Expectations switching in a DSGE model for the UK *

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Abstract

Rational expectations (RE) has been dominant both in the economic literature and in the macromodels routinely used in Central banks. The RE assumption has recently come under attack as one of the drawbacks of the Dynamic Stochastic General Equilibrium (DSGE modeling) paradigm. This study attempts to investigate whether other ways of modeling expectations would necessarily find a better support in the data.

We investigate the relevance of the RE assumption by introducing regime switching into the expectations formation of an otherwise standard DSGE model by Justiniano and Preston (2010). In our model, expectations switch between RE and Adaptive expectations (AE). The model is estimated on UK data using Bayesian techniques.

By introducing a switching mechanism, the model explains the data better than both the pure RE and the pure AE models. Expectation formation switches to AE during changes in monetary policy and the financial crisis.

The dynamics of the economic system is different under the two expectation regimes. Hence, should the UK economy switch to an AE regime after Brexit, the dynamics of the economic system could be substantially more uncertain than under RE, given the model.

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1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models are currently enjoying a leading role in the world of macroeconomic policy and research. However, after the financial crisis of 2007-08, their usefulness has been questioned due to their difficulty both to forecast the financial crisis and explain the depth and the slow recovery.1 Expectations are an essential part of the monetary policy transmission mechanism in DSGE models - with rational expectations as the standard assumption on how these are formed. Hence, the rational expectations assumption is one of the aspects of DSGE models that has been subject to criticism after the financial crisis.2 Under the assumption of rational expectations, the agents — firms, households and policy-makers — have complete knowledge of the economy, including the model and its parameters. Like many other assumptions made in economic models, this is not necessarily compatible with reality in all situations. Depending upon the invariance of the economic environment and the informational resources available, agents might in some situations have limited knowledge about the economy and insufficient ability to understand it, for example due to large shocks, temporary or permanent. Several deviations from rational expectations have been put forward, predominantly from the behavioral, learning and robust control literature.3 In addition, the concept of rational inattention has been introduced as the case where individual capacity for processing information is limited (Sims 2003).

A possible explanation of the failure of REH-models to match the inertia of macroeconomic variables, therefore, might be that the agents follow several strategies in their formation of expectations, where the strategy chosen depends upon the information set available. Consider the situation when they have to form their expectations based on limited information about their economic environment. This is where the case of learning expectations enters the picture, where agents are modelled as forming their expectations based on an econometric learning algorithm. Assuming expectations are based on learning, exogenous shocks to the model will lead to revisions of beliefs over time - drawing out the effects of a shock.

Milani (2007) finds that learning generates a persistence in DSGE models that can replace ‘mechanical’ sources of persistence like the above-mentioned habit formation and indexation. Slobodyan and Wouters (2012) use similar learning mechanisms in a larger new Keynesian (NK) model, and finds that learning fits the data equally well or better than rational expectations. Their results indicate that the persistence introduced by the learning process does not systematically alter the estimated structural parameters of the DSGE model, contradicting the claims made by Milani (2007) that habit formation and indexation become insignificant in the presence of learning. They also find that persistence of exogenous shocks mainly remain unchanged. Both these results are based on closed-economy models, using data for the US. Jääskelä and McKibbin (2010) introduces learning in a small open-economy model, estimating it using Australian data.

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1. See the collection in Vines and Wills (2018), in particular Hendry and Muellbauer (2018), and Lindé, Smets, and Wouters (2016) for various degrees of criticisms of DSGE models.
2. See, for example, the critique of Hendry and Mizon (2014).
3. Another aspect is the (lack of) modelling of the financial sector in these models — see for instance Negro, Giannoni, and Schorfheide (2015) — which is also the case of the model used in this paper.
4. An extensive survey is given by Woodford (2013). A shorter overview to different approaches to expectations formation is given in Driscoll and Holden (2014).
Comparing it with a version with rational expectations, they find that learning can replace some of the structural inertia in the model, and that the impulse response functions in the learning model exhibit more persistence than those of the REH model.

Behavioral and experimental economics also provide evidence against rational expectations. The literature survey on laboratory experiments on expectations in macroeconomics and finance, presented by Assenza et al. (2014), sum up important findings with regards to individuals’ expectations formation. With regard to New Keynesian DSGE models, evidence of individual learning taking the form of switching from one heuristics to another and subsequently heterogeneity in individual expectations are found by Assenza et. al (2012). Heterogeneity in expectations is one of the more popular alternatives to rational expectations, see for instance Galí and Gertler (1999), De Grauwe (2011) and Branch and Evans (2006). In experiments focusing on inflation expectations conducted by Pfajfar and Zakelj (2014), there is evidence of significant heterogeneity in subject’s forecasting strategies. They conclude that for about 40 % of subjects, rationality cannot be rejected, while more than 20 % of subjects are best described by adaptive learning models. However, they conclude that switching between expectations models are better at describing subjects’ behavior.

From a cognitive neuroscience perspective, the focus has been on the processing of information. Rolls (2019) argues that human decision processes are complex and lists several factors that need to be taken into account in macroeconomics:

- The utility function may be multidimensional
- the reasoning may be imperfect
- the individual decision-making process might be subject to noise, inducing uncertainty
- heterogeneity of individual value functions along different dimensions.

In particular he emphasizes the dual system approach in human decision making, with a slow, but accurate, system requiring much information and a quick, but more inaccurate, heuristic system requiring less information.

In this paper we use the financial crisis of 2007-08 as an example of an exogenous change in the information set, to analyse whether the expectation formation of agents in an economy shift from one expectations regime to another in the case of an exogenous shock to the economy. To investigate this hypothesis, we estimate a Markov-switching DSGE model (MS-DSGE) for the UK economy using Bayesian methods over the period from the introduction of the independence of the Bank of England (1997Q2) until after the referendum of leaving the European Union (2016Q4).

The implementation of switching expectations is done by building upon the small open economy DSGE model of Justiniano and Preston (2010), introducing Markov-switching in expectations allowing for shifts between rational expectations and simple constant-gain adaptive learning. We want to analyse whether the financial crisis may have induced a switch in expectations, as the large shock to the economy might force the economic agents of the model to alter how they form their expectations of the future⁵.

