International Resource Tax Policies
Beyond Rent Extraction

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Abstract

We study the incentives of selfish governments to tax tradable primary inputs under asymmetric trade. Using an empirically-consistent model of endogenous growth, we obtain explicit links between persistent gaps in productivity growth and the observed tendency of resource-exporting (importing) countries to subsidize (tax) domestic resource use. Assuming uncoordinated maximization of domestic welfare, national governments wish to deviate (i) from inefficient laissez-faire equilibria as well as (ii) from efficient equilibria in which domestic distortions are internalized. The incentive of resource-rich countries to subsidize hinges on slower productivity growth and is disconnected from the typical incentive of importers to tax resource inflows – i.e., rent extraction. The model predictions concerning the impact of resource taxes on relative income shares are supported by empirical evidence.

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1 Introduction

The recent up-surge in oil prices and the growing relevance of primary commodities in world trade have revived the interest in the international sharing of natural resource rents (WTO, 2011). Fiscal policies are central to the debate since uncoordinated taxation may influence trade outcomes to a great extent, and there are remarkable asymmetries in the fiscal treatment of primary resources. In particular, international comparisons between oil-rich and oil-poor countries reveal that while importers levy oil taxes with varying but often high rates, most oil exporters grant subsidies on domestic oil consumption (Gupta et al. 2002; Metschies, 2005). One crucial question is whether asymmetric trade – i.e., trade of primary resources versus final goods – creates incentives for national governments to impose strategic domestic taxes. A more specific question is why oil-rich countries do subsidize domestic oil use – a stylized fact that, beyond mostly political-economic arguments, is not explained by first principles like social welfare maximization.

In this paper, we tackle these issues in a two-country model of endogenous growth which draws an explicit link between persistent gaps in productivity growth and the observed tendency of resource-exporting (importing) countries to subsidize (tax) domestic consumption of primary resources.

The incentives behind resource taxation have traditionally been studied in two parallel strands of literature in international trade and in public economics. Bergstrom (1982) showed that, facing an inelastic world resource supply, importing countries may tax domestic use to extract rents that would otherwise accrue to exporters. The rent-extraction mechanism is reinforced by the introduction of pollution externalities (Amundsen and Schöb, 1999) and monopolistic behavior on the supply side (Brander and Djajic, 1983) since the importers’ incentive to tax is stronger the higher the rents to be potentially captured and the lower the social benefit from domestic resource consumption (Rubio and Escriche, 2001; Liski and Tahvonen, 2004). The existing literature on this topic neglects however two important aspects.

First, the rent-extraction mechanism does not explain why resource-exporting countries subsidize domestic resource use. The observed subsidies may reflect political convenience – e.g. providing benefits to well-organized groups (Tornell and Lane, 1999).
and bribing voters (Robinson et al., 2006) – but a clear economic rationale based on first principles and standard behavioral assumptions, like the pursuit of maximal social welfare, is still lacking. Second, most analyses of strategic trade policies hinge on partial equilibrium models that do not consider the role of economic growth and, especially, of international productivity gaps. In this respect, a number of empirical studies present cross-country evidence suggesting that specialization in resource production and exports is negatively correlated to domestic productivity (Lederman and Maloney, 2007). In particular, oil-exporting countries exhibited persistently slower growth in labor productivity but constant income levels relative to oil-importing countries during the last four decades, a plausible reason being the compensating effects of terms of trade (Bretschger and Valente, 2012). In this paper, we argue that persistent gaps in productivity growth influence the policymakers’ incentives to distort trade and may provide new rationales for both the rent-extracting taxes and the defensive subsidies that we observe in the real world.

We tackle the issue in a two-country model of endogenous growth where asymmetric trade is merged with country-specific engines of economic growth: persistent gaps in physical productivity between resource-rich and resource-poor economies originate in different investment rates since R&D productivity incorporates positive spillovers from past research. We assume that each national government selfishly aims at maximizing domestic welfare, observes an initial state of affairs and evaluates the welfare consequences of modifying the world resource allocation via national taxes. Since the rise of endogenous economic growth hinges on market failures within each economy, we are able to study two types of initial state of affairs, namely symmetric laissez-faire and symmetric efficient equilibria in which domestic distortions are internalized ex-ante.

Our analysis yields two main results. First, if the initial state of affairs is an efficient equilibrium, the government of the resource-importing country (labelled ‘Home’) can increase domestic welfare by raising the resource tax above the efficient level. Second, and most important, if the initial state is a laissez-faire equilibrium, technological differences determine asymmetric incentives consistent with the observed stylized facts: when productivity growth is faster in the resource-importing economy, Home’s incentive to raise the resource tax is reinforced whereas the government of the resource-exporting country
(labelled ‘Foreign’) has an incentive to subsidize domestic resource use. The general intuition is that resource taxes serve different purposes in the two economies. In Home, the resource tax increases Home’s share of world income by reducing the cost share of resource inputs purchased from Foreign firms – a variant of the rent-extraction mechanism. Therefore, Home’s government aims at distorting the world resource allocation, using the domestic resource tax to artificially reduce Home’s share of world resource use below the efficient level. The Foreign resource tax, instead, distorts the world resource allocation without influencing the world income distribution: since Foreign rents are maximized when relative resource use is efficient, the Foreign government will use the resource tax to keep an efficient proportion between the two countries’ levels of resource purchases. It follows that, starting from an efficient equilibrium, Foreign has no incentive to deviate whereas Home has an incentive to increase the resource tax in order to extract resource rents. Starting from laissez-faire, instead, productivity differences generate new incentives for both governments to deviate because relative resource use is determined by the ratio between the two countries’ investment rates.¹ When Home’s productivity in R&D is stronger and there is no public intervention to internalize R&D spillovers, the ratio between Home and Foreign investment rates is inefficiently high and implies an inefficiently high level of Home’s resource use relative to Foreign. This situation dissatisfies both governments. Foreign would like to subsidize domestic resource use in order to keep an efficient ratio with Home’s resource use. Home would like to tax domestic use for two reasons: eliminating its own over-consumption of resources induced by productivity differences, and pushing its relative resource use further below the efficient level in order extract rents. In a nutshell, the rent-extraction mechanism is logically disconnected from Foreign’s incentive to subsidize while it is not the unique driver of Home’s incentives to tax.

Our analysis is based on the two-country model developed in Bretschger and Valente (2012), where balanced growth yields a stable world income distribution consistently with the empirical evidence for oil-trading countries. A more specific prediction, which

¹This is an intermediate result of our analysis. For given taxes, the ratio between the two countries’ demand for resources reflects the ratio between the two countries’ final output and the latter ratio is higher the higher is the ratio between the two countries’ investment rates.
underlies all our theoretical conclusions, is that the relative income of oil-poor economies increases with domestic resource taxes and increases (decreases) with domestic (foreign) investment rates. Perform dynamic panel estimations on the determinants of relative income shares of oil-poor and oil-rich countries, we obtain results that confirm the positive impact of domestic resource taxes on the income share of oil-poor countries as well as the predicted impact of the respective investment rates.

At the theoretical level, our results shed further light on the conclusions of the existing literature. Among the few previous studies of asymmetric trade with endogenous growth,

the contribution that is closest to ours is Daubanes and Grimaud (2010), which however assumes identical R&D technologies and polluting resources. Daubanes and Grimaud (2010) show that, even if trading countries coordinate their policies to correct the global environmental problem, their divergent strategic interests cause a non-environmental distortion in the allocation of the resource – an outcome that is directly related to the rent-extraction mechanism in our model. While the two analyses differ in both aims and means, a specific value added of the present paper is to go beyond the rent-extraction mechanism by introducing persistent differences in productivity growth, which provides new rationales for the rise of both defensive subsidies and strategic taxes.

2 The Model

The model comprises two countries, called Home and Foreign and indexed by \( i = h, f \). Following Bretschger and Valente (2012), each economy produces a tradable final good, consumed by the residents of both countries, using man-made intermediate inputs and an exhaustible natural resource. Trade is asymmetric since the natural resource stock is exclusively owned by Foreign residents: Home only exports its final good whereas Foreign exports both final goods and resource units. Output growth is driven by R&D

\[ \text{Output growth is driven by R&D} \]

\[ \text{A related contribution is Peretto and Valente (2011), which presents an endogenous growth model to study the impact of resource booms – i.e., unexpected discoveries of new resource stocks – on innovation rates and relative welfare. This analysis is not related to strategic taxation and hinges on the assumption of identical R&D technologies.} \]

\[ \text{The present model adapts Bretschger and Valente (2012) to include the fiscal instruments that governments need to achieve efficient equilibria at the national level.} \]
activity that expands the varieties of intermediate inputs (Rivera-Batiz and Romer, 1991).

Two market failures affect the domestic equilibrium of each economy: the existence of monopoly rents in intermediates’ production, and knowledge spillovers enhancing the productivity of R&D firms over time. Accordingly, we will use two equilibrium concepts as possible ‘initial state of affairs’, studying whether selfish governments have incentives to deviate (a) from laissez-faire equilibria and (b) from efficient equilibria in which all domestic market failures are internalized ex-ante. In order to support efficient equilibria, each national government has access to three fiscal instruments: subsidies to R&D, taxes on final producers, and taxes on domestic resource use.4

2.1 Final Producers, Intermediate Sectors and R&D

Final Sector. Each country’s final sector produces $Y_i$ units of a tradable consumption good using $M_i$ varieties of differentiated intermediate products, $L_i$ units of labor, and $R_i$ units of an exhaustible resource, according to

$$Y_i = \int_0^{M_i} (X_i(m_i))^\alpha dm_i \cdot (v_iL_i)^\beta R_i^\gamma, \quad i = h, f,$$

(1)

where $X_i(m_i)$ is the quantity of the $m_i$-th variety of intermediate input, $v_i$ is labor productivity, and parameters satisfy $\alpha + \beta + \gamma = 1$ with $0 < \alpha, \beta, \gamma < 1$. The endogenous engine of growth is represented by increases in the mass $M_i$ of intermediates’ varieties, while labor efficiency grows at the exogenous rate $\dot{v}_i = \eta_i$. Labor is inelastically supplied: $L_h$ and $L_f$ are fixed amounts coinciding with the respective population sizes. The law of one price holds for all traded goods: the quantities $(Y_h, Y_f)$ are sold at the respective world prices $(P^h_Y, P^f_Y)$ and the exhaustible resource is sold to all final producers at the same world price $P_R$. Labor and intermediates are not traded so that the wage rate and the price of each intermediate, respectively denoted by $P^i_L$ and $P^i_{X(m_i)}$, are country-specific. Production costs in the final sector are affected by proportional taxes on the purchases of intermediate inputs and on resource use, respectively denoted by $b_i$ and $\tau_i$.

4Using conventional notation, the time-derivative and the growth rate of variable $g(t)$ are respectively denoted by $\dot{g}(t) = dg(t)/g(t)$ and $\ddot{g}(t) = \dot{g}(t)/g(t)$. All Propositions are proved in Appendix.
The resulting profit-maximizing conditions read\(^5\)

\[
P_R R_i (1 + \tau_i) = \gamma P_Y^i Y_i, \tag{2}
\]

\[
P_{X(m_i)}^i (1 + b_i) = \alpha P_Y^i (X_i (m_i))^{\alpha - 1} (v_i L_i)^{\beta} R_i^2, \tag{3}
\]

where (3) is valid for each variety \(m_i \in [0, M_i]\).

**Intermediate Sector.** Each variety of intermediate is produced by a monopolist who holds the relevant patent and maximizes profits \(\Pi_i (m_i)\) taking the demand schedule (3) as given. Producing one unit of intermediate requires \(\varsigma\) units of final good, where \(\varsigma > 0\) represents a constant marginal cost that equally applies to each variety. Profit maximization implies the mark-up rule

\[
P_{X(m_i)}^i = (\varsigma/\alpha) P_Y^i, \text{ for each } m_i \in [0, M_i],
\]

and therefore symmetric quantities and profits across monopolists.

**R&D Sector.** The mass of intermediates’ varieties \(M_i\) grows over time by virtue of R&D activity pursued by competitive firms that develop new blueprints and sell the relevant patents to new monopolists. We represent R&D firms as a consolidated sector earning zero profits due to free-entry.\(^6\) Developing blueprints requires investing units of the domestic final good, with marginal productivity \(\phi_i\). R&D investment is subsidized by the domestic government at rate \(a_i > 0\). Denoting by \(Z_i\) the total amount invested by R&D firms, aggregate R&D investment in country \(i\) is \(Z_i (1 + a_i)\), and the increase in the mass of varieties equals

\[
\dot{M}_i (t) = \phi_i (t) \cdot (1 + a_i) \cdot Z_i (t). \tag{5}
\]

The productivity of the R&D sector is affected by externalities that take the form of knowledge spillovers – exactly as in models à la Lucas (1988), where the productivity of

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\(^5\)Both \(b_i\) and \(\tau_i\) are assumed to be constant in order to preserve the balanced-growth properties of the world equilibrium. This assumption does not affect the generality of our results: as shown in section 3, both efficient allocations and laissez-faire equilibria exhibit balanced growth in each instant. Decentralizing efficient allocations thus requires implementing constant taxes.

