Porosity and Permeability Estimation by Integration of Production and Time-Lapse Near and Far Offset Seismic Data

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Abstract
This study presents a method based on the Gauss-Newton optimization technique for continuous reservoir model updating with respect to production history and time-lapse seismic data in the form of zero offset amplitudes and amplitude versus offset (AVO) gradients. The main objective of the study is to test the feasibility of using these integrated data as input to reservoir parameter estimation problems.

Using only production data or zero offset time-lapse seismic amplitudes as observation data in the parameter estimation process cannot properly limit the solution space. The emphasis of this work is to use the integrated data combined with empirical knowledge about rock types from laboratory measurements, to further constrain the inversion process.

The algorithm written for this study consists of three parts: the reservoir simulator, the rock physics petro-elastic model, and the optimization algorithm. The Gauss-Newton inversion is tested at a 2D semi-synthetic model inspired by real field data, from offshore Norway. The algorithm reduces the misfit between the observed and simulated data which make it possible to estimate porosity and permeability distributions.

The Gauss-Newton optimization technique is an efficient parameter estimation technique. However, the numerical estimation of the gradient is time-consuming, and it can be prohibitive for practical applications. This method is suitable for distributed computing which considerably reduces the total optimization time. The amount of reduction depends mainly on the number of available processors.

Keywords: time-lapse seismic, parameter estimation, simulation based optimization, AVO gradient, Zero offset amplitudes.

1. Introduction

• The use of amplitude versus offset (AVO) time-lapse data,
• The use of empirical rock physics relations.

The inversion process is non-unique, which is the primary reason for using time-lapse information, in addition to other data, to limit the solution space. Additionally, time-lapse seismic data provides information between wells. It is therefore, an important spatial, complementary data source. Time-lapse seismic technology was first introduced in the early 1980s, and many works have been published since that time concerning this area (Greaves and Fulp 1987, Lumley 1994, Landrø et al 1999, Koster et al 2000, Landrø 2001, Furre et al 2003, Vasco et al 2004).

Time-lapse seismic data are time dependent dynamic measurements whose aim is to determine reservoir changes that have occurred in the intervening time (Eastwood et al 1994, Lazaratos and Marion 1997, Landrø et al 1999, Burkhart et al 2000, and Behrens et al 2002). The majority of the production data used as observation data in the inversion process are mostly useful in the vicinity of the wells. These data do not represent accurate behaviour for the entire reservoir. Time-lapse seismic data have high resolution in either a horizontal or lateral direction. However, the measurements are associated with errors and uncertainties, which are related to the repeatability of the data acquisition, to the data processing sequences, to a low resolution in the vertical direction, to the lack of rock physics understanding, and to the scaling and cross-scaling of seismic and simulation data.

Porosity and permeability are two of the most important parameters in most reservoir simulation models, and they have a large impact on reserve estimates, production forecasts and the economical evaluation of the reservoir. Unfortunately, according to Landa and Horne (1997), these two parameters are among the most difficult to estimate. The main reasons for this are:

• Spatial variability of permeability and porosity;
• Very few sampling locations, compared to the extent of the reservoir;
• Different technologies used to obtain measurements;
• Complexity of the mathematical model of the reservoir (usually consisting of a numerical reservoir simulation).


The type of time-lapse seismic data considered to be observation data varies among different researchers. Some use indirect measurements such as wave velocities, saturation and pressure changes (Landa and Horne 1997, Arenas et al 2001). In many works, seismic elastic parameters are used as observation data (Waggoner et al 2002, Aanonsen 2003, Gosselin et al 2003, Dong and Oliver 2005). Huang et al (1997) used time-lapse amplitude differences as input data and a simple convolutional model for producing seismic traces.

In this project, we study zero offset amplitude and AVO gradient differences in conjunction with production data. We believe that this method is easy to implement since no heavy pre-processing is needed. Stovas and Arntsen (2006) show that the simple convolution procedure may produce incorrect results for thin-layer modelling/inversion. The matrix propagation technique for producing seismic traces from the seismic input parameters, as developed by Stovas and Arntsen (2006), can be successfully used in these situations.

The objective of this work is to develop an efficient procedure for the estimation of porosity and permeability distributions in the reservoir by integrating production and time-lapse seismic data. This work is an extension of our preliminary work on the estimation of saturation and pressure changes using time-lapse seismic data (Dadashpour et al 2008). This procedure is used
for the joint estimation of permeability and porosity, and is performed during hydrocarbon depletion and water injection of a 2D semi-synthetic model generated with field data from the Norne field, offshore Norway.

2. Reservoir flow simulation

The first step in the forward modelling is the flow simulation. This simulation entails the computation of fluid saturations and pore pressures for the entire reservoir. An efficient simulator is critical since the entire process is repeated multiple times.

As Romeu et al (2005) point out each reservoir flow modelling consists of two complementary components: a functional and a representation model. A set of differential flow equations and numerical methods for solving those are known as the functional model. The representation model mathematically describes a particular reservoir by spatial variable coefficients, external boundary conditions and initial conditions, which complete the formulation.

This study uses a commercial finite difference black oil reservoir simulator (ECLIPSE 100). It takes some rock and fluid properties, such as porosities and permeabilities as input, and calculates fluid saturations and pore pressures for each cell at the desired time intervals.