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⁵. Gerdrup et al. (2017) provide a different and interesting take to incorporate the financial crisis by including a large negative, but low probability, shock to consumption demand in the same model. The probability of the shock is endogenously given, and influenced by the financial imbalances in the model. However, where we model the impact of the crisis in such a model, they look at what influences the probability of the crisis happening.
The form of learning used in this paper, is of the simplest and most well-established form, namely adaptive expectations, a hypothesis of expectations formation tracing back to Irving Fisher (1930) and formally introduced in the 1950s by Cagan (1956), Friedman (1957), and Nerlove (1958). The hypothesis states that expectations in any period are revised (linearly) in the light of past errors of expectations, i.e. revisions of beliefs. This corresponds to a simple learning rule, where agents adjust their expectations of the future values of the forward-looking variables based on past forecast errors — the simplest example of constant-gain adaptive learning. This differs from statistical learning as in Milani (2007) and infinite-horizon learning adopted by Preston (2005a, 2005b).

The alternative to rational expectations chosen in this paper is therefore the simplest one possible and should not be interpreted to be the most realistic alternative, but rather as a simplified first attempt.

Note that the way in which the expectations shift is not obvious. A switch from rational to adaptive expectations could explain the slow recovery from the crisis — as the shock made the economy more difficult to predict, and agents then had to observe the development and learn adaptively what to expect of the future. On the other hand, before the crisis there was a period of great moderation possibly making economic agents ‘lazy’ and just expecting the economic variables to stay the way they were. A shift from adaptive to rational expectations during the crisis could then be explained with the crisis forcing economic agents to become more conscious of the effort required to form their expectations - and consequently becoming more ‘rational’.

The expectation formation of the agents will might also impact the dynamic properties and therefore also the forecasting performance of the model. When using a model to forecast an uncertain future, like Brexit, it might therefore be relevant take into consideration possible changes in the information sets available for the forecasts. One standard solution is therefore to produce forecasts conditional upon different information sets, like scenarios and structural breaks. Focusing on possible changes in expectations, the approach taken in this paper is therefore to investigate the forecasting properties of the model under different assumptions of the formation of expectations.

The Brexit vote took place on the 23rd of June 2016. We have therefore kept observations from 2017Q1-2019Q1 to use for forecasts, in order to evaluate how the Markov-switching model forecasts the domestic observables for the period after the referendum, allowing for possible changes in expectation formation schemes.

Finally, we investigate how the two alternative assumptions of expectations formation affects the model forecasts of domestic variables over the period 2019q2-2021q1.

The rest of the paper is organised as follows. The next section presents the open economy DSGE model. Section 3 introduces the Markov-switching aspects and the expectational regimes of the model in addition to the solution and estimation of the model. In section 4 the main findings are discussed, followed by an evaluation of the dynamic properties of the model, conditional upon the expectational regimes in section 5. Section 6 first evaluates the 1-step-ahead forecasts and then presents the ex-ante dynamic forecasts for the UK economy. The final section concludes.

2 Model setup

The model used as a basis for the analyses, is Justiniano and Preston’s (2010) model of a small, open economy (henceforth referred to as JP10), building on the work of Monacelli (2005) and Galí and Monacelli (2005). The model comprises two economies, one small domestic economy and one large foreign economy (the rest of the world), assumed exogenous to the domestic economy. Further, the model allows for incomplete asset markets, habit formation and indexation of prices to past prices.

For completeness, a brief presentation of the log-linearized equations of the model for the aggregate economy follows below. For the derivation of the entire model, the reader is referred to the original paper.

On the demand side, households choose the optimal allocation of consumption and labor to maximize utility subject to a budget constraint. Log-linearisation of the first order conditions yields a variant of the Euler equation with habit formation:

\[
\begin{align*}
ct - h_c ct - 1 &= E_t \{ct+1\} - h_c ct - \sigma^{-1}(1 - h_c)(rt - E_t \{\pi t+1\}) \\
&+ \sigma^{-1}(1 - h_c)(\varepsilon g,t - \varepsilon g,t+1) \\
\end{align*}
\]  

where consumption \(ct\) depends on both next period’s and the preceding period’s consumption, in addition to next period’s real interest rate \(rt - \pi t+1\). \(\sigma\) is the risk aversion parameter and \(h_c\) is the level of habit formation, where \(h_c = 0\) will result in the usual Euler equation. The notation \(E_t\) for forward-looking terms indicate that the expectations might shift, so the expression for expectations is written out following equation (13). \(\varepsilon g,t\) denotes a shock in preferences, and is assumed to follow an AR(1)-process.

On the supply side, there are two types of firms, producers and importers, that employ labor and maximize profits subject to costs and demand. The inflation dynamics for domestic goods results from the optimal price setting rule for producing firms resetting their price in period \(t\) and is given by:

\[
\begin{align*}
\pi H,t - \delta H \pi H,t-1 &= \frac{(1 - \theta H)(1 - \beta \theta H)}{\theta H}mc_t + \beta [E_t (\pi H,t+1) - \delta H \pi H,t] \\
mc_t &= \nu y_t - (1 + \nu)\varepsilon a,t + \alpha tot_t + \frac{\sigma}{1 - h_c}(ct - h_c ct - 1) \\
\end{align*}
\]

The model assumes Calvo-prices, where a fraction \(\theta H\) of firms resets their prices in period \(t\), while the remaining fraction \(0 < \theta H < 1\) adjust the prices according to price indexation, with \(\delta H\) being the indexation parameter. In the case of zero indexation \((\delta H = 0)\), the usual forward-looking Phillips curve applies. Domestic price inflation is consequently decided by current marginal costs \(mc_t\), expectations about inflation in the next period and the most recent observed inflation rate. Marginal costs given in equation (3), depends on output, where \(\nu\) is the inverse Frisch elasticity of labor supply, the technology disturbance \(\varepsilon a,t\), domestic consumption, and the terms of trade.

\[
\begin{align*}
\pi F,t - \delta F \pi F,t-1 &= \frac{(1 - \theta F)(1 - \beta \theta F)}{\theta F}\psi_t + \beta [E_t (\pi F,t+1) - \delta F \pi F,t] + \varepsilon c,t \\
\end{align*}
\]

Equation (4) results from the optimal price setting rule for importing firms resetting their prices in period \(t\). Calvo-pricing and price indexation are assumed also here, and a cost-push shock \(\varepsilon c,t\) has been added, meant to capture inefficient variations in mark-ups. Retailers import foreign differentiated goods for which the law of one price holds ‘at the dock’. When setting the domestic currency price of the imported goods, they are assumed to be monopolistically competitive, which
leads to deviations from the law of one price in the short run. These deviations can be seen as the current marginal cost conditions for the importing firms and are defined as:

$$\psi_t = e_t + p_t^* - p_{F,t}$$

The deviation from the law of one price $\psi_t$ is thus defined as the difference between the foreign consumer price index ($p_t^*$) in domestic currency ($e_t$ is the nominal exchange rate) and the domestic currency price index of imports ($p_{F,t}$).

