ISSN 1503-299X

WORKING PAPER SERIES

No. 3/2025

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September 2025

JEL classifications: C61 G11 G12

Key Words:

External habit

Dual function

Fixed point

I am greatly indebted to Fabian Andsem Harang and Silvia Lavagnini for pointing out to me the likelihood of a flaw in the solution commonly used for decision making with external habits. The responsibility for errors and omission is mine. Comments welcome.

Abstract

This paper points out a flaw in the solution method for dyanmic optimization with external habit formation, such as in Campbell and Cochrane (1999). Their method, which is common in the literature, is to impose the equality between the agent's and peers' decisions in the agent's optimization problem upon deriving the first-order conditions. This paper demonstrates the fallacy of this shortcut. Based on the dual function approach, I show that the results are likely to be different qualitatively as well as quantitatively if the above equality instead is imposed after optimization as the solution to a fixed-point problem in market equilibrium.

1 Introduction

In Campbell and Cochrane (1999), hereafter referred to as CC, the authors propose a solution to the equity premium puzzle, which was first pointed out by Mehra and Prescott (1985). According to Google Scholar, CC have been cited 6,872 times. The possibility of a major flaw in such a widely cited paper should thus hold considerable interest.

A key mechanism in the CC model is behavior based on external habits, that is, habits based on peers' past consumption rather than the agent's own. The idea of external habits was first introduced by Abel (1990), who dubbed it, "catching up with the Joneses." When embedded in a symmetric macrofinancial model, it means that each agent's preferences are shaped by everybody's identical history, but that no agent takes his or her own contribution to that history into account when making decisions on investment and consumption.

The common history is thus treated as exogenous in the derivation of the first-order conditions for consumption and portfolio composition. This is the essence of an external habit. However, once the first-order conditions are derived, CC impose the equality between the agent's and peers' decisions directly into them and use the formulae thus transformed in their subsequent analysis.

I claim that this shortcut is erroneous and that the correct procedure must be to treat peers' consumption as exogenous through the entire derivation of the respective policy functions. Only when these functions have been derived, the equality between the agent's and peers' consumption can be imposed as a the solution to a fixed-point problem in a symmetric market equilibrium. I furthermore claim that this correction has significant consequences for the solution, quantitatively as well as qualitatively.

I do not present a full reworking of CC's macrofinancial model. Rather, for clarity, I focus on the partial analysis of each agent's Merton (1969) problem and the resulting implications in a symmetric market equilibrium. For this purpose, I use the same specification as CC, with one exception, namely, that I replace their rather convoluted form of habit updating with the linear Markovian specification proposed by Constantinides (1990) for an internal habit. This simplification allows me to derive closed-form solutions for the policy functions with and without my proposed correction and demonstrate their difference. My solution uses the dual value function approach outlined in Section 1.3 of Rogers (2013). To my knowledge, this procedure has not previously been used to solve habit-formation models.

Because of the simpler form of the habit dynamics, I believe my specification avoids the issues debated by Ljungqvist and Uhlig (2015) and Campbell and Cochrane (2015). Like the latter, I carry out the analysis in continuous time.

2 Model Setup

Like CC and many others, I specify the instantaneous utility function with a linear habit as

$$u(c-x) = \frac{(c-x)^{1-\gamma}}{1-\gamma}, \gamma > 0, \tag{1}$$

where c is consumption and x the habit. As usual, the limiting case of $\gamma = 1$ is covered by the logarithmic function.

In order to highlight the consequences of the choice of solution method, I assume that the habit is a simple, linear functional of past consumption, as in Constantinides (1990). For the habit to be external, the past consumption underlying habits must be peers' consumption $\bar{c}$ rather than the agent's own consumption c. Thus, I assume the habit to build over time as

$$x_t = e^{-(\lambda+\mu)t}x_0 + \lambda \int_0^t e^{-(\lambda+\mu)(t-s)}\bar{c}_s ds, \tag{2}$$

where λ and μ are non-negative constants. This specification implies the following law of motion:

$$dx_t = \lambda(\bar{c}_t - x_t)dt - \mu x_t dt. \tag{3}$$

The non-negativity of μ helps prevent the habit from outgrowing consumption.