3. Petro-elastic model and forward seismic modelling

A petro-elastic model (PEM) is a set of equations which relates reservoir properties (such as pore volume, pore fluid, fluid saturation, reservoir pressures, and rock composition) to seismic elastic parameters (such as P-wave and S-wave velocities, $V_p$ and $V_s$, respectively and density). Forward seismic modelling produces seismic amplitudes from these elastic properties. A PEM can be used both in inversion and forward seismic modelling, and for the interpretation of seismic data in terms of lithology (Falcone et al 2004).

Time-lapse seismic is defined as repeated 2D/3D seismic data. Variations in seismic amplitudes and travel times are used to determine changes in saturation and reservoir pressure (Lumley 1994, Landrø 2001). Recently, Duffaut and Landrø (2007) analyzed how the $V_p/V_s$-ratio changes due to water injection as reservoir pressure increases. Variations in acoustic properties are a function of temperature, compaction, fluid saturation, and reservoir pressure (the effects of temperature and compaction are neglected in this study). The Gassmann equation (1951) and the Hertz-Mindlin contact theory (Mindlin 1949) are used to estimate seismic parameter changes caused by fluid saturation and reservoir pressure changes, respectively.

The Hertz–Mindlin model is used to describe seismic parameter changes caused by pressure changes. The effective bulk modulus and shear modulus of a dry random identical sphere packing are given by:

$$k_{HM} = k_{ma} \sqrt{\frac{P_{eff}}{(P_{ext} - P_i)}},$$

$$\mu_{HM} = \mu_{ma} \sqrt{\frac{P_{eff}}{(P_{ext} - P_i)}},$$

(1)

where $k_{HM}$ and $\mu_{HM}$ are the bulk and shear modulus at critical porosity, respectively. $P_{eff}$ is the effective pressure, which is the difference between the lithostatic $P_{ext}$ and the hydrostatic pressure $P$ (Christensen and Wang 1985). In this study, initial ($P_i$) and lithostatic ($P_{ext}$) pressure are set to 200 and 380 bar, respectively, while $k_{ma}$ and $\mu_{ma}$ are the bulk and shear modulus of the solid phase (Table 1), and $n$ is the coordination number.
The Hertz-Mindlin theory assumes that velocity varies with $P_{eff}$ raised to the $1/6$th power. Some laboratory measurements on samples gave other exponents. In this work, we set $n=5$ and $c = 9$.

Gassmann equations can be written as the following formula:

$$
k_{sat} = k_{fr} + \left( \frac{k_{HM} - k_{fr}}{k_{HM}} \right)^2 \left\{ 1 - \phi + \phi \frac{k_{HM}}{k_f} - \frac{k_{fr}}{k_{HM}} \right\}, \quad (2)
$$

where $k_{fr}$ is the bulk modulus of the solid framework, $k_{HM}$ is the bulk modulus of the Hertz-Mindlin formula (equation (1)), and $\phi$ is the effective porosity of the medium. $k_f$ is the bulk modulus of the pore fluid (water, oil and gas), and is estimated by Wood’s law for oil and water given as (Reuss harmonic average 1929):

$$
\frac{1}{k_f} = \frac{S_o}{k_o} + \frac{S_w}{k_w} + \frac{S_g}{k_g}, \quad (3)
$$

The conversion of reservoir properties to seismic amplitudes is conducted in two steps. First, reservoir parameters are converted to seismic elastic properties by using the petrophysical model (rock-physic relation). Next, seismic amplitudes are calculated from them based on the matrix propagation technique developed by Stovas and Arntsen (2006). We have used the Ricker wavelet with a central frequency of 30 Hz. Table 1 illustrates the input parameters for the PEM.

<table>
<thead>
<tr>
<th>Table 1: Input parameters for PEM.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shale properties</strong></td>
</tr>
<tr>
<td>P-wave Velocity $V_{PSH}$: 2450 m/s</td>
</tr>
<tr>
<td>S-wave Velocity $V_{SSH}$: 849 m/s</td>
</tr>
<tr>
<td>Shale Density $\rho_{SH}$: 2300 kg/m$^3$</td>
</tr>
<tr>
<td><strong>Rock (Sand) properties</strong></td>
</tr>
<tr>
<td>Matrix bulk modulus $k_{ma}$: 35 GPa</td>
</tr>
<tr>
<td>Matrix Density $\rho_{ma}$: 3500 kg/m$^3$</td>
</tr>
<tr>
<td>Frame bulk modulus $k_{fr}$: 8 GPa</td>
</tr>
<tr>
<td>Frame shear modulus $k_{fr}$: 3.5 GPa</td>
</tr>
<tr>
<td><strong>Fluid properties</strong></td>
</tr>
<tr>
<td>oil bulk modulus $k_o$: 1 GPa</td>
</tr>
<tr>
<td>water bulk modulus $k_w$: 2.7 GPa</td>
</tr>
<tr>
<td>Gas bulk modulus $k_g$: 0.1 GPa</td>
</tr>
<tr>
<td>oil density $\rho_o$: From the simulator</td>
</tr>
<tr>
<td>water density $\rho_w$: From the simulator</td>
</tr>
<tr>
<td>Gas density $\rho_g$: From the simulator</td>
</tr>
</tbody>
</table>

4. **Computer aided history matching**

History matching is the calibration or conditioning of reservoir simulation models with respect to historical production or survey. It is mathematically formulated by reducing the misfit between historical and simulated data. History matching is usually a problem with multiple solutions, and since it requires numerous model runs, it is normally a very expensive procedure.
Reservoir parameter estimation is a direct application of the history matching process. Every parameter estimation method consists of three parts:

1. A mathematical model,
2. An objective function based on the mathematical model,
3. A minimization algorithm.

The objective function which is used in this work is:

$$F(\theta) = \left\| M_p(\theta) - M_p(\theta^*) \right\| + \left\| M_s(\theta) - M_s(\theta^*) \right\|,$$

where $\theta$ is the vector of the unknown reservoir model parameters, $M_p(\theta)$ and $M_s(\theta)$ are the vectors of simulated reservoir production histories and time-lapse seismic differences, respectively, and $M_p(\theta^*)$ and $M_s(\theta^*)$ are the vectors of observed production historical data and time-lapse seismic differences respectively.