The consumer price inflation ($\pi_t$) is a weighted average of domestic and foreign inflation, weighted by their respective shares:

$$\pi_t = (1 - \alpha)\pi_{H,t} + \alpha \pi_{F,t}$$

where $\alpha$ can be understood as the openness of the domestic economy. The higher $\alpha$ is, the more weight on the foreign inflation. Terms of trade, $\text{tot}_t$, is defined as the difference between the domestic price level for foreign and domestic goods, given as $\text{tot}_t = p_{F,t} - p_{H,t}$. Time differencing the definition of terms of trade yields

$$\Delta \text{tot}_t = \pi_{F,t} - \pi_{H,t}$$

Using the expression for terms of trade, the relation between consumer price inflation and domestic inflation can be expressed as:

$$\pi_t = \pi_{H,t} + \alpha \Delta \text{tot}_t$$

Further the terms of trade, $\text{tot}_t$, and the real exchange rate, $q_t$, are related according to

$$q_t = e_t + p_t^* - p_t = \psi_t + (1 - \alpha)\text{tot}_t$$

where $p_t$ is the domestic consumer price index. Hence, the real exchange rate are influenced by deviations from the law of one price and also differences in the price indices for the domestic and foreign consumption bundles.

The clearing of the goods market requires that domestic production consists of domestic and foreign consumption of domestic goods. Log-linearizing the goods market clearing condition yields:

$$(1 - \alpha)c_t = y_t - \alpha\eta(2 - \alpha)\text{tot}_t - \alpha\eta\psi_t - \alpha y_t^*$$

Equation (8) states that equilibrium domestic consumption $(1 - \alpha)c_t$ depends on domestic output $(y_t)$, and three sources of foreign disturbance: the terms of trade (with $\eta$ being the elasticity between home and foreign goods), deviations from the law of one price and foreign output $(y_t^*)$.

A version of the Taylor (1993) monetary policy rule for the domestic economy is given by

$$r_t = \rho r_{t-1} + \phi_\pi \pi_t + \phi_\pi y_t + \phi_\Delta c_t + \varepsilon_{m,t}$$

The nominal interest rate is determined by past interest rate, in addition to being influenced by current consumer price inflation, output, output growth and the change in the nominal exchange rate. $\varepsilon_{m,t}$ can be interpreted as a monetary policy shock. The uncovered interest parity under incomplete asset markets is given by

$$(r_t - E_t \pi_{t+1}) - (r_t^* - E_t \pi_{t+1}^*) = E_t \Delta q_{t+1} - \chi d_t - \varepsilon_{r,t}$$

where $\varepsilon_{r,t}$ is a risk-premium shock and $d_t = \log(e_t B_t/(P_t \bar{Y}))$ is the log real net foreign asset position as a fraction of steady state output. The parameter $\chi$ controls the debt sensitivity of
the international markets. As foreign variables are assumed to follow a VAR(2)-process (see end of section), \( \pi_{t+1}^* \) will be determined by this process and not switch with the other forward-looking variables.

In addition to choose the optimal allocation of labor and consumption, households also choose to invest either in domestic bonds \( B_t \) or foreign bonds \( B_t^* \), yielding an Euler equation for \( d_t \):

\[
d_t - \frac{d_{t-1}}{\beta} = y_t - c_t - \alpha(tot_t + \psi_t)
\]

which can be interpreted as the evolution of the net foreign asset position.

Equations (1)-(11) are the equations for the domestic block of the open economy model in 10 unknowns \( \{c_t, y_t, r_t, q_t, tot_t, \pi_t, \pi_{H,t}, \pi_{F,t}, \psi_t, d_t\} \). The exogenous disturbances \( \{\varepsilon_{a,t}, \varepsilon_{g,t}, \varepsilon_{c,t}, \varepsilon_{p,t}\} \) are assumed to be independent AR(1)-processes, while the monetary policy shock \( \varepsilon_{m,t} \) is an i.i.d process.

The foreign block of the model consists of the variables \( y_t^*, \pi_t^* \) and \( r_t^* \), and as in JP10 we here assume their paths to follow a vector autoregressive process of order two:

\[
\begin{bmatrix}
y_t^* \\
\pi_t^* \\
r_t^*
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix} \begin{bmatrix}
y_{t-1}^* \\
\pi_{t-1}^* \\
r_{t-1}^*
\end{bmatrix} + \begin{bmatrix}
b_{11} & b_{12} & b_{13} \\
b_{21} & b_{22} & b_{23} \\
b_{31} & b_{32} & b_{33}
\end{bmatrix} \begin{bmatrix}
y_{t-2}^* \\
\pi_{t-2}^* \\
r_{t-2}^*
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{y,t}^* \\
\varepsilon_{\pi,t}^* \\
\varepsilon_{r,t}^*
\end{bmatrix}
\]

3 The MS-DSGE model

We now extend the model presented in the preceding section. Two features of the model are assumed to be regime-dependent, namely the expectations formation and the smoothing parameter of the Taylor rule.

The original model assumes rational agents, whereas this version is endogenizing expectations formation by Markov transition probabilities, allowing the switching between rational and adaptive expectations formation. The model hence has two regimes, 1 and 2, representing rational (RE) and adaptive (AE) expectations formation respectively. Illustrating with the auxiliary variable \( z_t \), the expectational term can be written out as:

\[
E_t z_{t+1} = \Upsilon(s_t) E_t^{RE} z_{t+1} + (1 - \Upsilon(s_t)) E_t^{AE} z_{t+1}
\]

The parameter \( \Upsilon(s_t) \) is governed by the 1.order Markov process \( s_t \), and takes the value 1 in the rational expectations regime (\( s_t = 1 \)), and 0 in the adaptive expectations regime (\( s_t = 2 \)).

