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A New Keynesian Framework and Wage and Price Dynamics in the US*

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Abstract

A New Keynesian model with wages and prices is introduced and estimated by maximum likelihood. The steady state relationships of the model are imposed as testable restrictions on the long-run cointegrating relationships in an equilibrium correction framework, giving an equilibrium correction model with imposed theoretical restrictions from a New Keynesian model. Short-run properties are also estimated and investigated. The results indicate that the main aspects of the theoretical model cannot be rejected. Hence, New Keynesian models may be combined with a more general vector autoregressive framework in order to conduct estimation.

JEL: C32, E24, E31.

Keywords: Cointegration, New Keynesian models, steady state, wages, prices, unemployment, equilibrium correction.

1 Introduction

New Keynesian models are founded on a microeconomic framework of optimizing agents, with structural parameters explaining the behavior of these agents. These models are often estimated via Bayesian estimation methods. So far, the main use of these models has been to analyze monetary policy. The focus in this paper is to extend the analysis to the labor market and to conduct estimation using other methods.

The New Keynesian model used here is mainly adopted from Blanchard and Galí (2010) with certain modifications. This yields a relatively transparent model with labor market frictions, real wage rigidities and staggered price setting, characterizing a New Keynesian model where some extensions to include certain aspects are added. Additionally, the oil price is included in the model, in order to account for effects of oil price changes on the general price level. Hence, a theoretical model which explains price- and wage setting is introduced, and rigidities which are often included in various New Keynesian models are also included.

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The theoretical model used here may be represented as a log-linear system, in log deviations from steady state. This is suitable for estimation under the null hypothesis of the correct model, where percentage deviations from the steady state are approximated by the log-linear system. The model is estimated by maximum likelihood, but only a subset of the structural parameters are possible to estimate, suggesting that it is hard to identify all of the structural parameters in the model.

If the observed variables are non-stationary, a filter such as e.g. the Hodrick-Prescott filter is often used on the variables prior to the analysis in order to isolate the cycle components of the data (which is a common way of operating when estimating New Keynesian models). However, an alternative procedure is to express the model as a vector autoregressive (VAR) model with cointegration or an equilibrium correction model (EqCM), where the cointegrating relations are the long-run equilibria which should be associated with the steady state of the theoretical model. The theoretical steady state relationships can then be tested in the EqCM framework, and should be imposed as the long-run equilibria for the system in order to encompass the steady state of the New Keynesian model with the long-run properties of the data.

A simultaneous equations model for the first differences of the observed variables with the inclusion of the identified cointegration relations is then constructed econometrically. The solution of the theoretical model, which consists of a system of log-linear equations, is in form of a restricted vector autoregressive (VAR) model such that the simultaneous equations model may be compared directly to the solution of the theoretical model.

This enables us to estimate the system of variables at a general level, and impose testable restrictions in the form of the long-run relations when the VAR model is transformed into an EqCM. Furthermore, a system of equations may then be estimated, where we may separate between the long-run and the short-run structure and compare the estimated model to the theoretical model. This removes the need to use maximum likelihood or Bayesian estimation methods directly on the theoretical model, where it is shown difficult to find the maximum of the likelihood function. Restrictions on the short-run structure may also be tested in this framework.

2 A Theoretical New Keynesian Model

The model used here is mainly adopted from the framework in Blanchard and Galí (2010). This involves a New Keynesian model which includes extensions such as labor market frictions, real wage rigidities and staggered price setting. Even though these extensions are added to the basic New Keynesian model, the model is relatively simple and transparent such that it is possible to analyze the model analytically in detail. The model consists of households which maximizes their utility, and perfectly competitive firms who each produce an intermediate good which is transformed into differentiated final goods by monopolistically competitive firms. Labor market frictions are included using hiring costs which depend on labor market tightness, such that there is a relationship between unemployment and wage and price dynamics.

2.1 Households

A representative households maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \frac{N_t^{1+\phi}}{1+\phi} \right\}, \tag{1}$$

where C_t is composite consumption (a composition of several goods with elasticity of substitution between them of ε) and N_t is employment or hours of work. β is the discount factor for the households and ϕ measures the inverse of the Frisch elasticity of labor supply. E_0 denotes the expectations operator given the information set at time period 0. This is maximized s.t. the budget constraint

$$P_t C_t + Q_t T_t \leq T_{t-1} + W_t N_t, \tag{2}$$

where W_t is the nominal wage and P_t is the price of the bundle of consumption goods. Furthermore, T_t denotes the quantity of a one-period, nominally riskless discount bond purchased in period t at price Q_t and maturing in period $t+1$, paying one unit of money. Maximizing (1) w.r.t (2) for labor/leisure, consumption and risk-free bonds gives

$$\frac{W_t}{P_t} = N_t^{\phi} C_t, \tag{3}$$

which shows the labor supply as a function of the real wage and consumption, and

$$Q_t = \beta E_t \left\{ \frac{C_t P_t}{C_{t+1} P_{t+1}} \right\}, \tag{4}$$

which may be interpreted as the stochastic discount factor[1] for one period ahead.

2.2 Intermediate goods firms

The j intermediate goods firms are assumed perfectly competitive and each produce $X_t(j)$ using the production function

$$X_t(j) = A_t N_t(j),$$

where A_t measures productivity, assumed to be equal for all firms, and $N_t(j)$ is employment in firm j.

Real profits for the intermediate goods firm are defined as

$$\Pi_t(j) = X_t(j) \frac{P_t(j)}{P_t} - G_t H_t(j) - \frac{W_t}{P_t} N_t(j), \tag{5}$$

where $P_t(j)$ is the price of the intermediate good sold to the final goods producers, G_t denotes real hiring costs, and is defined as

$$G_t = A_t B x_t^{\alpha}, \tag{6}$$

where we have the constants $B > 0$ and $\alpha \geq 0$, and x_t is a labor market tightness index defined below. $H_t(j)$ is the hiring in firm j at time t and W_t is the nominal wage, which is assumed equal across firms. Accordingly, profits in firm j are given as income from sales of the intermediate good X minus total hiring and wage costs.

[1] See e.g. Cochrane (2001) for an explanation of the stochastic discount factor approach, or Galí (2008) for applications of the stochastic discount factor in a New Keynesian framework.

Hiring in firm j evolves according to

$$H_t(j) = N_t(j) - (1-\delta) N_{t-1}(j), \tag{7}$$

where δ is a separation rate which measures the fraction of the employed in period $t-1$ who leaves their job in firm j prior to period t. Hence, hiring consists of the change in employment and the job separation. Furthermore, the labor market tightness index is defined as the ratio between aggregate hires and unemployment U_t,

$$x_t \equiv H_t/U_t = \frac{N_t - (1-\delta) N_{t-1}}{1-(1-\delta) N_{t-1}},$$

such that low unemployment or a high degree of hiring relatively to each other increase the labor market tightness, which leads to a higher hiring cost G_t in (6). The hiring costs are taken as given by the intermediate goods firms (following Blanchard and Galí (2010)).

As indicated by the dynamics of hiring, employment in firm j evolves according to

$$N_t(j) = (1-\delta)N_{t-1}(j) + H_t(j),$$

i.e. employment is made up of workers that stays in the firm and hiring.

For perfect competition when the intermediate goods firms maximize their profits by determining the optimal level of employment, the following expression must hold;

$$\frac{P_t(j)}{P_t} = \frac{W_t/P_t}{A_t} + \frac{G_t}{A_t} - Q_t(1-\delta) E_t\left\{\frac{G_{t+1}}{A_t}\right\}, \tag{8}$$

where $Q_t = \beta E_t\{(C_t P_t)/(C_{t+1}P_{t+1})\}$ is the stochastic discount factor for one period ahead as shown in (4), which is used for the intermediate goods firms since they are owned by the households.

2.3 Final goods firms

Final goods firms are monopolistically competitive such that they set the optimal price of the final goods by adding a markup over the price of the intermediate good (which implies the equivalent of adding a markup over their marginal cost). We define this (time varying) markup as

$$P_t = \mathcal{M}_t P_t(j)$$

for all t. Furthermore, the markup is defined as

$$\mathcal{M}_t \equiv \frac{\varepsilon}{\varepsilon - 1} V_t^{\xi}, \tag{9}$$

where V_t is the real oil price and ξ measures the weight by which the real oil price influences the markup and hence the price of the final good. A shock to the oil price will then be a shock to the supply side of the economy since it influences the cost of the final goods. As seen in (8), the real price of the intermediate good is given by real wages, real hiring costs, productivity and the stochastic discount factor, where the latter is taken as given by the intermediate goods firms since they are owned by the households. This implies that the oil price will only affect the price of the final good and not the optimizing behavior of the intermediate goods firms.

Blanchard and Galí (2007) introduces a non-produced input in the production function as an exogenous variable and interprets an increase in the real price of this input as an increase in the price of oil which increases the real marginal cost. By adding the real oil price to the markup instead, the transparency of the model is increased since the production function will not contain an additional factor. The effect of an increase in the real oil price will have a qualitatively similar effect on the aggregate price level.

As shown empirically in the case of Canada in Khan and Kim (2013), markup shocks are highly associated with current oil price movements. Changing markups of firms could therefore be an indicator of changes in the oil price such that the inclusion of oil prices in the markup may be a feasible way of measuring the markup and markup shocks. As shown below, the New Keynesian Phillips curve (NKPC) will consist of the log of the real oil price in deviation from its steady state value due to the inclusion of it in the markup. Roberts (1995) also includes the real oil price in the (NKPC), but he includes the change of the real oil price directly in the NKPC without including it structurally in the foundations of the model.