\(^6\)This is due to the symmetry in intermediate producers’ profits. See the Appendix for the derivation of the zero-profit condition in the R&D sector.
each worker increases with the average human capital in the society. In the present context, we assume that the current productivity of investment, $\phi_i$, is positively influenced by the importance of past research for the existing technology, measured by $M_i / Y_i$. The spillover function is

$$\phi_i (t) \equiv \varphi_i \cdot (M_i (t) / Y_i (t))$$

(6)

where $\varphi_i > 0$ is a constant parameter. From (5) and (6), the growth rate of intermediates’ varieties is proportional to the economy-wide rate of R&D investment,

$$\dot{M_i} (t) = \varphi_i (1 + a_i) \cdot (Z_i (t) / Y_i (t)),$$

(7)

a relationship that is empirically plausible (Barro and Sala-i-Martin, 2004: p.300-302) and has the desirable implication of eliminating scale effects.\footnote{The reason why relation (7) eliminates scale effects is that the cost of inventing a new variety of intermediate is proportional to the extra output that would be created by the new variety (Barro and Sala-i-Martin, 2004: p.301). The absence of scale effects is particularly desirable in the present context because production requires the use of exhaustible resources: a model exhibiting scale effects would predict that the growth rate of a resource-rich country is proportional to the size of the resource endowment, which is at odds with empirical evidence.}

2.2 Resource Extraction in Foreign

The owners of extracting firms are households in Foreign, each of whom earns the same fraction $1/L_f$ of rents. Extracting firms are competitive and costlessly extract the resource flow $R(t)$ from a non-renewable stock of resource $Q(t)$, taking the world resource price $P_R$ as given. Extraction equals the sum of the resource units employed in the two countries, $R(t) = R_h(t) + R_f(t)$, and firms maximize present-value profits

$$\int_0^\infty P_R(t) R(t) e^{-\int_t^\infty r_f(v) dv} dt,$$

(8)

subject to the dynamic resource constraint $\dot{Q}(t) = R(t)$. The solution to this dynamic problem is characterized by the conditions

$$\dot{P}_R(t) = r_f(t),$$

(9)

$$Q_0 = \int_0^\infty R(t) dt.$$  

(10)
Equation (9) is Hotelling’s rule: the resource price must grow at a rate equal to the rate of return to investment. Equation (10) is the intertemporal resource constraint requiring asymptotic exhaustion of the resource stock.

2.3 Governments, Households and Trade Balance

Governments. The public sector in country $i = h, f$ finances public R&D subsidies by means of the ad valorem taxes on intermediates’ purchases and resource use. Ruling out debt, the public budget is balanced by compensating possible imbalances with a lump-sum transfer $F_i$ imposed on each household:

$$a_i P_Y^i Z_i = F_i L_i + b_i M_i P_X^i X_i + \tau_i P_R R_i.$$  \hfill (11)

Households. Economy $i$ is populated by $L_i$ homogeneous households that solve a standard two-step consumer problem. First, agents decide how to allocate expenditures between imported and domestically-produced final goods. Denoting by $c^j_i$ the quantity of the good produced in country $j$ and individually consumed in country $i$, the instantaneous utility of each resident in country $i$ reads

$$u_i(c^h_i, c^f_i) = \ln \left[ (c^h_i)^{\epsilon} (c^f_i)^{1-\epsilon} \right], \quad 0 < \epsilon < 1,$$  \hfill (12)

where the weighting parameters, $\epsilon$ and $1 - \epsilon$, indicate the preference taste for Home and Foreign goods, respectively. Maximizing (12) subject to the expenditure constraint

$$E^c_i / L_i = P_Y^h c^h_i + P_Y^f c^f_i,$$  \hfill (13)

where $E^c_i$ is aggregate consumption expenditure in country $i$, we obtain the indirect utility function $\bar{u}_i = \ln[\omega \cdot (E^c_i / L_i)]$, where $\omega \equiv \omega(P_Y^h, P_Y^f)$ is a weighted average of final goods’ prices (see Appendix). In the second step, agents choose the time profile of expenditures by maximizing present-value utility

$$U_i \equiv \int_0^{\infty} e^{-\rho t} \cdot \ln[(\omega (t) \cdot (E^c_i (t) / L_i))] dt,$$  \hfill (14)

where $\rho > 0$ is the pure time-preference rate, and the path of $\omega (t)$ is taken as given by the household. Objective (14) is maximized subject to the dynamic wealth constraint of the household (see Appendix). The resulting optimality conditions yield

$$\dot{E}^c_i (t) = \tau_i (t) - \rho,$$  \hfill (15)
which is the standard Keynes-Ramsey rule.

**Trade.** Ruling out asset mobility, trade is balanced in each instant: the value of Foreign total exports – resources plus exported consumption goods – equals the value of final goods imported from Home,

\[ P_R R_h + P_Y^f L_h c_h^f = P_Y^h L_f c_f^h. \]  

(16)

The resource-rich economy exhibits a structural deficit in final-goods trade, and this asymmetric structure is the source of the rent-extraction mechanisms typically encountered in the related literature. Considering the aggregate constraints, we simplify the notation by denoting aggregate R&D expenditures of country \( i \) as \( E^d_i \equiv P_Y Z_i (1 + a_i) \) and aggregate expenditures in intermediates’ production as \( E^x_i \equiv P_Y \xi_i M_i X_i \). Consequently, the two economies satisfy

\[ E_h \equiv E^c_h + E^d_h + E^x_h = P_Y^h Y_h - P_R R_h, \]  

(17)

\[ E_f \equiv E^c_f + E^d_f + E^x_f = P_Y^f Y_f + P_R R_h, \]  

(18)

where \( E_i \equiv E^c_i + E^d_i + E^x_i \) may be interpreted as an index of gross aggregate expenditures in country \( i \).\(^8\) Equation (17), in particular, shows that total expenditures in Home equal the value of final output less the value of resource rents paid to Foreign resource owners.

### 2.4 World Equilibrium

The world equilibrium exhibits three fundamental properties: (i) interest rate parity, (ii) balanced growth with stable expenditure shares, and (iii) a constant equilibrium level of relative resource use. Results (i)-(ii) follow directly from the structure of the model: they also hold in the analysis of Bretschger and Valente (2012) and thus only require a brief summary here. In the present context, result (iii) deserves more emphasis because

\(^8\)If we subtract intermediate expenditures to the gross expenditure index, we obtain the national accounting definition of gross domestic income \( GDI_i = E_i - E^x_i \). In the present discussion of the theoretical model, we only consider the comprehensive measure of expenditure \( E_i \) because it considerably simplifies the calculations as well as the exposition. The accounting definition the national accounting definition of gross domestic income \( GDI_i \) obviously yields identical results (see Bretschger and Valente, 2012).
the analysis of strategic taxation is entirely based on the relationship between national welfare and relative resource use.

*Interest rate parity.* In each country \( i \), the rate of return to investment in terms of domestic final output is given by the growth rate of physical productivity in the domestic final sector, denoted by \( \Omega_i \) and equal to a weighted sum of the growth rates of the mass of varieties, of labor efficiency and of resource use (see Appendix):

\[
\begin{align*}
    r_i - \dot{P}_Y^i &= \Omega_i = \frac{\alpha(1 - \alpha)(1 + a_i)}{1 + b_i} \varphi_i + \frac{\beta}{1 - \alpha} \eta_i + \frac{\gamma}{1 - \alpha} \dot{R}_i, \\
    (19)
\end{align*}
\]

The country-specific terms in the right hand side of (19) imply that Home and Foreign may exhibit persistent gaps in productivity growth as a result of differences in structural parameters (\( \varphi_i, \eta_i \)) or in policy variables (\( a_i, b_i \)). Equilibrium in trade and symmetric preferences imply that physical productivity differentials are compensated by terms-of-trade dynamics (see Appendix):

\[
    \dot{P}_Y^h - \dot{P}_Y^f = \Omega_f - \Omega_h. \\
    (20)
\]

Consequently, the world equilibrium is characterized by interest rate parity: results (19) and (20) yield \( r_h = r_f \).

*Balanced growth.* Interest rate parity implies that consumption expenditures grow at the same rate in the two countries: by the Keynes-Ramsey rule (15), we have \( \dot{E}_h = \dot{E}_f = r_i - \rho \) with \( r_h = r_f \). The growth rates of physical final output, resource use, and mass of varieties, equal

\[
\begin{align*}
    \dot{Y}_h &= \Omega_h - \rho \quad \text{and} \quad \dot{Y}_f = \Omega_f - \rho, \\
    \dot{R}_h &= \dot{R}_f = -\rho, \\
    \dot{M}_i &= \varphi_i \alpha (1 - \alpha)(1 + a_i)(1 + b_i)^{-1} - \rho, \\
    (21), (22) \quad \text{and} \quad (23)
\end{align*}
\]

at each point in time. Equations (21), (22) and (23) are, respectively, the growth rates of physical final outputs implied by the Keynes-Ramsey rule, the growth rate of resource use implied by the Hotelling rule, and the equilibrium rate of varieties’ expansion implied by R&D activity. We stress two relevant implications of this balanced-growth equilibrium. First, being \( (P_Y^h Y_h)/(P_Y^f Y_f) \) constant, the two countries exhibit constant shares in the world market for final goods and thereby stable shares of world
income. Second, constant growth rates at each point in time allow us to obtain closed form solutions for both consumption paths and for present-value welfare levels: this property will allow us to calculate the welfare effects of discretionary tax policies.

Relative final output and relative resource use. Our analysis of taxation will hinge on two equilibrium relationships that link the two countries’ shares in final output and in world resource use to the respective propensities to invest. Formally, we use a gross index of investment rate, denoted by $I_i$ and defined as the sum of the shares of domestic final output invested in R&D and used in the production of intermediates in country $i$. In equilibrium, the investment rate equals (see Appendix)

$$I_i = \frac{E_i^d}{P_i Y_i} + \frac{E_i^x}{P_i Y_i} = \frac{\varphi_i \alpha (1 - \alpha) (1 + a_i) - \rho (1 + b_i)}{\varphi_i (1 + b_i)} + \frac{\alpha^2}{1 + b_i} \cdot$$

Considering market shares in final output, we can combine the expenditure constraints (17)-(18) with (24) to obtain (see Appendix)

$$\frac{P_h Y_h}{P_f Y_f} = \frac{\epsilon}{1 - \epsilon} \cdot \frac{1 - I_f}{1 - I_h}.$$  

Expression (25) shows that the value of Home’s final output (relative to Foreign) increases with the taste parameter of world consumers for Home’s final goods (relative to Foreign) and is positively related to Home’s investment rate (relative to Foreign). Now consider Home’s relative resource use, defined as $\theta (t) \equiv R_h (t) / R_f (t)$. Result (22) implies that, in the balanced growth equilibrium, relative resource use is constant over time. Importantly, this equilibrium level $\theta (t) = \bar{\theta}$ is directly affected by both countries’ investment rates through (25) because the countries’ relative demands for resources depend on the two countries’ relative output levels: combining (25) with the final sectors’ resource demand schedules (2), we obtain

$$\bar{\theta} = \frac{1 + \tau_f}{1 + \tau_h} \cdot \frac{\epsilon}{1 - \epsilon} \cdot \frac{1 - I_f}{1 - I_h}.$$  

The world resource allocation is thus determined by three components: the relative distortion induced by domestic taxes, the relative consumers’ taste for the countries’ final goods, and the relative investment rates. This result is crucial for understanding
the governments’ incentives to enact strategic resource taxes: we will indeed show that each government is tempted to use the domestic resource tax in order to achieve a specific level of relative resource use associated to maximal domestic welfare.

3 Efficiency and Policy

This section briefly describes the characteristics of two benchmark regimes: the *laissez-faire equilibrium* in which all taxes and subsidies are set to zero, and the (conditionally) *efficient allocation* in which domestic market failures are neutralized by fiscal authorities through the appropriate fiscal instruments.

3.1 Laissez-Faire Equilibrium

Suppose that taxes and subsidies are set to zero in each country: \( \tau_i = b_i = a_i = 0 \). The laissez-faire equilibrium is inefficient by construction since monopolistic competition and knowledge spillovers imply a misallocation of domestic output between consumption and investment within each country. The crucial aspect, however, concerns the implications for ‘aggregate efficiency in resource use’, that is, how laissez-faire changes the way in which the world resource supply is distributed between the two countries. From (24) and (26), relative resource use under laissez-faire equals

\[
\theta_{LF} \equiv \frac{\epsilon}{1 - \epsilon} \cdot \frac{1 - \alpha + (\rho/\varphi_f)}{1 - \alpha + (\rho/\varphi_h)}.
\] (27)

Although laissez-faire eliminates a direct source of distortion – that is, national resource taxes – the world resource allocation is affected by cross-country differences in R&D productivity because spillovers from past research distort Home and Foreign investment rates and thereby the two countries’ relative resource demand. The important information contained in expression (27) is that the extent by which the world’s allocation of resources is inefficient is determined by the size of the gap between the two countries’ parameters of R&D productivity, \( \varphi_h \) and \( \varphi_f \).
3.2 Conditional Efficiency

Suppose that one government internalizes all the domestic market failures generated by monopoly pricing and R&D spillovers. The resulting allocation is called “conditionally efficient” according to the following

**Definition 1** An allocation is conditionally efficient for country $i$ if domestic output is allocated so as to maximize present-value utility $U_i$ subject to the technology, income, and resource constraints faced by country $i$ at given international prices.

