FOURIER-WIGNER MULTIPLIERS IN QUANTUM HARMONIC ANALYSIS

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The Fourier-Wigner transform and convolution for operators were introduced in the seminal paper of Werner [3]. Werner laid the groundwork for quantum harmonic analysis, an operator variant of harmonic analysis in which the classical L^p-spaces are replaced by the Schatten p-classes of compact operators on L^2.

In recent years, several classical results from harmonic analysis have been shown to have counterparts in the setting of quantum harmonic analysis. An example is Fourier restriction, where the mapping properties of restricting the Fourier transform onto a subset $\Omega \subseteq \mathbb{R}^n$ are investigated. Fourier restriction for operators was recently considered in the context of quantum harmonic analysis [1].

Fourier restriction has been shown to follow from the Bochner-Riesz conjecture [2], an open problem on the mapping properties of the Fourier multiplier

$$T_{m_\delta}(\varphi)(x) = \int_{|\xi|<1} \left(1 - |\xi|^2\right)^\delta \widehat{\varphi}(\xi) e^{2\pi i x\cdot\xi}\, d\xi, \qquad \varphi \in \mathscr{S}(\mathbb{R}^n),$$

for $\delta > 0$. In light of [1], one might ask if there is an analogue of the Bochner-Riesz conjecture for operators.

In this talk, I will address how the Bochner-Riesz conjecture can be reformulated in the language of quantum harmonic analysis. For this, the framework of quantum harmonic analysis and the Fourier-Wigner multipliers will be introduced.

References

[1] F. Luef and H. J. Samuelsen, "Fourier Restriction for Schatten Class Operators and Functions on Phase Space," *Int. Math. Res. Not. IMRN*, no. 2, Paper No. 291, 22, 2025.

[2] T. Tao, "The Bochner-Riesz conjecture implies the restriction conjecture," *Duke Math. J.*, vol. 96, no. 2, pp. 363–375, 1999.

[3] R. Werner, "Quantum harmonic analysis on phase space," *J. Math. Phys.*, vol. 25, no. 5, pp. 1404–1411, 1984.

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