ISSN 1503-299X

WORKING PAPER SERIES

No. 4/2013

Livestock management at northern latitudes. Potential economic effects of climate change in sheep farming

Anne Borge Johannesen
Department of Economics, Norwegian University of Science and Technology

Anders Nielsen
Centre for Ecological and Evolutionary Synthesis (CEES), Department of Biology, University of Oslo

Anders Skonhoft
Department of Economics, Norwegian University of Science and Technology

Department of Economics

Norwegian University of Science and Technology
N-7491 Trondheim, Norway
www.svt.ntnu.no/iso/wp/wp.htm

Livestock management at northern latitudes.
Potential economic effects of climate change in sheep farming

Anne Borge Johannesen[1)]

Anders Nielsen[2)]

Anders Skonhoft[1)]

Abstract

We study the economy and ecology of sheep farming under future climate change scenarios. The analysis is at the farm level and includes two different categories of the animals, ewes (adult females) and lambs with a crucial distinction between the outdoors grazing season and the winter indoors season. The model is formulated in a Nordic economic and biological setting. During the outdoors grazing season, animals may experience growth constraints as a result of limited grazing resources. The available grazing resources are determined by animal density (stocking rate) and weather conditions potentially affecting the weight, and hence, the value of lambs. Because empirical evidence suggests that climate changes, e.g., increased temperature, have contrasting effects on lamb weights depending on the location of the farm, the spatial effects of such changes are analyzed.

Keywords: sheep farming, weather conditions, climate change, vegetation growth, stage model

1) Department of Economics, Norwegian University of Science and Technology, NO-7491 Dragvoll Campus, Trondheim, Norway

2) Centre for Ecological and Evolutionary Synthesis (CEES), Department of Biology, University of Oslo, P.O. 1066 Blindern, NO-0316 Oslo, Norway

1. Introduction

IPCC projections indicate that mean annual temperatures will increase and the increase will be strongest at higher latitudes (Solomon et al. 2007). However, summer temperatures are expected to increase more in southern Europe, while winter temperatures more in the north (Alcamo et al. 2007). Climate change is a major challenge to food and agriculture (FAO 2009) and has become a key issue for The Food and Agriculture Organization of the United Nations (FAO) (see: http://www.fao.org/climatechange). In particular, a slight warming in seasonally dry and tropical regions is expected to reduce crop yield, while the effect of elevated temperatures on pastoral systems in temperate regions is expected to be positive, at least up to a 3°C increase (Easterling et al. 2007). These projections indicate that Nordic sheep farmers will face novel climate conditions in the future. Nielsen et al. (2012) showed that in southern Norway increased spring temperature would have contrasting effects on lamb autumn body mass, depending on the location of the areas where the animals are kept during the outdoor grazing season. This indicates that any attempt to include weather conditions and climate change in optimization models for individual farmers has to be site specific. To illustrate the effect of the spatially inconsistency in climate effects, we include in our theoretical and numerical model two areas where the effect of increased spring temperature has been shown to have opposite effect. Our aim is to show how climate change may alter the body weight and the slaughter value of the animals, and how this will affect the stocking rate and profitability of the farmers.

Our sheep farming study is carried out with a crucial distinction made between the outdoors grazing season (spring, summer and fall) and the indoor winter feeding period, and between different categories of animals (lambs and ewes). Lambs are born in early spring, just before the outdoor grazing season starts, which is the typical situation found in many strongly seasonal environments at northern latitudes, such as in the Nordic countries, and at high altitudes in continental Europe, such as mountainous areas in France and Spain. The analysis essentially relates to the economic and biological setting found in Norway, but should also have relevance for sheep farmers in Iceland and Greenland, and possible also in mountainous areas in France and Spain. The problem analyzed here is to find the optimal number of animals to be fed and kept indoors during the winter season for a given farm capacity (i.e., farm size). A corollary of this problem is to find the effect that summer grazing sheep density has on vegetation productivity and hence on per-animal meat production. The problem is analyzed under the assumption that the farmer aims to do it 'as well as possible,' represented by present-value profit maximization.

The animal growth model presented in this paper builds on Skonhoft (2008). Skonhoft et al. (2010) extended this model to include a relationship between vegetation availability and lamb weight. Here we develop this relationship further by allowing lamb weights and slaughter values to be affected by weather and outdoors grazing conditions. Balancing the number of animals and weight of animals is indeed seen as a crucial management problem in the Nordic countries as well as other places (e.g., Olafsdottir and Juliusson 2000, Mysterud and Austrheim 2005, Thomson et al. 2005).

In the natural resource and agricultural economics literature, there is an increased focus on the potential effects of climate changes and weather uncertainty. Diekert et al. (2010), analyzing the Barents Sea cod fishery, assume that climate changes are channeled through a temperature variable affecting the recruitment of the cod stock, and where a higher temperature improves the recruitment. Hannesson (2007) also studies a situation where climate changes are materialized through sea temperature. His analysis is dealing with potential effects on the migration pattern of fish between the exclusive economic zones of different countries. Quaas and Baumgärtner (2012) study optimal livestock management in semi-arid rangelands with uncertain rainfall. Rainfall has no direct effect on livestock growth in their model, but affects the grazing capacity of the rangeland. They solve for the optimal stocking rate and demonstrate how it is influenced by the degree of risk aversion and amount of rainfall.

The present study differs from the above contributions in two ways. First, we consider climatic factors (i.e., temperature) as having no direct impact on animal recruitment as in Diekert et al. (2010), but as detrimental to lamb slaughter weights and hence, the per animal market values. Furthermore, we present and analyze an age-specific model consisting of adult animals and lambs. Second, along with empirical findings, we consider increased spring temperature as having a positive or negative effect on lamb slaughter weights depending on the specific site of consideration; that is, the spatial pattern and the location of the farm play a role. We focus on two mountain ranges and two scenarios; the Northern scenario, exemplified by Forollhogna in Trøndelag and the Southwestern scenario, exemplified by western side of Hardangervidda, where increased spring temperature has been shown to have a positive and negative effect, respectively, on lamb growth over summer (Nielsen et al. 2012). See Figure 1.

We analyze how temperature changes may alter the optimal slaughtering composition (lamb and ewes), the stocking rate, and profitability of the farmers. We therefore distinguish between the direct effect of a temperature change; that is, the effect on lamb weights, and the indirect effect which reflects that farmers may adapt to temperature changes by adjusting the size of the sheep population. This distinction adds new insight of potential effects of climate change on farm economy as climate studies usually focus only on the direct effect. No climate uncertainty is considered in the main modeling, but some possible effects of taking uncertainty and risk aversion into account are included in the Appendix.

Figure 1 about here

This paper is organized as follows. Section 2 describes briefly the Nordic sheep farming system. Section 3 provides information about sheep animal growth and presents the biological model. While animal population growth is unaffected by potential climate effects, weight growth per animal is affected and this relationship is discussed in Section 4. The revenue and cost functions follow in Section 5. The stocking problem of the farmer is then solved in Section 6, while Section 7 provides numerical results. Section 8 summarizes our findings.

2. The Nordic sheep farming system

There are approximately 16,000 sheep farms in Norway, all family farms. Because there are around 2.1 million animals during the outdoors grazing season, the average farm size only accounts for some 130 animals during the summer. Norwegian farms are located either close to mountain areas and other sparsely populated areas or along the coast, with a means to transport sheep to more distant alpine areas for summer grazing. The main product is meat, which accounts for about 80% of the average farmer's income. The remainder comes from wool, because sheep milk production is virtually nonexistent today (Nersten et al. 2003). On Iceland, there are about 450,000 winterfed and 1.2 million outdoor grazing animals today. Meat is also the most important product from sheep farming here. On Greenland, the available land for sheep grazing is much more restricted, and the population of ewes and outdoor grazing animals in 2007 was estimated at 25,000 and 65,000, respectively (Austrheim et al. 2008).