Letting \( p_{ij} = \text{Prob}(s_{t+1} = j|s_t = i) \), the transition probability from state \( i \) to state \( j \) is governed by the transition matrix

\[
P = \begin{bmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{bmatrix}
\]

The adaptive expectation formation is set up as follows in log-linear form\(^7\), letting \( f_t \) represent forward looking variables:

\[
E_t^{AE} \{f_{t+1}\} = E_t^{AE} \{f_t\} + \mu_i(f_t - E_t^{AE} \{f_t\})
\]

\(^7\) Lower case variables are defined as \( x_t = \log(X_t/X) \), where \( X \) is the non-stochastic steady state of the variable.
where $\mu_i \in [0,1]$ is the learning parameter. The rate of learning/adjustment rises as $\mu_i$ rises, with $\mu_i = 0$ being the case of no adaptive learning. Expectations regarding next period’s value of a variable depends on the expected value for this period and adjustments made due to deviations in these expectations from this period’s actual value. Solving iteratively we see that $E_t^{AE}\{f_{t+1}\} = \mu \sum_{k=0}^{\infty} (1 - \mu_i)^k x_{t-k}$, which is a distributed lag with exponentially declining weights. In JP10 there are five forward-looking variables of which expectations are formed, namely domestic consumption, $c_t$, domestic and imported inflation, $\pi_{H,t}$ and $\pi_{F,t}$, domestic CPI inflation, $\pi_t$, and the real exchange rate $q_t$. Hence, $f_t = [c_t \pi_t \pi_{H,t} \pi_{F,t} q_t]'$ and $\mu_i = [\mu_1 \mu_2 \mu_3 \mu_4 \mu_5]'$.

Also the smoothing parameter of the Taylor rule in (9) is allowed to switch following the same Markov-switching probabilities as the expectation formation processes:

$$r_t = \rho_r(s_t)r_{t-1} + \phi_x \pi_t + \phi_y y_t + \phi_{\Delta e} e_t + \varepsilon_{m,t}.$$  

(16)

This is to be able to model that interest rate smoothing became less important during the financial crisis as the lowering of the official bank rate was a mean to deal with the recession.

The $\Upsilon(s_t)$ parameter is calibrated in both regimes, taking the value 1 in the rational expectations regime and 0 in the adaptive expectations regime. $\rho_r(s_t)$ on the other hand, is estimated in both regimes.

The complete model can be more compactly summarised as

$$A_0 v_t = A_1(s_t) v_{t-1} + A_2(s_t) E_t^{AE}\{v_{t+1}\} + A_3 e_t$$  

(17)

for $s_t = 1, 2, l = RE, AE$, and where: $v_t$ is a vector of the model variables including the exogenous variables and $e_t$ is a vector of exogenous shocks.

$$v_t = [c_t, y_t, r_t, q_t, ln\pi_t, \pi_H, \pi_F, \psi_t, dt, y_t', \pi_t', \pi_H', \pi_F', \varepsilon_g, \varepsilon_y, \varepsilon_r, \varepsilon_{rp}, \varepsilon_{cp}]$$  

(18)

$$e_t = [\varepsilon_{g,t}, \varepsilon_{y,t}, \varepsilon_{r,t}, \varepsilon_{rp,t}, \varepsilon_{cp,t}, \varepsilon_{m,t}, \varepsilon_{\pi,t}, \varepsilon_{\pi_H,t}, \varepsilon_{\pi_F,t}]$$  

(19)

Due to the switching environment, standard techniques\(^8\) to find the solution of the model are no longer applicable. One of the solution techniques implemented in RISE is a functional iteration algorithm, which we use to solve our model. In the context of regime-switching DSGE models the technique has the advantages that it typically converges fast whenever it is able to find a solution, it can be used to solve larger systems and it is easy to implement (Maih 2015). A stable solution\(^{9,10}\) can be written as a VAR in $v_t$:

$$v_t(s_t) = \Omega(s_t) v_{t-1} + \Lambda(s_t) e_t,$$  

(20)

where $\Omega$ and $\Lambda$ are functions of the structural parameters of the model.

### 3.1 Data and estimation procedure

The model is estimated using Bayesian methods on quarterly UK (domestic economy) and US (foreign economy) time series spanning 1997Q2 : 2016Q4, while the observations 2017Q1 :


\(^{9}\) Following Hamilton (2016).

\(^{10}\) There might be more than one solution and functional iterations are not able to find all of them (Farmer, Waggoner, and Zha 2011).
2019Q1 are retained for forecasting evaluation. The aggregate variables used as observables include output, inflation, interest rate, terms of trade, real exchange rate, foreign output, foreign inflation, foreign interest rate, shown in Figure 1, with the start of the forecasting period indicated by the vertical line. The data have mainly been collected from Macrobond, while the exchange rate data and the data for the foreign economy were taken from the Database at the Federal Reserve Bank of St. Louis. More information on the observables used is found in the appendix.

In May 1997 the Monetary Policy Committee was formed under the independent Bank of England (BoE) and given the responsibility of setting the interest rate. This move of monetary policy from the Treasury to the BoE started a period of increased interest rates in the UK, and is the main reason for starting our sample period in 1997. Throughout most of the sample period, the UK interest rate is higher than the US interest rate as displayed in the lower left panel of figure 1. Both the BoE and the Federal Reserve (Fed) reacted to the financial crisis by lowering their rates. The Fed lowered its base rate to 0.25%, which is effectively zero - and kept it there until 2015. During the same period, BoE did not go lower than 0.5%. However, this changed in August 2016, when BoE set its base rate to 0.25% to boost quantitative easing after the Brexit vote two months earlier.

The top left panel of figure 1 shows the growth rates of the two countries’ real GDP per capita. The series are rarely corresponding, except for the financial crisis when both series show a negative growth rate of approximately -2.4% in the last quarter of 2008. The growth rates are mainly positive, with a couple of other exceptions with negative growth. For instance is there a couple of quarters in the early 2000s with negative growth in the US, presumably corresponding to the dot-com crash. Regarding the consumer price (CPI) inflation rates displayed in the top right panel of figure 1, they are mainly moving around a mean of approximately 0.5% corresponding well to an annual inflation target of 2%. The financial crisis led to a much steeper drop in the CPI of the US than of the UK (-3.48% to -0.70%).

The lower right panel of figure 1 displays the real exchange rate and the terms of trade variables. The real exchange rate shows some large fluctuations around the financial crisis. The British pound is first appreciated relatively to the U.S. dollar, presumably due to the fact that the Fed started to lower its base rate already in late 2007 while the BoE first cut its interest rate in late 2008. This is reflected in the real exchange rate growth which increases drastically before becoming negative for a short period during the financial crisis. Another curiosity of the real exchange rate growth rate plot, is the sharp decrease immediately following the Brexit vote in 2016. The terms of trade depicted in the same panel is relatively stable throughout the series, with the exception of the beginning of the sample period. The large fluctuations in the beginning of the sample of the terms of trade are caused by changes in the export price deflator - possibly reflecting the uncertainty following the Asian financial crisis of 97.