In our study, the vector of unknown reservoir model parameters includes porosity and permeability for all active cells. The $M_p(\theta^*)$ vector includes well bottom hole pressures for both injectors and producers (WBHPI and WBHPP, respectively), and well oil and water production rates (WOPR and WWPR, respectively). The $M_s(\theta^*)$ vectors include both time-lapse zero offset amplitudes and AVO gradient differences.

The Gauss-Newton algorithm is an iterative method regularly used in solving non-linear least squares problems. It requires the single (Jacobian,$\nabla F(\theta)$) and double differentiations (Hessian,$\nabla^2 F(\theta)$) of the objective function (Equation 4) which are usually performed numerically. The Jacobian matrix for the problem is $\nabla F(\theta) = J_{ij} = \partial \left(\frac{1}{2} (M_p(\theta) + M_s(\theta))\right) / \partial \theta$, where $I$ runs over data space and $j$ runs over model space. For special problems, it would be useful to analytically determine equations for these differentiations, but these are not generally performed. Numerical differentiation requires the calculation of the steepest descent direction and relative step size. This procedure consists of a sequence of linear least squares approximations to the non-linear problem, each of which is solved by an “inner” direct or iterative process. The program needs a criterion to decide when it has converged on the solution and it is time to stop.

The procedure is as follows (Bartholomew-Biggs 2005):

1. Choose $\theta_0$ as an estimate of $\theta^*$
2. Repeat for $k = 0,1,2, ...$
3. Set $f_k = \theta_{k+1}$ the vector with elements $f(\theta_{k+1})$
4. Set $J_k$ as the corresponding Jacobin
5. Obtain $\delta \theta_k$ by solving $(J_k^T J_k) \delta \theta_k = -J_k^T f_k$
6. Set $\theta_{k+1} = \theta_k + \delta \theta_k$
7. Until $\| J_k^T f_k \|$ is sufficiently small.

The vector $\delta \theta_k$ used in this algorithm approximates the Newton direction, since $2J_k^T f_k = \nabla F(\theta)$ and $2J_k^T J_k = \nabla^2 F(\theta)$. Helgesen and Landrø 1993 introduced two different scaling strategies were tested for various values of damping parameter:

1. Each parameter was scaled by the diagonal Hessian matrix element,
2. Each parameter class was scaled by the average of the corresponding diagonal Hessian matrix element.

We used the first method i.e. each parameter is scaled independently by the diagonal Hessian matrix element.
matrix elements in order to speed-up the algorithm. The scaling is then:

\[
\hat{H}_{ij} = \frac{H_{ij}}{\sqrt{\gamma_i \gamma_j}} \quad \hat{G}_{i} = \frac{G_{i}}{\sqrt{\gamma_i}} \quad \gamma_i = H_{ii} \quad \gamma_j = H_{jj},
\]

(5)

where \( H \) is the Hessian matrix and is defined as \( J^T_k J_k \), and \( G \) is the gradient matrix which is defined as \( J^T_k f_k \). The sign ‘\(^\wedge\)’ denotes the scaled parameters.

The main advantage and disadvantage of this type of non-linear optimization algorithm is as follows:

**Advantages:**
- Relatively efficient (direction and step size determined)
- Works especially well near the minimum,
- It does not require the evaluation of second-order derivatives in the Hessian of the objective function.

**Disadvantages:**
- May become lost with poor initial estimates
- Possibility of converging to a local minimum in the objective function,
- They provide a single solution, despite the fact there are multiple acceptable solutions that maybe significantly diverse,
- Long computing time required to perform the gradient calculation.

5. **Synthetic test case**

To test the efficiency and accuracy of the presented optimization approach, a complex semi-synthetic reservoir has been set up. This model is two-dimensional, and is based on field data from the Norne Field in offshore Norway.

The Norne field is located in the blocks 6608/10 and 6508/10 on a horst block in the southern part of the Nordland II in the Norwegian Sea. The horst block is approximately 9 km x 3 km. Norne Field, has been exposed for two periods of rifting; in Perm and Late Jurassic - Early Cretaceous. During the first rifting, faulting affected a wide part of the area. Especially normal faults, with NNE-SSW trends, are common from this period. The second rifting period can be subdivided into four phases ranged in age from Late Bathonian to Early Albian. The trend during this rifting was footwall uplift along the Nordland Ridge, and erosion of high structures. Between the two rifting periods the tectonic activity was limited, although some faulting occurred in the Mid and Late Triassic. This period was dominated by subsidence and transgression. The rocks within the Norne reservoir are of Late Triassic to Middle Jurassic age. The reservoir sandstones in the formations Tilje, Tofte, Ile and Garn, are dominated by fine-grained and well to very well sorted sub-arkosic arenites. Being buried between 2500 m and 2700 m these sandstones are affected by digenetic process. A sedimentological classification of the middle Jurassic Tilje, Tofte, Ile, Not and Garn formations in the reservoir have resulted in the definition of 14 different facies association. The deposition took place at (StatoillHydro 1998):