Housing and indoor feeding is required throughout winter because of snow and harsh weather conditions (Figure 2). In Norway, winter feeding typically consists of hay grown on pastures close to farms (80%), with the addition of concentrate pellets provided by the industry (20%) (Skonhoft et al. 2010). The spring lambing scheme is controlled by the farmers because of the In Vitro Fertilization protocol used to time the lambing to fit current climatic conditions. In late spring and early summer, the animals usually graze on fenced land close to the farm at low elevations, typically in the areas where winter food for the sheep is harvested during summer. When weather conditions permit ewes and lambs are released together into rough grazing areas in the valleys and mountains. In Norway, most sheep (about 75% of the total metabolic biomass) graze in the northern boreal and alpine region (Austrheim et al. 2008).

The outdoors grazing season ends between late August and the middle of September. The length of the outdoor grazing season is relatively fixed, partly because of local climatic conditions but also, at least in certain areas, because local traditions and historical reasons play a role in the timing. In general the outdoor grazing season does not exceed 130 days. Throughout the outdoor grazing season, lamb growth is affected by climate conditions, both directly, but also indirectly through climate effects on the vegetation (Nielsen et al. 2012). However, also weather conditions in winter and spring, before the lambs are released to their grazing areas, have been shown to affect lamb autumn weights. In particular, winter conditions affect lamb autumn weights indirectly through snow melt effects on the vegetation (Mysterud et al. 2011, Nielsen et al. 2012), while spring temperature and precipitation has an indirect effect through their effect on plant spring phenology (Nielsen et al. 2012). After the grazing season, the animals are mustered and the wool is shorn. Slaughtering takes place immediately or after a period of grazing on the farmland (more details are provided in Austrheim et al. 2008). The seasonal subdivision is similar in Iceland and Greenland.

Figure 2 about here

3. Biological model

The sheep animal growth model is formulated at a discrete time with a seasonal subdivision between the outdoors grazing period (spring, summer and fall) and indoors winter-feeding period. The sheep population is structured (e.g., Caswell 2001) as ewes and lambs. The farmers are in full control of the sheep population size, as fertility and the number of animals released in spring are unaffected by weather conditions. All natural mortality is supposed to occur during the grazing season and is also assumed to be independent of grazing and weather conditions. Accordingly, a change in the number of animals is independent of grazing and weather conditions. Natural mortality differs between adults and lambs, and is considered fixed and density independent. The rather low mortality rate of the lambs (see numerical section 7) is due to the presence of the ewes during the whole grazing season. Lambs not slaughtered, enters the adult (ewe) population after the slaughtering period (i.e., September–October). All male lambs are slaughtered because very few (or none when artificial insemination is practiced) are kept for breeding. Therefore, only female adults are considered. Demographic data on sheep are available in Mysterud et al. (2002).

The number of adult females in year ($t+1$) after the slaughter, consists of the previous year's adults and female lambs that have survived natural mortality and have not been slaughtered. This is written as $X_{t+1} = Y_t s^Y (1-h_t^Y) + X_t s^X (1-h_t^X)$, where Y_t is the number of female lambs, s^X and s^Y are the natural survival rates (fractions) of adult females and lambs, respectively, and h_t^X and h_t^Y are the fractions slaughtered. With the fecundity rate b (lambs per adult female) and ψ as the fraction of female lambs recruited (ψ is usually close to 0.5), $Y_t = \psi b X_t$ yields the number of female lambs. Therefore, the ewe population growth is governed by:

(1) $$X_{t+1} = \psi b X_t s^Y (1-h_t^Y) + X_t s^X (1-h_t^X).$$

Because the population growth equation (1) is linear for number of animals, there are infinite combinations of harvesting fractions that sustain a stable population. Therefore, for a constant number of animals $X_{t+1} = X_t = X$, we have:

(1') $$X = \psi b X s^Y (1-h^Y) + X s^X (1-h^X),$$

or simply $1 = \psi b s^Y (1-h^Y) + s^X (1-h^X)$ when $X > 0$ (see Figure 3). This isocline intersects with the h^X axis at $[1-(1-\psi b s^Y)/s^X]$, which may be above or below 1. Therefore, the highest adult slaughter rate compatible with zero animal growth is $\min\{1,[1-(1-\psi b s^Y)/s^X]\}$. For all realistic parameter values, it is below 1 (see numerical section), and this is assumed to hold in the subsequent analysis. The isocline intersects with the h_Y axis at $[1-(1-s^X)/\psi b s^Y] < 1$ and is hence the highest lamb-slaughtering rate compatible with equilibrium.

Figure 3 about here

4. Weather conditions, herbivore performance, and weight gain

High grazing pressure may cause a reduction in plant quality and/or quantity which in turn might affect herbivore growth (Mysterud and Austrheim 2005). Experimental studies show lower autumn weight of lambs at high sheep density as compared with low sheep density (Mysterud and Austrheim 2005). In the Norwegian sheep farming system, the major growth season of the animals is when they roam freely in the mountains. Consequently, the *per animal* value (autumn slaughter weight) is subject to among years variation in environmental conditions (e.g., temperature and precipitation) that influence vegetation quality and quantity. It has previously been shown that local weather conditions during winter, in spring (before the animals are released to the mountains) and during summer (the outfield grazing season) affect lamb weights (Nielsen et al. 2012). However, which weather variable (snow depth the previous winter, precipitation or temperature in spring or summer) that is most significant varies among Norwegian mountain ranges; not only in strength, but also in direction. Increased precipitation in spring and summer on the west side of Hardangervidda (high precipitation area) is found to be negative for lamb autumn weights, while the effect is positive on the drier Hardangervidda east (see map Figure 1). In Forollhogna in Trøndelag, increased spring temperature implies increased lamb autumn weights while the effect is negative in Setesdal in the south and on the west side of Hardangervidda (Nielsen et al. 2012).

Since the effect of certain changes in weather conditions are site specific, we choose to model two particular areas where the effect differs. We focus here on spring temperature (more precisely, mean temperature in May), but the exercise could be done on any measure of local weather conditions where its influence on lamb autumn weight is known. It is supposed that the spring population size indicates the grazing pressure during the grazing season. When in addition assuming similar grazing pressure among lambs and adults, the grazing pressure year t is hence defined by the number of animals $(1+b)X_t$. The relationship between the number of grazing animals, a certain change in mean spring temperature ΔT, and lamb weight gain during the grazing season year t w_t^Y is therefore formulated as:

(2) $$w_t^Y = w^Y((1+b)X_t, \Delta T)\,.$$

As already indicated, a negative relationship between the population size and the lamb autumn weight is well-established (Mysterud and Austrheim 2005, Mysterud et al. 2011), also in our focal areas (Nielsen et al. 2012); that is, $\partial w^{Y}((1+b)X_{t},\Delta T)/\partial((1+b)X_{t})=w^{Y,1}<0$. This relationship is further assumed concave, $\partial w^{Y,1}/\partial((1+b)X_{t})\leq 0$. $\Delta T = 0$ defines the situation as it is today and $\Delta T > 0$ hence indicates a positive shift in temperature in the future. As mentioned, the effect of $\Delta T > 0$ is site specific and can be positive, $\partial w^{Y}((1+b)X_{t},\Delta T)/\partial\Delta T=w^{Y,2}>0$, which will be the Northern scenario, exemplified by Forollhogna in Trøndelag, or negative, $w^{Y,2}<0$, which will be the Southwestern scenario, exemplified by western side of Hardangervidda. In the Northern scenario we assume that the marginal weight loss due to an increase in the sheep population is non-decreasing in the temperature, i.e., $\partial w^{Y,1}/\partial\Delta T\geq 0$, whereas the opposite is assumed for the Southern scenario, $\partial w^{Y,1}/\partial\Delta T\leq 0$) (see also numerical section 7).

For the adults, there is generally no weight change during the grazing season on productive pastures while there may be some loss in low productivity areas (Mysterud and Austrheim 2005). However, as a reasonably good approximation, we simply neglect any possible connection between the amount of vegetation and weight, and therefore also any effects of weather factors on ewe weight. The ewe slaughter weight is therefore simply fixed and determined outside the model and given as:

(3) $w_{t}^{X}=w^{X}$.