Optimization is also subject to the wealth dynamics constraint

$$dw_t = \big[(r + \alpha_t \pi) w_t - c_t\big] dt + \alpha_t w_t \sigma dB_t, \tag{4}$$

where w is wealth, r the riskless rate of return, α the equity share, π the equity premium, B a Brownian motion, and σ the standard error of equity returns. I thus assume, for simplicity, the presence of only one risky asset, equity, and that the riskless rate, the equity premium, and the standard error all are constant.

In this model, it is convenient to think of the agent as composed of two departments, A and B. The sole responsibility of Department B is to fund and carry out the habit part of consumption. For this purpose, it needs to maintain a portfolio X_t that is just large enough to fund the habit at each point in time. To keep utility from falling to minus infinity if the habit is not maintained, this portfolio must be invested in the riskless asset only. Thus, as noted by Lindset and Mork (2019) in their Appendix A.2, its movement can be described as

$$dX_t = (rX_t - x_t + \tau(\bar{c}_t - x_t))dt,$$

where τ denotes a rate of transfer to Department B's portfiolio from Department A that may be needed to keep the habit continually funded. Suppose further that the wealth set aside to fund the habit is proportional to the habit, so that

$$X_t = \varphi x_t,$$

say. Then, the movement of the habit over time must satisfy

$$dx_t = (r - 1/\varphi)dt + (\tau/\varphi)(\bar{c}_t - x_t)dt.$$

For this equation and (3) to hold at the same time, we must obviously have $\varphi = 1/(r + \mu)$

and $\tau = \lambda/(r+\mu)$.

We may thus define the "free" wealth to be managed by Department A as

$$\omega_t := w_t - X_t = w_t - \frac{x_t}{r+\mu}.$$

Similarly, define above-habit consumption

$$\kappa_t := c_t - x_t$$

and

$$\bar{\kappa}_t := \bar{c}_t - x_t.$$

Furthermore, let ξ_t denote the equity share of ω_t. The law of motion for the free wealth can then be written as

$$d\omega_t = \left[(r+\xi_t\pi)\omega_t - \kappa_t - \tau\bar{\kappa}_t\right]dt + \xi_t\omega_t\sigma dB_t. \tag{5}$$

Note that the term $\tau\bar{\kappa}_t$ can be interpreted as a tax to be paid by Department A to Department B. The task of Department A is now to determine a strategy for consumption in excess of the habit, κ_t, so as to maximize expected utility subject to (5). Then we can define $V(t,\omega)$ as the value function for department A. Letting subscripts denote partial derivatives, department A's HJB equation becomes

$$0 = \max_{\kappa_t,\xi_t}\left\{e^{-\rho t}u(\kappa_t) + V_t + V_\omega\left[(r+\xi_t\pi)\omega_t - \kappa_t - \tau\bar{\kappa}_t\right] + \frac{1}{2}\xi_t^2\omega_t^2\sigma^2 V_{ww}\right\}. \tag{6}$$

3 Two alternative solutions

What distinguishes an external from an internal habit is that, with an external habit, the first-order condition for κ is derived under the assumption that the agent ignores the effect that this choice has on future habits. In other words, for this purpose, $\bar{\kappa}$ is treated as exogenous, implying the first-order condition

$$e^{-\rho t} u'(\kappa_t) = V_\omega. \tag{7}$$

However, from there on, CC and the rest of the literature typically proceed to impose the constraint $\kappa = \bar{\kappa}$. Then, the tax just mentioned becomes a tax on above-habit consumption at the constant rate τ. Thus, the manager of department A can avoid part of the tax by consuming less. I argue that this shortcut is inconsistent with the exogeneity of $\bar{\kappa}$. If other agents' consumption is truly exogenous to our agent's behavior, the tax should instead be perceived as a lump-sum tax that cannot be avoided by lowering consumption.