- A fluvial dominated coastline in a shallow seaway (Lowe part of the Tilje Formation),
- A restricted lagoon (Middle part of the Tilje Formation),
- A tidal dominated coastline in a shallow seaway (Uppermost part of the Tilje Formation),
- A delta front on a fan-delta and/or a shoreline on a low-energy coast (Tofte and Ile
Formation),
- An open marine shoreline on a high energy coast (Not and Garn Formation)

5.1 Reservoir model description

The reservoir model is subdivided into four different formations from top to base: Garn, Ile, Tofte and Tilje. Hydrocarbons in this reservoir are located in the Lower- to Middle-Jurassic sandstones. The present geological model consists of 17 reservoir zones. Today’s reservoir-zonation is slightly altered from earlier subdivisions. An illustration of the zonation from 2001 can be seen in Figure 1. For simplicity we use nine different rock types based on relationship between porosity and permeabilities which are indicated by the ellipses (Figure 2).

![Figure 1: Stratigraphical sub-division of the Norne reservoir (Verlo and Hetland, 2008)]
The two-phase (oil and water), two-dimensional black-oil reservoir model has 26 layers. Each layer has 31 grid blocks of varying size between 48 m to 203 m. Thicknesses vary between 2 to 37 meters. The model contains 739 active cells. The parameters to be estimated are the porosity and horizontal permeability of each grid cell. For simplicity we assume an isotropic permeability (vertical and horizontal permeabilities are assumed to be equal within one cell) and the horizontal permeability and porosity of each cell are considered to be independent reservoir parameters. In total this study deals with 1478 unknown parameters. Four different faults subdivide this reservoir into four different compartments. Two synthetic wells are defined for injection and production purposes. The production well is located in grid block number 1 and perforated in blocks 1-12, and the water injection well is located in block number 31 and perforated in blocks 1-25. Three major faults are divided the reservoir to four sections. All faults assumed to be opened and transmissibility multipliers are equal one for all faults.

5.2 Time-lapse seismic and observed inversion data

Time-lapse seismic data are a set of time dependent dynamical measurements which are added to the inversion process to reduce the non-uniqueness problem. Time-lapse amplitudes and AVO gradients are the two time-lapse seismic inputs considered in this project. The value of such data increases away from the wells, since no other data is available between the wells.

In the real case, these data come directly from the field after processing. The main objective of this project is to check the accuracy and precision of a method for the estimation of the reservoir parameters. These model parameters are converted to time-lapse seismic data through the forward model and this data are used as observation data. Figure 3 shows the zero offset time-lapse amplitudes (top) and AVO gradient amplitudes (bottom), which are used as input data, to simulate field data.
5.3 **Constraints**

Estimated porosity and permeabilities are only valid when they satisfy given constraints. Two different types of bound constraints are used in this work.

**Type I.** Simple bound constraints.

Porosity and permeability values have to be between the following upper and lower boundaries:

\[ 0.15 \leq \phi \leq 0.40 \]
\[ 0 \leq K \leq 2000 \]

where \( \phi \) is porosity (in fraction) and \( K \) is permeability (in mD).

**Type II.** Geological bound constraints.

The present geological model consists of 16 reservoir zones but for simplicity we reduce it to the nine zones based on empirical knowledge about rock types which can be obtained from laboratory measurements (correlation between porosity and permeability from core measurements). Based on this knowledge nine different rock types are defined in this work from different geological environments during the deposition of sands. The rock types create new constraints for the parameters. Based on our new zonation formation Garn, Ile, and Tofte are divided to two zones while formation Tilje divided to three zones (Table 2).
Table 2: Geological bound constraints (Type II)

<table>
<thead>
<tr>
<th>Formation</th>
<th>Rock type</th>
<th>Limitation for Porosity (fraction)</th>
<th>Limitation for Permeability (mD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Garn</td>
<td>1</td>
<td>$0.22 \leq \phi \leq 0.32$</td>
<td>$150 \leq K \leq 1100$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$0.19 \leq \phi \leq 0.24$</td>
<td>$5 \leq K \leq 110$</td>
</tr>
<tr>
<td>Ile</td>
<td>1</td>
<td>$0.22 \leq \phi \leq 0.25$</td>
<td>$40 \leq K \leq 150$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$0.22 \leq \phi \leq 0.29$</td>
<td>$280 \leq K \leq 870$</td>
</tr>
<tr>
<td>Toftø</td>
<td>1</td>
<td>$0.20 \leq \phi \leq 0.31$</td>
<td>$70 \leq K \leq 1570$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$0.19 \leq \phi \leq 0.29$</td>
<td>$150 \leq K \leq 1950$</td>
</tr>
<tr>
<td>Tilje</td>
<td>1</td>
<td>$0.18 \leq \phi \leq 0.26$</td>
<td>$10 \leq K \leq 700$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$0.17 \leq \phi \leq 0.22$</td>
<td>$40 \leq K \leq 320$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$0.25 \leq \phi \leq 0.27$</td>
<td>$730 \leq K \leq 2000$</td>
</tr>
</tbody>
</table>

5.4 Sensitivity of time-lapse amplitude change to porosity and permeability

The study of the effect of reservoir parameter changes on seismic amplitude is vital to the parameter estimation problem. The sensitivity analysis is performed in this way:

1. Perturb reservoir parameters (porosity and permeability),
2. Run reservoir simulation to compute saturation and pressure at two different times (base and monitor seismic surveys),
3. Calculate the reflection amplitudes at these two different times by using petro-elastic model and forward seismic models,
4. Compute the amplitude changes by subtracting the results of the two computations,
5. The ratios of the change in the differential amplitude to the change in porosity and permeability provide the numerical sensitivities.