The conditionally efficient allocation (CE-allocation, hereafter) is similar to the welfare-maximizing allocation that characterizes social optimality in closed-economy models. However, in the present context, conditional efficiency and optimality are quite different concepts. In closed economies, the welfare-maximizing allocation is chosen by a social planner endowed with full control over all the elements of the allocation. The CE-allocation in country $i$, instead, postulates maximal domestic utility at given international prices. Since international prices are influenced by the fiscal policies of both countries, there is no general presumption that each government actually wishes to implement the CE-allocation. If a government actually takes international prices as given, achieving the CE-allocation is an overriding political target. If, instead, the government could infer all the general-equilibrium effects generated by domestic fiscal instruments, it may be desirable to deviate from conditional efficiency because non-efficient policies may increase domestic welfare to the detriment of the other country’s welfare. This is indeed the case for resource-importing countries, as we will show in the next section.

We characterize CE-allocations by denoting the relevant variables by tildas. In Home, the CE-allocation is represented by the paths of imported resource flows and expenditures (in consumption, intermediates’ production and R&D activity), that maximize Home’s indirect utility subject to the final-good technology, the intermediate-good technology, and Home’s expenditure constraint:

$$\{ \tilde{R}_h, \tilde{E}_c^o, \tilde{E}_x^o, \tilde{E}_d^o \}_{t=0}^\infty = \arg \max U_h \text{ s.t. } (1), (7), (17)$$

where $U_h$ in (14) is maximized taking international prices as given, and the R&D externality is fully taken into account through constraint (7). In Foreign, the CE-allocation
is represented by the paths of domestic resource use, exported resources, and expenditures that maximize Foreign utility subject to the technology constraints, the aggregate expenditure constraint, and the exhaustible resource constraint:

\[
\left\{ \tilde{R}_h, \tilde{R}_f, \tilde{E}_f^c, \tilde{E}_f^d \right\}_{t=0}^{\infty} = \arg \max U_f \text{ s.t. (1), (7), (17) and } \dot{Q} = -R_h - R_f.
\]

Solving these maximization problems, we obtain two results. First, if a government decentralizes the CE-allocation, it must implement an efficient policy that consists of the following subsidies and taxes (see Appendix):

\[
\begin{align*}
\tilde{a}_i & = (\varphi_i/\rho) - (1 - \alpha)^{-1} > 0, \\
\tilde{b}_i & = \alpha (1 - \alpha)(\varphi_i/\rho) - 1 > 0, \\
\tilde{\tau}_i & = (1 - \alpha)(\varphi_i/\rho) - 1 > 0.
\end{align*}
\]

The role of subsidies to R&D investment is intuitive: research activity generates positive externalities and must therefore be encouraged by public authorities through \( \tilde{a}_i > 0 \). This policy must be accompanied by positive taxes on resource use and intermediates’ purchases because private agents exhibit inefficiently low saving rates and therefore excessive demand for current production.\(^9\)

The second result follows directly from substituting the efficient tax rates (28)-(30) into (24)-(26): if both economies display conditional efficiency, relative resource use equals

\[
\theta_{CE} \equiv \frac{\epsilon}{1 - \epsilon}.
\]

Expression (31) shows that relative resource use is exclusively determined by preference parameters, with no role played by technology. The intuition is that, in a symmetric CE-equilibrium, technological spillovers are internalized and do not distort the countries’ relative demand for resources.\(^10\) Indeed, the notion of efficiency embodied in (31) applies to resource allocation at the world level: the relative demands for resources from the

---

\(^9\)The positive sign of the tax on intermediates’ purchases results from the absence of scale effects, as explained in Valente (2013).

\(^10\)Under laissez-faire, instead, technological R&D spillovers are not internalized and affect world resource allocation, as shown in expression (27) above.
two countries’ final sectors only reflect the relative tastes of world consumers for the two countries’ final goods.

Comparing the expression for $\theta_{CE}$ with the laissez-faire level $\theta_{LF}$ in (27), it follows that, under laissez-faire, a country’s relative resource use is inefficiently high (from the point of view of the world’s resource allocation) when its R&D productivity is higher than in the other country:

$$\varphi_h \geq \varphi_f \quad \text{implies} \quad \theta_{LF} \geq \theta_{CE}.$$  \hfill (32)

Result (32) implies different incentives for national governments to deviate from either regime, as we show below.

4 Taxation and Welfare

If national fiscal authorities recognize all the general-equilibrium effects induced by national resource taxes, Home and Foreign governments face different incentives to enact strategic policies. While Home wishes to deviate from both laissez-faire and efficient allocations in order to extract rents, lower (higher) R&D productivity creates an independent incentive for the Foreign government to deviate from laissez-faire by subsidizing (taxing) domestic resource use. These conclusions are formally derived below.

4.1 The Rent-Extraction Incentive

A basic property of the present model is that Home’s resource tax affects the world income distribution whereas the Foreign resource tax does change income shares. In every equilibrium, Home’s share of world total expenditures equals

$$\frac{E_h}{E_h + E_f} = \frac{(P^h_Y Y_h)/(P^f_Y Y_f)}{1 + (P^h_Y Y_h)/(P^f_Y Y_f)} \cdot \left(1 - \frac{\tilde{\gamma}_h}{\gamma (1 + \tau_h)^{-1}}\right),$$  \hfill (33)

where $\tilde{\gamma}_h \equiv \gamma (1 + \tau_h)^{-1}$ is the tax-adjusted resource elasticity in final production in Home. Expression (33) shows that Home’s expenditure share is the product of two factors. The first is Home’s share in world final output, which is independent of resource
taxes by result (25). The second factor represents the effect of Home’s resource dependence: from (2), domestic producers must use a fraction $\tilde{\tau}_h$ of revenues from final-good sales to purchase imported resources. By definition, $\tilde{\tau}_h \equiv \gamma (1 + \tau_h)^{-1}$ is affected by the Home resource tax whereas it is independent of Foreign resource tax. Therefore, an increase in $\tau_h$ increases Home’s share of world expenditures through a decline in $\tilde{\tau}_h$ whereas variations in $\tau_f$ leave expenditure shares unaffected. This result hinges on the asymmetric structure of trade and is a variant of Bergstrom’s (1982) rent-extraction mechanism whereby resource taxes in the importing country capture part of the rents that would otherwise accrue to foreign residents. The main consequence is that Home has a potential incentive to raise resource taxes, i.e. increasing its share of world income by artificially reducing its resource demand. As we show below, this incentive is not only potential since Home’s domestic welfare is indeed higher when Home’s relative resource use is inefficiently low (from the point of view of the world resource allocation).

4.2 Resource Taxes and Welfare Levels

The reaction of utility levels to variations in domestic resource taxes is represented by two welfare-tax relationships. Importantly, the balanced-growth property of the world equilibrium implies that consumption paths and welfare levels can be solved in closed form. Present-value utilities in the two countries equal (see Appendix)

$$U_h = \kappa_h + \frac{1}{\rho} \ln \left[ P_0^{1-\epsilon} \cdot Y_h (0) \cdot \tilde{\sigma}_h^c \right], \quad (34)$$

$$U_f = \kappa_f + \frac{1}{\rho} \ln \left[ P_0^{-\epsilon} \cdot Y_f (0) \cdot \tilde{\sigma}_f^c \right], \quad (35)$$

where $\kappa_i$ is a constant factor independent of resource taxes, $p_0 \equiv P_Y^h (0) / P_Y^f (0)$ is the initial relative price of the Home final good, and the constants $\tilde{\sigma}_i^c \equiv E_i^c / (P_Y^h Y_i)$ are indices of the respective consumption propensities.

The terms in square brackets in (34)-(35) imply that the marginal effect of an increase in the domestic resource tax on domestic welfare, $dU / d\tau$, generally incorporates

11 The intuition is that variations in $\tau_h$ or $\tau_f$ induce offsetting variations in physical output quantities and in physical output prices such that the ratio between the values of the two countries’ output is unchanged (Bretschger and Valente, 2012). These opposite price-quantity movements appear explicitly in expressions (36)-(37) below.
three effects: (i) on terms of trade, (ii) on domestic physical output, and (iii) on consumption propensity.\(^\text{12}\) The direction of the first two effects is intuitive: an increase in the Home (Foreign) resource tax increases the relative price of the Home (Foreign) good and reduces Home (Foreign) physical output. Instead, the direction of the consumption-propensity effect – i.e. the sign of \(d \ln \bar{\sigma}_i^c / d \tau_i\) – is asymmetric. In Home, the resource tax increases the ratio between consumption and final output:

\[
\rho \cdot \frac{d U_h}{d \tau_h} = (1 - \epsilon) \frac{d \ln p_0}{d \tau_h} + \frac{d \ln Y_h(0)}{d \tau_h} + \frac{d \ln \bar{\sigma}_h^c}{d \tau_h}.
\]

In Foreign, an increase in the domestic resource tax leaves the consumption-output ratio unchanged: since \(d \ln \bar{\sigma}_f^c / d \tau_f = 0\), the marginal welfare effect of the Foreign tax only depends on the relative strength of the variations in terms of trade and physical output,

\[
\rho \cdot \frac{d U_f}{d \tau_f} = \epsilon \frac{d \ln p_0^{-1}}{d \tau_f} + \frac{d \ln Y_f(0)}{d \tau_f}.
\]

The asymmetric effects of Home and Foreign taxes on the respective consumption propensities are directly linked to the rent-extraction mechanism described in the previous subsection: in Home, the resource tax increases domestic disposable income and thereby the value of consumption expenditures relative to domestic final output.

The contrasting effects of resource taxes on output prices and physical quantities imply that, in each country, the welfare-tax relationship \(U_i(\tau_i)\) is hump-shaped: there exists a unique level of the domestic resource tax, \(\tau_i^{\max}\), that maximizes domestic welfare for a given state of affairs in the other country. Importantly, the model structure implies that, for each country \(i\), the welfare-maximizing tax rate is always associated to a specific level of relative resource use, which we denote by \(\theta_i^{\max}\). The following Proposition establishes that the welfare-maximizing taxes of the two countries are necessarily associated with different equilibria: the two governments cannot simultaneously implement the respective \(\tau_i^{\max}\) because Home would prefer a lower level of relative resource use.

**Proposition 2** In Foreign, implementing the welfare-maximizing resource tax \(\tau_f^{\max}\) always implies

\[
\theta = \theta_f^{\max} = \frac{\epsilon}{1 - \epsilon}.
\]

\(^{12}\)See the Appendix for detailed proofs of the statements reported in this section.
In Home, implementing the welfare-maximizing resource tax $\tau_h^\text{max}$ always implies

$$\theta = \theta_h^\text{max} < \frac{\epsilon}{1 - \epsilon}. \quad (39)$$

Proposition 2 shows that resource taxes serve different purposes in the two countries. Result (38) establishes that Foreign welfare is maximal when relative resource use coincides with the efficient level (cf. expression (31) above). The reason is that Foreign firms act as price takers and thus earn maximal rents when total supply is split between the two countries in ‘efficient proportions’ from an aggregate perspective.\(^{13}\) The implication is that the Foreign government may use $\tau_f$ to induce an efficient level of relative resource use. Home’s government, instead, always has an incentive to deviate from efficiency in world resource allocation: from (39), domestic welfare is higher if Home pushes its relative resource use below the efficient level because a lower demand for primary imports raises Home’s income share via rent extraction. Consequently, Home may use $\tau_h$ to distort the world allocation of the resource in order to raise its disposable income. More generally, Proposition 2 implies that if both national governments fully recognize all the general-equilibrium effects of the respective resource taxes, the independent pursuit of maximal domestic welfare determines conflicting objectives: each government seeks a different equilibrium level of relative resource use. This is a very general conclusion since neither (39) nor (38) assume that the two economies are starting from a specific equilibrium.

### 4.3 Deviations from Efficiency and Laissez-Faire

Proposition 2 can be applied to any initial state of affairs. Our two reference benchmarks are the symmetric $CE$-equilibrium and the symmetric laissez-faire equilibrium.

In the symmetric $CE$-equilibrium, both governments implement the efficient taxes (28)-(29)-(30) in the respective countries. Under this state of affairs, relative resource use is given by (31) and the following result holds.

\(^{13}\)Note that this does not mean that $\tau_f^\text{max}$ is always associated to an “efficient general equilibrium”. Relative resource use may be equal to the efficient level $\epsilon/(1 - \epsilon)$ in equilibria featuring inefficiencies in some submarkets. For example, when $\phi_h = \phi_f$, laissez-faire conditions would imply $\theta = \epsilon/(1 - \epsilon)$ – see equation (27) above – but the general equilibrium of the economies is inefficient due to R&D externalities and monopolistic competition.
**Proposition 3** In a symmetric CE-equilibrium, $dU_h/d\tau_h > 0$ and $dU_f/d\tau_f = 0$.