Using the markup defined in (9) and inserting for the stochastic discount factor $Q_t = \beta E_t\{(C_t P_t)/(C_{t+1}P_{t+1})\}$, we may rewrite the optimality condition in (8) to

$$\frac{1}{\mathcal{M}_t} = \frac{W_t/P_t}{A_t} + \frac{G_t}{A_t} - \beta\left(1-\delta\right)E_t\left\{\frac{C_t P_t}{C_{t+1}P_{t+1}}\frac{G_{t+1}}{A_t}\right\}. \tag{10}$$

Inserting for the hiring costs defined in (6), we get the steady state relationship

$$\frac{1}{\mathcal{M}} = \frac{W/P}{A} + Bx^{\alpha}\left(1-\beta\left(1-\delta\right)\right),$$

which may be approximated to (assuming $Bx^{\alpha}\left(1-\beta\left(1-\delta\right)\right)$ is small)

$$p = w - a + \xi v + \ln\left(\varepsilon/\left(\varepsilon-1\right)\right), \tag{11}$$

where lower case letters denote logarithms of the respective variable. This shows that we should have a linear relationship between the price, wage, productivity and the real oil price in steady state. Even though the labor market tightness index is included in the price equation (before approximating it), it is taken as given by the firm when they maximize their profits since hiring costs are taken as given. Labor market tightness is therefore considered a given parameter in the price setting.

Furthermore, Calvo pricing (Calvo, 1983) introduces nominal rigidities, which yields the New Keynesian Phillips curve,

$$\pi_t = \frac{\left(1-\theta\right)\left(1-\beta\theta\right)}{\theta}\widehat{mc}_t^v + \beta E_t\pi_{t+1}, \tag{12}$$

where θ measures the fraction of the final goods producers who are not able to reset their prices in each period, π_t is inflation and $\widehat{mc}_t^v$ is the log deviation from the steady state value of the real marginal cost, which consists of the standard marginal cost (i.e. the textbook New Keynesian marginal cost, see e.g. Galí (2008)) and the real oil price which now enters the marginal cost since it is a time varying part of the markup. The log deviation of the total real marginal cost may be decomposed into

$$\widehat{mc}_t^v \equiv mc_t^v - mc^v = mc_t^v - \ln\left(\frac{\varepsilon-1}{\varepsilon}V^{\xi}\right) = mc_t + \xi v_t - \xi v - \ln\frac{\varepsilon-1}{\varepsilon} = \xi\hat{v}_t + mc_t - \ln\left(\frac{\varepsilon-1}{\varepsilon}\right).$$

Inserting this into (12) yields

$$\pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}(\xi\hat{v}_t + mc_t) + \beta E_t \pi_{t+1} - \frac{(1-\theta)(1-\beta\theta)}{\theta}\ln\left(\frac{\varepsilon - 1}{\varepsilon}\right), \qquad (13)$$

such that we have a New Keynesian Phillips curve which describes inflation as a function of the real oil price gap, the real marginal cost, the constant component of the markup and future expected inflation. This is in line with e.g. Hooker (2002) who includes the growth rate for crude oil relative to the inflation rate, and Roberts (1995) who includes the log difference of the real oil price in the NKPC.

The effect of oil price movements on core inflation in the US has e.g. by Hooker (2002) been shown to be much larger before 1981 than after. This indicates that the parameter for the effect of oil price movements on inflation may be non-constant. However, the sample used in the estimation conducted below starts in 1982Q3 such that this problem is less relevant.

2.4 Real wage rigidities

The real wages ($w_t - p_t$ or W_t/P_t) are assumed to be respond slowly to changes in the labor market, and are modeled as in Blanchard and Galí (2007) as a partial adjustment model for the log of the real wage:

$$(w_t - p_t) = \gamma(w_{t-1} - p_{t-1}) + (1-\gamma) mrs_t, \qquad (14)$$

where $0 \leq \gamma \leq 1$ measures the degree of real wage rigidity. Hence, changes in the marginal rate of substitution, mrs_t, is not reflected fully in real wages as long as $\gamma > 0$. The equilibrium wage is therefore defined to be equal to the households' marginal rate of substitution between labor and consumption. This marginal rate of substitution is shown in (3), such that $MRS_t = C_t N_t^{\phi}$. Inserting for this, we get

$$(w_t - p_t) = \gamma(w_{t-1} - p_{t-1}) + (1-\gamma)(c_t + \phi n_t).$$

In steady state, the rigidities are absent since they are assumed to represent distortions. Hence, this amounts to the same steady state relationship as the steady state of (3);

$$\frac{W}{P} = CN^{\phi}.$$

Inserting for the aggregate resource constraint (as defined in Blanchard and Galí (2010) and consistent with the first part of the model outlined above),

$$C_t = A_t(N_t - Bx_t^{\alpha} H_t), \qquad (15)$$

we get the steady state relationship

$$\frac{W}{P} = A(N - Bx^{\alpha})N^{\phi}$$

The steady state expression for employment is $N = 1 - u$ (the sum of aggregate employment and aggregate unemployment rate is defined to equal unity), for hiring $H = \delta N = \delta(1-u)$

and for labor market tightness $x = \delta(1-u)/(u+\delta(1-u))$. Inserting these, we have the steady state relationship

$$\frac{W/P}{A} = (1-u)^{1+\phi}\left(1-\delta B\left(\frac{\delta(1-u)}{u+\delta(1-u)}\right)^{\alpha}\right).$$

Taking logs yields

$$w-p-a = \log\left((1-u)^{1+\phi}\right) + \log\left(1-\delta B\left(\frac{\delta(1-u)}{u+\delta(1-u)}\right)^{\alpha}\right),$$

which may be approximated to (assuming u and $\delta B\left(\frac{\delta(1-u)}{u+\delta(1-u)}\right)^{\alpha}$ are small)

$$w-p-a = -(1+\phi)u - \delta B\left(\frac{\delta(1-u)}{u+\delta(1-u)}\right)^{\alpha},$$

where u is the unemployment rate (not in logs). A Taylor approximation of $\delta B(\delta(1-u)/(u+\delta(1-u)))^{\alpha}$ around $u=0$ yields $(B\delta - B\alpha u)$, such that we obtain

$$w = p + a - (B\alpha + 1 + \phi)u - B\delta. \tag{16}$$

This implies that there should be a linear relationship between the log of the wage, log of the price log of productivity and the unemployment rate in steady state. We denote this relationship the long-run wage equation.

2.5 Log-linearized model

The first equation of the log-linearized model is the New Keynesian Phillips curve shown in (13). The real marginal cost is given by the right-hand side of (10);

$$MC_t = \frac{W_t/P_t}{A_t} + Bx_t^{\alpha} - \beta(1-\delta)E_t\left\{\frac{C_t}{C_{t+1}}\frac{A_{t+1}}{A_t}Bx_{t+1}^{\alpha}\right\},$$

which yields

$$\begin{aligned}\widehat{mc}_t \;=\; & \frac{\varepsilon}{\varepsilon-1}\left(\frac{x}{x+\delta-x\delta}\right)^{1+\phi}(1-g\delta)(\hat{w}_t-\hat{p}_t) - \frac{\varepsilon}{\varepsilon-1}\left(\frac{x}{x+\delta-x\delta}\right)^{1+\phi}(1-g\delta)\hat{a}_t \\ & + \frac{\varepsilon}{\varepsilon-1}g\alpha\hat{x}_t - \frac{\varepsilon}{\varepsilon-1}\beta(1-\delta)g(\hat{c}_t-\hat{c}_{t+1}+\hat{a}_{t+1}-\hat{a}_t+\alpha\hat{x}_{t+1}),\end{aligned} \tag{17}$$

where variables denoted with a hat are log deviations from their steady state values (except for unemployment where it is the deviation from steady state unemployment). Log-linearization of the real wage rigidities in (14) gives

$$\hat{w}_t - \hat{p}_t = \gamma(\hat{w}_{t-1}-\hat{p}_{t-1}) + (1-\gamma)\phi\hat{n}_t + (1-\gamma)\hat{c}_t, \tag{18}$$

and the log-linear expression for labor market tightness is given by

$$\delta\hat{x}_t = \hat{n}_t - (1-\delta)(1-x)\hat{n}_{t-1}. \tag{19}$$

Furthermore, the aggregate resource constraint given in (15) can be log-linearized to yield

$$(1-\delta g)\hat{c}_t = (1-g)\hat{n}_t + (1-\delta g)\hat{n}_t + (1-\delta g)\hat{a}_t + (g-\delta g)\hat{n}_{t-1} - (\delta g\alpha)\hat{x}_t \tag{20}$$

The first order conditions of the consumer shown in (4) yields a log-linear IS equation

$$\hat{c}_t = E_t\hat{c}_{t+1} - (i_t - E_t\pi_{t+1} - \rho)\,, \tag{21}$$

where $i_t \equiv -\ln Q_t$ is the nominal interest rate and $\rho \equiv -\ln\beta$ is its steady state value.