Figure 4 shows the time-lapse amplitude sensitivity with respect to porosity and permeability in the fifth column of the reservoir. The amount of perturbation is represented as DELTA in the figure. There is a direct relationship between permeability and porosity and the time-lapse seismic amplitude sensitivity. It is difficult to detect reservoir parameters for low permeable layers with low porosity due to low amplitude sensitivity. In the figure DELTA Perm and DELTA POR represent the amount of perturbation in permeability and porosity, respectively. In this study, the degree of perturbation has a significant effect in permeability, but less of an effect in porosity. A similar experiment for changes in saturation and pressure (Dadashpour et al 2008) shows similar results.
Figure 4: Porosity (upper left) and permeability (upper right) and amplitude sensitivities for porosity (bottom left) and permeability (bottom right) in column 5 of the reservoir model. The vertical scale is the dimensionless ratio of the fractional change in amplitude to the fractional change in pore pressure.

5.5 Data matching

In the first part of this section, we discuss the effect of using empirical knowledge of rock types as an inversion constraint, and we also study the use of both zero offset amplitudes and AVO gradient differences as input data in the inversion. To study the effect of noise in the inversion process two different kinds of noise, white and real noise, are added to the observation data. This study is tested with multiple initial guess (Figure 5 and 6, Initial guess a - d). In the second part of this section we analyse integrating of production and time-lapse seismic data in model inversion. This study also tested with the initial guess (a).
This type of model inversion requires an enormous number of forward simulations for the numerical approximation of the gradient which made this method difficult to implement, although
it has a good capability to work in a distributed manner. We have implemented a distributed computing workflow for the estimation of reservoir properties from time-lapse seismic and production data using a Gauss-Newton optimization procedure. The case studied in this work has 1478 estimation parameters. It should be noted that each forward simulation requires the numerical solution of discretized multiphase flow equations, the calculation of the elastic parameters at each cell by rock physics equations, and finally the obtaining of the seismic traces.

The computing resources are based on 20 processors from an IBM P690, 32 CPU (Power-4) with 32GB RAM under AIX version 5.3. (Using the 32 CPU was not possible, due to the reduced number of flow simulation software licences). This will give us a speed-up factor of approximately 20 while each forward simulation take 26 second which 10 second is related to flow simulation and 18 second is related to the calculation of seismic amplitudes and AVO gradients.

There are many different methods to speed-up these kinds of algorithms. One method is to reduce the order of estimated parameters which is done in several ways. Some of these methods are using simple zonation techniques (Brun et al 2001, Harb 2004) while some of them using complex mathematical transforms (Shah et al 1978, Scheeval and Payrazyan 2001, Sarma et al 2007). In this work, we did not try to speed-up algorithm by reparameterization techniques.

**Part 1: Effect of constraints (geological constraints) and use of zero offset amplitudes and AVO gradient differences.** To study the effect of constraints, we consider two different cases which are using only free noise seismic data: case SZ-SC and case SZ-GC. Case SZ-SC uses the Seismic Zero offset with Simple Constraints and case SZ-GC uses the Seismic Zero offset with Geological Constraints. Case SZG is used to look at the effect of adding AVO gradients to the optimization loop. SZG represents the use of the Seismic Zero offset and AVO Gradients (see section 5.3 and Table 3). In Figure 7, we compare the mismatch between real and calculated time-lapse seismic data for all the cases in four different initial guess (Initial guess a-d). In every situation the objective function is significantly reduced. The fact that the cost function in case SZG is different than in cases SZ-CS and SZ-GC explains why the initial cost function value in Figure 7 is not the same for all three cases.

**Table 3: Case definitions for testing the effect of constraints and observation data in the parameter inversion.**

<table>
<thead>
<tr>
<th>Part</th>
<th>Case</th>
<th>Zero Offset Amplitude</th>
<th>AVO gradient</th>
<th>Production Data</th>
<th>Simple constraints</th>
<th>Geological</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SZ-SC</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>SZ-GC</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SZG</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SZG+P</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2</td>
<td>ALT*</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

* Production data added after iteration 15
Figure 8 and 9 show the average error in the estimation of porosity and permeability respectively. Figure 10 shows the correlation for all cases between real and final permeability and porosity values.

Based on figures 8 to 10, it is clear that SZ-SC gives a worse estimate than the other two cases. Although it reduces the objective function, the parameters estimated from this method are far from reality (the degree of the error is the same). This is a clear case of non-uniqueness in the inversion problem. The error reduction in porosity and permeability is not significant. There is a reduction from 3.35\% to 3.02\% for the average error in porosity. By using initial guess (a) (the standard deviation (true standard deviation of the error) increases from 2.3\% to 3.4\%). Analogously, the average error in permeability is reduced from 655.3 mD to 545.11 mD (the standard deviation decreases from 543.7 mD to 506.0 mD).

Limiting the solution space with some additional information from laboratory measurements about the rock types increases the accuracy of the estimation of the reservoir parameters (see Figure 10). The average error in porosity estimation uses initial guess (a) decreases from 3.35\% to 0.91\% (standard deviation decreases from 2.3\% to 1.1\%). Moreover, the average error in permeability estimation decreases from 655.3 mD to 323.1 mD (standard deviation decreases from 543.7 mD to 362.6 mD). This trend is also repeated for other tests with different initial guesses.