The Home government has an incentive to deviate from conditional efficiency because it may improve domestic welfare by increasing $\tau_h$ above the efficient level. The Foreign government, instead, has no incentive to deviate since, by (38), the welfare-maximizing resource tax is associated to an equilibrium in which the world resource allocation is efficient. Hence, starting from conditional efficiency, the crucial source of deviations is the rent-extraction mechanism.

Now consider a symmetric laissez-faire, i.e. an initial state of affairs in which all taxes and subsidies are set to zero. Starting from this equilibrium, the scheme of incentives falls in three possible cases depending on the sign of productivity gaps:

**Proposition 4** Given a laissez-faire equilibrium, higher R&D productivity in Home creates an incentive for Foreign to subsidize domestic resource use and exacerbates Home’s incentive to tax domestic resource use. The general scheme is:

i. If $\varphi_h > \varphi_f$ then $dU_h/d\tau_h > 0$ and $dU_f/d\tau_f < 0$;

ii. If $\varphi_h = \varphi_f$ then $dU_h/d\tau_h > 0$ and $dU_f/d\tau_f = 0$;

iii. If $\varphi_h < \varphi_f$ then $dU_h/d\tau_h \geq 0$ and $dU_f/d\tau_f > 0$;

Focusing on result (i), the intuition follows from expression (32). Higher R&D productivity in Home implies that Home’s relative resource use strictly exceeds the efficient level. In this situation, both countries have incentives to deviate. On the one hand, Foreign would gain from subsidizing domestic resource use since this would contrast Home’s over-consumption of the resource and move the world resource allocation towards the efficiency condition that maximizes Foreign welfare (cf. expression (38) above). On the other hand, Home would gain from taxing domestic resource use even more intensively than starting from efficiency conditions: under laissez-faire, Home’s resource use is strictly above the efficient level whereas maximal welfare would require it to be strictly below the efficient level. Therefore, persistent gaps in productivity growth originating in R&D externalities matter for both countries: they create the incentive to implement subsidies in Foreign, and exacerbate the incentive to raise taxes in Home. Since the
hypothesis \( \varphi_h > \varphi_f \) is empirically plausible, this conclusion suggests a novel potential explanation for the stylized facts that characterize world oil trade: the observed subsidies (taxes) on domestic oil consumption in oil-rich (oil-poor) economies may be partly induced by the fact that oil-poor countries exhibit faster growth in R&D productivity with stronger spillovers from past research.\(^{14}\)

5 Resource Taxes and Income Shares: Evidence

All the main results of the theoretical analysis hinge on the fact that world income shares depend on the Home resource tax (via the rent-extraction mechanism) as well as on the country-specific levels of R&D productivity (via the two countries’ rates of investment). In this section, we test whether these determinants of relative income shares find empirical support in international data for oil-rich and oil-poor countries.

5.1 Income Shares: A Reformulation

Equations (33) and (25) imply that Home’s share of world income can be written as a function of the domestic resource tax and of investment rates: denoting by \( s_h \) the income share of the resource-poor economy, and substituting the definitions of \( I_h \) and \( I_f \) from (24), we obtain the function\(^{15}\)

\[
s_h = \Psi(I_h, I_f, \tau_h) \quad \text{with} \quad \Psi_{I_h} > 0, \quad \Psi_{I_f} < 0, \quad \Psi_{\tau_h} > 0.
\]

Expression (40) shows that the income share of the resource-poor economy is positively related to the domestic investment rate, negatively related to the investment rate of the

\(^{14}\)The other cases (ii)-(iii) reported in Proposition 4 are easily interpreted. If R&D technologies are identical in the two countries, relative resource use coincides with the efficient level and this implies, similarly to Proposition 3, an incentive to raise a tax in Home but no incentive to deviate from laissez-faire in Foreign. Finally, if R&D productivity is higher in Foreign, relative resource use falls short of the efficient level: Foreign would gain from raising a resource tax whereas Home would gain by implementing either a resource tax or a subsidy, depending on the width of the productivity gap.

\(^{15}\)The definition of income share \( s_h \) may be equivalently expressed in terms of total expenditure indices – that is, \( s_h = E_h / (E_h + E_f) \) – or in terms of gross domestic income – that is, \( s_h = GDI_h / (GDI_h + GDI_f) \) where \( GDI_i = E_i - E_i^r \). The properties of the crucial relationship \( \Psi \) do not change (see Bretschger and Valente, 2012).
resource-rich economy, and positively related to the national tax on domestic resource use.

As noted by Bretschger and Valente (2012), the present two-country framework can be directly applied to two sets of countries. The ‘Foreign economy’ resembles the world’s top net exporters of oil – henceforth labelled as $OEX$ group – comprising countries that (i) have never been a net oil importer and (ii) steadily appeared in the top exporters list in the last three decades. The ‘Home economy’ is the set of the world’s top net importers of oil that do not produce oil domestically – henceforth labelled as $OIM$ group – comprising the countries that, since 1980, steadily appeared in the list of top oil importers and relied heavily on imported oil for domestic use (this definition excludes, e.g., oil-importing countries that produce more than 10% of the oil they consume domestically). Starting from the country sample compiled in Bretschger and Valente (2012), we can perform a direct empirical test of equation (40) using a dynamic panel-estimation technique.

5.2 Empirical Test and Results

We collected data for the time period 1980-2008, sixteen OIM countries – namely Belgium, France, Germany, Greece, Italy, Japan, Korea, Netherlands, Philippines, Poland, Portugal, Singapore, Spain, Sweden, Switzerland, Turkey – and the ten OEX countries – Algeria, Canada, Iran, Kuwait, Mexico, Norway, Oman, Saudi Arabia, United Arab Emirates, and Venezuela. This is the country sample for which the relevant data are nearly completely available, except for taxes in the Philippines and Singapore. In order to focus on long-run effects and to avoid the impact of business cycles, we build five-year averages considering the periods 1980-84, 1985-89, 1990-94, 1995-99, 2000-04, and 2005-08. To capture autonomous dynamic components, we include lags of the dependent variable. By construction, the emerging unobserved panel-level effects are correlated with the lagged dependent variables, which makes standard estimators inconsistent. For this reason, we use the Arellano-Bond dynamic panel-data estimation, which provides a consistent generalized method-of-moments (GMM) estimator for the parameters.

We use data from the World Bank (2009) for the macroeconomic variables, and from
the International Energy Agency (EIA, 2009) for resource taxes. Specifically, income shares are calculated as the ratio between an oil-importing country’s GNP and the sum of the GNPs of all oil-exporting countries, and are labeled as \( \text{shareoim} \). For the investment rates, we take gross capital formation as a percentage of GDP for both oil-importing and oil-exporting countries.\(^{16}\) For oil importers, the variable is denoted by \( \text{investoim} \). For oil exporters, we calculate the average investment rate across the OEX group – with population size used as the weighting factor – denoted by \( \text{investoex} \). Resource taxes are measured by taxes on light fuel oil and labelled as \( \text{oiltax} \). Further control variables are education expenditures as a percentage of GDP (\( \text{eduexp} \), the investment rate for human capital), research expenditures as a percentage of GDP (\( \text{rdexp} \), the investment rate for knowledge capital), population size (\( \text{pop} \)), and central government debt as a percentage of GDP (\( \text{cgovdebt} \)).

The results are presented in Table 1, which includes six representative equations [1]-[6]. In all equations we include the (first) lag of the endogenous variable which is significant at the 1%-level in all specifications. This confirms that the estimation method is appropriate. In equation [1], we test the impact of investment shares in both types of countries. The results confirm the model predictions: domestic investment affects the oil-importers’ income share positively while the opposite holds true for the impact of foreign investment rates. Equation [2] shows that also the impact of domestic investment rates in human capital, \( \text{eduexp} \), on the income shares of OIM countries is positive, in line with our theoretical results. Equation [3] includes oil taxes: in line with the prediction of the model, national taxes on domestic resource use have a positive impact on the income share. This is a remarkable finding which, to our knowledge, has not been studied in the previous literature. Resource-poor countries seem to be able to raise their income

\(^{16}\)The reason for using capital formation instead of R&D expenditures is twofold. First, in the theoretical model we concentrate on R&D investment and abstract from physical capital in order to keep the analysis tractable: if we introduce physical capital, the role of overall investment in determining income shares would be the same as that of specific R&D investment in the current setup. Second, our empirical analysis of income shares requires to build an average investment rate for all the OEX group, and the lack of data on R&D for several oil-exporting economies suggests using the variable for which we have homogeneous data, i.e., capital formation. Nonetheless, we use R&D data for OIM countries as an additional control variable – see specification [5] in Table 1.
share by raising national taxes on domestic resource use.

Population size $\text{pop}$, which measures the scale of the economy, has no significant effect in any specification. Similarly, research expenditures $\text{rdexp}$ as well as central government debt $\text{cgovdebt}$ have no significant impact and do not change our general results.

The Wald test statistics show that the independent variables provide a significant contribution to the regression model. The ‘Sargan overid’ statistics report tests of over-identifying restrictions, that is, of whether the instruments, as a group, appear exogenous. The obtained test values do not reject the null hypothesis of a valid specification. We also report difference-in-Sargan statistics showing that the subsets of instruments are valid and thus the right-hand variables are not endogenous.

6 Conclusion

Asymmetric trade structures may provide national governments with different types of incentives to enact strategic taxes at the national level. Our analysis shows that, introducing endogenous growth in a two-country model with uneven resource endowments, structural gaps in productivity growth create incentives to deviate from both laissez-faire equilibria and domestically efficient allocations. Stronger spillovers from past research in resource-poor economies exacerbate the importers’ willingness to tax resource use while prompting exporters to subsidize domestic consumption independently of the rent-extraction mechanism. This conclusion is consistent with the stylized facts that characterize world oil trade and, in our view, deserves further empirical scrutiny. At the theoretical level, our results concerning the welfare effects of taxation suggest that, if domestic welfare represents the payoff of each government in a political game, inefficient equilibria may well be the outcome and this is a very interesting issue for future research. More generally, the argument that growth differentials matter for strategic trade policies is under-researched so that further research in this direction is certainly warranted.
References


A Appendix

Monopoly rents. Maximization of $\Pi_i (m_i) = (P^i_{X(m_i)} - \zeta P^i_{Y}) \cdot X_i (m_i)$ s.t. (3) gives

\[ X_i (m_i) = X_i = \left\{ \alpha^2 (v_i L_i)^{\beta} R_i \left[ \zeta (1 + b_i) \right]^{-1} \right\}^{1-\alpha}, \]  
\[ \Pi_i (m_i) = \Pi_i = (1 - \alpha) P^i_{X_i} X_i. \]  

Substituting (A.1) in (3) yields (4). For future reference, expressions (A.1) and (1) imply

\[ Y_i = \left( \alpha^2 / \zeta \right)^{\frac{\alpha}{1-\alpha}} \cdot [1 + b_i]^{-1} \cdot M_i (v_i L_i)^{\frac{\beta}{1-\alpha}} (R_i)^{\frac{\alpha}{1-\alpha}}. \]  

(A.3)
**R&D sector.** Denoting by $V_i$ the value of a patent, the zero-profit condition is\(^{17}\)

\[
V_i = P_i^h / \left[ \phi_i (1 + a_i) \right]. \tag{A.4}
\]

Denoting by $r_i$ the interest rate in country $i$, the no-arbitrage condition is

\[
r_i (t) V_i (t) = \Pi_i (t) + \dot{V}_i (t), \tag{A.5}
\]

**Derivation of (9)-(10).** Maximize (8) subject to $\dot{Q} = -R$ using the Hamiltonian $P_R R - \chi R$, where $\chi$ is the dynamic multiplier. The optimality conditions read

\[
P_R (t) = \chi (t), \tag{A.6}
\]

\[
\dot{\chi} (t) = r_f (t) \chi (t), \tag{A.7}
\]

\[
\lim_{t \to \infty} \chi (t) Q (t) e^{- \int_t^\infty r_f (v) dv} = 0, \tag{A.8}
\]

Plugging (A.6) in (A.7), we have (9). Integrating (A.7) and substituting the resulting expression in (A.8), we have $\lim_{t \to \infty} \chi (0) Q (t) = 0$, which implies $\lim_{t \to \infty} Q (t) = 0$. Integrating $\dot{Q} (t) = -R (t)$ between time zero and infinity thus yields (10).