Additionally, we need to add an equation for the interest rate in order to close the model. This is done by including a simple Taylor rule as in Blanchard and Galí (2010);

$$i_t = \rho + \phi_\pi\pi_t - \phi_u\hat{u}_t. \tag{22}$$

We also assume autoregressive processes of order one (AR(1) processes) for the oil price and productivity;

$$\hat{v}_t = \rho_v\hat{v}_{t-1} + \varepsilon_t^v \tag{23}$$

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_t^a. \tag{24}$$

The equations (13) and (17)-(22) may be combined to yield the following system of three equations:

$$\begin{aligned}
\hat{w}_t - \hat{p}_t &= \gamma(\hat{w}_{t-1} - \hat{p}_{t-1}) + \gamma_1\hat{u}_t + \gamma_2\hat{u}_{t-1} + (1-\gamma)\,\hat{a}_t \\
\beta_0\pi_t &= \beta_1 E_t\pi_{t+1} + \beta_2\hat{u}_{t-1} + \beta_3\hat{u}_t + \beta_4 E_t\hat{u}_{t+1} + \beta_5\hat{a}_t + \beta_6 E_t\hat{a}_{t+1} + \kappa\xi\hat{v}_t + \beta_7(\hat{w}_t - \hat{p}_t) \\
\delta_0\hat{u}_t &= \delta_1\hat{u}_{t-1} + \delta_2 E_t\hat{u}_{t+1} - \phi_\pi\pi_t + E_t\pi_{t+1} - \hat{a}_t + E_t\hat{a}_{t+1},
\end{aligned}$$

where $\delta_0 = \delta\,(x-1)\,\frac{g(\alpha(-\delta-x+\delta x+2)-\delta+2)-1}{x(g\delta-1)} - \phi_u$, $\delta_1 = -g\frac{\delta}{x}\,(\delta-1)\,\frac{x-1}{g\delta-1}\,(\alpha-\alpha x+1)$, $\delta_2 = \delta\,(x-1)\,\frac{g+g\alpha-1}{x(g\delta-1)}$, $\gamma_1 = \delta\,(x-1)\,(\gamma-1)\,\frac{-g(\alpha+\delta\phi+1)+\phi+1}{x(g\delta-1)}$, $\gamma_2 = g\delta\,(x-1)\,(\delta-1)\cdot(\gamma-1)\,\frac{\alpha-\alpha x-1}{x(g\delta-1)}$, $\beta_0 = \frac{1}{\varepsilon-1}\,(\varepsilon + g\kappa\beta\varepsilon\phi_\pi\,(\delta-1)-1)$, $\beta_1 = \frac{\beta(\varepsilon+g\kappa\varepsilon(\delta-1)-1)}{\varepsilon-1}$, $\beta_2 = -g\alpha\kappa\frac{\varepsilon}{x}\frac{\delta-1}{\varepsilon-1}\cdot(x-1)^2$, $\beta_3 = -g\kappa\varepsilon\frac{\alpha-\alpha x+\beta(\delta-1)\left(\alpha(x-1)^2(\delta-1)-\phi_u x\right)}{x(\varepsilon-1)}$, $\beta_4 = g\alpha\kappa\beta\frac{\varepsilon}{x}\frac{\delta-1}{\varepsilon-1}\,(x-1)$, $\beta_5 = -\kappa\varepsilon\cdot\frac{(W/P)/A+g\beta(\delta-1)}{\varepsilon-1}$, $\beta_6 = g\kappa\beta\varepsilon\frac{\delta-1}{\varepsilon-1}$, $\beta_7 = \kappa\frac{\varepsilon}{\varepsilon-1}\frac{(W/P)}{A}$, $M = \frac{\varepsilon}{\varepsilon-1}$, $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, $g = Bx^\alpha$ and $(W/P)/A = (1-\delta g)\,(x/(\delta+x-\delta x))^{1+\phi}$. This, together with the autoregressive processes for the oil price and productivity as shown in (23) and (24) yields the entire log-linear model, which may be written in matrix notation as

$$\begin{aligned}
&\begin{pmatrix} 1 & 0 & -\gamma_1 & (\gamma-1) & 0 \\ -\beta_7 & \beta_0 & -\beta_3 & -\beta_5 & -\kappa\xi \\ 0 & \phi_\pi & \delta_0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} \hat{w}_t - \hat{p}_t \\ \pi_t \\ \hat{u}_t \\ \hat{a}_t \\ \hat{v}_t \end{pmatrix}
= \begin{pmatrix} \gamma & 0 & \gamma_2 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 \\ 0 & 0 & \delta_1 & 0 & 0 \\ 0 & 0 & 0 & \rho_a & 0 \\ 0 & 0 & 0 & 0 & \rho_v \end{pmatrix}
\begin{pmatrix} \hat{w}_{t-1} - \hat{p}_{t-1} \\ \pi_{t-1} \\ \hat{u}_{t-1} \\ \hat{a}_{t-1} \\ \hat{v}_{t-1} \end{pmatrix} \\
&+ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1 & \beta_4 & \beta_6 & 0 \\ 0 & 1 & \delta_2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}
E_t \begin{pmatrix} \hat{w}_{t+1} - \hat{p}_{t+1} \\ \pi_{t+1} \\ \hat{u}_{t+1} \\ \hat{a}_{t+1} \\ \hat{v}_{t+1} \end{pmatrix}
+ \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}
\begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^v \end{pmatrix},
\end{aligned} \tag{25}$$

i.e. giving a system for the real wage, inflation, unemployment, productivity and the real oil price.

3 Estimation

3.1 Data

Five observable variables are used in the analysis below. These are variables for wage, price, the unemployment rate, productivity and the oil price, such that none of the variables in (25) are unobserved. The data set includes the civilian unemployment rate for persons 16 years of age and older from the Bureau of Labor Statistics (BLS), the non-farm business sector's implicit price deflator, compensation per hour for the non-farm business sector and output per hour of all persons for the non-farm business sector from BLS' Productivity and Costs release, as well as the West Texas Intermediate Spot Oil Price. The log of these series (except the unemployment rate which is in levels) are plotted in figure 1 for the sample used in the estimation (1982Q3-2011Q1). The real oil price is the nominal oil price deflated by the implicit price deflator used in the data set.

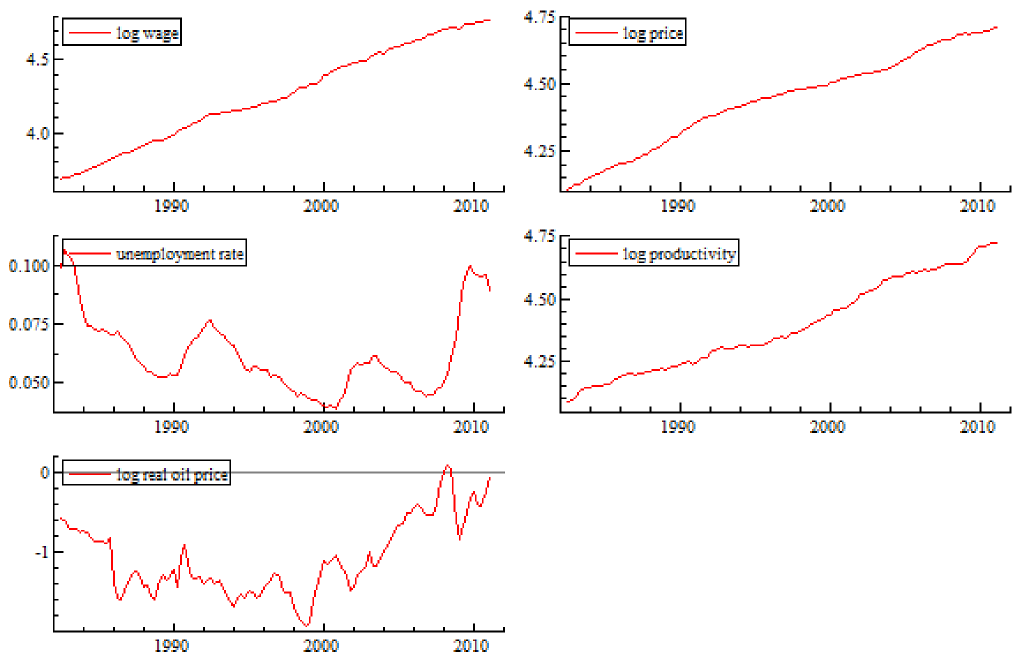

Figure 1: Logarithms of the data series (except the unemployment rate in levels).

3.2 Maximum likelihood estimation

One of the benefits of New Keynesian models is that the parameters in the model explain structural behavior of the agents in the economy such that by estimating the New Keynesian model, we are able to estimate the structural parameters.

In order to estimate the complete model, we may use maximum likelihood estimation (MLE). This is done by using Dynare (Adjemian et al., 2011), where the log-linear model which consists of equations (13) and (17)-(24) are inserted. Maximum likelihood estimation is then conducted by using the observable variables which corresponds to inflation, real wage gap, productivity gap, oil price gap and unemployment gap. In order to connect the data to the theoretically defined variables in the log-linear model, demeaned log differences (approximating demeaned growth rates) of the data series are used, corresponding to the log deviation from the theoretical steady state.[2] Using the log differences is also in line with the econometric model used in section 3.3. Hence, the data set provides the same information when using MLE and when using the econometric model. If different filtering methods were used, the data could have given different information in the two cases.

The log-difference of the implicit price deflator is used as a measure of inflation, and it is demeaned to have an average value of zero in order to match the theoretical model which is defined as having a zero inflation steady state. We should also include the real oil price and the real wage (not the nominal variables) since the model is expressed using these variables. Compensation per hour and the spot oil price is therefore divided by the implicit price deflator in order to create the variables in real terms, and the log difference of these express their growth rates.[3] Furthermore, the log difference of output per hour is used to measure the growth rate of productivity. The unemployment rate is divided by 100 and demeaned, and used as a measure for the deviation between actual and steady state unemployment rate, which should be zero in steady state. We then have observations for all of the variables in (25). The data consists of quarterly observations, and I use the sample 1982Q3-2011Q1, which implies 115 observations. This sample is chosen because of the inflation regime break found for 1982Q3 by Caporale and Grier (2005), which cites other works that also finds a break close to this date.