5. Revenue and costs

We disregard income from wool production, so meat sales are the only revenue component for the farmer. Slaughtering takes place in the fall after the outdoors grazing season (Figure 2). Therefore, the number of ewes and female lambs removed are $X_{t}s^{X}h_{t}^{X}$ and $\psi bX_{t}s^{Y}h_{t}^{Y}$, respectively. As mentioned above, the entire male lamb subpopulation $(1-\psi)bX_{t}s_{Y}$ is slaughtered. The number of animals removed year t is then defined as $H_{t}=bX_{t}s^{Y}(\psi h_{t}^{Y}+1-\psi)+X_{t}s^{X}h_{t}^{X}$. With p^{X} as the net (of slaughtering costs) ewe slaughtering price (NOK per kg) and p^{Y} as the lamb net slaughtering price, both assumed to be fixed and independent of the number of animals supplied at the farm level, the current meat income of the farmer is given by $R_{t}=[p^{Y}w_{t}^{Y}bX_{t}s^{Y}(\psi h_{t}^{Y}+1-\psi)+p^{X}w_{t}^{X}X_{t}s^{X}h_{t}^{X}]$.

The cost structure differs sharply between the outdoor grazing season and the indoor feeding season, the indoor costs being substantially higher. Throughout this analysis, we assume a given farm capacity (but see Gauteplass and Skonhoft 2012). Therefore, the costs of buildings, machinery and so forth are fixed. The indoor season variable costs include labor (typically an opportunity cost), electricity, and veterinary costs in addition to fodder. It depends on the indoors stock size and is given as $C_t = C(X_t)$. The cost function is assumed to be increasing and convex; that is, $C' > 0$ and $C'' \geq 0$.

During the grazing period the sheep may graze on communally owned lands ('commons') or private land. Here we assume private land, so we are neglecting any possible grazing externalities.There may be some transportation and maintenance costs, but such costs are neglected because they are generally rather low. The total yearly variable cost is hence simply assumed to be the indoor season cost. Therefore, when ignoring discounting within the year, the current (yearly) profit of the farmer is described by:

(4) $$\pi_t = R_t - C_t = \left[p^Y w_t^Y b X_t s^Y (\psi h_t^Y + 1 - \psi) + p^X w^X X_t s^X h_t^X \right] - C(X_t) .$$

6. The optimal program

6.1 Optimality conditions

We assume that the farmer is well informed and rational, and aims to maximize the present value of profit over an infinite time horizon, $\sum_{t=0}^{\infty} \rho^t \pi_t$, given the biological growth constraint (1). $\rho = 1/(1+\delta)$ is the discount factor with $\delta \geq 0$ as the (yearly) fixed discount rate. The Lagrange function of this problem may be written as

$$L = \sum_{t=0}^{\infty} \Big\{ \rho^t \left[p^Y w_t^Y b X_t s^Y (\psi h_t^Y + 1 - \psi) + p^X w^X X_t s^X h_t^X - C(X_t) \right] \\ - \rho^{t+1} \lambda_{t+1} \left[X_{t+1} - X_t s^X (1 - h_t^X) - \psi b X_t s^Y (1 - h_t^Y) \right] \Big\}$$

where $\lambda_t > 0$ is the animal shadow value. Following the Kuhn-Tucker theorem, the first-order necessary conditions of this problem (when $X_t > 0$) are:

(5) $$\partial L / \partial h_t^X = X_t (p^X w^X - \rho \lambda_{t+1}) \begin{smallmatrix} \leq \\ = \\ > \end{smallmatrix} 0 ; \quad 0 \leq h_t^X \leq 1 ,\ t = 0,1,2... ,$$

(6) $$\partial L / \partial h_t^Y = X_t [p^Y w_t^Y - \rho \lambda_{t+1}] \begin{smallmatrix} \leq \\ = \\ > \end{smallmatrix} 0 ; \quad 0 \leq h_t^Y \leq 1 ,\ t = 0,1,2...$$

and

$$(7)\ \partial L / \partial X_t = p^Y b s^Y (\psi h_t^Y + 1 - \psi)(w_t^Y + (1+b) X_t w^{Y,1})$$
$$+ p^X w^X s^X h_t^X - C' + \rho \lambda_{t+1} \left[s^X (1 - h_t^X) + \psi b s^Y (1 - h_t^Y) \right] - \lambda_t = 0,\ t = 1, 2, 3\ldots\ .$$

The control condition (5) indicates that slaughtering of the adults should take place up to the point where the per animal value is below, equal or above the cost of reduced growth in animal numbers, evaluated at the shadow price. The lamb control condition (6) is analogous. Equation (7) is the portfolio condition and states that the number of female adults is determined such that the immediate net return on adult females equals the shadow price of natural growth. The first term in the first bracket reflects that increased animal numbers increases the total meat weight, whereas the second term accounts for the marginal cost of increased animal numbers due to reduced weight per lamb. These conditions are also sufficient when the Lagrangean is concave in the state and control variables. Since the Langrangean is linear in the controls, the sufficiency conditions boil down to $\partial^2 L / \partial X_t^2 \leq 0$ (the weak Arrow sufficiency condition). With strictly convex cost function, $C'' > 0$, and concave, decreasing lamb weight gain function, i.e., $w^{Y,1} < 0$ and $\partial w^{Y,1} / \partial((1+b)X_t) \leq 0$ (section 4), we find this condition to be satisfied.

From the control conditions (5) and (6) it is evident that the per animal slaughter value steers the optimal slaughter composition. If the demand and market conditions are in favor of lambs, which is the typical situation (see numerical section 7), then $p^Y > p^X$. If, in addition, the climatic conditions are favorable and the sheep population level is such that the weight of the lambs $w_t^Y = w^Y((1+b)X_t, \Delta T)$ is 'high', we find that the per animal slaughter value of the lambs will exceed that of the ewes, $p^Y w^Y((1+b)X_t, \Delta T) > p^X w^X$. The control conditions then indicate a higher harvesting fraction of the lambs than the ewes. This can be satisfied in three ways: i) $h_t^Y = 1$ and $0 < h_t^X < 1$, ii) $h_t^Y = 1$ and $h_t^X = 0$ and iii) $0 < h_t^Y < 1$ and $h_t^X = 0$. On the contrary, if the demand conditions are in favor of ewes, the climate conditions are unfavorable, and/or the sheep population level is 'high', so that lamb weight is 'low', then $p^X w^X > p^Y w^Y((1+b)X_t, \Delta T)$. In this situation a more aggressively harvesting of the adults is optimal, and the control conditions (5) and (6) can be satisfied either as iv) $h_t^X = 1$ and $0 < h_t^Y < 1$, v) $h_t^X = 1$ and $h_t^Y = 0$, or as vi) $0 < h_t^X < 1$ and $h_t^Y = 0$.

6.2 Steady state analysis

In a steady state where all variables are constant over time with a 'high' lamb weight and hence $p^Y w^Y((1+b)X, \Delta T) > p^X w^X$ (the time subscript is omitted when considering steady state), we find the above control conditions to be satisfied only as possibility iii) with $0 < h^Y < 1$ and $h^X = 0$ because slaughtering all the lambs is not an option in a possible steady state. See equation (1') and Figure 3. A corollary of $h^X = 0$ is that (female) lamb slaughtering should take place at the highest level compatible with the sheep population equilibrium; that is, $h^Y = 1-(1-s^X)/\psi b s^Y < 1$. Therefore, the optimal slaughtering rate depends on biological conditions only, and such that higher fertility b and higher survival rates indicate that it is beneficial to slaughter a higher fraction of the lambs.

Lambs not slaughtered enter the ewe population next spring. When inserting $h^X = 0$, $h^Y = 1-(1-s^X)/(\psi b s^Y)$, and additionally $\lambda = p^Y w^Y((1+b)X, \Delta T)/\rho$ from condition (6) into equation (7) and rearranging, the optimal equilibrium number of animals to be kept during the indoor season is determined by $p^Y(bs^Y + s^X - 1 - \delta)w^Y = C' - p^Y(bs^Y + s^X - 1)(1+b)Xw^{Y,1}$. The left hand side is the marginal benefit of keeping animals for next season lamb slaughtering net of the discount rate, and reflects that saving an additional animal increases the total number of lambs available for slaughtering next year. The right hand side is the marginal cost of keeping animals for the next season, and equalizes the cost of an additional animal indoor plus the weight loss an additional animal imposes on all lambs. Note that economic as well as biological parameters influence the optimal steady state number of adult animals.