**Consumer problem (step 1).** Maximization of (12) s.t. (13) implies

\[
c_i^f / c_i^h = \frac{1 - \epsilon}{\epsilon} (P_i^h / P_i^f), \tag{A.9}
\]

\[
P_i^h c_i^h = \epsilon \cdot E_i^c / L_i \text{ and } P_i^f c_i^f = (1 - \epsilon) \cdot E_i^c / L_i, \tag{A.10}
\]

\[
\bar{u}_i = \ln \left\{ \frac{\epsilon}{(P_i^h)^{\epsilon} (P_i^f)^{1-\epsilon}} \left( \frac{1 - \epsilon}{\epsilon} \right)^{1-\epsilon} \cdot E_i^c / L_i \right\}, \tag{A.11}
\]

where (A.9) holds in each country $i = h, f$, expressions (A.10) follow from plugging (A.9) in (13), and expression (A.11) follows from substituting (A.10) in (12). Denoting the term in square brackets in (A.11) as $\omega = \omega (P_i^h, P_i^f)$, indirect utility is $\bar{u}_i = \ln [\omega \cdot (E_i^c / L_i)]$ in each country $i = h, f$.

**Consumer problem (step 2).** Individual wealth is $(1 / L_i)$ times the value of all domestic assets $V_i M_i$. Defining $n_i \equiv (V_i M_i) / L_i$, the wealth constraints read

\[
\dot{n}_h = r_h n_h + P_i^h - (E_i^c / L_h) - F_h, \tag{A.12}
\]

\[
\dot{n}_f = r_f n_f + P_i^f - (E_i^c / L_f) - F_f + P_R (R / L_f), \tag{A.13}
\]

\(^{17}\)Aggregate profits of the R&D sector equal $V_i M_i - P_i^h Z_i = V_i \phi_i Z_i (1 + a_i) - P_i^h Z_i$, so that condition (A.4) maximizes R&D profits for a given marginal productivity $\phi_i$. Condition (A.4) can be equivalently obtained assuming free entry in the R&D business (see Barro and Sala-i-Martin, 2004).
where \( r_i n_i + P^i_L \) is income from assets and labor in country \( i \), and \( P_R (R/L_f) \) is resource income for each Foreign resident. Agents in country \( i \) maximize (14) subject to the relevant constraint, (A.12) or (A.13), using consumption expenditure \( (E^c_i / L_i) \) as control variable. Denoting by \( \lambda_i \) the multiplier, the optimality conditions \( L_i/E_i = \lambda_i \) and \( \lambda_i = \lambda_i (\rho - r_i) \) imply (15).

**Derivation of (19).** From (A.2) and (A.4), we have

\[
\frac{\Pi_i}{V_i} = \phi_i \frac{(1 + a_i) (1 - \alpha) P^i_X X_i}{P^i_Y} = \varphi_i \cdot \frac{(1 + a_i) (1 - \alpha) \alpha}{1 + b_i}, \tag{A.14}
\]

where the last term follows from substituting \( \phi_i \) by (6) and \( (P^i_X M_i X_i) / (P^i_Y Y_i) = \alpha / (1 + b_i) \) by (3). Equations (A.4) and (6) yield \( V_i = (P^i_Y Y_i) / [\varphi_i \cdot M_i (1 + a_i)] \), so that

\[
\hat{V}_i (t) = \hat{P}^i_Y + \hat{Y}_i - \hat{M}_i. \tag{A.15}
\]

Substituting (A.14) and (A.15) in (A.5), we get

\[
r_i = \varphi_i \alpha (1 - \alpha) (1 + a_i) (1 + b_i)^{-1} + \hat{P}^i_Y + \hat{Y}_i - \hat{M}_i. \tag{A.16}
\]

Time-differentiation of (A.3) yields

\[
\dot{Y}_i = \dot{M}_i + \frac{\beta}{1 - \alpha} \eta_i + \frac{\gamma}{1 - \alpha} \dot{R}_i. \tag{A.17}
\]

Plugging (A.17) in (A.16), we obtain equation (19).

**Propensities to spend.** For future reference, define the propensities

\[
\sigma^c_i \equiv E^c_i / (P^i_Y Y_i), \quad \sigma^d_i \equiv E^d_i / (P^i_Y Y_i), \quad \sigma^x_i \equiv E^x_i / (P^i_Y Y_i). \tag{A.18}
\]

Two equilibrium relationships characterize both countries. First, from (4), we have \( \sigma^x_i = (P^i_Y M_i X_i) / (P^i_Y Y_i) = \alpha (P^i_X M_i X_i) / (P^i_Y Y_i) \), where we can substitute \( (P^i_X M_i X_i) / (P^i_Y Y_i) = \alpha / (1 + b_i) \) from (3) to obtain

\[
\sigma^x_i = E^x_i / (P^i_Y Y_i) = \alpha (1 + b_i)^{-1} \quad \text{for each } i = h, f. \tag{A.19}
\]

Second, from (15), the growth rate of \( \sigma^c_i = E^c_i / (P^i_Y Y_i) \) equals

\[
\dot{\sigma}^c_i = r_i (t) - \rho - \dot{P}^i_Y - \dot{Y}_i = \varphi_i \alpha (1 - \alpha) (1 + a_i) (1 + b_i)^{-1} - \dot{M}_i - \rho,
\]
where we have substituted $r_i$ by (A.16). Plugging $E_i^d = P_i^i Z_i (1 + a_i)$ in (7) and using $\sigma_i^d = E_i^d / (P_i^i Y_i)$, the growth rate of varieties equals $\dot{M}_i = \phi_i \sigma_i^d$, which can be substituted in the above expression to obtain

$$\dot{\sigma}_i^d = \phi_i (1 - \alpha) (1 + a_i) (1 + b_i)^{-1} - \phi_i \sigma_i^d - \rho$$

for each $i = h, f$. \hfill (A.20)

**Derivation of (20).** Consider Home. Using (A.18), and defining $\gamma_h \equiv (1 + \tau_h)^{-1}$, we can write (17) as

$$\dot{\sigma}_h^c + \dot{\sigma}_h^d + \dot{\sigma}_h^x = 1 - (P_R R_h) / \left( P_h^h Y_h \right) = 1 - \gamma_h,$$

where the last term follows from (2). A standard stability analysis based on (A.20) shows that $\dot{\sigma}_h^c$ and $\dot{\sigma}_h^d$ are constant and equal to (see proofs in the Supplementary Material)

$$\dot{\sigma}_h^c = (1 - \gamma_h) - \frac{\phi_h (\alpha (1 - \alpha) (1 + a_h) + \alpha^2) - \rho (1 + b_h)}{\phi_h (1 + b_h)},$$

$$\dot{\sigma}_h^d = 1 - \gamma_h - \dot{\sigma}_h^c - \dot{\sigma}_h^x = \frac{\phi_h (1 - \alpha) (1 + a_h) - \rho (1 + b_h)}{\phi_h (1 + b_h)}.$$

Given (A.18), constant values of $(\dot{\sigma}_h^c, \dot{\sigma}_h^d, \dot{\sigma}_h^x)$ imply that $P_h^h Y_h$ grows at the same rate as all expenditure shares, $\dot{E}_h^c = \dot{E}_h^d = \dot{E}_h^x$. From (17) and (2), the ratio

$$E_h/P_h^h Y_h = (1 - \gamma_h)$$

is constant, so that Home’s growth rate is determined by the Keynes-Ramsey rule (15):

$$\dot{\gamma}_h = \dot{\gamma}_h^c = \dot{P}_h^h + \dot{Y}_h = r_h - \rho.$$ \hfill (A.25)

Now use (A.10) to eliminate $P_Y^f c_h^f$ and $P_Y^h c_h^f$ from (16), obtaining

$$P_R R_h + (1 - \epsilon) E_h^c = \epsilon \dot{E}_f^c.$$ \hfill (A.26)

Substituting $P_R R_h = \gamma_h P_h^h Y_h$ from (2), and $E_h^c = \sigma_h^c P_h^h Y_h$ from (A.18), we get

$$E_f^c = \frac{1}{\epsilon} \left[ \gamma_h + (1 - \epsilon) \sigma_h^c \right] \cdot P_h^h Y_h,$$

where the term in square brackets is constant, implying that $E_f^c / (P_h^h Y_h)$ is constant. Since $P_h^h Y_h$ grows at the same rate as $E_h^c$ by (A.25), we have $\dot{E}_f^c = \dot{E}_h^c$. By the Keynes-Ramsey rules (15), this implies $r_h = r_f$. Imposing $r_h = r_f$ in (19) yields (20).
Derivation of (21). Combining the conditions (2) for Home and Foreign, we obtain

\[
\theta(t) = \frac{R_h(t)}{R_f(t)} = \frac{\bar{\gamma}_h}{\bar{\gamma}_f} \cdot \frac{P^h_Y(t) Y_h(t)}{P^f_Y(t) Y_f(t)} \quad \text{in each } t \in [0, \infty), \tag{A.28}
\]

where \(\bar{\gamma}_i \equiv \gamma(1 + \tau_i)^{-1}\) is the tax-adjusted resource elasticity in final production. Using the definition \(R_h = \theta R_f\) and condition (2) for country \(i = f\), constraint (18) implies

\[
E_f = P^f_Y Y_f + P_h R_h = P^f_Y Y_f + \theta P_R R_f = P^f_Y Y_f (1 + \bar{\gamma}_f \theta). \tag{A.29}
\]

Recalling definitions (A.18), result (A.29) and the central term in (18) imply

\[
\bar{\sigma}_f = \frac{\alpha(1 - \alpha)(1 + a_f)}{1 + b_f} - \bar{\gamma}_f \theta = \frac{\alpha^2}{1 + b_f} - \bar{\sigma}_f. \tag{A.30}
\]

Plugging (A.30) in (A.20) for country \(i = f\) we obtain

\[
\bar{\sigma}_f = \varphi_f \frac{\alpha(1 - \alpha)(1 + a_f)}{1 + b_f} - \varphi_f \left[1 + \bar{\gamma}_f \theta - \frac{\alpha^2}{1 + b_f} - \bar{\sigma}_f\right] - \rho. \tag{A.31}
\]

Dividing both sides of (A.27) by \(P^f_Y Y_f\) and solving for \(\bar{\sigma}_f \equiv E_f^c/(P^f_Y Y_f)\), we obtain

\[
\bar{\sigma}_f = \frac{1}{\epsilon} \left[\bar{\gamma}_h + (1 - \epsilon) \bar{\sigma}_h^c\right] \cdot \frac{P^f_Y Y_h}{P^f_Y Y_f} = \frac{1}{\epsilon} \left[\bar{\gamma}_h + (1 - \epsilon) \bar{\sigma}_h^c\right] \cdot \frac{\bar{\gamma}_f}{\bar{\gamma}_h} \tag{A.32}
\]

where we have used (A.28) to get the last term. Next, define

\[
\chi \equiv \frac{1}{\epsilon} + \frac{1 - \epsilon}{\epsilon} \cdot \frac{\bar{\sigma}_h^c}{\bar{\gamma}_h} > 1. \tag{A.33}
\]

Since \(\bar{\sigma}_h^c\) is constant by (A.22), \(\chi\) is also constant and (A.32) implies

\[
\bar{\sigma}_f = \chi \bar{\gamma}_f \theta \quad \text{and} \quad \bar{\sigma}_f = \bar{\theta}. \tag{A.34}
\]

Substituting the second expression in (A.34) into (A.31) we obtain

\[
\bar{\theta}(t) = \varphi_f (\chi - 1) \bar{\gamma}_f \cdot \theta(t) + \varphi_f \left[\alpha(1 - \alpha)(1 + a_f) + \alpha^2\right] (1 + b_f)^{-1} - (\varphi_f + \rho). \tag{A.35}
\]

Since \(\varphi_f (\chi - 1) \bar{\gamma}_f > 0\), equation (A.35) is globally unstable around \(\bar{\theta}(t) = 0\). Ruling out explosive dynamics implying unbounded propensities to consume in Foreign, we have \(\theta(t) = \bar{\theta}\) in each \(t \in [0, \infty)\), where \(\bar{\theta}\) is the steady-state level in (A.35):

\[
\bar{\theta} \equiv \frac{(\varphi_f + \rho) (1 + \tau_f) - \varphi_f \left[\alpha(1 - \alpha)(1 + a_f) + \alpha^2\right]}{\bar{\gamma}_f (\chi - 1) \varphi_f (1 + \tau_f)}. \tag{A.36}
\]
From (A.28), a constant $\theta$ implies

$$\dot{P}_Y^h - \dot{P}_Y^f = \dot{Y}_f - \dot{Y}_h,$$

(A.37)

where we can substitute (20) and (A.25) to obtain (21). Also note that, from (A.29), a constant $\theta$ also implies that $E_f$ grows at the same rate as $P_Y^f Y_f$, which coincides with the growth rate of $E_h$ and $P_Y^h Y_h$ by (A.37) and (A.25). We thus have

$$\dot{E}_h = \dot{E}_f = r_i - \rho \quad \text{with} \quad r_h = r_f.$$

(A.38)

**Derivation of (22).** Given $P_R R_h = \tilde{\gamma}_h P_Y^h Y_h$, the Hotelling rule (9) and result (A.25) imply that $P_R R_h$ grows at the rate $r_h - \rho$, so that $\tilde{R}_h = -\rho$. A constant $\theta$ then implies $\tilde{R}_f = -\rho$, which proves (22).