In order to match the data with the model, since the model uses variables in deviations from steady state and the data set consists of approximated growth rates, I add observational errors to some of the variables. This is also necessary in order to avoid stochastic singularity, which occurs if we have more observables than shocks in the model. Stochastic singularity may be avoided by having at least as many shocks in the model as observable variables either by adding more structural shocks or shock in the form of measurement errors. Since the autoregressive processes for productivity and the real oil price contains the shocks ε_t^a and ε_t^v, we need at least three additional shocks in the model. These are added as measurement errors for the observed variables for inflation, the real wage and unemployment in the following way:

$$\begin{aligned} \hat{u}_t^{obs} &= \hat{u}_t + \nu_t^u \\ \hat{w}_t^{R,obs} &= \hat{w}_t^R + \nu_t^w \\ \pi_t^{obs} &= \pi_t + \nu_t^p, \end{aligned}$$

such that the shocks ν_t^u, ν_t^w and ν_t^p are added as measurement errors by including these equations in the log-linear model. Hence, superscript 'obs' denotes the variables in the data set.

When applying MLE on the log linear model in Dynare, there seems to be problems related to finding the maximum of the likelihood function. This may be due to problems with identification of some of the parameters in the model or misspecification of the model. The various optimization routines for finding the maximum of the likelihood function available in Dynare seems to have problems finding estimates for all of the parameters in the model. I

[2] This is known as a first order difference filter. See e.g. Canova and Ferroni (2011) for an overview of filtering methods used when estimating New Keynesian models.

[3] Using the log deviation of the real oil price as an approximation for the growth in the real oil price may be an inaccurate approximation since there are quite large movements in the oil price.

therefore choose to calibrate the parameters that seems to be most difficult to obtain estimates for, and estimate the remaining parameters. This results in 8 of the 14 parameters being estimated, while the remaining 6 parameters are calibrated. Additionally, the standard deviations of the shocks and measurement errors are estimated.

I calibrate $\alpha = 1$ following Blanchard and Galí (2010), $\beta = 0.99$ is the assumed discount factor, $\phi = 1$ assumes a unitary Frisch labor supply elasticity, $\varepsilon = 6$ implies a gross steady state markup of $\mathcal{M}^v = 1.2$, the weight of the real oil price in the markup is set to $\xi = 0.1$ and the steady state value of the job finding rate is calibrated to $x = 0.7$ which corresponds to a monthly rate of 0.3. Hence, the parameters estimated by maximum likelihood is conditioned on these calibrated values, and the results are shown in table 1.

Table 1: Results from maximum likelihood estimation

parameters	Estimate	s.d.	t-stat
δ	1.4626	0.1195	12.24
γ	-0.5818	0.4416	1.3175
ρ_a	0.4978	0.0822	6.0531
θ	-0.9951	8.2717	0.1203
ρ_v	0.2048	0.0914	2.2419
B	-73.9446	43.6676	1.6934
ϕ_π	1.2645	0.0947	13.3492
ϕ_u	0.3525	0.1915	1.8407
s.d. of shocks	Estimate	s.d.	t-stat
ε_a	0.0077	0.0005	14.9537
ε_v	0.1445	0.0096	15.1319
s.d. of measurement errors	Estimate	s.d.	t-stat
ν_t^u	0.0166	0.0011	15.1027
ν_t^p	0.0026	0.0002	14.7449
ν_t^{wr}	0.0068	0.0005	14.8715

A lot of the estimated parameter values are quite far from what we would expect to be 'reasonable parameter estimates', i.e. consistent with the definitions in the theoretical model. Additionally, some of these parameters are insignificant, so we should be careful in drawing conclusions about how the data explains these deep parameters since this implies that the data does not seem to provide information about them.

The separation rate δ is estimated to be significantly above unity. This makes little sense theoretically, since this parameter should measure the fraction of the employed in period $t-1$ who leave their job during the period. Additionally, the Calvo parameter, θ, and the parameter value describing the degree of real wage rigidity, γ, are both estimated to be negative although we would expect them to be between zero and unity. However, they are not significant. The parameter B, which measures the impact of labor market tightness on hiring costs is estimated to be a large negative number. This is also counter-intuitive, but the degree of significance for this parameter is quite low, so it does not seem to be a lot of explanatory power in the data about this parameter either. The significantly estimated parameter values are δ, ρ_a, ρ_v, ϕ_π and ϕ_u, which all seems to have reasonable results except for δ.

The likelihood functions of New Keynesian models are often full of local maxima and minima and nearly flat surfaces due to a relatively small data set when using quarterly data and the fact that the model contains a lot of parameters (Fernández-Villaverde, 2010). In order to circumvent this issue, bayesian estimation is often used in order to estimate New Keynesian models. Calibrating some parameters prior to MLE may be considered a special case of Bayesian estimation, since this is analogous to imposing a very strict prior for these parameters while having a very loose prior for the remaining parameters. Pure MLE is therefore not actually conducted here, but rather a special case of Bayesian estimation.

In order to get more information on how the differences between the NK model and the data cause problems when estimating using maximum likelihood, we should estimate the model by imposing the assumptions of the theoretical model on the data rather than assuming the NK model is the real data generating process (DGP) and estimate conditional on this assumption. By using an equilibrium correction model (EqCM), this is possible. This enables us to estimate an unrestricted model which shows the behavior of the data, and then we may impose the restrictions of the theoretical NK model and see whether these restrictions are found in the data, as well as where the differences between the data and the assumed DGP lies. The steady state relations of the NK model may be imposed as long-run restrictions and tested, and the dynamic specification of the NK model may be compared to the dynamic specification of the EqCM in order to see what the differences between the model and the data are.

3.3 More general framework

In order to avoid imposing the entire structure of the theoretical New Keynesian model prior to estimation, we may use a more general framework for estimating the economy, and impose certain features of the theoretical NK model as restrictions on an econometrical model. These restrictions will be testable as restrictions on a more general model. Two relationships that we may impose on the long-run structure of the VAR model are the long-run relationship for the price in (11) and for the wage in (16).

The following procedure follows the framework for empirically modeling a system for wages and prices as e.g. in Bårdsen et al. (2007) for Australia, Bårdsen and Fisher (1999) for the UK or Bårdsen et al. (1998) for Norway and the UK. However, while these mainly use a theoretical model of wage bargaining, the theoretical model which will be implemented in the econometric framework here is the New Keynesian model in section 2.

The system outlined in (25) can be solved such that we get a restricted VAR(1) model. Going the opposite way, an estimated unrestricted VAR may be restricted in order to obtain the VAR(1) representation of the solution of the log-linearized model.

The same observable variables as in the estimation of the New Keynesian model by maximum likelihood are used, but logarithms of the variables are used rather than the approximated growth rates. Additionally, the nominal wage is used as the observable variable instead of the real wage since we may impose a relationship between the price level and the nominal wage which describes the real wage implied by the theoretical model. This implies that we have observable variables for the nominal wage, w_t, the price level, p_t, the real oil price, v_t, productivity, a_t and the unemployment rate, u_t as outlined in section 3.1. Using the log of the four former series and the level of the unemployment rate is in line with the theoretical variables in the steady state relations shown in (11) and (16).

The system expressed as a VAR model with k lags may be written as

$$Z_t = \Pi_1 Z_{t-1} + \cdots \Pi_k Z_{t-k} + \Phi D_t + \varepsilon_t. \tag{26}$$

If all of the five variables in (25) are observable, the log-linearized NK model may be solved in order to yield a purely backward-looking solution such as the VAR in (26), though with only one lag and various cross-equation restrictions based on non-linear relations between the parameters.[4] Hence, it is possible to compare an estimated VAR with the solution of the log-linearized forward-looking model (using calibrated or estimated structural parameters) in (25). The dynamic properties of the NK model may therefore be compared with those of the estimated VAR in order to investigate the differences between the data and the theoretical model.

Alternatively, (26) may be reformulated to a vector equilibrium correction model, such as

$$\Delta Z_t = \Gamma_1 \Delta Z_{t-1} + \cdots + \Gamma_{k-1} \Delta Z_{t-k+1} + \alpha \tilde{\beta}' \tilde{Z}_{t-1} + \Phi D_t + \gamma_0 + \gamma_1 t + \varepsilon_t, \tag{27}$$

where $Z_t = [w_t, p_t, u_t, a_t, v_t]'$, $\tilde{\beta}' = [\beta, \beta_0, \beta_1]$, $\tilde{Z}_{t-1} = [Z_{t-1}, 1, t]'$, $\varepsilon_t \sim IN(0, \Omega)$ for $t = 1, \ldots, T$ and Z_{-1}, Z_0 is given. D_t is a vector of dummy variables and γ_0 is a constant.

