BOOK OF ABSTRACTS: CHALLENGES IN PRESERVATION OF STRUCTURE, WORKSHOP JUNE 27TH- 30TH 2017

Tuesday 27th of June

Time	**Speaker**	**Title**
Tue 27th, 8:45		Opening
Tue 27th, 9:00	Reinout Quispel	Geometric integrability properties of Kahan's method
Tue 27th, 9:45		**Coffee break**
Tue 27th, 10:15	Klas Modin	Global results for geodesic equations on smooth probability densities
Tue 27th, 11:00	Alexander Schmeding	Shape analysis on Lie groups and beyond
Tue 27th, 12:00		**Lunch**
Tue 27th, 13:30	Milo Viviani	Casimir preserving discretization of incompressible Euler equations on S2
Tue 27th, 14:15	Hans Munthe-Kaas	Lie-Butcher theory and Rough Differential Equations on homogeneous manifolds
Tue 27th, 15:00		**Coffee break**
Tue 27th, 15:30	Antonella Zanna	A modified trigonometric integrator for highly oscillatory problems

Wednesday 28th of June

Time	**Speaker**	**Title**
Wed 28th, 9:00	Schönlieb	Discrete Gradients for Non-Smooth Bi-Level Learning
Wed 28th, 9:45		**Coffee break**
Wed 28th, 10:15	Lazic	MATLAB for Image Processing with Machine Learning Applications
Wed 28th, 11:00	Grasmair	A nonlocal, nonconvex functional for coherence enhancing image restoration
Wed 28th, 12:00		**Lunch**
Wed 28th, 13:30	Iserles	It takes a wave packet to catch a wave packet
Wed 28th, 14:15	Bogfjellmo	Spline interpolation on Riemannian symmetric spaces
Wed 28th, 15:00		**Coffee break**
Wed 28th, 15:30	Curry	Invariant measures and stochastic flows: a surprising appearance of geometric integration

Thursday 29th of June

Time	**Speaker**	**Title**
Thu 29th, 9:00	Kværnø	Quadratic invariant preserving methods for stochastic differential equations
Thu 29th, 9:45		**Coffee break**
Thu 29th, 10:15	Malinnikova	Discrete harmonic functions: stability and growth properties
Thu 29th, 11:00	Owren	Structure preserving model reduction
Thu 29th, 12:00		**Lunch**
Thur 29th, 13:30	Eidnes	Energy preserving moving mesh methods for PDEs
Thu 29th, 14:15	Verdier	The spherical midpoint method
Thu 29th, 15:00	**Coffee break**	
Thu 29th, 15:30	Sato	Numerical Integration of Higher-Order Lagrangians

Friday 30th of June

Time	**Speaker**	**Title**
Fri 30th, 9:00	Jakobsen	On numerical methods for nonlocal nonlinear PDEs.
Fri 30th, 9:45		**Coffee break**
Fri 30th, 10:15	Larsson	Strong convergence of a fully discrete finite element approximation of equation the stochastic Cahn-Hilliard equation
Fri 30th, 11:00	Holden	On traffic modeling and the Braess paradox

Geometric integrability properties of Kahan's method

Reinout Quispel, Tue 27th, 9:00.

TBA

Global results for geodesic equations on smooth probability densities

Klas Modin, Tue 27th, 9:45.

The geometric approach to optimal transport and information theory has triggered the interpretation of probability densities as an infinite-dimensional Riemannian manifold. The most studied Riemannian structures are Otto's metric, yielding the L2-Wasserstein distance of optimal mass transport, and the Fisher–Rao metric, predominant in the theory of information geometry. On the space of smooth probability densities, none of these Riemannian metrics are geodesically complete—a property desirable for example in imaging applications. That is, the existence interval for solutions to the geodesic flow equations cannot be extended to the whole real line. Here we study a class of Hamilton–Jacobi-like partial differential equations arising as geodesic flow equations for higher-order Sobolev type metrics on the space of smooth probability densities. We give order conditions for global existence and uniqueness, thereby providing geodesic completeness.

Shape analysis on Lie groups and beyond

Alexander Schmeding, Tue 27th, 11:00.