When a higher temperature yields higher lamb weight $w^{Y,2} > 0$, we find $\partial X / \partial \Delta T > 0$ by using the sufficiency conditions and in addition the assumption that the marginal lamb weight loss function is non-decreasing in the temperature effect, $\partial w^{Y,1} / \partial \Delta T \geq 0$. Because the steady state harvesting fraction is determined by biological parameters alone, we hence also find that more lambs should be slaughtered. In this case increased temperature thus represents a 'double dividend' for sheep farmers; it increases the value per lamb slaughtered and increases also the optimal number of lambs slaughtered. In the opposite case when a higher temperature yields lower lamb weight, it will be beneficial for the farmers to reduce the number of sheep. Other comparative static results may also be deduced. For example, with a higher slaughter price the farmer will find it rewarding to keep more animals, $\partial X / \partial p^Y > 0$. As the summer stocking rate then also increases, the lamb weight reduces accordingly. The effect of a higher discount rate δ is a smaller sheep population and higher lamb value.

In the opposite case of a 'low' lamb weight and more valuable ewes than lambs, the control conditions in a possible steady state can generally be satisfied either as case iv) with $h^x = 1$ and $0 < h^Y < 1$, case v) with $h^X = 1$ and $h^Y = 0$, or case vi) with $0 < h^X < 1$ and $h^Y = 0$. However, as already indicated, steady state slaughtering of all adults can be ruled out as an option because of the actual demographic parameter values (numerical section 7). Therefore case vi) with $h^X = 1 - (1 - \psi b s^Y) / s^X < 1$ and $h^Y = 0$ will be the only steady state possibility when adults are more valuable than lambs. That is, (female) lamb slaughtering equals zero whereas adult slaughtering should take place at the highest level compatible with the sustainable sheep population equilibrium cf. equation (1') and Figure 3. Also now only biological parameters influence the optimal harvesting rate. When inserting for the optimal steady state slaughtering values into equation (7) and rearranging, the optimal animal population is now determined by $p^Y b s^Y (1-\psi) w^Y + p^X w^X (s^X + \psi b s^Y - (1+\delta)) = C' - p^Y b s^Y (1-\psi)(1+b) X w^{Y,1}$. The interpretation is similar to the above lamb only slaughtering case, although now animals kept over winter add to future male lamb and adult slaughtering. When a higher temperature yields lower lamb weight, and we in addition assume $\partial w^{Y,1} / \partial \Delta T \leq 0$, we now find $\partial X / \partial \Delta T < 0$. We also find that a higher slaughter price, this time of the ewes, means that it is beneficial for the farmer to increase the sheep population and hence also increase the number of animals slaughtered.

In our example from two mountain ranges in Norway an increase in temperature implies more favorable vegetation growth conditions in the Northern scenario and less favorable vegetation growth conditions in the Southwestern scenario. If all farmers initially face market and climate conditions favoring lamb slaughtering only, then increased temperature will have no impact on the slaughtering composition for farmers in north. However, as demonstrated, the sheep population increases. In south, on the other hand, farmers are less likely to slaughter lambs only when faced with a temperature increase. Furthermore, increased temperature motivates southern farmers to reduce the sheep population.

6.3 The dynamics

Above some properties of a possible steady state with a constant number of animals through time was studied. As the profit function is linear in the controls, economic theory suggests that harvest should be adjusted such as to lead the population to steady state as fast as possible; that is, Most Rapid Approach Path (MRAP) dynamics, but not necessarily exactly a MRAP-path as two controls are included. Hence, if the initial stocking rate is below the optimal steady state, and the per lamb value is above that of the ewes, it will for sure be no ewe harvesting, but some (small) lamb harvesting such that the optimal control conditions (5) and (6) are satisfied. On the other hand, if the initial stock is above the steady state and still with the per lamb value above that of the ewes, the stock should be slaughtered down to the optimal state level as fast as possible. This strategy may include slaughtering all lambs as well as some ewe slaughtering, or it may include a high lamb slaughtering while no ewe slaughtering. The steady state may be reached the first year, but it can also take a somewhat longer time. The complexity of analyzing the approach paths in multi-dimensional models is exemplified by the predator – prey model of Mesterton-Gibbons (1996). The dynamics is further considered in the numerical section 7.

7. Numerical results

7.1 Data and specific functional forms

We now present some numerical results. The sheep biological data are based on a large set of observations from Norwegian sheep farming, and the baseline parameter values are shown in Table 1. The ewe weight is set to 30 (kg/animal) with a meat market slaughter value of 35 (NOK/kg). Therefore, the fixed ewe slaughter value is $p^x w^x = 35 \cdot 30 = 1{,}050$ (NOK/animal). The lamb meat value is $p^Y = 60$ (NOK/kg). We assume a strictly concave maintenance cost function, $C(X_t) = (c/2)X_t^2$, with $c = 10$ (NOK/animal^2).

Table 1 about here

As already indicated, several aspects of climate conditions have been shown to affect lamb weights in autumn (Nielsen et al. 2012). We use mean temperature in spring as the projection for the climate variable because it is spatially more synchronous as compared to e.g. precipitation and that the temperature change is expected to be larger in spring than in summer (Christensen et al. 2007 and Hanssen-Bauer et al. 2003). Predicted future changes in climate conditions are based on output from global climate models (e.g. Christensen et al.

2007). The simulated annual mean warming from 1980 to 1999 to 2080 to 2099 in Northern Europe varies from 2.3°C to 5.3°C, with the largest warming occurring in winter (Christensen et al. 2007). These models are, however, rather imprecise in predicting exact changes in e.g. seasonal average temperatures in particular areas. A few attempts have been made to down scale global climate projections to Norwegian conditions (e.g., Hanssen-Bauer et al. 2003 and Benestad 2011). These studies estimated mean spring temperature to increase approximately 1° C in the period 2030-2049 as compared to the period 1980-1999. They found no significant difference in temperature increase between the two areas included in our study.

As discussed, we focus on two mountain ranges; the Northern and the Southwestern areas (see Figure 1), where increased spring temperature has been shown to have a positive and negative effect, respectively, on lamb autumn weight. In the baseline calculations with no climate change and $\Delta T = 0$ °C, the lamb slaughter weight function (2) is specified linear as $w_t^Y = w^Y((1+b)X_t, 0) = k^0 - k^1(1+b)X_t$ with $k^0 = 22$ (kg/animal) and $k^1 = 0.01$ (kg/animal^2). Accordingly, with a number of grazing animals of, say, $(1+b)X_t = 250$, we find $w_t^Y = 22 - 0.01 \cdot 250 = 19.50$ (kg/animal) and $p^Y w_t^Y = 60 \cdot 19.50 = 1{,}170$ (NOK/animal) and therefore a substantial higher slaughter value of the lambs than the ewes (see above). With climate change we assume a uniform shift of the weigh function such that equation (2) now reads $w_t^Y = w^Y((1+b)X_t, \Delta T) = k^0 - k^1(1+b)X_t + k^2\Delta T$. Under this assumption climate change thus has no effect on the marginal weight – stock relationship, $\partial w^{Y,1} / \partial \Delta T = 0$ .This simple shift is not necessarily realistic as climate effects might be stronger at higher stock levels (additive effects). We do, however, find it as a reasonable simplification.

Nielsen et al. (2012) found that for an increase in average spring temperature of 1°C ($\Delta T = 1$) the average lamb autumn weight would increase with 0.37 kg in the north and decrease with 0.69 kg in southwest. Though they modeled lamb autumn body mass, we use the same estimates to illustrate the effects on lamb slaughter weight. That is, k^2 is assumed to be 0.37 and -0.69 ((kg/animal)/°C) in the Northern and Southwestern scenario, respectively. However, we still only model the current conditions as compared to a down scaled projected climate change scenario expected to represent climate conditions in 2050. A more realistic approach would have been to use a dynamic ΔT representing a continuous change in temperature over time. We do not however, find this to be necessary to illustrate the potential effects of future climate change on the economy of the sheep farming.