First, an unrestricted VAR model is estimated, where the number of lags are chosen to be as few as is possible while still having a well specified model. Additionally, dummy variables pertaining to periods with extraordinary events not explained by the model may need to be added to account for residual outliers from the normal distribution such that we get a well specified model. These quarters are 1982Q4, 1983Q1 (where the dummy is set to +1 in the former and -1 in the latter), 1986Q1, 1990Q3, 2000Q1, 2008Q4 and 2009Q1. The peak of unemployment in 1982Q4 and the rapid decline the following period after the recession of 1981-82 may be the institutional event behind the first two dummy variables. The VAR model also indicates this, since the dummy variable is only significant for the unemployment equation. The increase in oil production in Saudi Arabia in late 1985 caused a drop in the crude oil price, which is indicated by the 1986Q1 dummy. This dummy is only significant in the oil price equation. The Iraqi invasion of Kuwait in august 1990 caused a lot of uncertainty in the oil market, and the oil price peaked in this period. The dummy for 1990Q3 is significant for the oil price, so the oil price peak may be a reasonable explanation for this dummy variable. The recession in 1990 caused by the savings and loans crisis, caused a rapid increase in unemployment. This may also be an explanation behind the dummy for 1990Q3, since it is also significant for the unemployment equation. The dummy for 2000Q1 is significant in the wage equation, and controls for the large increase in wages in this period which may be due to the inclusion of employee stock options in the wage compensation variable in the late 1990s (Mehran and Tracy, 2001). In the quarters 2008Q4 and 2009Q1, dummies are added to take account for the financial crisis starting in this period. The residual analysis is shown in table 2, indicating that for a significance level at 3.5%, the model is well specified.

Table 2: Residual analysis for the VAR(2) with dummy variables

Multivariate tests (p-values in brackets)	
Residual autocorrelation	$F(125, 319) = \underset{[0.035]}{1.300}$
Test for normality	$\chi^2(10) = \underset{[0.051]}{18.257}$
Test for ARCH	$F(160, 366) = \underset{[0.285]}{1.076}$

[4] The analytical solution of the model outlined in section 2 is not presented here since a unique solution to the system was not found.

Table 3: Determination of lag length

Model	T	p		log-likelihood	SC	HQ	AIC
VAR(3)	115	110	OLS	2155.7995	-32.954	-34.513	-35.579
VAR(2)	115	85	OLS	2132.4207	-33.578	-34.784	-35.607
Tests of lag reduction:							
VAR(3) $\rightarrow$ VAR(2): F(25,332) = 1.5366 [0.0507]							

The lag length was set to $k = 2$ as indicated by various information criteria and an F test of reducing the lag length from $k = 3$ to $k = 2$, as shown in table 3.

The steady state relationships for wages and prices in the New Keynesian model indicates that we should expect at least two cointegrating vectors in the estimated VAR and a therefore rank of $r = 2$ or more.

Table 4: Eigenvalues of the model and the trace test of the cointegration rank.

eigenvalue	log-likelihood	rank	$H_0 : r \leq$	Trace test	[Prob]
	2085.643	0	0	93.556	[0.000]**
0.259	2102.858	1	1	59.125	[0.003]**
0.222	2117.301	2	2	30.239	[0.044]*
0.161	2127.397	3	3	10.047	[0.282]
0.081	2132.234	4	4	0.37355	[0.541]
0.003	2132.421	5			

Using a 4.4% level of significance or higher, a rank of $r = 3$ seems plausible, according to the trace test shown in table 4. The small eigenvalue for a rank of $r = 3$ also suggests this.

Testing for weak exogeneity given a rank of $r = 3$ suggests that productivity and the real oil price are (jointly) weakly exogenous.[5] The oil price and productivity should be assumed weakly exogenous according to the theoretical model in section 2, since they are modeled as AR(1) processes. Restricting the model to have weakly exogenous productivity and oil price could therefore be assumed prior to estimating the model by using a partial VAR instead of a standard VAR model.

A partial VAR, as introduced by Johansen (1992), which assumes weak exogeneity of oil price and productivity may be used to analyze the theoretical model. We may use the partial VAR(2) model in equilibrium correction form,

$$\Delta ZZ_t = \alpha \tilde{\beta}' \tilde{Z}_{t-1} + \Gamma_1 \Delta Z_{t-1} + \Phi D_t + \gamma_0 + \gamma_1 t + \varepsilon_t, \tag{28}$$

where $ZZ_t = [w_t, p_t, u_t]'$ and $\tilde{Z}_{t-1}$ is as defined above, such that productivity and the oil price is assumed weakly exogenous.

When estimating the partial VAR model, many of the dummy variables pertaining to institutional events are superfluous and the number of dummy variables needed in order to have a well specified model is reduced to three; 1983Q1, 2000Q1 and 2009Q1. The excluded

[5]Restricting the α matrix to have weak exogenity for productivity and the real oil price suggests that the hypothesis cannot be rejected at a 2.5% level of significance (test value of $\chi^2(6) = 14.471$).

dummy variables represented institutional events explaining events influencing the oil price, such that they are not needed in the partial VAR for wage, prices and unemployment.

Testing the rank in the partial VAR model indicates full rank, i.e. $r = 3$ when comparing the trace test statistics in table 5 with the critical values in table 3 in Harbo et al. (1998) (the critical value when testing $H_0 : r \leq 2$ at a 5% significance level is 15.1, so we see that H_0 is rejected, indicating a rank of $r = 3$).

Table 5: Determining the rank in the partial VAR

H0:rank $\leq$	Trace test
0	171.80
1	87.320
2	36.650

This implies that we should have three cointegrating relationships. Based on the theoretical model, the first two would be the long-run wage and the long-run price equation. The third cointegrating relationship may be a relationship for unemployment, and this may be that unemployment is stationary such that there is a steady state of unemployment.

Imposing restrictions according to the steady state relations, i.e. the price equation in (11) and the wage equation in (16) (except for imposing productivity being fully restricted in wages), and stationarity of unemployment, yields the results in table 6.

Table 6: Estimated long run structure of restricted partial VAR for the three α and β vectors. Standard errors in parentheses below estimated coefficients. Test of over-identifying restrictions: $\chi^2(4) = 10.84$ (p-value 0.029).

	β_1	β_2	β_3
w	1.00	−1.00	0.00
p	−1.00	1.00	0.00
u	3.128	0.00	1.00
a	−0.827	1.00	0.00
v	0.00	−0.053	0.00
	α_1	α_2	α_2
w_t	−0.136 (0.038)	−0.072 (0.028)	0.252 (0.098)
p_t	−0.015 (0.016)	−0.047 (0.012)	0.049 (0.042)
u_t	0.0064 (0.014)	−0.021 (0.010)	−0.057 (0.035)

The three cointegration relations in the restricted partial VAR are shown in figure 2, where the first vector represents the long-run wage equation, the second the price equation and the third the unemployment rate. These seems to be stationary, indicating that they may represent the long-run equilibrium relationships of the data.

The restriction of productivity being fully reflected in wages (a unity coefficient for productivity in (16)) is strongly rejected, so this parameter is set to vary freely. As shown in table 6, it is estimated to be −0.827 which indicates that 82.7% of changes in productivity

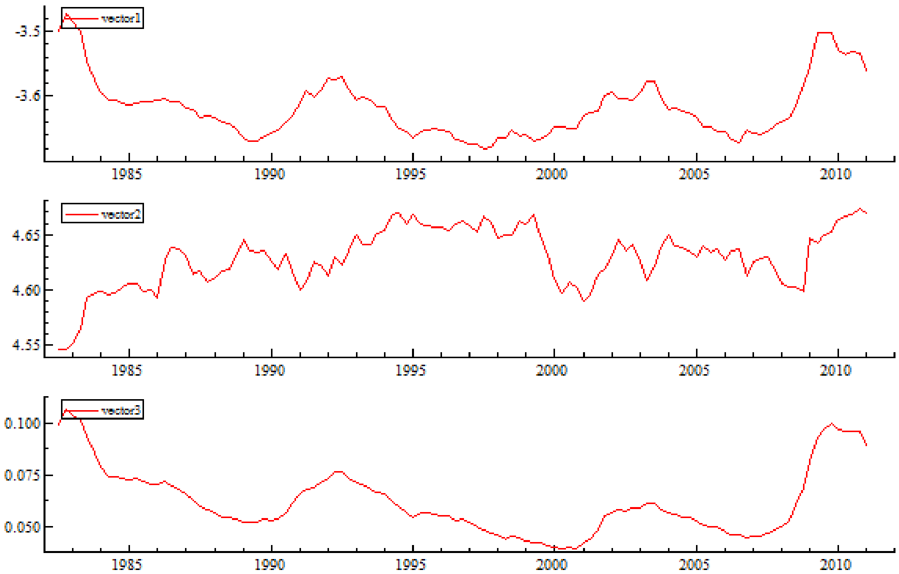

Figure 2: Cointegration relations, with restrictions on all vectors.

are reflected in wages (in contrast to the theoretical model in section 2 which suggests that 100% of changes in productivity should be reflected in wages). The theoretical NK model could be modified to incorporate this e.g. by including a proportional distortionary coefficient in the production function. Furthermore, the estimated long-run parameter for unemployment indicates that an increase in unemployment by one percentage point decreases wages by 3.18%.[6] The long-run elasticity of the real oil price of 0.053 indicates that increases in the real oil price has a small positive effect on the price level. This interpretation assumes that approximating real oil price growth by the first difference of the log of the real oil price holds. Hence, we should be careful in intepreting the estimated value of this coefficient since the growth of the real oil price has been quite large (regarding both positive and negative growth) for the sample period.

The long-run elasticity of the unemployment rate estimated in Bårdsen et al. (2007) for Australia, Bårdsen and Fisher (1999) for the UK or Bårdsen et al. (1998) for Norway is 0.1, 0.065 and 0.08, respectively. These estimates indicates the percentage decrease in wages when unemployment increases by 1%. E.g. for Australia, this indicates that if unemployment increase from 5% to 6% (a 20% increase), wages decrease by 2%. This indicates that the effect of unemployment on wages is larger in the US than in Australia, the UK and Norway, where the log of unemployment is used in the estimation. However, the unemployment rate is used here rather than the log of the unemployment rate such that non-linear effects of the unemployment rate on wages is not caputred to the same degree as in the case of Australia, the UK and Norway. However, estimations using the log of the unemployment rate in the model in this paper (not shown here) also indicate a small effect of unemployment on wages in the case of the US.

