Martijn Caspers

Title:

On the best constants of Schur multipliers of second order divided difference functions

Abstract

We give a new proof of the boundedness of bilinear Schur multipliers of second order divided difference functions, as obtained earlier by Potapov, Skripka and Sukochev in their proof of Koplienko's conjecture on the existence of higher order spectral shift functions. Our proof is based on recent methods involving bilinear transference and the Hörmander-Mikhlin-Schur multiplier theorem. Our approach provides a significant sharpening of the known asymptotic bounds of bilinear Schur multipliers of second order divided difference functions. Furthermore, we give a new lower bound of these bilinear Schur multipliers.

More precisely, we prove that for $f \in C^2(\mathbb{R})$ and $1 < p, p_1, p_2 < \infty$ with $\frac{1}{p} = \frac{1}{p_1} + \frac{1}{p_2}$ we have

$$\|M_{f^{[2]}} : S_{p_1} \times S_{p_2} \to S_p\| \lesssim \|f''\|_\infty D(p, p_1, p_2),$$

where the constant $D(p, p_1, p_2)$ is explicit and $D(p, 2p, 2p) \approx p^4 p^*$ with p^* the Hölder conjugate of p. We further show that for $f(\lambda) = \lambda|\lambda|$, $\lambda \in \mathbb{R}$, for every $1 < p < \infty$ we have

$$p^2 p^* \lesssim \|M_{f^{[2]}} : S_{2p} \times S_{2p} \to S_p\|.$$

Here $f^{[2]}$ is the second order divided difference function of f with $M_{f^{[2]}}$ the associated Schur multiplier. In particular it follows that our estimate $D(p, 2p, 2p)$ is optimal for $p \searrow 1$. If time permits or if I have more results at the time of the conference I will speculate about the asymptotics for $p \to \infty$. This is joint work with Jesse Reimann (arXiv: 2405.00464).