The solution to the model under that assumption has not, to my knowledge, been derived before. However, I show in the Appendix how the dual value function approach in Section 1.3 of Rogers (2013) can be used to derive the following solution to the PDE defined by (6):

$$V(t, \omega) = \frac{e^{-\rho t} \eta^{-\gamma}}{1 - \gamma} \big[\omega - (\tau/r)\bar{\kappa}\big]^{1-\gamma}, \tag{8}$$

where

$$\eta := \rho/\gamma + (1 - 1/\gamma)\big[r + \frac{1}{2\gamma}(\frac{\pi}{\sigma})^2\big].$$

By the argument presented in Rogers (2013), this solution is unique. The constant η is easily recognized as the optimal consumption-wealth ratio for an agent with CRRA preferences and no habit. I assume $\eta > 0$, for which $\gamma \geq 1$ is a sufficient (though by no means necessary) condition, provided ρ and r are both positive. I also assume

$$\omega - (\tau/r)\bar{\kappa} > 0.$$

The resulting policy functions for this solution are

$$\kappa_t^* = \eta\big[\omega - (\tau/r)\bar{\kappa}_t\big] \tag{9}$$

and

$$\xi_t^* = \left[1 - \frac{\tau\bar{\kappa}_t}{r\omega_t}\right](\frac{\pi}{\gamma\sigma^2}). \tag{10}$$

Even though $\bar{\kappa}_t$ is exogenous, it must equal κ_t in a symmetric market equilibrium. Solving this fixed-point problem is now trivial, yielding the solutions for κ and ξ in market equilibrium as

$$\kappa_t^{fp} = \frac{\eta\omega_t}{1 + (\eta/r)\tau} \tag{11}$$

and

$$\xi_t^{fp} = (\frac{r}{r + \eta\tau})(\frac{\pi}{\gamma\sigma^2}), \tag{12}$$

where the superscript fp stands for "fixed point".

The alternative solution follows the CC procedure. Expressed in the language of my two-department interpretation, it amounts to optimizing the HJB equation (6) with respect to ξ_t, subjecct to (7) and $\bar{\kappa}_t = \kappa_t$ as joint constraints. Then, the same method as above implies the following policy functions:

$$\kappa_t^{im} = \frac{\eta\omega}{1 + (1 - 1/\gamma)\tau}, \tag{13}$$

and

$$\xi_t^{im} = \frac{\pi}{\gamma\sigma^2}, \tag{14}$$

where the superscript im stands for "imposed".

4 Comparison

The two solutions are obviously different. A Department A manager who views the tax $\tau\bar{\kappa}$ as a lump sum will keep risk low. In fact, the equity share ξ^{fp} is likely to be *much* smaller than ξ^{im}. To see that, note that λ, which roughly represents the speed at which habits catch up with actual consumption, is likely to be at least one order of magnitude greater than the riskless rate r. The other catch-up parameter μ should at most be of the same order of magnitude as λ. Thus, the implicit tax rate τ is likely to be at least one order of magnitude greater than r. That means that the ratio between ξ^{fp} and ξ^{im} is likely to be much closer to zero than to unity.

With lower risk taking, and hence more modest harvesting of the risk premium, above-habit consumption relative to wealth will also be kept lower, so that $\kappa^{fp} < \kappa^{im}$ for the same level of free wealth ω. However, for comparison with data, we are interested in total consumption c and equity as a share of total wealth α. To parallel CC's analysis, I focus on the solutions for the equity share

$$\alpha = \xi\omega/w = \xi\left[1 - \frac{x/w}{r+\mu}\right]. \tag{15}$$

Clearly, $\alpha < \xi$ under either solution. As the habit moves sluggishly in response to financial surprises, α is not constant. However, simulation of the model suggests that its probability distribution converges to a stochastic steady state rather quickly. Figures 1 and 2 display the distributions implied by the respective solution methods for the ratio of free to total wealth, ω/w, as the simulated distribution for the last period after simulation over 50 years with 10,000 duplications and time steps $\Delta t = 1/12$, corresponding to months.