Adding AVO gradients to the inversion do not yield significant improvement in the permeability estimation (by using initial guess (a) the error is reduced from 323.1 mD to 310.5 mD, approximately a 2.5 \% improvement compared to case SZ-GC), but the average error in porosity estimation in case SZG is worse than in case SZ-GC. By considering both AVO gradient and zero offset amplitudes, the estimation improvement in porosity is reduced from 73\% to 56\% (still acceptable). But the effect of using AVO gradients is going to be significant by using initial guess (b) and (d). Figures 8 - 10 show that adding AVO gradients to the objective function can stabilize the inversion (limiting the solution space), but might influence the parameter estimation.
Figure 25 in the appendix illustrates the effect of P-wave velocity changes with respect to variation in the effective pressure and water saturation. Based on this figure, we find that for our case example that effect of water saturation change is approximately 10 times higher than the effect of pressure changes. This means that the saturation effect is dominating over the pressure effect. Therefore it is reasonable to assume that the extra value of using AVO data (as proposed by Landrø, 2001) is somewhat limited. Table 4 and 5 summarized the results for all cases with multiple initial guesses.

Table 4: Errors in porosity estimation for cases SZ+SC, SZ+GC, SZG with multiple initial guesses.

<table>
<thead>
<tr>
<th>Case</th>
<th>Initial guess (a)</th>
<th>Initial guess (b)</th>
<th>Initial guess (c)</th>
<th>Initial guess (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volumetric average</td>
<td>Error in porosity estimation (%)</td>
<td>Volumetric average</td>
<td>Error in porosity estimation (%)</td>
</tr>
<tr>
<td></td>
<td>Initial</td>
<td>Best Predicted</td>
<td>Initial</td>
<td>Best Predicted</td>
</tr>
<tr>
<td>SZ+SC</td>
<td>3.35</td>
<td>3.02</td>
<td>2.3</td>
<td>3.40</td>
</tr>
<tr>
<td>SZ+GC</td>
<td>0.91</td>
<td>0.91</td>
<td>1.10</td>
<td>1.52</td>
</tr>
<tr>
<td>SZG</td>
<td>1.49</td>
<td>1.49</td>
<td>1.52</td>
<td>1.52</td>
</tr>
</tbody>
</table>

Table 5: Errors in permeability estimation for cases SZ+SC, SZ+GC, SZG with multiple initial guesses.

<table>
<thead>
<tr>
<th>Case</th>
<th>Initial guess (a)</th>
<th>Initial guess (b)</th>
<th>Initial guess (c)</th>
<th>Initial guess (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volumetric average</td>
<td>Error in permeability estimation (mD)</td>
<td>Volumetric average</td>
<td>Error in permeability estimation (mD)</td>
</tr>
<tr>
<td></td>
<td>Initial</td>
<td>Best predicted</td>
<td>Initial</td>
<td>Best predicted</td>
</tr>
<tr>
<td>SZ+SC</td>
<td>655.30</td>
<td>545.11</td>
<td>543.70</td>
<td>506.00</td>
</tr>
<tr>
<td>SZ+GC</td>
<td>323.10</td>
<td>323.10</td>
<td>362.06</td>
<td>362.06</td>
</tr>
<tr>
<td>SZG</td>
<td>310.49</td>
<td>310.49</td>
<td>349.57</td>
<td>349.57</td>
</tr>
<tr>
<td>SZ+SC</td>
<td>544.35</td>
<td>520.41</td>
<td>494.01</td>
<td>494.01</td>
</tr>
<tr>
<td>SZ+GC</td>
<td>533.51</td>
<td>533.51</td>
<td>618.40</td>
<td>618.40</td>
</tr>
<tr>
<td>SZG</td>
<td>628.39</td>
<td>607.39</td>
<td>726.32</td>
<td>726.32</td>
</tr>
</tbody>
</table>

To check the effect of noise in the algorithm two different kind of noise are added to the observation data: white random noise which is a Gaussian distribution with no time and spatial correlation and real noise which is generated based on noise spectrum from real 4D seismic data.
in the Norne Field. The amplitude spectrum was computed for several seismic traces from the Norne Field. The separation between the signal and the noise spectrum was performed by estimating noise by doing the following operation: 

$$r_j(t) = r'_j(t) - (r_{j-1}(t) + r_{j+1}(t))/2.$$ 

Then we compute 

$$N_j(\omega) = FT[r_j(t)]$$ (see Figure 25 in the appendix). The amplitude spectra for the noise averaged and used to produce the “colour” noise data. Signal to noise ration is defined as the RMS value of the signal divided by RMS value of the noise and it is set to be equal 1 for both noises. White noise is added only to the 4D-seismic amplitudes while real noise are added to both 4D seismic amplitude and time shift. The parameter estimation process uses initial guess (a) for this study (Figure 11).

Adding AVO gradients for noisy data does not reduce the estimation error (see figures 12 and 13). However, the approach that uses AVO gradients in addition to zero offset amplitude gives more accurate forecasts for both clean and noisy data (Figure 14).

Figure 8: Average error in estimation of porosity using multiple initial guess (Part 1).
Figure 9: Average error in estimation of permeability using multiple initial guess (Part 1).

Figure 10: Correlation between real and estimated porosity and permeability (Initial guess (a)). a) initial. b) SZ-SC. c) SZ-GC. d) SZG.
Figure 11: White (left) and real (right) noise added to the observed zero offset amplitudes (top) and AVO gradients (bottom).