**Derivation of (23).** From (A.34), substitute $\tilde{\sigma}_f^c = \chi \tilde{\gamma}_f \tilde{\theta}$ in (A.30) to obtain $\tilde{\sigma}_f^d = 1 - \frac{\alpha^2}{1 + b_f} - (\chi - 1) \tilde{\gamma}_f \tilde{\theta}$, and eliminate $(\chi - 1) \tilde{\gamma}_f \tilde{\theta}$ by (A.36) to obtain

$$\tilde{\sigma}_f^d = \frac{\varphi_f \alpha (1 - \alpha) (1 + a_f) - \rho (1 + b_f)}{\varphi_f (1 + b_f)}.$$

(A.39)

From (7), both countries exhibit $\tilde{M}_i = \varphi_i \tilde{\sigma}_i^d$, and results (A.39) and (A.23) imply (23).

**Derivation of (24).** Defining $I_i \equiv \tilde{\sigma}_i^x + \tilde{\sigma}_i^d$ and substituting $\tilde{\sigma}_i^c$ by (A.19) and $\tilde{\sigma}_i^d$ by (A.23)-(A.39), we obtain (24).

**Derivation of (25)-(26).** Substituting the definition $E_f^c = \tilde{\sigma}_f^c P_Y^f Y_f$ in (A.27), we have

$$\frac{P_Y^h Y_h}{P_Y^f Y_f} = \frac{\epsilon \tilde{\sigma}_f^c}{\tilde{\gamma}_h + (1 - \epsilon) \tilde{\sigma}_h^c}.$$

(A.40)

Substituting $\tilde{\sigma}_f^c = 1 + \tilde{\gamma}_f \tilde{\theta} - \tilde{\sigma}_f^x - \tilde{\sigma}_f^d$ from (A.30), $\tilde{\sigma}_h^c = 1 - \tilde{\gamma}_h - \tilde{\sigma}_h^x - \tilde{\sigma}_h^d$ from (A.21), and $\frac{P_Y^h Y_h}{P_Y^f Y_f} = \frac{\tilde{\gamma}_f}{\tilde{\gamma}_h}$ from (A.28), equation (A.40) yields $\frac{\tilde{\gamma}_f}{\tilde{\gamma}_h} \tilde{\theta} = \frac{\epsilon (1 - \tilde{\sigma}_f^x - \tilde{\sigma}_f^d)}{(1 - \epsilon) (1 - \tilde{\sigma}_h^x - \tilde{\sigma}_h^d)}$, which can be solved for $\tilde{\theta}$ to get

$$\tilde{\theta} = \frac{\tilde{\gamma}_h}{\tilde{\gamma}_f} \cdot \frac{\epsilon (1 - \tilde{\sigma}_f^x - \tilde{\sigma}_f^d)}{(1 - \epsilon) (1 - \tilde{\sigma}_h^x - \tilde{\sigma}_h^d)}.$$

(A.41)

Substituting $I_i \equiv \tilde{\sigma}_i^x + \tilde{\sigma}_i^d$ in (A.41) and recalling that $\frac{\tilde{\gamma}_h}{\tilde{\gamma}_f} = \frac{1 + \tau_f}{1 + \tau_h}$, we get (26). Substituting (26) in (A.28), we obtain (25).
Closed-form solutions. For future reference, the closed-form solutions for output levels and prices are (see proofs in the Supplementary Material)

\[ Y_h(t) = \frac{(\alpha^2/\zeta) \delta}{1 + b_h} \cdot M_h(0)(v_h(0)L_h) \frac{\rho Q_0\theta}{(1 + \theta)} \cdot e^{(\Omega_h - \rho)t}, \]  

(A.42)

\[ Y_f(t) = \frac{(\alpha^2/\zeta) \delta}{1 + b_f} \cdot M_f(0)(v_f(0)L_f) \frac{\rho Q_0\theta}{(1 + \theta)} \cdot e^{(\Omega_f - \rho)t}, \]  

(A.43)

\[ \frac{Y_h(t)}{Y_f(t)} = \theta \frac{\gamma}{1 - \alpha} \cdot \psi_0 \cdot e^{(\Omega_h - \Omega_f)t}, \]  

(A.44)

\[ \frac{P_h(t)}{P_f(t)} = \frac{e}{1 - \epsilon} \cdot \frac{1 - I_f}{1 - I_h} \cdot \psi_0^{-1} \cdot \bar{\theta} \frac{\gamma}{1 - \alpha} \cdot e^{-(\Omega_h - \Omega_f)t}, \]  

(A.45)

where we have defined \( \psi_0 \equiv \left[ \frac{M_h(0)}{M_f(0)} \left( \frac{1 + b_f}{1 + b_h} \right) \left( \frac{v_h(0)L_h}{v_f(0)L_f} \right) \right]^{\frac{\beta}{1 - \alpha}} \).

**Derivation of (28)-(29)-(30).** See proofs in the Supplementary Material.

**Derivation of (33).** Expression (33) directly follows from (17)-(18).

**Derivation of (34)-(35).** Defining the constant \( \bar{\epsilon}_i \equiv (\epsilon/L_i) \left( \frac{1 - \epsilon}{\epsilon} \right) \) and recalling that \( E_i^c = \bar{\sigma}_i^c P_i^Y Y_i \) by (A.18), present-value utility (14) reads

\[ U_i = \int_0^\infty e^{-\rho t} \cdot \ln \left[ \bar{\epsilon}_i \frac{\bar{\sigma}_i^c P_i^Y Y_i}{(P_i^Y)^c (P_i^Y)^{1 - \epsilon}} \right] dt. \]  

(A.46)

Plugging the respective country indices, we obtain

\[ U_h = \int_0^\infty e^{-\rho t} \cdot \ln \left[ \bar{\epsilon}_h \left( \frac{P_h^Y}{P_f^Y} \right)^{1 - \epsilon} \bar{\sigma}_h^c Y_h \right] dt \text{ and } U_f = \int_0^\infty e^{-\rho t} \cdot \ln \left[ \bar{\epsilon}_f \left( \frac{P_f^Y}{P_h^Y} \right)^\epsilon \bar{\sigma}_f^c Y_f \right] dt. \]

Substituting \( P_h^Y(t)/P_f^Y(t) = [P_h^Y(0)/P_f^Y(0)]e^{(\Omega_f - \Omega_h)t} \) from (20), and \( Y_i(t) = Y_i(0) e^{(\Omega_i - \rho)t} \) from (21), and collecting the terms to isolate the initial values, we can define

\[ \bar{z}_h \equiv \int_0^\infty e^{-\rho t} \cdot \ln \left[ e^{[\Omega_h - \rho + (1 - \epsilon)(\Omega_f - \Omega_h)]t} \right] dt + \frac{1}{\rho} \ln \bar{\epsilon}_h, \]

\[ \bar{z}_f \equiv \int_0^\infty e^{-\rho t} \cdot \ln \left[ e^{[\Omega_f - \rho + \epsilon(\Omega_h - \Omega_f)]t} \right] dt + \frac{1}{\rho} \ln \bar{\epsilon}_f, \]

and rewrite \( U_h \) and \( U_f \) as in (34)-(35).
Derivation of results (36)-(37). From (A.42), (A.43) and (A.45), we have

\[
\frac{d \ln Y_h(0)}{d \tau_f} = \frac{\gamma}{1 - \alpha} \cdot \frac{d \ln [\tilde{\theta} / (1 + \tilde{\theta})]}{d \tau_f} = \frac{\gamma}{1 - \alpha} \cdot \frac{1}{1 + \tilde{\theta}} \cdot \frac{d \ln \tilde{\theta}}{d \tau_f} < 0, \tag{A.47}
\]

\[
\frac{d \ln Y_f(0)}{d \tau_f} = \gamma \cdot \frac{d \ln [1 / (1 + \tilde{\theta})]}{d \tau_f} = - \frac{\gamma}{1 - \alpha} \cdot \frac{\tilde{\theta}}{1 + \tilde{\theta}} \cdot \frac{d \ln \tilde{\theta}}{d \tau_f} < 0, \tag{A.48}
\]

\[
\frac{d \ln p_0}{d \tau_h} = - \frac{\gamma}{1 - \alpha} \cdot \frac{d \ln \tilde{\theta}}{d \tau_f} > 0, \tag{A.49}
\]

\[
\frac{d \ln p_0}{d \tau_f} = - \frac{\gamma}{1 - \alpha} \cdot \frac{d \ln \tilde{\theta}}{d \tau_f} < 0, \tag{A.50}
\]

where \( p_0 \equiv P^h_Y(0) / P^f_Y(0) \). The signs in (A.47)-(A.50) come from \( d \tilde{\theta}/d \tau_h < 0 \) and \( d \tilde{\theta}/d \tau_f > 0 \) as implied by (26). These results imply the signs of terms-of-trade effects and physical-output effects reported in (36)-(37). Considering the consumption-share effect in Home, expression (A.22) implies

\[
\frac{d \ln \bar{\sigma}^c_h}{d \tau_h} = \frac{1}{1 + \tau_h} \cdot \frac{\tilde{\gamma}_h}{\bar{\sigma}^c_h} = \frac{1}{1 + \tau_h} \cdot \frac{\tilde{\gamma}_h}{1 - \tilde{\gamma}_h - I_h} > 0, \tag{A.51}
\]

where the last term comes from substituting \( I_h = \bar{\sigma}^c_h + \bar{\sigma}^d_h \) in (A.22). In Foreign, equation (A.34) implies

\[
\frac{d \ln \bar{\sigma}^c_f}{d \tau_f} = \frac{d \ln (\chi \tilde{\gamma}_f \tilde{\theta})}{d \tau_f} = \frac{d \ln \chi}{d \tau_f} + \frac{d \ln (\tilde{\gamma}_f \tilde{\theta})}{d \tau_f} = 0, \tag{A.52}
\]

where \( d \ln \chi/d \tau_f = 0 \) is implied by (A.33) and \( d \ln (\tilde{\gamma}_f \tilde{\theta})/d \tau_f = 0 \) follows from (26).\(^{18}\)

**Proof of Proposition 2 (Foreign).** Substituting (A.48) and (A.50) in (37), and using \( d \ln \tilde{\theta}/d \tau_f = (1 + \tau_f) \) from (26), we have

\[
\rho \cdot \frac{d U_f}{d \tau_f} = \frac{\gamma (1 + \tau_f)}{1 - \alpha} \cdot \left[ \frac{\varepsilon}{1 + \tilde{\theta}} \right], \tag{A.53}
\]

the sign of which is determined by the term in square brackets. As \( \tilde{\theta} \) is monotonously increasing in \( \tau_f \) by (26), the condition \( d U_f/d \tau_f = 0 \) is univocously associated to a Foreign tax \( \tau_f^{\max} \) associated to a relative resource use \( \tilde{\theta}^{\max} = \varepsilon / (1 - \varepsilon) \). The condition \( d U_f/d \tau_f = 0 \) identifies a maximum of \( U_f \) because (A.53) implies \( d U_f/d \tau_f > 0 \) when \( \tilde{\theta} < \varepsilon / (1 - \varepsilon) \) and \( d U_f/d \tau_f < 0 \) when \( \tilde{\theta} > \varepsilon / (1 - \varepsilon) \).

\(^{18}\)In (A.33), all terms to the right hand side are independent of \( \tau_f \), which implies \( d \chi/d \tau_f = 0 \). In (26), we can multiply both sides by \( \tilde{\gamma}_f \) and obtain an expression for \( \tilde{\gamma}_f \tilde{\theta} \) that is independent of \( \tau_f \) because the terms \((1 + \tau_f)^{-1}\) cancel out, which implies \( d (\tilde{\gamma}_f \tilde{\theta})/d \tau_f = 0 \).
Proof of Proposition 2 (Home). Substituting (A.47), (A.49) and (A.52) in (36),

$$\rho \cdot \frac{dU_h}{d\tau_h} = -\gamma \frac{(1 - \epsilon)}{1 - \alpha} \cdot \frac{d\ln \tilde{\theta}}{d\tau_h} + \gamma \frac{1}{1 - \alpha} \cdot \frac{d\ln \tilde{\theta}}{d\tau_h} + \frac{1}{1 + \theta} \cdot \frac{1}{1 - \frac{\tilde{\gamma}_h}{\tilde{\gamma}_h - I_h}}.$$  

From (26), we have $d\ln \tilde{\theta}/d\tau_h = -(1 + \tau_h)^{-1}$ and the above expression reduces to

$$\rho \cdot \frac{dU_h}{d\tau_h} = \frac{\gamma}{1 + \tau_h} \cdot \left\{ \frac{1}{(1 + \tau_h)(1 - \tilde{\gamma}_h - I_h)} - \frac{1}{1 - \alpha} \cdot \left[ \frac{1}{1 + \theta} - (1 - \epsilon) \right] \right\}, \quad (A.54)$$

the sign of which is determined by the term in curly brackets: defining $\Upsilon^a(\tau_h) \equiv 1/[(1 + \tau_h)(1 - \tilde{\gamma}_h - I_h)]$ and $\Upsilon^b(\tau_h) \equiv \frac{1}{1 - \alpha} \cdot \left[ \frac{1}{1 + \theta} - (1 - \epsilon) \right]$, we have

$$\rho \cdot \frac{dU_h}{d\tau_h} = \frac{\gamma}{1 + \tau_h} \cdot \left[ \Upsilon^a(\tau_h) - \Upsilon^b(\tau_h) \right]. \quad (A.55)$$

where $\Upsilon^a(\tau_h)$ is strictly decreasing in $\tau_h$ and satisfies $\lim_{\tau_h \to \infty} \Upsilon^a(\tau_h) = 0$, while $\Upsilon^b(\tau_h)$ is strictly increasing in $\tau_h$ and satisfies $\lim_{\tau_h \to \infty} \Upsilon^b(\tau_h) = \frac{\epsilon}{1 - \alpha} > 0$. Therefore, $U_h$ is a hump-shaped function of $\tau_h$, with a unique maximum in $\tau_h = \tau_h^{\text{max}}$ associated to $\Upsilon^a(\tau_h^{\text{max}}) = \Upsilon^b(\tau_h^{\text{max}}) \to dU_h/d\tau_h = 0$. Consider any level $\tilde{\tau}_R^h$ of the Home tax such that relative resource use is $\tilde{\theta} = \epsilon/(1 - \epsilon)$: from (A.54) and (A.55), we have $\Upsilon^a(\tilde{\tau}_R^h) > \Upsilon^b(\tilde{\tau}_R^h) = 0$ and $dU_h/d\tau_h > 0$. Hence, the condition $dU_h/d\tau_h = 0$ is associated to a resource tax $\tau_h^{\text{max}} > \tilde{\tau}_R^h$ and a level of relative resource use $\theta_h^{\text{max}} < \epsilon/(1 - \epsilon)$.