Parameter	**Symbol**	**Value**
Equity premium	π	0.06
Standard error of equity returns	σ	0.165
Riskless rate	r	0.01
Subjective discount rate	ρ	0.037
Curvature parameter	γ	2.1

Table 1: Parameter values used in simulations

For these simulations, I use the same parameter values as Constantinides (1990), listed in Table 1, except for λ and μ, which govern the habit updating. Because their values cannot easily be inferred from data, I use the central case in Constantinides (1990), which in my terminology translates as $\lambda = 0.4$ and $\mu = 0.07$. However, to indicate parameter

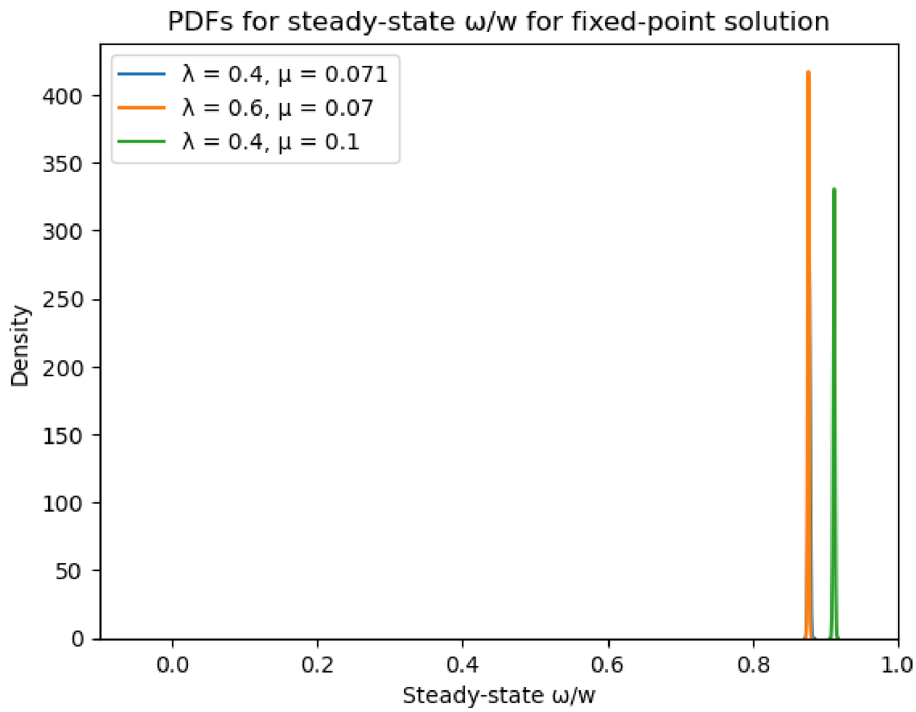

Figure 1: Simulated steady-state probability distributions of the ratio of free to total wealth for the fixed-point solution. Parameter values: $\pi = 0.06, \sigma = 0.165, r = 0.01, \rho = 0.037, \gamma = 2.1$.

sensitivity, I also display the results for a higher $\lambda = 0.6$ and a higer $\mu = 0.1$.

The difference between the two solutions is striking. Because the fixed-point solution implies lower above-habit consumption relative to wealth, the habit also tends to be low relative to wealth. As a result, the ratio in (15) is close to unity and hardly variable at all. It is also quite insensitive to variations in the habit parameters λ and μ. In fact, the difference between the cases of $\lambda = 0.4$ and $\lambda = 0.6$ is barely visible to the naked eye.

For the solution where the equality $\kappa = \bar{\kappa}$ is imposed in the HJB equation, the ratio x/w is not only greater, but also more variable. The sensitivity to parameter values is also greater, especially in response to changes in μ because a higher μ slows the growth of the habit even as it raises the denominator in (15).