In the following, we first calculate the optimal management policy for the baseline parameter values, including no climate change. We then study the effects of climate change through temperature shifts given as ΔT as 3°C in addition to1°C, as well as changes in some of the key parameter values like the discount rate and the meat prices.

7.2 Results

We start with presenting the basic dynamic results. While we solve the model for a long time horizon (50 years), we only report the results for the first 35 years. This long time horizon ensures that the reported solutions will be numerically indistinguishable from the infinite horizon solution over the reported period of 35 years. As already indicated, because the profit function is linear in the controls, MRAP dynamics, but not necessarily exactly a MRAP-path, is supposed to describe the optimal transitional dynamics. Figure 3 seems partly to confirm this where the steady state stock size approaches the steady state value of 123 animals after about 3 years with the baseline parameter values and where the discount rate is 3%, $\delta = 0.03$. During the transitional phase, as well as in the steady state, the value per lamb exceeds the value per adult. In the first year, all lambs are slaughtered before it is gradually reduced to its optimal steady state harvesting rate of $h^Y = 1-(1-s^X)/(\psi b s^Y)$ = 0.93. See also Table 2. No ewes are slaughtered. Not surprisingly, we find that increasing the discount rate results in progressively smaller populations with corresponding higher harvesting rates of lambs during the transitional phase, but still no ewes slaughtered, while the dynamics do not change qualitatively. We have also studied the effects of changing initial stock size, and all the time we find that the stock size and harvest approach the same steady state (ergodic dynamics).

Figure 4 about here

Next we study the effect of climate changes exemplified by an increase in mean spring temperature. Table 2 reports steady state animal numbers and profit for the different temperature increase shifts. Consider first the Northern scenario where a higher temperature increases the lamb weight and is profitably for the farmer. Increased weight shifts up the net income of lamb slaughtering for a fixed stock size. At the same time, higher weight and hence higher slaughter value means that it is beneficial for the farmer to keep more animals. This imposes an additional positive effect on farm profitability. At a temperature increase of 1°C, the direct effect of increased weight (from 18.88 kg to 19.25 kg) adds 3,663 NOK to the yearly gross slaughtering income, while the indirect effect due to the increased stock size and the corresponding reduction in lamb weight adds 4,606 NOK. See Figure 5. Thus, as indicated, increased temperature represents a 'double dividend' for the sheep farmer, and the indirect economic effect of the increased stocking rate is stronger than the direct effect. The net effect on yearly profit as reported in Table 2 is, however, dampened due to increased maintenance cost following the higher number of animals. At a temperature increase of 3°C, we also find that the indirect effect exceeds the direct effect.

Figure 5 about here

The Southwestern scenario where increased temperature affects the lamb weight negatively is then considered. The low temperature increase of 1°C reduces the lamb weight but not sufficient to give a smaller per animal value of the lambs than that of the ewes. Therefore, the optimal steady state slaughtering composition is unchanged. However, the gross slaughtering income reduces due to the direct negative effect of reduced lamb weight and the indirect negative effect working through a lower stocking rate. With a temperature increase of 1°C, the direct effect of reduced weight reduces the yearly farm gross income by 6,732 NOK, whereas the indirect effect on slaughtering income working through a smaller stock size reduces the yearly gross income by 4,531 NOK. Therefore, also in this Southwestern scenario the indirect effect is strong. The total negative effect on yearly profit as reported in Table 2 is, however, smaller due to reduced maintenance cost. A further increase in the temperature may drive the slaughtering value per lamb below that of the ewes and hence shift the optimal steady state slaughtering composition from lamb slaughtering only to adult slaughtering only. Table 2 shows that this happens when ΔT is 3°C. Thus, in this case the less favorable vegetation growth conditions mean that the lamb slaughter weight reduces such that we find $p^Y w^Y = 60 \cdot 16.84 = 1{,}010 < p^x w^x = 35 \cdot 30 = 1{,}050$ (NOK/animal).

Table 2 about here

The results in Table 2 indicate that temperature changes have crucial spatial effects. For example, when comparing two equally sized farms located in our two areas where lamb weights (or productivity) are affected in an opposite manner, the farmer that benefits from high productivity (Northern scenario) will find it rewarding to keep significantly more animals than the other one (Southwestern scenario). In case of a 1°C temperature increase, the farmer that gains from climate change will earn some 10% higher profit per year than the farmer located in the negatively affected area (115,267 NOK vs. 104,596 NOK). With an even higher temperature change the profit discrepancy increases further, and with 3°C the difference becomes about 30% (123,014 vs. 94,176).

7.3 Sensitivity analysis

Table 3 reports some steady state sensitivity results. First, we study the effects of reducing the discount rate. Ignoring discounting and $\delta = 0$ without any temperature change has no impact on the steady state slaughtering composition and hence, no impact on the slaughtering rates which are determined by biological factors only. However, as also seen in Figure 4, the farmer will find it beneficial to keep more animals. The profit also increases compared to the baseline scenario of positive discounting. This effect is well known as the steady state solution of present value profit maximizing with zero discounting coincides with the solution of maximizing current profit in biological equilibrium. Ignoring discounting with higher temperature has the same qualitative effects as with discounting. For both levels of temperature change, it is beneficial to keep more animals in North as well as in Southwest. Increasing the lamb slaughter price to $p^Y = 70$ (NOK/kg), has no impact on the steady state slaughtering composition compared to the baseline scenario. However, as also indicated (section 6), a higher lamb meat value increases the marginal benefit of saving animals for next season and hence, the animal stock increases. Because higher temperature increases the value of lambs through increased weight in the Northern scenario, the impacts on population size and profit are strengthened in this area with $\Delta T > 0$. The opposite occurs in Southwest where increased temperature dampens the impact of a higher lamb meat price. The spatial effects of temperature changes are of more or less similar strength compared to the baseline lamb price. Table 3 also demonstrates the effects of increasing the ewe slaughter value, and for $p^X = 40$ (NOK/kg) it is beneficial for farmers in both areas to change the slaughter strategy and only slaughter ewes. This strategy is even beneficial with $\Delta T = 3$ (°C) in the Northern scenario because $p^X w^X = 40 \cdot 30 > p^Y w^Y = 60 \cdot 19.55$. The spatial effect of increased temperature now reduces because only ewe slaughtering becomes beneficial and the slaughter value of this animal category is not related to temperature changes.

Table 3 about here

Finally, Table 3 reports some steady state effects when changing the lamb weight - grazing density relationship. Not surprisingly, with $k^1 = 0.005$ (kg/animal) and making this relationship less sensitive, the optimal lamb slaughter weights increase compared to the baseline value of $k^1 = 0.01$, and are accompanied by more animals and higher profits. The spatial effects of changing temperature are more or less similar as with the baseline parameter values. With $k^1 = 0.015$ and making the lamb weight more sensitive to stock changes, the optimal slaughter weight and profit reduce compared to the baseline scenario. However, the spatial effect of increased temperature is still significant. At the 1°C change the profit of the farmer in the Northern scenario is about 8% higher than that of the farmer in the Southwestern scenario (100,692 NOK compared to 93,400 NOK) while the difference with 3°C temperature increase is 24% (and 107,442 NOK compared to 86,638 NOK).

8. Concluding remarks

This paper has analyzed the economics of sheep farming under future climate change scenarios in a two stage model of lambs and adult females (ewes). The analysis is at the farm level in a Nordic context with a crucial distinction between the outdoor grazing season and the winter indoor feeding season. The farmer is assumed to be 'rational' and well informed, and aims to find the number of animals slaughtered that maximize present value profit. The outdoor grazing season makes the autumn weight of the lambs subject to changes in environmental conditions and possible climate change effects. Several aspects of climate conditions have been shown to affect lamb weights in autumn (Nielsen et al. 2012), and we used mean temperature in spring as the future projections for the climate variable. According to IPCC, the simulated annual mean warming from 1980 to 1999 to 2080 to 2099 in Northern Europe varies from 2.3°C to 5.3°C (Christensen et al. 2007), while downscaling have indicated an increase of ~1°C in spring temperature in our focal areas (Hanssen-Bauer et al. 2003 and Benestad 2011). In our modeling we focused on spring temperature increases in the range of 1°C to 3°C.