Proof of Proposition 3. By (31), in a symmetric $CE$-allocation we have $\tilde{\theta} = \epsilon/(1 - \epsilon), implying $dU_f/d\tau_f = 0$ from (A.53) and $dU_h/d\tau_h > 0$ from (A.55).

Proof of Proposition 4. In a laissez-faire equilibrium, $\tilde{\theta}$ is given by (27). If $\varphi_h = \varphi_f$ we have $\tilde{\theta} = \epsilon/(1 - \epsilon)$, in which case $dU_f/d\tau_f = 0$ and $dU_h/d\tau_h > 0$ by Proposition 3. If $\varphi_h > \varphi_f$, we have $\tilde{\theta} > \epsilon/(1 - \epsilon)$, implying $dU_f/d\tau_f < 0$ from (A.53) and $dU_h/d\tau_h > 0$ from (A.54). If $\varphi_h < \varphi_f$, we have $\tilde{\theta} < \epsilon/(1 - \epsilon)$, which implies $dU_f/d\tau_f > 0$ from (A.53) whereas, from (A.55), the sign of $dU_h/d\tau_h$ is generally ambiguous.
Supplementary Material

Aggregate Constraints: derivation of (17)-(18). Equation (17) is derived as follows. Substituting \( n_i \equiv (V_i M_i) / L_i \) and (A.5) in (A.12), we obtain

\[
V_h \dot{M}_h = \Pi_h M_h + P_L^h L_h - E_h^c - F_h L_h.
\]

Plugging \( V_i \dot{M}_i = P_Y^i Z_i \) from (5)-(A.4), and \( M_i \Pi_i = M_i X_i (P_X^i - \zeta P_Y^i) \) from (A.2), in the above equation, we obtain

\[
P_Y^h Z_h + E_h^c + P_Y^h \varsigma M_h X_h = M_h P_X^h X_h + P_L^h L_h - F_h L_h,
\]

where we substitute \( F_i L_i = a_i P_Y^i Z_i - b_i M_i P_X^i X_i - \tau_i P_R R_i \) from (11) to get

\[
P_Y^h Z_h (1 + a_h) + E_h^c + P_Y^h \varsigma M_h X_h = M_h P_X^h X_h (1 + b_h) + P_L^h L_h + \tau_h P_R R_h.
\]

From the final sectors’ profit-maximizing conditions, we can substitute \( P_L^h L_i = \beta P_Y^i Y_i \) and \( M_i P_X^i X_i (1 + b_i) = \alpha P_Y^i Y_i \) in the above equation, obtaining

\[
E_h^c + P_Y^h Z_h (1 + a_h) + P_Y^h \varsigma M_h X_h = (\alpha + \beta) P_Y^h Y_h + \tau_h P_R R_h,
\]

where we can plug \( \alpha + \beta = 1 - \gamma \), and condition (2), to obtain

\[
E_h^c + P_Y^h Z_h (1 + a_h) + P_Y^h \varsigma M_h X_h = P_Y^h Y_h - P_R R_h. \tag{B.1}
\]

Substituting \( E_h^d \equiv P_Y^h Z_h (1 + a_h) \) and \( E_h^c \equiv P_Y^h \varsigma M_h X_h \) we obtain (17). Repeating the above steps for the Foreign economy starting from constraint (A.13), and recalling that \( R - R_f = R_h \), we obtain (18).

Derivation of (A.22)-(A.23). Consider Home. From (A.21), substitute \( \delta_h^d = 1 - \gamma_h - \delta_h^c - \delta_h^f \) in (A.20), and eliminate \( \delta_h^f \) by (A.19), to obtain

\[
\hat{\delta}_i^c (t) = \varphi_h \delta_h^c (t) + \varphi_h \frac{\alpha (1 - \alpha) (1 + a_h) + \alpha^2}{1 + b_h} - \varphi_h (1 - \gamma_h) - \rho, \tag{B.2}
\]

Since \( \varphi_h > 0 \), equation (B.2) is globally unstable around the unique stationary point: ruling out by standard arguments explosive dynamics in the consumption propensity, we have

\[
\delta_h^c = (1 - \gamma_h) - \frac{\varphi_h [\alpha (1 - \alpha) (1 + a_h) + \alpha^2] - \rho (1 + b_h)}{\varphi_h (1 + b_h)} \text{ in each } t. \tag{B.3}
\]
From (A.19) and (B.3), constant values of \( \bar{c}_h \) and \( \bar{x}_h \) imply a constant \( \bar{d}_h \) which, from (A.21), equals
\[
\bar{d}_h = 1 - \bar{c}_h - \bar{x}_h = \frac{\varphi_h \alpha (1 - \alpha) (1 + a_h) - \rho (1 + b_h)}{\varphi_h (1 + b_h)}.
\]
(B.4)

**Derivation of (A.42)-(A.45).** Equation (A.3) and result (21) imply
\[
Y_i(t) = \left( \frac{\alpha^2}{\zeta} \right)^{\frac{\beta}{\alpha}} \cdot M_i(0) (v_i(0) L_i)^{\frac{\beta}{1-\alpha}} (R_i(0))^{\frac{2}{1-\alpha}} \cdot e^{(\Omega_i - \rho)t},
\]
(B.5)
where \( M_i(0) \) and \( v_i(0) \) are exogenously given. Initial resource use \( R_i(0) \) is determined by the solution of the optimal extraction problem:
\[
R_h(0) = \frac{\bar{\theta}}{1 + \bar{\theta}} \rho Q_0 \quad \text{and} \quad R_f(0) = \frac{1}{1 + \bar{\theta}} \rho Q_0.
\]
(B.6)
Substituting (B.6) in (B.5) for each \( i = h, f \), we obtain (A.42) and (A.43). Taking the ratio between (A.42) and (A.43), and defining \( \psi_0 \equiv \left[ \frac{M_h(0)}{M_f(0)} \left( \frac{1 + b_f}{1 + b_h} \right) \left( \frac{v_h(0)L_h}{v_f(0)L_f} \right)^{\frac{\beta}{1-\alpha}} \right] \), we obtain (A.44). Re-writing (A.28) as
\[
\frac{P^h_Y(t)}{P^f_Y(t)} = \frac{1 + \tau_h Y_f(t)}{1 + \tau_f Y_h(t)},
\]
and using (A.44) to eliminate \( Y_h(t) / Y_f(t) \), we obtain (A.45).

**Conditional efficiency in Home.** By definition, the CE-allocation in Home solves
\[
\max_{\{E^h, E^f, E^{rh}, R_h\}} \int_0^\infty e^{-\rho t} \cdot \ln((\omega / L_h) \cdot E^h_h)dt \text{ subject to}
\]
\[
Y_h = M_hX_h^{\alpha} (v_hL_h)^{\beta} R_h^{\gamma},
\]
\[
E^{\bar{r}}_h = P^h_Y \varphi M_hX_h,
\]
\[
P^h_Y Y_h = E^c_h + E^d_h + E^{\bar{r}}_h + P_R R_h,
\]
\[
\bar{M}_h = M_h \varphi \bar{h} \cdot \left[ E^{\bar{d}}_h / (P^h_Y Y_h) \right],
\]
where \( \omega = \omega(P^h_Y, P^f_Y) \) is taken as given and symmetry across varieties is already imposed without any loss of generality. The first constraint is the final-good technology (1), the

\[\text{Q0} = \int_0^\infty R_f(t) (1 + \bar{\theta}) dt \text{ and directly integrated to obtain } R_f(0) \text{ in (B.6), from which } R_h(0) \text{ can be obtained as } \bar{\theta} R_f(0).\]
second is the intermediate-good technology with linear cost, the third is (17), the fourth is the R&D technology (7) with knowledge spillovers taken into account. Recalling that \( \sigma^d_h \equiv E^d_h / (P^b_Y Y_h) \) and combining the first three constraints, the problem becomes

\[
\max_{\{E^c_h, X_h, \sigma^d_h, R_h\}} \int_0^\infty e^{-\mu t} \cdot \ln((\omega/L_h) \cdot E^c_h) dt \quad \text{subject to}
\]

\[
P^b_Y M_h X_h \alpha (v_h L_h)^\beta R_h \gamma \left(1 - \sigma^d_h\right) = E^c_h + P^b_Y \varsigma M_h X_h + P_R R_h, \tag{B.7}
\]

\[
\dot{M}_h = M_h \varphi_h \sigma^d_h, \tag{B.8}
\]

where the controls are \( \{E^c_h, X_h, \sigma^d_h, R_h\} \) and the only state variable is \( M_h \). The current-value Hamiltonian is

\[
\ln \left[\frac{\omega_h}{L_h} \cdot E^c_h\right] + \mu_h' \cdot M_h \varphi_h \sigma^d_h + \mu''^n \cdot \left[P^b_Y M_h X_h \alpha (v_h L_h)^\beta R_h \gamma \left(1 - \sigma^d_h\right) - E^c_h - P^b_Y \varsigma M_h X_h - P_R R_h\right]
\]

where \( \mu_h' \) is the dynamic multiplier associated to (B.8) and \( \mu''_h \) is the static multiplier attached to (B.7). The optimality conditions read

\[
\frac{\partial}{\partial E^c_h} = 0 \quad \Rightarrow \quad \frac{1}{E^c_h} = \mu''_h, \tag{B.9}
\]

\[
\frac{\partial}{\partial X_h} = 0 \quad \Rightarrow \quad (1 - \sigma^d_h) \alpha P^b_Y Y_h = P^b_Y \varsigma M_h X_h, \tag{B.10}
\]

\[
\frac{\partial}{\partial \sigma^d_h} = 0 \quad \Rightarrow \quad \mu'_h M_h \varphi_h = \mu''^n P^b_Y Y_h, \tag{B.11}
\]

\[
\frac{\partial}{\partial R_h} = 0 \quad \Rightarrow \quad (1 - \sigma^d_h) \gamma P^b_Y Y_h = P_R R_h \tag{B.12}
\]

\[
\rho \mu'_h - \dot{\mu}'_h = \frac{\partial}{\partial M_h} \quad \Rightarrow \quad \rho \mu'_h - \dot{\mu}'_h = \mu'_h \varphi_h \sigma^d_h + \mu''^n P^b_Y \left[Y_h \frac{1}{M_h} (1 - \sigma^d_h) - \varsigma K_h\right], \tag{B.13}
\]

and imply\(^{20}\)

\[
\tilde{E}_h = \left[1 - \gamma \left(1 - \sigma^d_h\right)\right] \cdot P^b_Y Y_h, \tag{B.14}
\]

\[
\tilde{E}^x_h = \alpha \left(1 - \sigma^d_h\right) \cdot P^b_Y Y_h, \tag{B.15}
\]

\[
\tilde{E}^c_h = \beta \left(1 - \sigma^d_h\right) \cdot P^b_Y Y_h, \tag{B.16}
\]

\[
E^d_h = \sigma^d_h \cdot P^b_Y Y_h. \tag{B.17}
\]

\(^{20}\)Plugging (B.12) in constraint (17) we have (B.14). Plugging (B.10) in technology \( E^c_h = P^b_Y \varsigma M_h K_h \) yields (B.15). Plugging (B.10) and (B.12) in (B.7) we have (B.16). Equation (B.17) is determined residually by \( \tilde{E}^d_h = \tilde{E}_h - \tilde{E}^x_h - \tilde{E}^c_h \).
Substituting (B.10) and (B.11) in (B.13) we have
\[
\frac{\dot{\mu}_h}{\mu_h} = \rho - \varphi_h \left[ 1 - \alpha \left( 1 - \sigma^d_h \right) \right].
\] (B.18)