[6]The percentage change in the dependent variable following a one percentage point increase in the independent variable is measured as $100 \cdot (\exp(\beta_i) - 1)$ if the dependent variable is in logs and the independent variable is in levels.

The small estimated values in the alpha matrix are in line with the rigidities implied by the theoretical model, such as real wage rigidities introduced in (14), price rigidities modeled by Calvo pricing and labor market frictions modeled by hiring costs and job separation. Additionally, wages adjust faster to disequilibrium than prices and unemployment, according to the estimated alpha coefficients, indicating that wages are not as rigid as prices and unemployment. The more rapid adjustment of wages to disequilibrium may be anticipated in the US, since the labor market in the US is considered to be quite fluid.

The estimated long-run relationships are then imposed on the model as equilibrium expressions in an equilibrium correction model (EqCM). They may be expressed as

$$\begin{aligned} ciw_{t-1} &= w_{t-1} - [p_{t-1} + 0.83a_{t-1} - 3.13u_{t-1}] + constant \\ cip_{t-1} &= p_{t-1} - [w_{t-1} - a_{t-1} + 0.053v_{t-1}] + constant \\ ciu_{t-1} &= u_{t-1} + constant \end{aligned} \tag{29}$$

These cointegrating relationships are then used as long-run values for the system $\{\Delta w_t, \Delta p_t, \Delta u_t\}$ which is estimated conditional on $\{ciw_{t-1}, cip_{t-1}, u_{t-1}, \Delta w_{t-1}, \Delta p_{t-1}, \Delta u_{t-1}, \Delta a_{t-1}, \Delta v_{t-1}\}$, where the oil price is excluded from the wage and unemployment equation and unemployment is excluded from the price equation, in order to identify the estimated parameters. Additionally, only the long-run relation pertaining to the dependent variable in each equation is kept. The results from the estimation is given as

$$\begin{aligned} \Delta w_t = & - \underset{(0.46)}{0.5} \Delta w_{t-1} + \underset{(3.3)}{3.7} \Delta p_t - \underset{(1.1)}{0.63} \Delta p_{t-1} - \underset{(5.5)}{5.6} \Delta u_t + \underset{(4.3)}{4.5} \Delta u_{t-1} \\ & + \underset{(0.45)}{0.65} \Delta a_t + \underset{(0.27)}{0.26} \Delta a_{t-1} + \underset{(0.017)}{0.024} D_{2000Q1} + \underset{(0.037)}{0.016} D_{2009Q1} \\ & - \underset{(0.038)}{0.033} D_{1983Q1} - \underset{(0.33)}{0.42} - \underset{(0.087)}{0.11} ciw_{t-1} \\[2ex] \Delta p_t = & \underset{(0.1)}{0.024} \Delta w_t + \underset{(0.038)}{0.056} \Delta w_{t-1} + \underset{(0.096)}{0.21} \Delta p_{t-1} - \underset{(0.043)}{0.18} \Delta a_t - \underset{(0.041)}{0.036} \Delta a_{t-1} \\ & + \underset{(0.0014)}{0.0019} \Delta v_t + \underset{(0.0014)}{0.00023} \Delta v_{t-1} + \underset{(0.0038)}{0.00044} D_{2000Q1} + \underset{(0.0032)}{0.0024} D_{2009Q1} \\ & - \underset{(0.0025)}{0.0036} D_{1983Q1} + \underset{(0.05)}{0.22} - \underset{(0.011)}{0.046} cip_{t-1} \\[2ex] \Delta u_t = & - \underset{(0.12)}{0.029} \Delta w_t - \underset{(0.046)}{0.063} \Delta w_{t-1} + \underset{(0.24)}{0.45} \Delta p_t - \underset{(0.12)}{0.069} \Delta p_{t-1} + \underset{(0.066)}{0.76} \Delta u_{t-1} \\ & + \underset{(0.066)}{0.031} \Delta a_t + \underset{(0.044)}{0.0083} \Delta a_{t-1} + \underset{(0.004)}{0.00053} D_{2000Q1} + \underset{(0.0033)}{0.0057} D_{2009Q1} \\ & - \underset{(0.0025)}{0.0065} D_{1983Q1} + \underset{(0.0015)}{0.00099} - \underset{(0.014)}{0.039} u_{t-1}, \end{aligned} \tag{30}$$

where standard errors of the coefficients are in parenthesis below the estimated parameter values.

In order to provide a simpler and more robust model, the insignificant parameters in the three equations may be removed. Additionally, the variables in the steady state solutions should be dated at their longest lag, see e.g. Bårdsen (1992), Bårdsen and Fisher (1999) or Bårdsen et al. (2005). This is done in order to be able to facilitate the interpretation of the short-run parameters. It also provides the possibility of reducing the model further since this often enables removing additional insignificant dynamic terms. The equilibrium correction relations will then be defined as

$$\begin{aligned} ecw_t &= w_{t-1} - [p_{t-1} + 0.83a_{t-1} - 3.13u_{t-2}] + constant \\ ecp_t &= p_{t-2} - [w_{t-1} - a_{t-1} + 0.053v_{t-1}] + constant \\ ecu_t &= u_{t-2} + constant, \end{aligned} \tag{31}$$

since the longest lag of unemployment in the wage equation and the unemployment equation and of price in the price equation is $t-2$ after removing insignificant parameters. This gives

$$\begin{aligned} \Delta w_t \;=\; & -\underset{(1.24)}{3.93}\;\Delta u_t + \underset{(0.998)}{3.09}\;\Delta u_{t-1} + \underset{(0.00531)}{0.0264}\;D_{2000Q1} - \underset{(0.0145)}{0.0259}\;D_{1983Q1} \\ & -\underset{(0.0841)}{0.287} - \underset{(0.0232)}{0.0817}\;ecw_t \end{aligned}$$

$$\Delta p_t = -\underset{(0.033)}{0.183}\;\Delta a_t - \underset{(0.034)}{0.0623}\;\Delta a_{t-1} + \underset{(0.00165)}{0.00446}\;\Delta v_t + \underset{(0.0374)}{0.271} - \underset{(0.00808)}{0.0571}\;ecp_t \tag{32}$$

$$\begin{aligned} \Delta u_t \;=\; & \underset{(0.0611)}{0.757}\;\Delta u_{t-1} - \underset{(0.0196)}{0.0658}\;\Delta a_t - \underset{(0.0157)}{0.0334}\;\Delta a_{t-1} + \underset{(0.00151)}{0.00533}\;D_{2009Q1} \\ & -\underset{(0.00205)}{0.00712}\;D_{1983Q1} + \underset{(0.000671)}{0.00263} - \underset{(0.00993)}{0.0349}\;u_{t-2}. \end{aligned}$$

Additionally, some constraints may be added on the parameters in order to further simplify the model. These are exclusion of the 1983Q1 dummy in the wage equation and restrictions on the parameters for the two lags of productivity in the price equation and in the unemployment equation. The coefficient on period t productivity is restricted to be twice the size of that on period $t-1$, such that we have $2\beta_i\Delta a_t + \beta_i\Delta a_{t-1} = \beta_i\Delta a_t + \beta_i\Delta^2 a_t$. This implies that price growth (Δp_t) and change in unemployment (Δu_t) are affected by the productivity growth as well as the speed of productivity growth. These restrictions are added, and the estimated model now becomes

$$\begin{aligned} \Delta w_t \;=\; & -\underset{(0.839)}{2.897}\;\Delta u_t + \underset{(0.655)}{2.203}\;\Delta u_{t-1} + \underset{(0.00523)}{0.0260}\;D_{2000Q1} - \underset{(0.0755)}{0.266} - \underset{(0.0208)}{0.07597}\;ecw_t \\ \Delta p_t \;=\; & -\underset{(0)}{0.1704}\;\Delta a_t - \underset{(0.014)}{0.08518}\;\Delta a_{t-1} + \underset{(0.0016)}{0.00480}\;\Delta v_t + \underset{(0.037)}{0.2654} - \underset{(0.00801)}{0.0559}\;ecp_t \\ \Delta u_t \;=\; & \underset{(0.0616)}{0.7373}\;\Delta u_{t-1} - \underset{(0)}{0.07748}\;\Delta a_t - \underset{(0.00944)}{0.03874}\;\Delta a_{t-1} + \underset{(0.00158)}{0.006451}\;D_{2009Q1} \\ & -\underset{(0.00136)}{0.003757}\;D_{1983Q1} + \underset{(0.000678)}{0.002897} - \underset{(0.0103)}{0.03827}\;u_{t-2} \end{aligned} \tag{33}$$

This model cannot be rejected wen tested against the dynamic unrestricted model in (30) at a 1.4% significance level ($\chi^2(27) = 45.52$) and at a 13.6% significance level when tested against the dynamic model with correctly dated error correction terms shown in (32) ($\chi^2(3) = 5.54$). Model reduction test are summarized in table 7. The model is also well specified as shown by the residual analysis in table 8, and explains the data quite well as indicated by the fit of the model in figure 3.