Figures 3 and 4 display the corresponding steady-state probability distributions for the two solutions for equity as the share α of total wealth. Despite the variability and the sensitivity to paramater values, the overall impression is very clear: The two solution

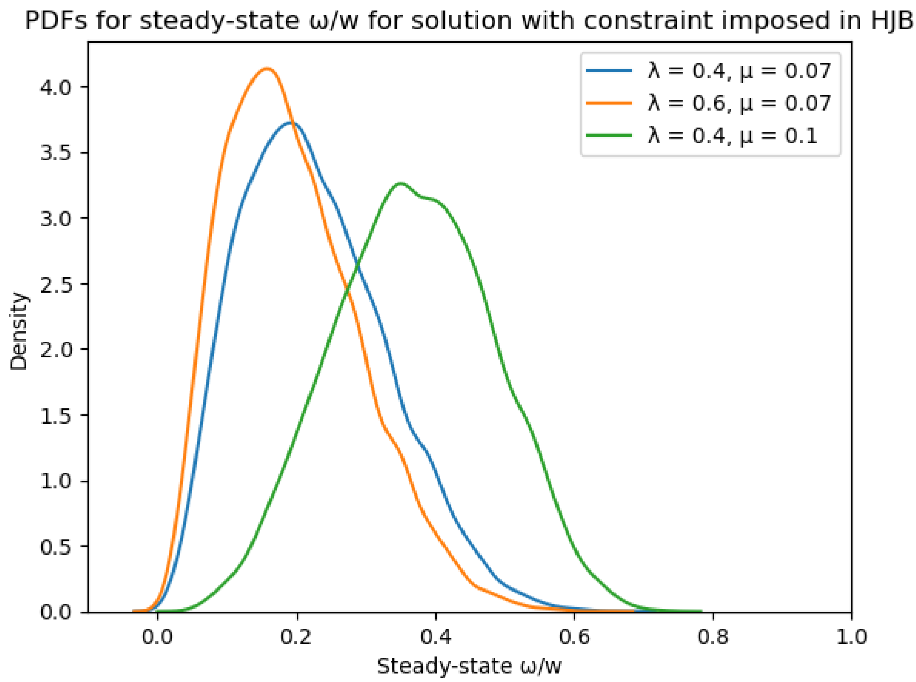

Figure 2: Simulated steady-state probability distributions of the ratio of free to total wealth for the solution where the equality $\kappa = \bar{\kappa}$ is imposed in the HJB equation. Parameter values: $\pi = 0.06, \sigma = 0.165, r = 0.01, \rho = 0.037, \gamma = 2.1$.

methods imply very different results. When the exogeneity of other agents' consumption is taken seriously in the optimization procedure and their equality imposed as the solution to a fixed-point problem, the implied equity share is typically considerably lower, much less variable, and quite robust in regard to parameter values.

Conclusion

The model used in this paper is both simpler and more partial than the one employed by CC. Their conclusions depend heavily on the special process they assume for habit updating as well as their inclusion of a supply side for goods, which allows derivation of general-equilibrium results. Whereas their model derives a result for the equity premium, which can then be compared to the empirical data, I assume a fixed equity premium. Thus, my results are not directly comparable to theirs. However, my finding that a correct treatment