In this two-stage model of lambs and ewes, the steady state harvesting decision is basically shaped by economic and climate factors. For the given price and climate conditions with more valuable lambs than ewes, lamb only slaughtering at the highest possible level represents the optimal steady state harvesting strategy. On the other hand, the optimal lamb slaughter fraction is determined by sheep biological factors alone. The reason for this sharp distinction between the effects of economic and biological forces is the lack of any density-dependent factors regulating sheep population growth.

We find that higher temperature represents a ‘double dividend’ for the farmer experiencing increased lamb weight; it increases both the slaughter value per animal and the number of lambs the farmer will find it beneficial to slaughter. Both the direct effect, represented by the increased lamb weight and higher slaughter value, and the indirect effect, working through increased number of animals slaughtered, may contribute significantly to increased profitability for the farmer. The numerical illustrations also indicate that shifting temperature has crucial spatial effects. For example, when comparing two equally sized farms located in areas in which temperature affect lamb weight in different directions, the farmer that benefits from higher temperature will find it rewarding to keep a higher stocking rate than the other one. The farmer experiencing increased lamb weight will receive substantial higher economic benefits as well. At a realistic temperature increase of 1°C the farmer benefiting from increased lamb weight will earn some 10% higher profit than the farmer facing reduced lamb weight with our baseline parameter values. With 3°C increase, the profit gain increases to 30%. The spatial effect of increased temperature is of less importance when adult slaughter is optimal.

References

Alcamo, J., J. M. Moreno, B. Nováky, M. Bindi, R. Corobov, R. J. N. Devoy, C. Giannakopoulos, E. Martin, J. E. Olesen, and A. Shvidenko. 2007. Europe. Pages 541-580 in M. L. Parry, O. F. Canziani, J. P. Palutikof, P. J. v. d. Linden, and C. E. Hanson, editors. Climate Change 2007: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge UK.

Austrheim, G., L.J. Asheim, G. Bjarnason, J. Feilberg, A.M. Fosaa, Ø. Holand, K. Høegh, I.S. Jónsdóttir, B. Magnússon, L.E. Mortensen, A. Mysterud, E. Olsen, A. Skonhoft, G.

Steinheim, and A.G. Thórhallsdóttir 2008. Sheep grazing in the North Atlantic region: A long term perspective on management, resource economy and ecology. Report Zoology Ser. 2008, 3 Vitenskapsmuseet NTNU.

Benestad R.E. 2011. A New Global Set of Downscaled Temperature Scenarios. Journal of Climate, 24, 2080-2098.

Caswell, H. 2001. Matrix Population Models. Sinauer Boston.

Christensen J.H., B. Hewitson and A. Busuioc 2007. Pp. 847-940 in Solomon et al.

Diekert, F. K., D. Ø. Hjermann, E. Nævdal, and N. C. Stenseth 2010. Spare the young fish: Optimal harvesting policies for North-East Artic Cod. Environmental and Resource Economics 47: 455-475.

Easterling, W. E., P. K. Aggarwal, P. Batima, K. M. Brander, L. Erda, S. M. Howden, A. Kirilenko, J. Morton, J.-F. Soussana, J. Schmidhuber, and F. N. Tubiello. 2007. Food, fibre and forest products. Pages 273-313 in M. L. Parry, O. F. Canziani, J. P. Palutikof, P. J. v. d. Linden, and C. E. Hanson, editors. Climate Change 2007: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge UK.

FAO 2009. Profile for Climate Change. Food and Agriculture Organization of the United Nations, Rome.

Gauteplass, A. and A. Skonhoft 2012. Optimal exploitation of a renewable resources with capital limitations. Working paper Department of Economics NTNU, Trondheim

Hannesson, R. 2007. Global warming and fish migrations. Natural Resource Modeling 20: 301-319.

Hanssen-Bauer I, E. J. Forland, J. E. Haugen and O. E. Tveito 2003. Temperature and precipitation scenarios for Norway: comparison of results from dynamical and empirical downscaling. Climate Research 25: 15-27.

Mesteron-Gibbons, M. 1996. A technique for finding optimal two-species harvesting policies. Ecological Modeling 92: 235-244

Mysterud, A., G. Steinsheim, N. Yoccoz, O. Holand, and N.C. Stenseth. 2002. Early onset of reproductive senescence in domestic sheep. Oikos 97: 177–183.

Mysterud, A. and G. Austrheim 2005. Ecological effects of sheep grazing in alpine habitats, short term effects. Utmarksnæringen i Norge 1-05: 1-91.

Mysterud, A., Hessen, D.O., Mobæk, R., Martinsen, V., Mulder, J., and G. Austrheim. 2011. Plant quality, seasonality and sheep grazing in an alpine ecosystem. Basic and Applied Ecology 12: 195-206.

Nersten, N., A. Hegrenes, O. Sjelmo, and K. Stokke. 2003. Saueholdet i Norge ('Sheep Farming in Norway'). Report Norwegian Agricultural Economic Research Institute Oslo.

Nielsen A., N. G. Yoccoz, G. Steinheim, G. O. Storvik, Y. Rekdal, M. Angeloff, N. Pettorelli, Ø. Holand, and A. Mysterud. 2012. Are responses of herbivores to environmental variability spatially consistent in alpine ecosystems? Global Change Biology 18: 3050-3062.

Ólafsdóttir, R. and Á.D. Júlíusson. 2000. Farmers' perception of land-cover changes in NE Iceland. Land Degradation & Development 11: 439-458.

Quaas, M. and S. Baumgärtner. 2012. Optimal grazing management rules in semi-arid rangelands with uncertain rainfall. Natural Resource Modeling 25: 364-387.

Skonhoft, A. 2008. Sheep as capital and farmers as portfolio managers: A bioeconomic model of Scandinavian sheep farming. Agricultural Economics 38: 193–200.

Skonhoft, A., G. Austerheim and A. Mysterud 2010. A bioeconomic sheep – vegetation trade-off model: An analysis of the Nordic sheep farming system. Natural Resource Modeling 23: 354-380.

Solomon, S. D., Q. M. Manning, Z. Chen, M. Marquis, K. B. Averyt, M. Tignor, and H. L. Miller (eds.) 2007. Climate change 2007 – the physical science basis. Cambridge University Press, Cambridge.

Appendix

Uncertainty and risk aversion

Equation (2) in the main text indicates that lamb weight can be predicted exactly from the current stocking rate and climate conditions. However, these changes are in fact partly random to the farmer, and in this Appendix it is shown how uncertainty and risk aversion may affect the optimal slaughtering composition and the animal stock. Therefore, we now specify the lamb weight as stochastic:

(A1) $w_t^Y = w^Y((1+b)X_t, \Delta T, \varepsilon_t)$

where ε_t is a stochastic variable, assumed to be independent and identically distributed (i.i.d.) over time with mean zero and variance σ^2.