Shape analysis methods have in the past few years become very popular, both for theoretical exploration as well as from an application point of view. Originally developed for planar curves, these methods have been expanded to higher dimensional curves and surfaces. To solve problems arising in applications one endows the (infinite-dimensional) shape spaces with the structure of a Riemannian manifold and studies the associated geometry. We will discuss one possible construction of such a geometry (using the so called "square root velocity transform") for shape spaces of curves with values in a Lie group or a homogeneous manifold. This allows us to treat situations of interest to mechanics while preserving desirable structural properties of the shapes. To achieve this, we exploit the additional geometry imposed on the shape space by the geometry on the target space. This is joint work with E. Celledoni, S. Eidnes and M. Eslitzbichler.

Casimir preserving discretization of incompressible Euler equations on S2

Milo Viviani, Tue 27th, 13:30.

The Euler equations for an incompressible fluid on a sphere are the basic model to describe and study the oceanic and the atmospheric dynamics on the Earth. Their solution has been a long standing challenge which concerns several different approaches both from an analytical and numerical point of view. From a numerical point of view, the main issue is to take as much as possible of the geometry of the problem into account when discretizing the original system. We will present a numerical method based on the geometric quantization of the Poisson algebra of the smooth functions on a sphere, which gives a solution of the Euler equations with a consistent number of discrete first integrals with respect to the level of discretization.

Lie-Butcher theory and Rough Differential Equations on homogeneous manifolds

Hans Munthe-Kaas, Tue 27th, 14:15.

Part 1: A quick introduction to Lie-Butcher series and the MKW Hopf algebra. Part 2: We discuss a new theory for rough differential equations on homogeneous manifolds. Terry Lyons 1998 defined geometric rough paths and RDEs in terms of the character group on the shuffle Hopf algebra. This was generalised in Massimiliano Gubinellis "branched rough paths" (2010), based on the Butcher-Connes-Kreimer Hopf algebra. BCK is associated with the geometry of Euclidean spaces. The MKW Hopf algebra is associated with homogeneous spaces (Klein geometries). We show how Gubinellis theory generalises to this setting. Ongoing work joint with Charles Curry, Kurusch Ebrahimi-Fard and Dominique Manchon.

A modified trigonometric integrator for highly oscillatory problems

Antonella Zanna Munthe-Kaas, Tue 27th, 16:00.

I will introduce a new modified trigonometric integrator for highly oscillatory problem. It is a symplectic second order method derived from a discrete Lagrangian. The methods will be compared with well known trigonometric integrators and the IMEX method (which is a modified trigonometric integrator).

Discrete Gradients for Non-Smooth Bi-Level Learning

Carola-Bibiane Schönlieb, Wed 28th, 9:00.

In this talk we study the learning of parameters in a non-smooth variational model by means of a bi-level optimisation problem. Variational regularisation models in imaging usually have a number of free parameters that might encode a class of appropriate forward models or different types of image regularisation. Bi-level optimisation problems are usually non-convex and in the special case considered in this talk they are also non-smooth. Therefore algorithms that are based on classical derivatives are not applicable. To circumvent this problem we make use of the discrete gradient framework which allows a descent of the energy without the computation of classical derivatives. We numerically verify that this methodology is suitable to solve non-smooth bi-level learning problems with applications in image restoration.

MATLAB for Image Processing with Machine Learning Applications

Jasmina Lazic, Wed 28th, 10:15.

We demonstrate how MATLAB can accelerate algorithm development and exploration when working with image data and vision systems. We will cover the use of machine learning and deep learning methods for advanced Image Processing and Computer Vision applications. You will learn how to:

- Analyze and process (e.g. enhance, measure, segment) images
- Acquire live video from external cameras and deal with large collections of image files
- Categorize and retrieve images using advanced clustering and classification techniques
- Train models using large image datasets:
  1. Train deep neural networks from scratch – Using transfer learning to re-use trained deep networks for new tasks
  2. Explore the tradeoffs between machine learning and deep learning – Leverage multicore CPU and GPU computing for processing large image

A nonlocal, nonconvex functional for coherence enhancing image restoration

Markus Grasmair, Wed 28th, 11:00.

In this talk we will propose and analyse a variational method for image restoration that is based on a nonconvex and nonlocal integral functional. The main idea is to increase the local coherence of the restored images by penalising local variations of the direction of the gradient, which can be achieved using a nonconvex although separately convex regularisation functional. Because of the separate convexity one obtains lower semicontinuity of the resulting regularisation functional and, as a consequence, the existence of minimisers; uniqueness, however, cannot be guaranteed because of the lack of convexity. In the talk we will briefly present the underlying theory and some numerical results in order to demonstrate the conherence enhancing effects of the proposed method. In addition, we will discuss the relation to PDE based anisotropic methods due to Weickert. This is joint work with Peter Elbau (Vienna) and Lasse ÃĚsmot (NTNU).