Table 7: Model reduction tests

Model	log-likelihood	Test	χ^2(d.f.)	d.f.	p-value
M1: Full dyn. model	1561.77				
M2: Identified model, (30)	1553.41	$LR(M2\|M1)$	16.73	6	0.010
M3: Reduced model, (32)	1541.79	$LR(M3\|M2)$	39.97	24	0.022
		$LR(M3\|M2)$	23.24	18	0.182
M4: Final model, (33)	1539.01	$LR(M4\|M1)$	45.52	27	0.014
		$LR(M4\|M2)$	28.79	21	0.119
		$LR(M4\|M3)$	5.54	3	0.136

Table 8: Residual analysis for final restricted dynamic model.

Multivariate tests (p-values in brackets)	
Residual autocorrelation	$F(45, 277) = \underset{[0.08]}{1.34}$
Test for normality	$\chi^2(6) = \underset{[0.32]}{7.00}$
Test for ARCH	$F(120, 521) = \underset{[0.04]}{1.28}$

Δu_t and Δu_{t-1} enters the wage equation with opposite signs. The coefficients on these are almost identical (taking into account the size of their standard errors) with opposite signs which would indicate that the speed of change of unemployment affects wage growth rather than the change in the unemployment rate[7], as e.g. in Bårdsen et al. (2007). Adding this restriction can be rejected at a 0.5% significance level, so it does not seem plausible to add this restriction. However, the data for the unemployment rate here is in rates rather than in logs, so the speed of change in unemployment could matter for wage growth, but it may be conditional on the level of the unemployment rate such that both need to be included in the equation. Thus, we may consider the case where we have Δu_t and $\Delta^2 u_t$ as variables in the wage equation. Since the estimated value for the coefficient for Δu_t is -2.897 and for Δu_{t-1} is $+2.203$, this could be interpreted as $\Delta w_t = -0.694\Delta u_t - 2.203\Delta^2 u_{t-1}$, such that both the change and the speed of change of unemployment matters for wage growth, capturing a non-linear effect of unemployment on wages in the short run. These estimated parameter values would imply that the negative effect of unemployment on wage growth is smaller in the short run than in the long run, indicating labor market frictions.

The final model shows that wages and prices have no short run effects on each other. This may be a sign of rigidities, in line with the theoretical model which suggests real wage rigidities and price rigidities. This also implies that there is no short-run homogeneity between

[7] $-\Delta u_t + \Delta u_{t-1} = -(u_t - u_{t-1}) + (u_{t-1} - u_{t-2}) = \Delta^2 u_t$

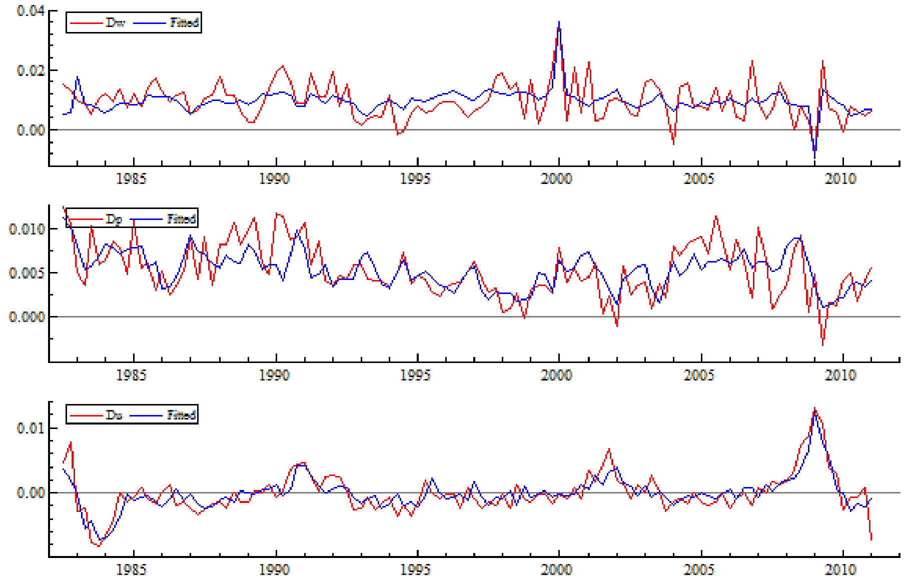

Figure 3: Actual and fitted values for the three equations.

wages and prices or vice versa in the short run, which also may indicate rigidities of wages and prices.

The low values for the estimated parameters of the speed of adjustment indicates rigidities is in line with the estimates in the α matrix in the EqCM. The speed of adjustment is lowest for unemployment and highest for wages, which also was indicated by the α matrix as shown in table 6. The high value of the autocorrelation parameter for unemployment change (0.74) also indicates that the speed of adjustment to disequilibrium for unemployment is low.

In the short run, the effect of oil price changes is smaller than in the long run (0.005 and 0.053, respectively). Since an increase in the oil price affects the cost for the firms, it takes time before firms change their prices and in effect influences the general price level. The difference between the short run and the long run parameter for the oil price as seen here should therefore be expected.

3.4 Impulse response functions

Impulse response functions (IRFs) for the calibrated[8] NK model and the estimated dynamic model (the final restricted dynamic model shown in (33)) are compared in figures 4 and 5. The variables being affected are log deviations from steady state in the NK model and log differences (approximating growth rates) in the EqCM. All of the shocks are transitory such that the error correction terms (i.e. the estimated long-run relations) are assumed not to be affected by the shocks. Additionally, the growth rates are assumed to equal zero in the long run according to the IRFs, which is not the case for the non-stationary variables

[8]The parameters estimated by maximum likelihood estimation in section 3.2 are now calibrated. The parameters are calibrated to $\delta = 0.12$, $B = 0.12$, $\gamma = 0.5$, $\phi_\pi = 5$, $\phi_u = 0.8$ and $\rho_a = 0.9$, following the calibration in Blanchard and Galí (2010). Additionally, the persistence of the oil price shock is set to $\rho_v = 0.5$, and $\theta = 0.4$.

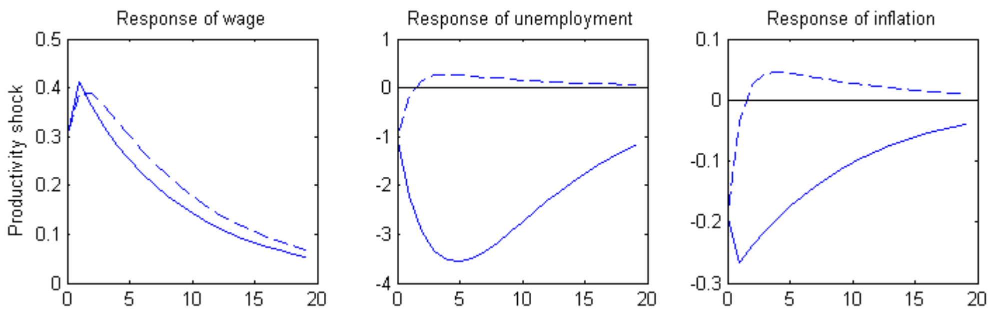

Figure 4: Impulse response functions for the estimated dynamic model (solid lines) and the calibrated NK model (dashed lines) following a positive productivity shock.

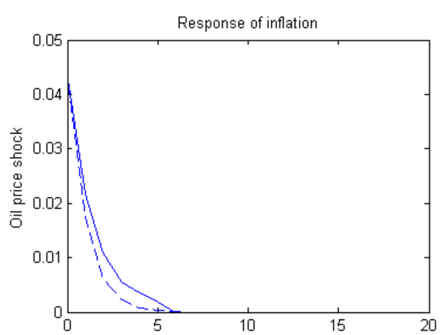

Figure 5: Impulse response functions for the estimated dynamic model (solid lines) and the calibrated NK model (dashed lines) following a positive oil price shock.

in the EqCM where the long-run relationship indicates the long-run growth of the variable. Hence, we should view the path which each variable returns to as the return to the long-run equilibrium. Furthermore, the effect on wages concerns nominal wages in the dynamic model and real wages in the NK model. The shocks are scaled such that the initial effect in the estimated dynamic model and the NK model are of equal size for each impulse response function in order to make the comparisons of the impulse response functions more transparent. Additionally, AR(1) processes are added for the productivity and oil price shocks in the EqCM in order to have the same dynamics for the shocks as in the NK model. In the dynamic model, a transitory oil price shock will only affect inflation in the short run since inflation in that case does not affect wages or unemployment as seen in (33). We are therefore only able to compare the effect of an oil price shock on inflation in this case.

As seen by the IRFs in figure 4, the response to a productivity shock seems to be quite similar for the nominal wage in the estimated model and the real wage in the NK model. The response of inflation to an oil price shock as shown in figure 5 also seems to be similar for the two models, since there is a direct effect on inflation from a change in the oil price. This shows that the theoretical assumption of oil prices being a part of the markup for the final goods firms, as defined in (9), is in line with what is shown by the data. A positive oil price shock will increase the markup, which will lead to a higher price of final goods. Inflation is therefore directly affected by the oil price such as shown by the data in the EqCM.

A positive productivity shock has a similar negative initial effect on inflation and unemployment as seen in the second and third panel of figure 4. However, the negative inflation and unemployment growth then increase and thereafter return back to equilibrium in the EqCM, while they return quickly back to equilibrium in the NK model and then become slightly positive before coming back to equilibrium again. This suggests that there are some differences between the NK model and the dynamic properties of the data.