It can be verified that uncertainty together with the assumption of risk neutrality yields the same solution as in section 6. We therefore solve the model by assuming that the optimizing farmer is risk averse. That is, we assume that farmer utility increases with the current profit at a decreasing rate, i.e., $U'(\pi_t) > 0$ and $U''(\pi_t) < 0$. Under risk aversion, the farmer now aims to maximize expected present value utility over an infinite time horizon, $E_0\left\{\sum_{t=0}^{\infty} \rho^t U(\pi_t)\right\}$,

given the biological constraint (1) and equations (A1) and (3). E_0 is expectation given information at time 0. The Lagrange function of this problem may be written as

$$L = E_0\left[\begin{array}{l}\sum_{t=0}^{\infty}\left\{\rho^t U\left[p^Y\left[w^Y((1+b)X_t,\Delta T,\varepsilon_t)\right]bX_t s^Y(\psi h_t^Y + 1-\psi) + p^X w^X X_t s^X h_t^X - C(X_t)\right]\right. \\ \qquad \left. -\rho^{t+1}\lambda_{t+1}\left[X_{t+1} - X_t s^X(1-h_t^X) - \psi b X_t s^Y(1-h_t^Y)\right]\right\}\end{array}\right]$$

The first order conditions are now given by:

(A2) $\partial L/\partial h_t^X = X_t\left[E_t[U'(\pi_t)]p^X w^X - \rho\lambda_{t+1}\right] \substack{\le \\ = \\ >} 0;\ 0 \le h_t^X \le 1,\ t = 0,1,2...$,

(A3) $\partial L/\partial h_t^Y = X_t\left[E_t[U'(\pi_t)w^Y((1+b)X_t,\Delta T,\varepsilon_t)]p^Y - \rho\lambda_{t+1}\right] \substack{\le \\ = \\ >} 0;\ 0 \le h_t^Y \le 1,\ t = 0,1,2...$,

and

(A4) $$\partial L / \partial X_t = p^Y b s^Y (\psi h_t^Y + 1 - \psi) \Big\{ E_t[U'(\pi_t) w^Y ((1+b) X_t, \Delta T, \varepsilon_t)] + (1+b) X_t (\partial w^Y / \partial X_t) E_t[U'(\pi_t)] \Big\} + p^X w^X s^X h_t^X E_t[U'(\pi_t)] - C'(X_t) E_t[U'(\pi_t)] + \rho \lambda_{t+1} \left[s^X (1 - h_t^X) + \psi b s^Y (1 - h_t^Y) \right] - \lambda_t = 0 , \; t = 1, 2, 3 \dots$$

It is assumed that the weather conditions at time t are known when h_t^Y and h_t^X are determined. Therefore, the expectation operator in (A2) - (A4) at period t is E_t .The control conditions (A2) and (A3) can be given similar interpretations as the control conditions (5) and (6) in the main text, except that the marginal gain now is represented by expected values. Equation (A4) states that the population size is determined such that the immediate expected marginal utility of ewes equals the shadow price of natural growth.

We only look at the steady state solution in this Appendix. The first term in the bracket in (A3) may be rewritten as $p^Y \left[E[U'(\pi)] \cdot E[w^Y ((1+b) X, \Delta T, \varepsilon)] + \mathrm{cov}(U'(\pi), w^Y ((1+b) X, \Delta T, \varepsilon)) \right]$. The covariance term is negative as higher lamb weight, and hence higher profit, yields reduced marginal utility for the risk adverse farmer. The expected marginal utility of lamb slaughtering is therefore smaller the larger absolute value of the covariance term. When combining this expression with (A2), we find that the farmer in presence of uncertainty will slaughter a higher fraction of lambs than ewes suggested that $p^Y E[w^Y ((1+b) X, \Delta T, \varepsilon)] - p^X w^X > -\mathrm{cov}(U'(\pi), w^Y ((1+b) X, \Delta T, \varepsilon)) / E[U'(\pi)] > 0$. That is, with risk attached to lamb weight, the expected slaughtering value per lamb should exceed the slaughtering value per adult by more than required in the deterministic case for a higher fraction of lamb slaughtering to be optimal. More precisely, the difference in the expected slaughtering values should exceed the absolute value of the covariance term divided by the expected marginal utility of income, i.e., the sensitivity rate of the marginal utility to lamb weight changes. If this condition is fulfilled, a higher harvesting rate of lambs than adults can only be satisfied as the above case iii) in the main text section 6 with $0 < h^Y < 1$ and $h^X = 0$ because slaughtering all the lambs is not a possible option at steady state. Hence, as in the deterministic case, optimal slaughtering rate then equals $h^Y = 1 - (1 - s^X) / \psi b s^Y < 1$. However, with uncertainty, the likelihood for lamb slaughtering only to be optimal is smaller.

When inserting $h^X = 0$, $h^Y = 1-(1-s^X)/\psi bs^Y$, and $\lambda = E[U'(\pi)w^Y((1+b)X, \Delta T, \varepsilon)]/\rho$ from (A2) into (A3), inserting the covariance, and rearranging, the optimal number of animals is determined through

$$p^Y(bs^Y + s^X - 1 - \delta)\Big[E[w^Y((1+b)X, \Delta T, \varepsilon)] + \mathrm{cov}(U'(\pi), w^Y((1+b)X, \Delta T, \varepsilon))/E[U'(\pi)]\Big]$$

$= C'(X) - p^Y(bs^Y + s^X - 1)(1+b)X(\partial w^Y / \partial X)$. The left hand side is the expected marginal benefit of keeping lambs for next season slaughtering net of the discount rate. The right hand side is the marginal cost of saving animals for the next season when taking the loss weight of lambs into account. Consequently, a larger covariance (in absolute value) reduces the expected marginal benefit of keeping animals for the next season relatively to the marginal cost, and hence, reduces the optimal number of animals. That is, the more sensitive the marginal utility of income is to lamb weight changes, the smaller is the optimal sheep stock.

The other cases with a higher slaughter value of the ewes than that of the expected value of the lambs can be analyzed in a parallel manner.

Figure 1: Norway and the focal areas. The Northern scenario (Forollhogna) and the Southwestern scenario (Hardangervidda) are presented in white, encapsulated with solid lines. The two other areas referred to in the text (Setesdal in the south and the eastern side of Hardangervidda) are presented in dark grey, encapsulated with dotted lines.

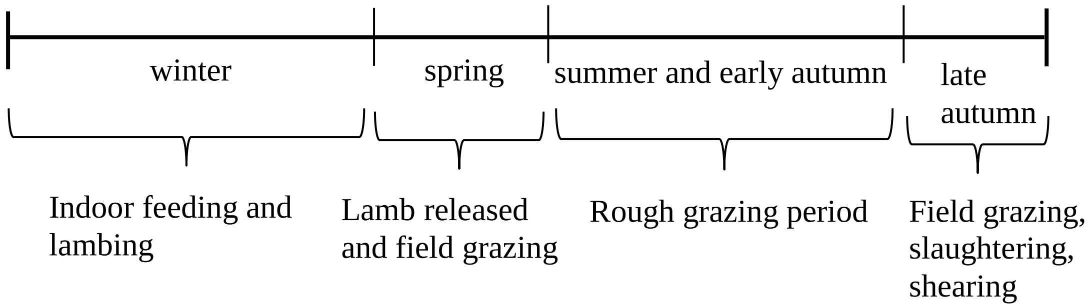

Figure 2: Seasonal s subdivision in the Nordic sheep farming system.

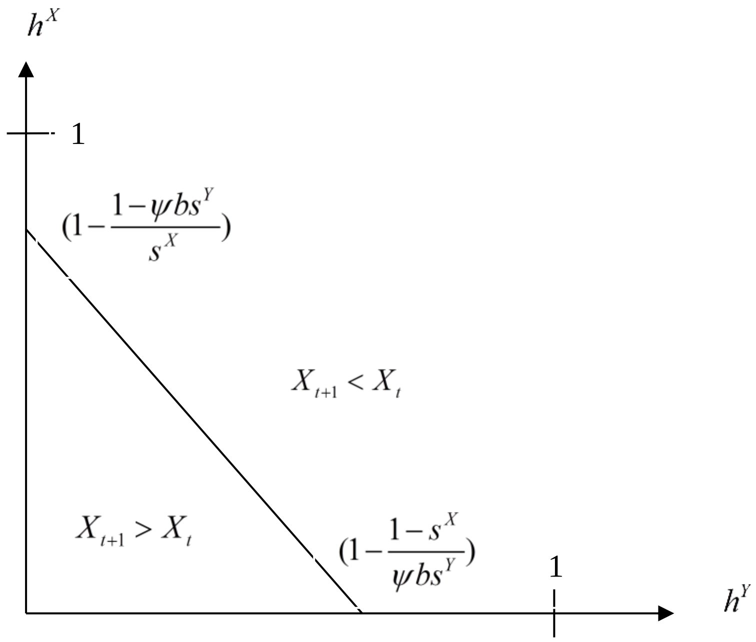

Figure 3: Equilibrium (constant animal population) harvesting relationship (Eq. 1'). h^Y, female lamb slaughtering fraction; h^X, ewe (adult female) slaughtering fraction.