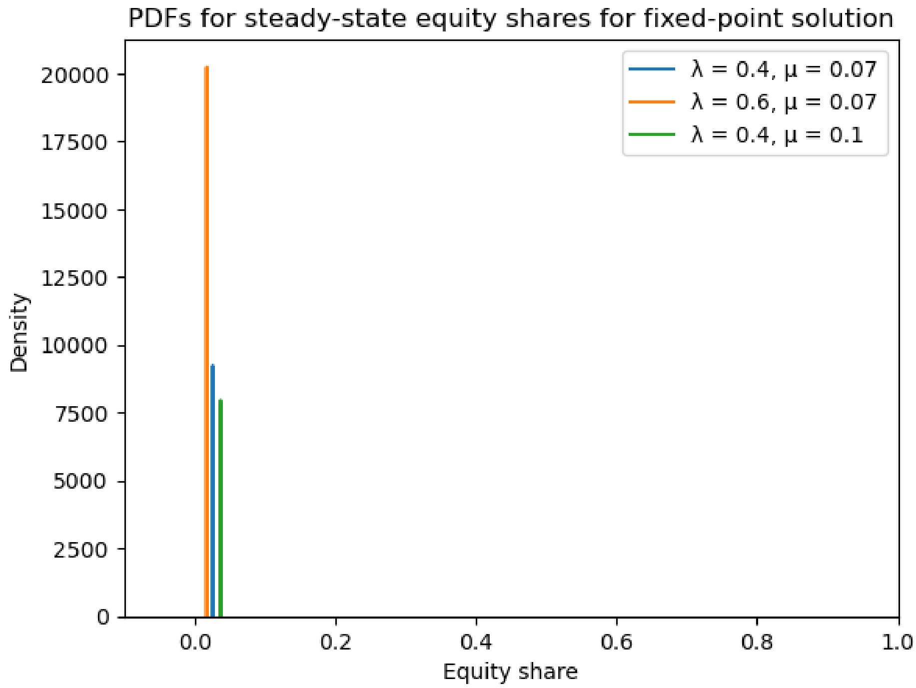

Figure 3: Simulated steady-state probability distribution of the equity share for the fixed-point solution. Parameter values: $\pi = 0.06, \sigma = 0.165, r = 0.01, \rho = 0.037, \gamma = 2.1$.

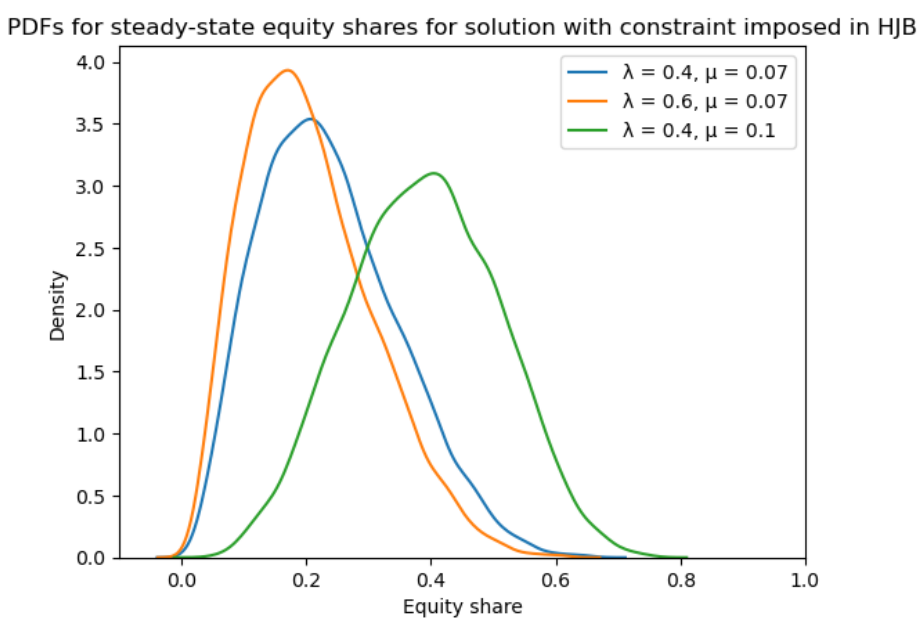

Figure 4: Simulated steady-state probability distribution of the equity share for the fixed-point solution. Parameter values: $\pi = 0.06, \sigma = 0.165, r = 0.01, \rho = 0.037, \gamma = 2.1$.

of the exogeneity of peers' consumption typically implies a significantly lower equity share for a given equity premium suggests that their model should have predicted a higher equity premium in general equilibrium. In other words, my results suggest that their model would have overexplained the observed equity premium if correctly solved.