The 8 observable variables are represented by the following vector

\[ z_t = \begin{bmatrix} \Delta \log (rGDP)_t \\ \Delta \log (CPI)_t \\ \Delta \log \text{(Real exch. rate)}_t \\ \Delta \log \text{(Terms of trade)}_t \\ LIBOR_t/4 \\ \Delta \log (rGDP)_t^* \\ \Delta \log (CPI)_t^* \\ \text{FEDFUNDS}/4 \end{bmatrix} \]
Figure 1: The observables of the model, seasonally adjusted quarterly data

which is connected to the model state variables through a measurement equation of the form

$$z_t = H v_t$$  \hspace{1cm} (21)$$

where no measurement errors are needed, as the eight stochastic shocks of the model help avoid the potential problem of stochastic singularity.

The solution of the model (20) combined with the measurement equation (21) form the regime-switching state-space form. We apply a modification of the Kim and Nelson filter to this state-space form to compute the (approximate) likelihood of the data given a parameter vector $\theta$, which includes both the structural model parameters and the parameters pertaining to the transition matrix$^{11}$.

The likelihood is then combined with a prior distribution on the parameters to form the posterior kernel that is maximized to find the mode of the posterior distribution. For this optimization process we use a trust-region reflective algorithm based on the method described in Coleman and Liu (1994,1996).

$^{11}$ The difference between the Kim and Nelson filter and the filtering algorithm used in this paper resides in the collapsing rule. The latter algorithm is computationally more efficient but yields the same results as the Kim and Nelson algorithm.
We take a sample from the posterior distribution using a standard Markov Chain Monte Carlo (MCMC) technique, namely the Metropolis-Hastings algorithm.

To evaluate the model fit to the data and to compare it to rival models in a Bayesian context, one needs the marginal data density defined as

\[ p(Y_T) \equiv \int p(Y_T|\theta) p(\theta) d\theta \]  

(22)

All those algorithms mentioned are coded in the RISE toolbox, which we use for implementation.

3.2 Prior information

The priors are mainly the same as those used by Justiniano and Preston (2010), and described in the four first columns in Table 1. The discount rate and the debt sensitivity parameter \( \beta \) and \( \chi \) are calibrated and set equal to 0.99 and 0.01 respectively. For the inverse Frisch elasticity and the elasticity of substitution between home and foreign goods, loose Gamma priors with large tails are adopted. The risk aversion parameter is also given a Gamma prior, accommodating values from 0.5 to well above 1, as all are plausible estimates in a model of this type. Parameters that are restricted to lie between 0 and 1 are mainly given Beta priors. For the learning parameters, the Beta priors are in accordance with the work of Levine et al. (2012), while the other Beta priors are the same as in Justiniano and Preston (2010). The switching probabilities are given a uniform distribution, restricting the values to be between 0 and 1. The standard deviations of the shocks are given an inverse gamma distribution with a mean of 0.5 and infinite variance (by fixing the degrees of freedom to 2), reflecting that we have little prior information on the shocks. Regarding the VAR(2)-priors which are not included in table 1, we also here follow Justiniano and Preston (2010), as they also use US data for the foreign economy.

4 Results

The switching model is estimated as described earlier in RISE, finding a posterior mode. Following the estimation, a MCMC procedure is initiated with the inverse Hessian estimated at the posterior mode. Two chains of 200,000 draws are simulated with 10,000 initial draws discarded and every 20th draw afterwards saved for a total volume of 10,000 draws for each block. The two chains converged towards the same mean, which together with standard deviation and the 90% credibility intervals are reported in Tables 1 and 2. Convergence diagnostics are found in the appendix, with plots of prior and posterior distributions, trace plots, recursive mean plots in addition to Gelman-Rubin plots.

Figure 2 shows the data together with shadowed fields for the smoothed regime probabilities of the AE-regime. The model is in the adaptive expectations regime in the start of the sample (from 1997Q2 to 1998Q2) and during the financial crisis in the last quarter of 2008 and the first quarter of 2009. The shift corresponds well to a sharp decrease in the UK interest rates. From October 2008 to January 2009 the 3-month LIBOR decreased from 4.82% to 1.17%. In figure 2, it is also shown that the growth of both UK and US real output are at their lowest at the time of the shift to the AE-regime. The real exchange rate growth is also unusually high, indicating that the British pound weakened against the US dollar due to the crisis.
The transition probabilities $p_{12}$ and $p_{21}$ are estimated to be 0.02 and 0.30 respectively. This indicates that the probability of staying in the RE-regime is 0.98, and the probability of leaving the RE-regime cannot be said to be significantly different from zero. As can be seen from Figure 2 however, the model is in the AE-regime two times during the sample period. The regime-dependent smoothing parameter in the Taylor rule is estimated to be relatively higher in the RE-regime than in the AE-regime with mean values of 0.93 and 0.64 respectively. This reflects that the Bank of England lowered its interest rate to deal with the recession following the financial crisis.

Some of the (non-switching) parameters are unusually low. The openness parameter $\alpha$ is estimated to be 0.05.\footnote{This parameter was calibrated in Justiniano and Preston (2010). We found that calibrating $\alpha$ led to the risk aversion parameter $\sigma$ to be close to zero, and hence chose to estimate both.} The risk aversion parameter $\sigma$ is estimated to 0.28, which is somewhat lower.

\textbf{Figure 2:} Data (left-hand scale) and regime probability (right-hand scale) for the AE-regime.
Table 1: Prior densities and posterior estimates of structural parameters for the RS-DSGE model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior density</th>
<th>Prior</th>
<th>Mean</th>
<th>SD</th>
<th>Mean</th>
<th>SD</th>
<th>5%</th>
<th>95%</th>
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<td>Openness parameter α</td>
<td>Beta</td>
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<td>0.05</td>
<td>0.02</td>
<td>0.02</td>
<td>0.10</td>
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<td>Risk aversion σ</td>
<td>Gamma</td>
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<td>0.10</td>
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<td>0.95</td>
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<td>Calvo dom. prices θ_H</td>
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<td>0.10</td>
<td>0.30</td>
<td>0.06</td>
<td>0.20</td>
<td>0.40</td>
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</tr>
<tr>
<td>Calvo import prices θ_F</td>
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<td>0.76</td>
<td>0.04</td>
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<td>Elasticity H-F goods η</td>
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<td>0.25</td>
<td>0.22</td>
<td>0.17</td>
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<tr>
<td>Indexation foreign δ_F</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
<td>0.03</td>
<td>0.02</td>
<td>0.003</td>
<td>0.08</td>
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<td>Taylor smoothing RE regime φ_r1</td>
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<td>0.25</td>
<td>0.93</td>
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<td>0.84</td>
<td>0.99</td>
<td></td>
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<tr>
<td>Taylor smoothing AE regime φ_r2</td>
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<td>0.25</td>
<td>0.64</td>
<td>0.20</td>
<td>0.27</td>
<td>0.91</td>
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<tr>
<td>Taylor inflation φ_π</td>
<td>Gamma</td>
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<td>0.10</td>
<td>1.51</td>
<td>0.10</td>
<td>1.36</td>
<td>1.68</td>
<td></td>
</tr>
<tr>
<td>Taylor output φ_y</td>
<td>Gamma</td>
<td>0.25</td>
<td>0.13</td>
<td>0.13</td>
<td>0.06</td>
<td>0.05</td>
<td>0.24</td>
<td></td>
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<td>Taylor exch.rate φ_Δe</td>
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<td>0.13</td>
<td>0.06</td>
<td>0.02</td>
<td>0.03</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>Learning consumption μ_1</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.81</td>
<td>0.11</td>
<td>0.61</td>
<td>0.95</td>
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</tr>
<tr>
<td>Learning CPI inflation μ_2</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.30</td>
<td>0.13</td>
<td>0.13</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>Learning dom. inflation μ_3</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.51</td>
<td>0.19</td>
<td>0.20</td>
<td>0.83</td>
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</tr>
<tr>
<td>Learning import inflation μ_4</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.90</td>
<td>0.05</td>
<td>0.81</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>Learning real exch. rate μ_5</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.70</td>
<td>0.08</td>
<td>0.56</td>
<td>0.81</td>
<td></td>
</tr>
</tbody>
</table>