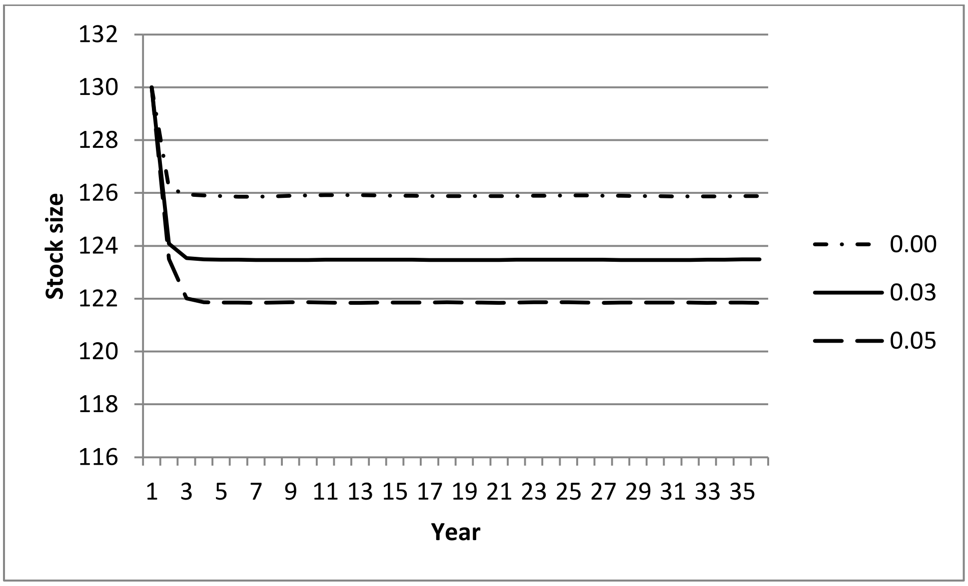

Figure 4: Stock dynamics X_t for different discount rate values. Baseline parameter values and $\Delta T = 0$.

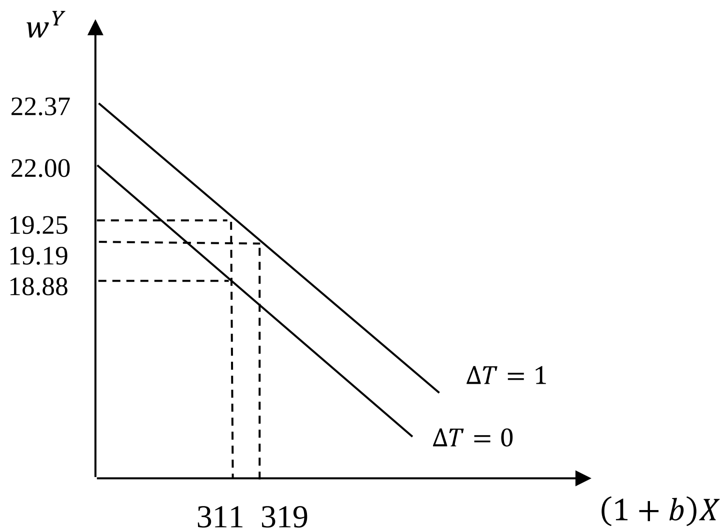

Figure 5: The lamb weight – stock size relationship for different temperature changes, $\Delta T = 0$ and $\Delta T = 1$. The Northern scenario.

Table 1: Baseline ecological and economic parameter values.

Parameter	Parameter description	Value	Source
s^Y	Natural survival fraction lamb	0.91	Mysterud et al. (2002)
s^X	Natural survival fraction ewe	0.95	Mysterud et al. (2002)
b	Fertility rate	1.53 (lamb/ewe)	Mysterud et al. (2002)
ψ	Proportion of female lambs	0.50	Mysterud et al. (2002)
k^0	Interaction term lamb weight function	22 (kg/animal)	Mysterud and Austrheim (2005)
k^1	Slope term lamb weight function, stocking rate	0.01 (kg/animal^2)	Derived from Nielsen et al. (2012)
k^2	Shift term lamb weight function, temperature	0.37 ((kg/animal)/°C) (North) -0.69 ((kg/animal)/°C) (Southwest)	Nielsen et al. (2012)
w^X	Adult (ewe) slaughter weight	30 (kg/animal)	Skonhoft et al. (2010)
p^X	Adult (ewe) slaughter price	35 (NOK/kg)	Skonhoft et al. (2010)
p^Y	Lamb slaughter price	60 (NOK/kg)	Skonhoft et al. (2010)
c	Cost coefficient	10 (NOK/animal^2)	Calibrated (scales the farm size)
δ	Discount rate	0.03	Assumption

Table note: Exchange rate: 1 Euro = 7.50 NOK (Aug. 2012).

Table 2: Steady state results. Changing temperature.

Case	ΔT (° C)	Winter stock X	#animals slaughtered	h^X	h^Y	w^Y	π (NOK)
Baseline[1)]	0	123	165	0.00	0.93	18.88	111 483
North	1	126	169	0.00	0.93	19.19	115 267
	3	130	175	0.00	0.93	19.83	123 014
Southwest	1	120	161	0.00	0.93	18.28	104 596
	3	122	164	0.68	0.00	16.84	94 176

1) See Table 1 for baseline parameter values.

Table 3: Steady state sensitivity results. Changing economic and biological conditions.

		Winter stock X	# animals slaughtered	h^X	h^Y	w^Y	π (NOK)
Baseline[1)]		123	165	0.00	0.93	18.88	111 483
North							
$\delta = 0.0$	$\Delta T = 0$	126	169	0.00	0.93	18.82	111 520
	$\Delta T = 1$	128	172	0.00	0.93	19.13	115 304
	$\Delta T = 3$	132	177	0.00	0.93	19.76	123 058
$P^Y = 70$	$\Delta T = 0$	137	184	0.00	0.93	18.52	144 760
	$\Delta T = 1$	140	188	0.00	0.93	18.83	149 659
	$\Delta T = 3$	144	193	0.00	0.93	19.46	159 772
$P^X = 40$	$\Delta T = 0$	137	184	0.68	0.00	18.54	118 430
	$\Delta T = 1$	138	185	0.68	0.00	18.87	120 590
	$\Delta T = 3$	141	189	0.68	0.00	19.55	125 011
$k^1 = 0.005$	$\Delta T = 0$	144	191	0.00	0.93	20.18	130 348
	$\Delta T = 1$	147	195	0.00	0.93	20.52	134 773
	$\Delta T = 3$	151	201	0.00	0.93	21.19	143 816
$k^1 = 0.015$	$\Delta T = 0$	108	144	0.00	0.93	17.90	97 371
	$\Delta T = 1$	110	146	0.00	0.93	18.20	100 692
	$\Delta T = 3$	113	150	0.00	0.93	18.81	107 442
Southwest							
$\delta = 0.0$	$\Delta T = 0$	126	169	0.00	0.93	18.82	111 520
	$\Delta T = 1$	122	164	0.00	0.93	18.23	104 611
	$\Delta T = 3$	125	168	0.68	0.00	16.77	94 262
$P^Y = 70$	$\Delta T = 0$	137	184	0.00	0.93	18.52	144 760
	$\Delta T = 1$	133	179	0.00	0.93	17.94	135 824
	$\Delta T = 3$	125	168	0.00	0.93	16.79	118 803
$P^X = 40$	$\Delta T = 0$	137	184	0.68	0.00	18.54	118 430
	$\Delta T = 1$	135	181	0.68	0.00	17.91	114 442
	$\Delta T = 3$	130	174	0.68	0.00	16.65	106 663
$k^1 = 0.005$	$\Delta T = 0$	144	191	0.00	0.93	20.18	130 348
	$\Delta T = 1$	140	186	0.00	0.93	19.54	122 280

	$\Delta T = 3$	131	174	0.00	0.93	18.28	106 986
$k^1 = 0.015$	$\Delta T = 0$	108	144	0.00	0.93	17.90	97 371
	$\Delta T = 1$	117	157	0.68	0.00	16.88	93 400
	$\Delta T = 3$	112	150	0.68	0.00	15.68	86 638

1) See Table 1 for baseline parameter values.