cos \alpha(t_1 - t_2)}{1 - r^2},
\]

\[
x'(\alpha) = \frac{(t_1 + t_2) \sin \alpha(t_1 + t_2) - r^2(t_1 - t_2) \sin \alpha(t_1 - t_2)}{1 - r^2}. \tag{A.12}
\]

In the following discussion we will use

\[
\chi(\alpha) = -\frac{\ln(g + \sqrt{1 + g^2})}{x'(\alpha)} \sqrt{1 - x^2(\alpha)}. \tag{A.13}
\]

with \( g \) defined in equation (A.10).

We have shown that if \( \alpha \neq 0, \beta \rightarrow \chi(\alpha)/M \) and \( \sin Mp \rightarrow 0 \), i.e. \( U_{M-1}(x(\alpha)) \rightarrow 0 \) if \( p \neq 0 \). Therefore, for large \( M \), the approximated roots of equation (A.1) satisfy the equation, \( \omega = \alpha + i \chi(\omega)/M \).
Let us consider the case when $\alpha = 0$. If $\alpha \to 0$, then $\omega \to 0$ at $M \to \infty$. Therefore, it gives

$$b \to \frac{2\sqrt{t_2}}{1 - r^2}, \quad \varphi \to \omega t_p,$$

where $t_p$ is the wave packet travel time given by

$$t_p^2 = \frac{(t_1 + t_2)^2 - r^2(t_1 - t_2)}{1 - r^2},$$

(A.15)

which is equivalent to the Backus (1962) equation. Therefore, equation for $\chi(0)$ takes the form

$$\chi(0) = -\ln\left(q + \sqrt{1 + q^2}\right) = -\ln\left|\frac{(1 - r^2)t_p}{2\pi t_2}\right|,$$

(A.16)

where

$$q = \frac{(1 - r^2)t_p}{2\pi t_2}.$$

(A.17)

Note that $U_0 = 1$ and, therefore, for $M = 1$, the following equation is valid

$$\omega_k = \frac{1}{t_2}(\pi k + i \ln |r|), \quad k \in \mathbb{N}.$$  

(A.18)

All derivations and discussions above were performed for the series of periodical media with different number of periods $M$ with other parameters being fixed. Let us consider the case when the layer thickness is also changed with change in $M$ such that the total thickness being constant, i.e. $d_i^{(M)}M = \text{const.}$ Therefore, $t_i^{(M)}M = \text{const}$. The circular frequency $\omega = 2\pi f$ enters the equations for the secular equation roots by the products $\omega t_i^{(M)}$. For accurate comparison of the roots from the models with different $M$, they have to be scaled such that they correspond to the periodical medium with fixed layer thicknesses $d_i$, for example, from the reference model $M_{64}$. This scaling can be performed by multiplication of the roots $\omega^{(M)}$ by the reference number $64/M$. The scaled roots have to approximately satisfy the following equality:

$$\omega^{(M)} = \frac{M}{64}\alpha + \frac{i}{64}\chi(\alpha).$$

(A.19)

The roots for the reference model $M_{64}$ satisfy

$$\omega^{(64)} = \alpha + \frac{i}{64}\chi(\alpha).$$

(A.20)

Comparing equations (A.19) and (A.20), one can see that the position of the root $\omega^{(M)}$ can be obtained by scaling the Re $\omega$ axis on the reference number inverse $M/64$.

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