The initial effect of the productivity shock on inflation and unemployment goes in the same direction for the EqCM and the NK model. This implies that the differences in the dynamics of the NK model and the EqCM explains the differences in the IRFs. Firtstly, since there is a larger degree of persistence for the technology shock in the data than in the NK model, the rigidities seems to be stronger than explained by the theory. This may therefore be related to Calvo pricing as introduced through the New Keynesian Phillips curve, real wage rigidities as included in the partial adjustment model in (14) or labor market frictions as included by hiring costs and job separation in (6) and (7). The implementation of rigidities in the theoretical model does not correspond to the rigidities found by the persistence of shocks in the data, because of the magnitude or the structure of the theoretically modeled rigidities.

For a fluid labor market such as in the US, the persistence of shocks should be quite low (Blanchard and Galí, 2010). This seems to be the opposite of what is found in the data here, since there is a large persistence of shocks. Real wage rigidities, measured by the parameter γ in (14) is set to $\gamma = 0.5$ in the calibrated model, following the calibration in Blanchard and Galí (2010). This parameter measures to what extent the real wage depends on the real wage in the previous period relative to the marginal rate of substitution, i.e. factors that suggest a change in wages such as e.g. increased unemployment. Since the wage rigidities were found to be large in the estimated alpha matrix as shown in table 6 (although not as large as for prices and unemployment), the calibrated value of 0.5 may be too low in order to account for the strong degree of rigidities as shown in the IRFs for the EqCM. The estimated parameters for the equilibrium correction relations in the EqCM in (33) show that the least significant

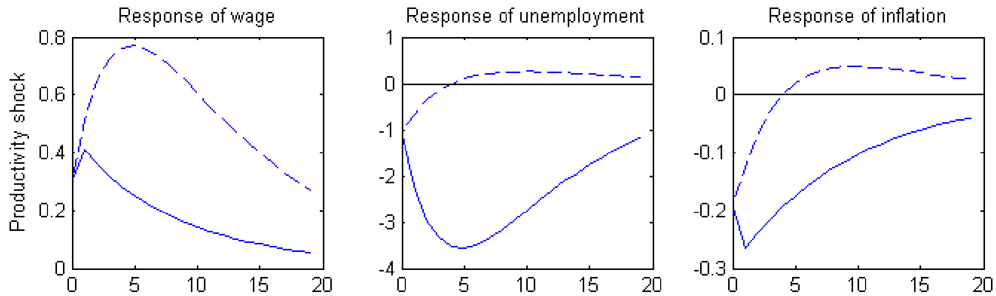

Figure 6: Impulse response functions for the estimated dynamic model (solid lines) and the calibrated NK model (dashed lines) following a positive productivity shock. The parameter for real wage rigidities is calibrated to $\gamma = 0.9$ in the NK model.

error correction term is the term for the wage equation indicating that there is a large degree of wage rigidities.

By calibrating the parameter measuring real wage rigidities to $\gamma = 0.9$ in order to increase the degree of real wage rigidities in the NK model, we get the IRFs for the productivity shock in figure 6. The dynamic response of the real wage in the NK model is now much higher than the response of the nominal wage in the EqCM as seen in the first panel of figure 6. This is reasonable since decreased inflation following a productivity shock should lead to a larger positive effect on the real wage than the isolated effect on the nominal wage. Furthermore, we see that the response of unemployment and inflation to the productivity shock are now returning more slowly back to equilibrium after the negative initial effect, compared to the case where we used a smaller calibrated value for the real wage rigidities. This implies that the real wage rigidities should be larger than what was assumed in the initial calibrated NK model in order for the model to replicate the results found in the data. It also highlights the need of including wage rigidities such as modeled in (14) into NK models in order to match what is observed in the data.

However, the persistence of the productivity shock on unemployment and inflation is still not as large in the NK model as it is in the IRFs from the EqCM. The hump-shaped IRFs in the EqCM are not replicated by the NK model and may be the explanation of this larger degree of persistence of the productivity shock. By using a hybrid New Keynesian Phillips curve (i.e. a forward-looking and a backward-looking inflation term in the NKPC due to partial indexation as in Galí et al. (2001)) rather than the basic version used in (13), a hump-shaped response of inflation to a productivity shock may be modeled. The standard New Keynesian Phillips curve used in the NK model in (12) indicates that prices change as a result of some firms resetting their prices while other firms are not able to and have to keep the price they also used the previous period. The hybrid New Keynesian Phillips curve is developed by modeling that the firms who are not able to reset their price in the period change their price according to some indexation rule depending on past aggregate prices. This enables a Phillips curve where inflation depends on both future expected inflation and previous inflation. The system will then depend on past prices, allowing the hump-shaped response of variables to shocks because of an amplification of shocks the first few periods after the shock. This is because firms that set prices based on previous prices react to the shock and set prices according to it after some periods rather than to react immediately. Since some firms may react slowly and gradually to shocks, this is more reasonable than assuming that firms either optimize fully or

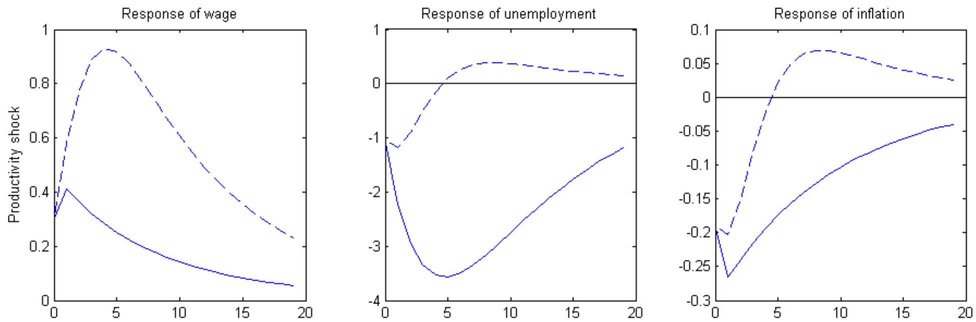

Figure 7: Impulse response functions for the estimated dynamic model (solid lines) and the calibrated NK model (dashed lines) following a positive productivity shock. A hybrid New Keynesian Phillips curve is used, and the parameter for real wage rigidities is calibrated to $\gamma = 0.9$ in the NK model.

not at all in a given period. The parameter for the forward-looking term is calibrated to 0.6 and the backward-looking term to 0.4. This yields the IRFs following the productivity shock as shown in figure 7 (where the parameter for the real wage rigidity is still set to $\gamma = 0.9$).

After replacing the basic NKPC with the hybrid version, it is possible to obtain hump-shaped responses of the IRFs. However, there is overshooting in the NK model, while it is not found in the data. This overshooting response to the productivity shock may be because of the interest rate rule included in the NK model which reacts to deviations from steady state for unemployment and inflation (Friedman and Woodford, 2010, p.285-367). The estimated parameters in the EqCM show the relation between the five observable variables in the sample such that monetary policy conducted in the sample period is included implicitly by the estimated model. Hence, this could indicate that the monetary policy in the NK model through a Taylor rule as introduced in (22) is not equivalent to the monetary policy actually conducted in the US for the sample in the estimated EqCM. Assuming that the central bank follows an instrument rule such as the Taylor rule may be a too strong assumption since central banks rather use a more complex decision making process that is hard to model. Particularly, the interest rate may react more slowly than in the Taylor rule which assumes an instant reaction to a change in inflation or the unemployment gap.

Additionally, we know that the solution of the log-linearized model has a VAR(1) representation. Since the dynamic model following the EqCM consists of more lags, this may also be a reason for why the EqCM captures more dynamics than the theoretical model. However, we see that by making some adjustments to the NK model in section 2 such as increasing the degree of real wage rigidities and using backward-looking price setting firms, the IRFs of the theoretical model becomes more similar to what is found in the data. The results therefore indicate that the relatively simple NK model outlined here with some additional extensions may explain what is found in the data. A larger degree of rigidities is of particular importance, which is shown after increasing the degree of real wage rigidities and using the hybrid Phillips curve as shown by the IRFs in figure 7.

4 Conclusion

By estimating an econometrical model for wages, prices and unemployment, it is possible to investigate what is implied by the observed data for the economy. However, a theoretical framework is needed in order to analyze the data, either in terms of a theoretical structural model or by comparing the data to some stylized facts. The theory we are interested in may be tested in the econometrical framework as over-identifying restrictions, provided that the estimated model is well specified. Furthermore, the dynamic model which is estimated with the restricted long run relations yields a backward-looking model which also typically is the representation of a solution of a theoretical New Keynesian model. This also indicates that a New Keynesian model may be estimated through an equilibrium correction framework.

The New Keynesian model used here as a theoretical background is tested in the estimated econometric model, and most of the restrictions on the long-run structure of the data in form of the steady states of the theoretical model are identified and cannot be rejected as restrictions on the econometric model. This implies that the theoretical model seems to be a good description of the data in the long run.

The estimates of both the long-run vectors and the dynamic model which shows the short-run effects, indicate rigidities in prices, wages and unemployment. This is a clear indication of rigidities being present in the data, which are important factors in New Keynesian models such as rigid price setting, wage rigidities and labor market frictions.

When we look at the effects of simulated shocks to the economy, the estimated model and the calibrated theoretical model seems to produce quite similar results, as indicated by the impulse response functions. This suggests that the results from the calibrated theoretical model are to a large extent found in the data, especially when we increase the degree of real wage rigidities and include the hybrid New Keynesian Phillips curve in the theoretical model.

Combining a New Keynesian model with an econometric framework as done here is an alternative to estimate the complete model using maximum likelihood or bayesian estimation methods. The advantage of using the method outlined here is that properties of the theoretical model may be tested, while maximum likelihood and bayesian estimation of the log-linearized model assumes the theoretical model being the data generating process prior to estimation. Hence, this method enables us to combine observed data with a New Keynesian model without preferring one over the other.

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