It takes a wave packet to catch a wave packet

Arieh Iserles, Wed 28th, 13:30.

We are concerned with structure-preserving spectral methods for solutions composed of wave packets, e.g. in quantum mechanics. The traditional approach is to use periodic boundary conditions, in which case standard Fourier methods are more than adequate, except that in long-term integration wave packets might reach the boundary and non-physical behaviour ensues. This motivates us to consider approximations on the entire real line. We consider and analyse four candidates: Hermite polynomials, Hermite functions, stretched Chebyshev expansions and stretched Fourier expansions. And the winner is ...

Spline interpolation on Riemannian symmetric spaces

Geir Bogfjellmo, Wed 28th, 14:15.

Cubic spline interpolation on Euclidean space is a standard topic in numerical analysis, with countless applications in science and technology. In several emerging fields, for example computer vision and quantum control, there is a growing need for spline interpolation on curved, non-Euclidean space. The generalization of cubic splines to manifolds is not self-evident, with several distinct approaches. One possibility is to mimic the acceleration minimizing property, which leads to Riemannian cubics. This, however, require the solution of a coupled set of non-linear boundary value problems that cannot be integrated explicitly, even if formulae for geodesics are available. Another possibility, which we pursue, is to mimic De Casteljau's algorithm, which leads to generalized BÃľzier curves. To construct C^2-splines from such curves is a complicated non-linear problem, until now lacking numerical methods. Here we provide an iterative algorithm for C^2-splines on Riemannian symmetric spaces, and we prove convergence of linear order. In terms of numerical tractability and computational efficiency, the new method surpasses those based on Riemannian cubics. Each iteration is parallel, thus suitable for multi-core implementation. We demonstrate the algorithm for three geometries of interest: the n-sphere, complex projective space, and the real Grassmannian. Joint work with Klas Modin, Chalmers, and Olivier Verdier, Western Norway University of Applied Sciences.

Invariant measures and stochastic flows: a surprising appearance of geometric integration

Charles Curry, Wed 28th, 16:00.

We discuss the importance of geometric integration in simulating certain stochastic flows, and show its relevance in sampling from probability measures via the Hamiltonian Monte Carlo method. Applications to Bayesian statistics are outlined. The talk is based on joint work with Olivier Verdier.

Quadratic invariant preserving methods for stochastic differential equations

Anne Kværnø Thu 29th, 9:00.

Sanz-Serna and Abia (1991) have proved that for Runge-Kutta methods preserving quadratic invariants only order conditions related to rootless trees have to be satisfied. The same authors proved a similar result for partitioned Runge-Kutta methods (1993). In this talk, we discuss how these ideas can be extended to stochastic Runge-Kutta methods.

Discrete harmonic functions: stability and growth properties

Eugenia Malinnikova, Thu 30th, 10:15.

In this talk we discuss a very simple "five-point" discretization of the Laplace equation and focus on two discrete phenomena that have no continuous counterparts. One concerns the local behavior of harmonic functions and can be considered as a discrete non-uniqueness result. Another one is about the global behavior, we will formulate an improvement of the classical Liouville theorem for discrete harmonic functions. The talk is partly based on joint results with L. Buhovsky, A. Logunov and M. Sodin.

Structure preserving model reduction

Brynjulf Owren, Thu 29th, 11:00.

Model reduction via Principal Orthogonal Decomposition (POD) is widely used in fluid dynamics, but it has also been applied in other areas, including control. POD, also known as the Karhunen-Loewe expansion and Principal Component analysis, produces a least-squares optimally ordered basis for a given set of data. In model reduction applications, POD is used to identify an ordered basis of (linear) phase space from a set of data points called snapshots. Truncating the POD basis and projecting the dynamics onto that subspace provides a lower-dimensional model of the system.

In this work we shall seek to adapt this approach to mechanical systems. It is of essence to find a reduced model which inherits the geometric features of the original problem, either in the Lagrangian or Hamiltonian setting. If the phase space in question is a manifold there are additional difficulties. Lall, Krysl and Marsden (2003) proposed to accomplish this by first embedding the manifold into a larger Euclidean space, perform standard model reduction there and finally intersecting the reduced (linear) model space by the original manifold. We have not been able to develop this strategy in a way such that efficient methods can be obtained, but have instead opted for an approach where local coordinates is applied. This causes however other difficulties, in particular related to the switching of coordinate charts. There are various ways to address the problem and we shall present at least one approach here.