*Calibrated β = 0.99 and χ = 0.01.

than what is usual (0.5-1.0). The inverse Frisch elasticity of labor supply ν is 3.19 with a rather large credible interval. The elasticity between home and foreign goods, η is 1.98. The estimates for the Calvo-parameters are 0.30 and 0.76 for domestic and import prices respectively. This indicates price contract durations of approximately 1.5 and 4 quarters.

The habit parameter h_c is not significantly different from zero, estimated to be 0.05, with a standard error of 0.04. As can be seen from the posterior distribution of h_c in figure 8, most of the distribution is concentrated close to zero. The two other potential sources of persistence in the RE model, the price indexation parameters δ_H and δ_F have mean estimates 0.22 and 0.03, respectively. As with the habit-parameter, the price indexation parameters are not significantly different from zero with their large standard errors. The Taylor-parameters for output and exchange rate are estimated to be around 0.10 and 0.06, respectively. The learning parameters range between 0.30-0.90, introducing varying degrees of persistence to the model when it shifts to the AE-regime. The mean estimates of learning for CPI inflation and domestic inflation μ_2 and μ_3 are both very imprecise, with the 90% credible interval ranging from 0.13 to 0.52 for the former and from 0.20 to 0.83 for the latter. Figure 8) also show that the posterior distribution for μ_2 has its mean to the right of its mode which is as low as 0.2.
Table 2: Prior densities and posterior estimates of structural parameters for the RS-DSGE model (continued).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior density</th>
<th>Prior</th>
<th>Posterior</th>
<th>Posterior</th>
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</thead>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td>Technology ρ_a</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.72</td>
</tr>
<tr>
<td>Preferences ρ_g</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.92</td>
</tr>
<tr>
<td>Risk premium ρ_rp</td>
<td>Beta</td>
<td>0.80</td>
<td>0.10</td>
<td>0.63</td>
</tr>
<tr>
<td>Import cost-push shock ρ_cp</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
<td>0.05</td>
</tr>
<tr>
<td>1st lag y∗_t</td>
<td>Normal</td>
<td>0.90</td>
<td>0.01</td>
<td>0.90</td>
</tr>
<tr>
<td>1st lag π∗_t</td>
<td>Normal</td>
<td>0.59</td>
<td>0.04</td>
<td>0.53</td>
</tr>
<tr>
<td>1st lag r∗_t</td>
<td>Normal</td>
<td>0.90</td>
<td>0.01</td>
<td>0.90</td>
</tr>
<tr>
<td>sd foreign output σ_y∗_t</td>
<td>Inv. Gamma</td>
<td>0.50</td>
<td>inf.</td>
<td>0.65</td>
</tr>
<tr>
<td>sd foreign inflation σ_π∗_t</td>
<td>Inv. Gamma</td>
<td>0.50</td>
<td>inf.</td>
<td>0.77</td>
</tr>
<tr>
<td>sd foreign interest rate σ_r∗_t</td>
<td>Inv. Gamma</td>
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<td>sd technology σ_a</td>
<td>Inv. Gamma</td>
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<td>inf.</td>
<td>0.43</td>
</tr>
<tr>
<td>sd preferences σ_g</td>
<td>Inv. Gamma</td>
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<td>inf.</td>
<td>1.68</td>
</tr>
<tr>
<td>sd Taylor rule σ_m</td>
<td>Inv. Gamma</td>
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<td>inf.</td>
<td>0.75</td>
</tr>
<tr>
<td>sd risk premium σ_rp</td>
<td>Inv. Gamma</td>
<td>0.50</td>
<td>inf.</td>
<td>1.05</td>
</tr>
<tr>
<td>sd import cost push σ_cp</td>
<td>Inv. Gamma</td>
<td>0.50</td>
<td>inf.</td>
<td>1.41</td>
</tr>
<tr>
<td>Transition probability, p_{12}</td>
<td>Uniform</td>
<td>0.00</td>
<td>1.00</td>
<td>0.02</td>
</tr>
<tr>
<td>Transition probability, p_{21}</td>
<td>Uniform</td>
<td>0.00</td>
<td>1.00</td>
<td>0.30</td>
</tr>
</tbody>
</table>

* For the uniform distribution min and max value for the prior density is reported.

Regarding the estimates of the foreign block, the priors were based on estimating pre-sample individual autoregressions for the three variables y∗_t, π∗_t and r∗_t (see Justiniano and Preston (2010)). The posteriors for the first lags of the VAR(2)-estimates in figure 8 are identical to the priors, indicating that the dataset did not bring additional information to the estimation. The other exogenous shocks are mostly highly persistent, with estimates ranging from 0.63-0.72. This excludes the import cost-push shock parameter, which is estimated to as low as 0.05, not significantly different from zero. The data otherwise looks informative about the volatility of all the exogenous shocks, judging from Figure 8 in the Appendix.

In addition to the suggested model with switching in expectations, we have also estimated the rational expectations model version as well as a model version with adaptive expectations. The filtered outcomes from the various versions, together with the observables, are shown in Figure 3.