Figure 12: Mismatch reduction for the cases of noisy data using initial guess (a) (100% Gaussian noise added (left) and Real noise added (right)). Dashed line indicates the misfit of the true solution.
Figure 13: Average error in estimation of porosity (top) and permeability (bottom) in the noisy data using initial guess (a) (100% Gaussian noise added (left) and Real noise added (right)).

Figure 14: Oil production forecast for different cases Part 1 with without noise (top-left) and with white noise (top-right) and real noise (bottom).
Part 2: Effect of integration of historical production and time-lapse seismic data. Three new cases (P, SZG+P, and ALT) are defined to test the effect of integrating production and time-lapse seismic data. No noises are added to the data. These cases differ in the input data. Case P represents the case which only uses production data. These production data are well bottom-hole pressure for injector and producer (WBHPI and WBHPP, respectively), well oil production rate (WOPR), and well water production rate (WWPR). Case SZG+P represent the case which uses production data, together with Seismic Zero offset and AVO Gradients, and case ALT represents ALternating technique. It starts the optimization with only seismic data, and then at iteration 15 it includes production data as well.

In Figure 15, we compare the mismatch reduction for the four different cases. In this section Initial guess (a) has been used (see figures 5 and 6). In all cases, the objective function is acceptably reduced for practical purposes.

![Figure 15: Mismatch reduction for the cases in Part 2.](image)

Figure 16 refers to the production data (bottom-hole pressures and fluid rates) and Figure 17 refers to the time-lapse seismic mismatch functions (zero offset amplitudes and AVO gradients.)
Figure 16: Best predicted, historical and initial values for case ALT concerning the well bottom-hole pressure in the injector (left top) and in the producer (left bottom), and the well oil production (right top) and water production rates (right bottom) rates.

Figure 17: Initial (top) and final (bottom) mismatch function for the 4D-seismic in the form of zero offset amplitude (left) and AVO gradients (right), Case ALT.

Figure 18 shows the average error in the estimation of permeability and porosity for case ALT. Figure 19 depicts the forecasts based on these estimations. In all cases, the initial guess mismatch error is reduced, and it appears that the reservoir parameters can be estimated sufficiently well.
Case ALT, which starts with using only time-lapse seismic data and then switches to both time-lapse seismic and production data, yields the best porosity and permeability estimation and the most accurate forecast. The integration of production and time-lapse seismic data as observation data during all iterations (case SZG+P,) seems to be less effective in the estimation. As far as this study is concerned, we conclude that using time-lapse seismic data as observation data for the permeability and porosity estimation gives better results than only considering production historical data.

In case ALT, the average error in the estimation of porosity decreases from 3.35% to 1.58% (the standard deviation decreases from 2.3% to 1.52%) and in the estimating of permeability from 655.29 mD to 294.45 mD (the standard deviation decreases from 543.7 mD to 333.1 mD).

Average errors (for cases SZP, P, SZP+P and ALT) in the estimation of porosity and permeability are summarized in tables 6 and 7, respectively.

![Figure 18: Average error in the estimation of porosity (top) and permeability (bottom) distributions for cases P, SZG, SZG+P, and ALT.](image-url)
Table 6: Errors in porosity estimation for cases SZG, P, SZG+P and ALT.

<table>
<thead>
<tr>
<th>Case</th>
<th>Error in porosity estimation (%)</th>
<th>Volumetric average</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Initial</td>
<td>Best predicted</td>
</tr>
<tr>
<td>SZG</td>
<td></td>
<td>1.49</td>
<td>3.35</td>
</tr>
<tr>
<td>P</td>
<td></td>
<td>2.54</td>
<td>1.96</td>
</tr>
<tr>
<td>SZG+P</td>
<td></td>
<td>1.73</td>
<td>1.63</td>
</tr>
<tr>
<td>ALT</td>
<td></td>
<td>1.58</td>
<td>1.52</td>
</tr>
</tbody>
</table>

Table 7: Errors in permeability estimation for cases SZG, P, SZG+P and ALT.

<table>
<thead>
<tr>
<th>Case</th>
<th>Error in permeability estimation (mD)</th>
<th>Volumetric average</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Initial</td>
<td>Best predicted</td>
</tr>
<tr>
<td>SZG</td>
<td></td>
<td>655.3</td>
<td>310.5</td>
</tr>
<tr>
<td>P</td>
<td></td>
<td>355.3</td>
<td>356.7</td>
</tr>
<tr>
<td>SZG+P</td>
<td></td>
<td>356.7</td>
<td>356.7</td>
</tr>
<tr>
<td>ALT</td>
<td></td>
<td>294.4</td>
<td>294.4</td>
</tr>
</tbody>
</table>

In Figure 20, we compare the correlation for cases P, SZG+P and ALT between real and estimated porosity and permeability distributions. Figures 21 and 22 represent real and estimated porosity and permeability distributions for case ALT.
Figure 20: Correlation between real and estimated porosity (left) and permeability (right) distributions for case P (top), SZG+P (middle) and ALT (bottom).

Figure 21: Real (top) and estimated (bottom) porosity distributions after 20 iterations for case ALT.
Figures 23 and 24 show the error in the estimation of porosity and permeability in the first and last iterations for case ALT. We can see in these figures how the error in the initial guess is reduced.