Collaborators: Elena Celledoni, Helen Parks and Vanje Rebni Kjer.

Energy preserving moving mesh methods for PDEs

Sølve Eidnes, Thu 29th, 13:30.

A general framework for the energy preserving discrete gradient methods on fixed grids is presented, and their relationship to the disrete variational derivative methods is discussed. This framework is then extended to also incorporate adaptive meshes. The application of the methods is demonstrated through numerical results. Joint work with B. Owren and T. Ringholm.

The spherical midpoint method

Olivier Verdier, Thu 29th, 14:15.

Spin systems model spinning black holes binaries, ferromagnetic materials (Laudau Lifshitz equation), or hurricanes on gas planets. Their phase spaces, products of spheres, have a natural symplectic structure for which those systems are Hamiltonian. Is there a method with no extra variable which preserves symplecticity? The answer is yes, and it is a remarkable modification of the classical midpoint method. Joint work with Robert I McLachlan and Klas Modin.

Numerical Integration of Higher-Order Lagrangians

Rodrigo T. Sato Martín de Almagro, Thu 29th, 16:00.

In [1] the authors develop a discrete theory for higher order Lagrangians and proceed to apply this to obtain variational integrators for higher order problems such as the spline or a fully actuated optimal control problem. In my talk I will give an overview of the main idea from that paper and show how to obtain Runge-Kutta methods for second order Lagrangian problems. Joint work with David Martín de Diego.

[1] L. Colombo, S. Ferraro, and D. Martin de Diego. *Geometric integrators for higher-order variational systems and their application to optimal control.* Journal of Nonlinear Science, 26(6):1615–1650, 2016.

On numerical methods for nonlocal nonlinear PDEs.

Espen Jakobsen, Fri 30th, 9:00.

In this talk we focus on numerical methods for nonlocal diffusion equations of porous medium type

$$u_t - \mathcal{L}\varphi(u) = g,$$

and nonlocal convection-diffusion equations

$$v_t + \nabla \cdot f(v) - \mathcal{L}\varphi(v) = g.$$

Some of these methods will be structure preserving as we explain in the end of this abstact. Here $\mathcal{L}$ is a nonlocal or anomalous diffusion operator and the generator of a Levy process. Our admissible class of operators is very large and includes e.g. the fractional Laplacian $(-\Delta)^{\frac{\alpha}{2}}$ for $\alpha \in (0, 2)$. In the first part of the talk, we will explain and motivate these nonlocal models. In the second part of the talk we briefly mention several different numerical methods for these problems that we have proposed and analyzed in recent years: Difference-quadrature methods, discontinuous Galerkin methods, and spectral vaninishing viscosity methods. Then in the main part of the talk, we explain the latest results on monotone difference-quadrature approximations for nonlinear diffusion equations of porous medium type. These results include the construction of the methods, well-posedness, a priori estimates, and convergence for the schemes. The convergence result follows from a compactness argument, stability results, and recent uniqueness results for the limit equation. These latter results are joint work with Felix del Teso and Jørgen Endal.

Monotonicity is a structure preserving property that plays a cruicial role in these convergence proofs. Monotoncity means that the numerical method preserves the ordering of the initial data, just like the solution of the exact equation. It implies that the maximum principle holds and that the numerical solutions are non-oscillatory. The latter property is very advantagous since too much oscillations in nonlinear problems like these can destroy the strong convergence of method.

Strong convergence of a fully discrete finite element approximation of the stochastic Cahn-Hilliard equation

Stig Larsson, Fri 30th, 10:15.

We consider the stochastic Cahn–Hilliard equation driven by additive Gaussian noise in a convex domain with polygonal boundary in dimension $d \leq 3$. We discretize the equation using a standard finite element method in space and a fully implicit backward Euler method in time. By proving optimal error estimates on subsets of the probability space with arbitrarily large probability and uniform-in-time moment bounds we show that the numerical solution converges strongly to the solution as the discretization parameters tend to zero.

Joint work with D. Furihata, M. Kovacs, and F. Lindgren.

On traffic modeling and the Braess paradox

Hege Holden, Fri 30th, 11:00.

We will discuss models for vehicular traffic flow on networks. The models include both the Lighthill-Whitham-Richards model and Follow-the-Leader models. The emphasis will be on the Braess paradox in which adding a road to a traffic network can make travel times worse for all drivers.