Table 3: Comparing model fit

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>AE model</th>
<th>RE model</th>
<th>Switching model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chib-Jeliazkov</td>
<td>-848.73</td>
<td>-727.42</td>
<td>-696.07</td>
</tr>
<tr>
<td>Laplace</td>
<td>-826.56</td>
<td>-722.76</td>
<td>-685.26</td>
</tr>
<tr>
<td>Laplace MCMC</td>
<td>-829.4</td>
<td>-723.74</td>
<td>-685.66</td>
</tr>
<tr>
<td>Modified Harmonic Mean</td>
<td>-821.76</td>
<td>-714.51</td>
<td>-677.11</td>
</tr>
<tr>
<td>Reciprocal Importance Sampling</td>
<td>-839.97</td>
<td>-714.4</td>
<td>-677.01</td>
</tr>
</tbody>
</table>
With the exception of UK inflation, the filtered model seems to be tracking the observables quite well. This lack of fit of the inflation specification corresponds well with the the insignificance of the indexation parameter $\delta_H$ and the very low estimate of the openness parameter $\alpha$ and could therefore be a source of uncertainty when using the model for forecasting.

Table 3 reports the log marginal data densities (MDD) using different algorithms, including Modified Harmonic Mean, Chib-Jeliazkov, Reciprocal Importance Sampling, Laplace approximation using the normal distribution and Laplace approximation using the covariance matrix of the MCMC draws. For all the different measures of log-MDDs used here, the switching model is preferred to the two alternatives.

![Figure 3: Filtered variables from different model versions with the actual observables](image)

### 5 Impulse response functions

Figures 4 and 5 shows the impulse responses following a monetary policy shock and a technology shock under the two regimes. Variables respond similarly in the different regimes, but in the adaptive expectations regime the responses are much stronger. In many cases the variables do not seem to return to their steady state values following a shock in the adaptive expectations regime. For instance, the terms of trade variable initially responds positively to a monetary policy shock in both regimes. A positive shock normally leads to higher interest rate, lower...
inflation and hence the terms of trade also responds positively. The higher interest rate leads to an appreciation of the domestic currency, the interest rate will be lowered again to prevent this and the terms of trade will hence move back to its steady state in the RE-regime. In the AE-regime however, it is not clear that it does - as the variables moves back and forth across its steady state not really settling during the fifteen periods illustrated.

For the domestic interest rate, the initial responses in the two regimes are very different. The initial response in the AE-regime is large and negative - while it is relatively small and positive in the RE-regime. This is likely due to the learning mechanism in the domestic CPI inflation and real exchange rate variable in the UIP condition (see equation (10)). A monetary policy shock and a higher interest rate can normally be ‘neutralised’ in the UIP condition by raising the expected CPI inflation or expected real exchange rate for the next period. Due to the learning mechanisms in the adaptive expectations regime, the expectations cannot simply be raised as easily, as they depend (rather strongly) on the expectation past period. A monetary policy shock ceteris paribus in the AE-regime will therefore force the interest rate down to keep the UIP condition stable. As can be seen from the impulse responses also the CPI inflation and real exchange rate react negatively and stronger than in the RE regime, allowing for the interest rate to become positive the next period (before moving back to its steady state).

There are other examples of variables that do not return to their steady states in the AE-regime. For instance the imported inflation $\pi_F$ has an initial positive response to a technology shock, before becoming negative and then positive again. During the fifteen periods illustrated, the variable does not return to its steady state. As can be seen from the price-setting mechanism of the imported price inflation, it is forward-looking and hence following a learning mechanism in the AE-regime. The lower the learning parameter is, the longer time the variable will use to get back to its steady state. If the learning parameter is low enough, it might never get back to its steady state after a shock.

The volatility of the estimated AE-regime indicates that the model cannot stay in that regime for too long, which is also reflected in the estimated probability of staying in that regime ($p_{22} = 0.70$)

Figure 4: Impulse responses of a monetary policy shock.
Figure 5: Impulse responses of a technology shock corresponding to 9 quarters).

6 Forecasts

6.1 1-step ahead forecasts

The Brexit vote took place on the 23rd of June 2016. We have therefore kept observations from 2017Q1-2019Q1 to use for forecasts, in order to evaluate how the Markov-switching model forecasts the domestic observables for the period after the referendum. The forecasts are 1-step ahead, so the model is re-estimated for each quarter and a forecast for the next period is made.

The distributions of the forecasts are generated by drawing parameters from the posterior distribution, using 1000 replications for each time period. Of particular interest in the present setting is to allow for changes in expectation formation as new information accrues when forming the 1-step-ahead forecasts. We therefore use forecasts with uncertainty in regimes and shocks, dependent upon transition probabilities and shock distributions, in addition to allowing for parameter estimation uncertainty.

Not surprisingly, therefore, the 1-step ahead forecasts, shown in Figure 6 are characterised by substantial uncertainty, as documented by the prediction intervals. However, using this model, we find that the Brexit referendum result in 2016 has not lead to a new shift from rational to adaptive expectations as of 2019q1, as shown in Figure 12. Also, the smoothed regime probabilities are invariant to the extension of the sample, indicating that the earlier result of shifts from rational expectations to adaptive expectations during the change in monetary policy and the financial crisis is robust.

This is perhaps not so surprising, since only the real exchange rate seem to have been heavily influenced by the outcome of the Brexit-vote, as shown in Figure 6.
6.2 Ex-ante dynamic forecasts

The actual consequences of the Brexit vote will presumably be clear by the end of 2019. Given the uncertainty of the outcome and the resulting economic environment, it could therefore be interesting to use the model to forecast the period post Brexit. We therefore compute two sets of dynamic forecasts for the period 2019q2-2020q4, conditional upon the two regimes of rational and adaptive expectations formation. The main results are reported in Figure 7. Regardless of regime, the model forecasts suffer from considerable uncertainty. The main difference between the two regimes, however, is that the non-rational regime induces higher uncertainty with often substantially wider prediction bands for the forecasted outcomes, corresponding to less information about the economic environment. This is in particular evident for variables including the exchange rate, with a substantial higher prediction uncertainty affecting both real exchange rate and terms of trade growth under an adaptive expectations regime.

Figure 6: 1-step-ahead forecasts for the domestic variables
7 Conclusions

We have used the past financial crisis to investigate the possibility of allowing for switching in expectation formation in a DSGE model. The model we used is based on the work of Justiniano and Preston (2010), where we introduced Markov switching between rational expectations and adaptive expectations. The model has then been estimated using U.K. and U.S. data to analyse whether an economic shock in the scale of the past financial crisis or the possible consequences of Brexit could induce a shift in the expectation formation.