This parameter estimation problem is underdetermined, which explains that adding extra information in the algorithm for limiting the solution space has a positive effect in improving the estimation process. The main difficulties in this problem can be attributed to two thin highly permeable layers (see Figure 22, layers 13 and 20) and to an area close to the injection well. It is very difficult to detect thin highly permeable zones by seismic methods since seismic has limited vertical resolution. Therefore, the algorithm tends to spread the estimation in the vertical direction. In this study, we detect some layers with medium permeability instead of just two with high permeability. A more accurate bound constraints selection in problematic zones, as those described, can be beneficial in the estimation. In any case, including geological information in the process is, according to the results presented here, indispensable.
Figure 23: Difference between real and estimated porosity for first (top) and last (bottom) iteration for case ALT.

Figure 24: Difference between real and estimated permeability for first (top) and last (bottom) iteration for case ALT.
Discussion

In this paper we have introduced a systematic approach for estimating reservoir parameters such as porosity and permeability by integrating production and time-lapse seismic data in the form of zero offset amplitudes and AVO gradient differences. This methodology is based on the Gauss-Newton optimization technique for reducing the mismatch between modelled and best predicted data. The results of this algorithm for a 2D synthetic reservoir, based on field data from a complex reservoir offshore Norway (Norne Field), suggest that this approach can be applied to parameter estimation problems of practical relevance.

With this method, it is possible to estimate porosity and permeability distributions by integrating production and time-lapse seismic data. Correlation between porosity and permeability, which comes from laboratory measurements, could help to subdivide the reservoir into different regions with distinct rock types. By using this empirical knowledge as additional constraints the estimation can be improved.

As far as sensitivity analysis is concerned, we have observed that layers with low porosity and permeability have low amplitude sensitivity. As can be expected the perturbation size in the analysis has a significant effect on the computed sensitivity.

Since porosity and permeability are estimated for every single grid cell in the reservoir model, the number of model parameters is high, and therefore the inversion problem is undetermined. Thus, a good fit with the observation data is not sufficient for a good estimation of the unknown reservoir properties.

The major weaknesses of this algorithm are:

- The gradients are estimated numerically. If first order finite difference schemes are used the number of additional simulation runs is equal to the number of unknown parameters,
- As in any local optimization algorithm, there is a strong dependency on the initial guess taken,
- It is not straight-forward to find constraints that properly limit the solution space.

This approach is suitable for distributed computing and for time demanding forward models (in this case the gradient estimation will be accelerated considerably). A distributed computing optimization framework is developed applied successfully.

The main advantage of this method respect to the other optimization techniques are: right descent direction, the procedure converges quadratically at the optimal point, no need to compute second derivative of objective function for computing hessian matrix and it requires less simulation run respect to global optimizers like genetic algorithm.

Future work is to increase the speed of the optimization. One method is reducing the number of parameters which called reparameterization (Dadashpour et al 2009). Reparameterization can be done by simple zonation or by mathematical transforms techniques. We will try to use gradzone and principal component analysis which incorporated in a distributed computing environment to speed up the parameter estimation. We hope with using these techniques it will be possible to use these type of optimization in the large and realistic problems.

Conclusions

A Gauss-Newton optimization technique is used to estimate porosity and permeability distributions. This algorithm is conditioned to production and time-lapse near and far offset data.
The result shows that AVO gradient can increase the stability of the parameter estimation. Empirical knowledge about rock types can improve the estimation by means of some additional constraints in the optimization routine. Increasing the sensitivity of amplitudes to the estimated parameters can increase the accuracy of the estimates.

Integrating production history and time-lapse seismic data as observation data is better implemented in two steps:

1) Limiting the solution space by only using time-lapse seismic data,
2) After a number of optimization iterations, consider both production history and time-lapse seismic data in the misfit function.

In this study we have observed that using only time-lapse seismic data is more effective for permeability and porosity estimation than using only production performance historical data.

It is very difficult to detect thin high permeable zones by seismic methods. This is probably associated with less vertical resolution in the seismic data.

Distributed computing can significantly accelerate the parameter estimation process.

Acknowledgments

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Nomenclature

c: the average number of contacts per grain
Cp: data noise covariance matrix
Cs: data noise covariance matrix
dp: vector of observed production historical data
dS: vector of observed time-lapse seismic differences
F: objective function
G: Gradient matrix
H: Hessian matrix
J: Jacobin matrix
K: permeability, mD
kf: the bulk modulus of the saturating fluid
kfr: the bulk modulus of the solid framework
kHM: bulk modulus of Hertz–Mindlin formula
kma: matrix bulk modulus
ko: oil bulk modulus
kw: water bulk modulus
kg: gas bulk modulus
M_p(θ): the vectors of simulated reservoir production histories
M_s(θ): the vectors of simulated reservoir production histories
MS(θ): the vectors of simulated time-lapse seismic differences
MS(θ): the vectors of simulated time-lapse seismic differences
n: the coordination number
P: hydrostatic pressure
Pe: effective pressure
Pex: the lithostatic pressure
So: oil saturation
Sw: oil saturation
Sg: oil saturation
VPSH: shale P-wave Velocity
VSSH: shale S-wave Velocity
WBHPI: well bottom hole pressure for injector
WBHP: well bottom hole pressure for producer
WOPR: well oil production rate
WWPRI: well water production rate
ρma: Matrix Density
ρo: oil Density
ρw: water Density
ρg: gas Density
ρSH: shale Density
φ: effective porosity
φc: critical porosity
θ: Vector of unknown reservoir model parameters
μ: shear modulus of the solid phase
μHM: shear modulus of Hertz–Mindlin formula
ν: Poisson’s ratio
^: scaled (Hessian, gradient,…)
Appendix

Figure 25: The effect of P-wave velocity changes with respect to variation in the effective pressure (top) and water saturation (bottom).

Figure 26: Noise spectrum generated from Norne Field.