These suggestions may perhaps be considered speculative as long as they are based on a partial-equilibrium model. A general-equilibrium analysis lies beyond the scope of this paper. However, my analysis suggests that the authors just mentioned should review their analysis in view of my findings. Such an exercise should indeed be quite interesting.

Appendix A: Closed-form PDE solutions

Optimization of 6 with respect to ξ_t, while treating $\bar{\kappa}_t$ as exogenous, gives the first-order condition

$$\xi_t^* = -\frac{V_\omega}{V\omega\omega}\frac{\pi}{\sigma^2},$$

which implies

$$V_\omega \xi_t \pi \omega_t + \frac{1}{2}\xi_t^2 \omega_t^2 \sigma^2 V_{\omega\omega} = -\frac{1}{2}(\frac{V_\omega^2}{V_{\omega\omega}})(\frac{\pi}{\sigma})^2.$$

For consumption in excess of habit, the first-order condition becomes

$$e^{-\rho t}\kappa_t^{-\gamma} = V_\omega.$$

Section 1.3 in Rogers (2013) now suggests the following change of variables:

$$(t, z) = (t, V_\omega).$$

Furthermore, define $y := e^{\rho t} z$. The first-order condition for κ then clearly implies

$$\kappa_t^* = y^{-1/\gamma},$$

so that

$$e^{-\rho t}u(t,\kappa_t^*) = e^{-\rho t}\frac{y^{1-1/\gamma}}{1-\gamma}.$$

Similarly,

$$V_\omega \kappa_t^* = e^{-\rho t}y^{1-1/\gamma},$$

so that

$$e^{-\rho t}u(t,\kappa_t^*) - V_\omega \kappa_t^* = -e^{-\rho t}\frac{y^{1-1/\gamma}}{1-1/\gamma}.$$

Now, define the dual value function

$$J(t,z) := V(t,\omega) - \omega z.$$

As shown by Rogers (2013), this function has the convenient properties

$$J_z = -\omega, J_t = V_t, J_{zz} = -1/V_{\omega\omega}.$$

Furthermore, time homogeneity allows us to write $V(t,\omega) = e^{-\rho t}v(\omega)$, so that

$$J(t,z) = e^{-\rho t}v(\omega) - \omega z = e^{-\rho t}[v(\omega) - \omega y] = e^{-\rho t}j(y).$$

The time derivative of J can thus be written as

$$J_t = -\rho e^{-\rho t}j(y) + e^{-\rho t}j'(y)\frac{\partial y}{\partial t} = -\rho e^{-\rho t}\big[-j(y) + j'(y)y\big].$$

Thus, after division through by $e^{-\rho t}$, the HJB equation becomes the following linear ODE:

$$-\rho j(y) + (\rho - r)j'(y)y + \frac{1}{2}(\frac{\pi}{\sigma})^2 y^2 j''(y) = \frac{y^{1-1/\gamma}}{1-1/\gamma} + \tau\bar{\kappa}y.$$

The arguments in Rogers (2013) imply that the ODE has the unique solution

$$j_0(y) = -\frac{y^{1-1/\gamma}}{\eta(1-1/\gamma)} - (\tau/r)\bar{\kappa}y,$$

where η is defined as in the main text. Now, substitute for y to obtain the J function:

$$J(t,z) = e^{-\rho t} j(ze^{\rho t}) = -\frac{e^{-(\rho/\gamma)t} z^{1-1/\gamma}}{\eta(1-1/\gamma)} - (\tau/r)\bar{\kappa}z.$$

The negative of the z-derivative of this function equals free wealth at the optimum:

$$\omega = -J_z = e^{-(\rho/\gamma)t} z^{-1/\gamma}/\eta + (\tau/r)\bar{\kappa}.$$

Because $z = V_\omega$, we can invert this function to obtain

$$V_\omega = z = e^{-\rho t}\eta^{-\gamma}\big[\omega - (\tau/r)\bar{\kappa}\big]^{-\gamma}.$$

Straightforward integration now gives the value function in (8). Optimizing the HJB equation (6) with this value function then gives (9) and (10).

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