When estimating the model on a sample from 1997Q2-2016Q4, we find evidence of switching, in particular during the financial crisis. Based on the log marginal data densities, using different algorithms, our model also was an improved fit compared to non-switching model alternatives. The model is mainly in the rational expectations regime, but switches to the adaptive expectations regime during the financial crisis possibly reflecting less information about the economic environment.

The switching of expectations is also robust to an expanded estimation sample, where we have estimated the model adding more observations at the end of the sample from 2016Q4-2019Q1.

We have also investigated the properties of the model in terms of impulse responses and forecasting in the different expectations regimes. The impulse response functions indicated that the adaptive expectations regime is more volatile and that the learning mechanism might prevent variables from returning to their steady states after a shock. In the dynamic forecasting exercise, there were considerably higher uncertainty associated with the forecasts in the adaptive expectations regime, being in line with an interpretation of lack of information.
The alternative hypothesis to rational expectations chosen in this paper is simplest one possible and should therefore not be interpreted to be the most realistic alternative, but rather as a simplified first attempt. It would therefore be of interest to investigate both more realistic alternatives to expectation formation in a switching environment, as well as alternative proxies for information switching mechanisms.
References


A Appendix

A.1 Data description

**Output** ($y_t$): Real seasonally adjusted GDP per capita for the UK. Log differenced.

**Nominal interest rate** ($r_t$): The average three-month LIBOR expressed as net quarterly interest rate.

**Real exchange rate** ($q_t$): The real exchange rate is calculated using the nominal exchange rate between the UK and the U.S., and the consumer price indices from the two countries. Log differenced.

**Terms of trade** ($s_t$): Calculated using the import and export price deflators from the UK. Log differenced.

**CPI Inflation** ($\pi_t$): The seasonally adjusted consumer price index for all goods in the UK. Log differenced.

**Foreign output** ($y^*_t$): Seasonally adjusted real US GDP per capita. Log differenced.

**Foreign inflation** ($\pi^*_t$): The seasonally adjusted consumer price index for all goods in the U.S. Log differenced.

**Foreign interest rate** ($r^*_t$): The Federal Funds interest rate expressed as net quarterly interest rate.

All variables are in logs and demeaned unless otherwise specified. The data were collected from Macrobond and the Federal Reserve Bank of St. Louis.

A.2 Convergence diagnostics for 2016

See figures 8, 9, 10 and 11.
Figure 8: Prior and posterior distributions of parameters from UK model (2016 sample). Dashed curves are the prior distributions, while the solid curves are the posterior distributions of the respective parameters.
Figure 9: Traceplot UK model (2016 sample)
Figure 10: Recursive means of the parameters in the UK model (2016 sample)
Figure 11: Gelman-Rubin plots for selected parameters of the UK model (2016 sample)
### A.3 Results and convergence diagnostics for 2019

**Table 4:** Prior densities and posterior estimates of structural parameters for the RS-DSGE model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior density</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td>Openness parameter: ( \alpha )</td>
<td>Beta</td>
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<td>0.20</td>
</tr>
<tr>
<td>Risk aversion: ( \sigma )</td>
<td>Gamma</td>
<td>1.20</td>
<td>0.40</td>
</tr>
<tr>
<td>Inverse Frisch: ( \nu )</td>
<td>Gamma</td>
<td>1.50</td>
<td>0.75</td>
</tr>
<tr>
<td>Calvo dom. prices: ( \theta_H )</td>
<td>Beta</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>Calvo import prices: ( \theta_F )</td>
<td>Beta</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>Elasticity H-F goods: ( \eta )</td>
<td>Gamma</td>
<td>1.50</td>
<td>0.75</td>
</tr>
<tr>
<td>Habit: ( h_c )</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>Indexation domestic: ( \delta_H )</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>Indexation foreign: ( \delta_F )</td>
<td>Beta</td>
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<td>0.25</td>
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<tr>
<td>Taylor smoothing: ( \phi_{r1} )</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>RE regime: ( \phi_{r2} )</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>Taylor inflation: ( \phi_{\pi} )</td>
<td>Gamma</td>
<td>1.50</td>
<td>0.10</td>
</tr>
<tr>
<td>Taylor output: ( \phi_y )</td>
<td>Gamma</td>
<td>0.25</td>
<td>0.13</td>
</tr>
<tr>
<td>Taylor exch rate: ( \phi_{\Delta e} )</td>
<td>Gamma</td>
<td>0.25</td>
<td>0.13</td>
</tr>
<tr>
<td>Learning consumption: ( \mu_1 )</td>
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<tr>
<td>Learning CPI inflation: ( \mu_2 )</td>
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<tr>
<td>Learning dom. inflation: ( \mu_3 )</td>
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<td>0.20</td>
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<tr>
<td>Learning import inflation: ( \mu_4 )</td>
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<td>Learning real exch. rate: ( \mu_5 )</td>
<td>Beta</td>
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<td>0.20</td>
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</table>

\(a\) Sample 1997Q2-2019Q1  
\(b\) Log-MDD (Laplace): -705.16  
\(c\) Calibrate \( \beta = 0.99 \) and \( \chi = 0.01 \).

For convergence diagnostics, see figures 13, 14, 15 and 16.
Table 5: Prior densities and posterior estimates of structural parameters for the RS-DSGE model (continued).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior density</th>
<th>Mean</th>
<th>SD</th>
<th>Mean</th>
<th>SD</th>
<th>5%</th>
<th>95%</th>
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<td>Preferences, $\rho_g$</td>
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<td>0.10</td>
<td>0.92</td>
<td>0.02</td>
<td>0.89</td>
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<td>0.25</td>
<td>0.06</td>
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<tr>
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<td>sd import cost push, $\sigma_{\sigma}$</td>
<td>Inv. Gamma</td>
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<td>inf.</td>
<td>1.38</td>
<td>0.18</td>
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<td>0.29</td>
<td>0.17</td>
<td>0.07</td>
<td>0.61</td>
</tr>
</tbody>
</table>

\[\textsuperscript{a}\] Sample 1997Q2-2019Q1
\[\textsuperscript{b}\] Log-MDD (Laplace): -705.16
\[\textsuperscript{c}\] For the uniform distribution min and max value for the prior density is reported.
Figure 12: Data and regime probability for the AE-regime, sample 1997Q2-2019Q1.
Figure 13: Prior and posterior distributions of parameters from UK model (2019 sample)

Dashed curves are the prior distributions, while the solid curves are the posterior distributions of the respective parameters.
Figure 14: Traceplot UK model (2019 sample)
Figure 15: Recursive means of the parameters in the UK model (2019 sample)
Figure 16: Gelman-Rubin plots for selected parameters of the UK model (2019 sample)