Time-differentiating (B.11) and using (B.18) we have
\[
\frac{\ddot{\mu}_h}{\mu_h} = \rho - \varphi_h (1 - \alpha) \left( 1 - \sigma^d_h \right) - \frac{\dot{P}_hY_h}{P^hY_h},
\] where we can substitute \( \dot{\mu}_h = 1/E^c_h \) from (B.9) to obtain
\[
\frac{\dot{E}^c_h}{E^c_h} - \frac{P^{bh}_hY_h}{P^hY_h} = \varphi_h (1 - \alpha) \left( 1 - \sigma^d_h \right) - \rho. \quad \text{(B.19)}
\]

From (B.16) we have \( \dot{E}^c_h - \frac{P^{bh}_hY_h}{P^hY_h} = -\frac{\dot{\sigma}^d}{1 - \sigma^d_h} \) which can be combined with (B.19) to get
\[
\dot{\sigma}^d_h = \rho \left( 1 - \sigma^d_h \right) - \varphi_h (1 - \alpha) \left( 1 - \sigma^d_h \right)^2. \quad \text{(B.20)}
\]

Equation (B.20) is globally unstable around its unique steady state: ruling out explosive dynamics by standard arguments, the conditionally-efficient rate of investment in R&D is
\[
\dot{\sigma}^d_h = \frac{\varphi_h (1 - \alpha) - \rho}{\varphi_h (1 - \alpha)} \quad \text{and} \quad 1 - \dot{\sigma}^d_h = \frac{\rho}{\varphi_h (1 - \alpha)} \quad \text{(B.21)}
\]
in each point in time. Substituting (B.21) in (B.15)-(B.16) we obtain
\[
\tilde{\sigma}^x_h = \frac{\alpha \rho}{\varphi_h (1 - \alpha)} \quad \text{and} \quad \tilde{\sigma}^c_h = \frac{\beta \rho}{\varphi_h (1 - \alpha)}. \quad \text{(B.22)}
\]

**Conditional efficiency in Foreign.** Following the same preliminary steps of the Home problem, the \( CE \)-allocation in Foreign solves

\[
\max_{\{E^c_f, X_f, \sigma^d_f, R_h, R_f\}} \int_0^\infty e^{-\rho t} \cdot \ln(\omega/L_f) \cdot E^c_f \, dt \text{ subject to}
\]
\[
P^d_f M_f X_f^\gamma (v_f L_f)^\beta R_f^\gamma \left( 1 - \sigma^d_f \right) = E^c_f + P^d_y M_f X_f - P_R R_h,
\] (B.23)
\[
M_f = M_f \varphi_f \sigma^d_f,
\] (B.24)
\[
\dot{Q} = -R_h - R_f
\] (B.25)
where (B.23) follows from (18) and, differently from Home, we have the resource constraint (B.25) and also exported resources $R_h$ as an additional control. The state variables are $M_f$ and the resource stock $Q$. The Hamiltonian is

$$\ln \left(\frac{\omega}{L_f}\cdot E_f^c\right) + \mu_f' \cdot M_f \varphi_f \sigma_f^d + \mu_f'' \cdot P_Y^f M_f X_f^\gamma \left(1 - \sigma_f^d\right) - E_f^c - P_Y^f M_f X_f + P_R R_h$$

$$+ \mu_f''' \cdot (-R_h - R_f)$$

where $\mu_f'$ is the dynamic multiplier associated to (B.24), $\mu_h'$ is the Lagrange multiplier attached to (B.23), and $\mu_f'''$ is the dynamic multiplier associated to (B.25). The first order conditions read

$$\frac{\partial}{\partial E_f^c} = 0 \rightarrow \frac{1}{E_f^c} = \mu_f''$$

(B.26)

$$\frac{\partial}{\partial X_f} = 0 \rightarrow \left(1 - \sigma_f^d\right) \alpha P_Y^f Y_f = P_Y^f M_f X_f$$

(B.27)

$$\frac{\partial}{\partial \sigma_f^d} = 0 \rightarrow \mu_f' M_f \varphi_f = \mu_f'' P_Y^f Y_f$$

(B.28)

$$\frac{\partial}{\partial R_h} = 0 \rightarrow \mu_f'' \cdot P_R = \mu_f'''$$

(B.29)

$$\frac{\partial}{\partial R_f} = 0 \rightarrow \mu_f'' \cdot \left(1 - \sigma_f^d\right) \gamma P_Y^f Y_f = \mu_f''' R_f$$

(B.30)

$$\rho \mu_f' - \mu_f' = \frac{\partial}{\partial M_f} \rightarrow \rho \mu_f' - \mu_f' = \mu_f' \varphi_f \sigma_f^d + \mu'' P_Y^f \left[\frac{Y_f}{M_f} \left(1 - \sigma_f^d\right) - \zeta M_f\right]$$

(B.31)

$$\rho \mu_f''' - \mu_f''' = \frac{\partial}{\partial Q} \rightarrow \rho \mu_f''' - \mu_f''' = 0.$$  

(B.32)

Notice that, from (B.29)-(B.30) and definition $R_h = \theta R_f$, we have

$$P_R R_f = \left(1 - \sigma_f^d\right) \gamma P_Y^f Y_f$$

(B.33)

$$P_R R_h = \left(1 - \sigma_f^d\right) \gamma \tilde{\theta} \cdot P_Y^f Y_f$$

(B.34)
so that expenditures equal \(^{21}\)

\[
\tilde{E}_f = \left[ 1 + \left( 1 - \tilde{\sigma}_d^f \right) \gamma \tilde{\theta} \right] \cdot P_Y^f \tilde{Y}_f, \\
\tilde{E}^p_f = \alpha \left( 1 - \tilde{\sigma}_d^f \right) \cdot P_Y^f \tilde{Y}_f, \\
\tilde{E}_c^c = \left( 1 - \alpha + \gamma \tilde{\theta} \right) \left( 1 - \tilde{\sigma}_d^f \right) \cdot P_Y^f \tilde{Y}_f, \\
\tilde{E}_d^d = \tilde{\sigma}_d^f \cdot P_Y^f \tilde{Y}_f. 
\]  

(B.35)  

(B.36)  

(B.37)  

(B.38)  

Before deriving the explicit value of \(\tilde{\sigma}_d^d\) we show that the efficient relative resource use \(\tilde{\theta}\) is constant over time. From the balanced trade condition (A.26), we have \(P_R R_h + (1 - \epsilon) E_h^c = \epsilon E_f^c\) where we can use (B.16) and (B.37) to eliminate \(E_h^c\) and \(E_f^c\), respectively, and also use (B.12) to eliminate \(P_R R_h\), obtaining

\[
\frac{1 - \tilde{\sigma}_d^d}{1 - \tilde{\sigma}_f^d} \cdot \frac{P_h^h \tilde{Y}_h}{P_Y^f \tilde{Y}_f} = \frac{\epsilon \left( 1 - \alpha + \gamma \tilde{\theta} \right)}{\gamma + (1 - \epsilon) \beta}, 
\]  

(B.39)  

where tildas denote conditionally-efficient values. Taking the ratio between (B.12) and (B.34) we have

\[
\tilde{\theta} = \frac{1 - \tilde{\sigma}_h^d}{1 - \tilde{\sigma}_f^d} \cdot \frac{P_h^h \tilde{Y}_h}{P_Y^f \tilde{Y}_f}. 
\]  

(B.40)  

Combining (B.40) with (B.39) we obtain

\[
\tilde{\theta} = \frac{\epsilon}{1 - \epsilon} \cdot \frac{1 - \alpha}{\gamma + \beta} = \frac{\epsilon}{1 - \epsilon}. 
\]  

(B.41)  

Result (B.41) implies that \(\tilde{\theta}\) is constant. Now go back to (B.31) and substitute (B.27)-(B.28) to re-write it as

\[
\frac{\dot{\mu}_f^f}{\mu_f^f} = \rho - \varphi_f \left[ 1 - \alpha \left( 1 - \tilde{\sigma}_d^f \right) \right]. 
\]  

(B.42)  

Time-differentiating (B.28) and substituting (B.26)-(B.24), we obtain

\[
\frac{\dot{\mu}_f^f}{\mu_f^f} = -\frac{\dot{E}_c^c}{E_c^c} + \frac{P_Y^f \dot{Y}_f}{P_Y^f Y_f} - \varphi_d^d \tilde{\sigma}_d^d 
\]

\(^{21}\)Plugging (B.34) in (18) yields (B.35). Plugging (B.27) in technology \(E_f^p = P_Y^f M_f K_f\) yields (B.36). Plugging (B.27) and (B.34) in (B.23) we have (B.37). Equation (B.38) is determined residually by \(\tilde{E}_d^d = \tilde{E}_f - \tilde{E}^p_f - \tilde{E}_h^c\).
which can be combined with (B.42) to obtain
\[
\frac{\dot{\mathcal{E}}_f}{\mathcal{E}_f} - \frac{P_{Y_f}^{f}}{P_{Y_f}^{f}} = \varphi_f \left[ (1 - \alpha) \left( 1 - \dot{\sigma}_f \right) \right] - \rho. \tag{B.43}
\]
Since \( \dot{\theta} \) is constant by (B.41), time-differentiation of (B.37) yields
\[
\frac{\dot{\mathcal{E}}_f}{\mathcal{E}_f} - \frac{P_{Y_f}^{f}}{P_{Y_f}^{f}} = -\frac{\dot{\sigma}_f}{1 - \sigma_f}. \tag{B.44}
\]
Plugging this result in (B.44) we obtain the usual equilibrium relation (see (B.20) above for Home) which can be solved for the steady-state level
\[
\dot{\sigma}_f = \frac{\varphi_f (1 - \alpha) - \rho}{\varphi_f (1 - \alpha)} \quad \text{or} \quad 1 - \dot{\sigma}_f = \frac{\rho}{\varphi_f (1 - \alpha)}. \tag{B.44}
\]
Substituting (B.44) in (B.36)-(B.38) we obtain
\[
\dot{\sigma}_f^x = \frac{\alpha \rho}{\varphi_f (1 - \alpha)} \quad \text{and} \quad \dot{\sigma}_f^c = \frac{\rho (1 - \alpha + \gamma \dot{\theta})}{\varphi_f (1 - \alpha)}. \tag{B.45}
\]

Derivation of (31). Equation (31) is proved in (B.41).

Derivation of (28)-(30). Efficient taxes are obtained by equalizing efficient and equilibrium values of \((\sigma_f^x, \sigma_f^c, \sigma_f^e)\). First, results (B.22) and (B.45) imply \(\dot{\sigma}_f^x = \frac{\alpha \rho}{\varphi(1 - \alpha)}\) in both countries. Imposing the equality between the efficient values \(\dot{\sigma}_f^x\) and the competitive-equilibrium values \(\dot{\sigma}_f^x = \frac{\alpha^2}{1 + b_i}\) derived in (A.19), we obtain the efficient tax on intermediates’ purchases \(\tilde{b}_i\) in (29). Second, results (B.21) and (B.44) imply \(\dot{\sigma}_f^d = \frac{\varphi(1 - \alpha) - \rho}{\varphi(1 - \alpha)}\) in both countries. Imposing the equality between \(\dot{\sigma}_f^d\) and the competitive-equilibrium values \(\dot{\sigma}_f^d = \frac{\alpha \rho}{\varphi(1 - \alpha)}\) derived in (A.39), and substituting \(\tilde{b}_i\) by (29), we obtain the efficient subsidy \(\tilde{a}_i\) in (28). Now consider Home: from (B.22) we have \(\dot{\sigma}_h^x = \frac{\beta \rho}{\varphi_h(1 - \alpha)}\), whereas (B.4) implies \(\dot{\sigma}_h^c = 1 - \dot{\gamma}_h - \dot{\sigma}_h^d - \dot{\sigma}_h^x\). Setting \(\dot{\sigma}_h^c = \dot{\sigma}_h^c\) and imposing that \(\dot{\sigma}_h^x = \dot{\sigma}_h^x\) and \(\dot{\sigma}_h^d = \dot{\sigma}_h^d\) by virtue of (28)-(29), we obtain
\[
\frac{\beta \rho}{\varphi_h(1 - \alpha)} = 1 - \dot{\gamma}_h - \dot{\sigma}_h^d - \dot{\sigma}_h^x = 1 - \dot{\gamma}_h - \frac{\varphi_h (1 - \alpha) - \rho}{\varphi_h (1 - \alpha)} - \frac{\alpha \rho}{\varphi_h (1 - \alpha)}
\]
where the last term follows from \(\dot{\sigma}_h^d\) and \(\dot{\sigma}_h^x\) derived in (B.21) and (B.22). Rearranging terms and solving for \(\dot{\gamma}_h\) we obtain \(\dot{\gamma}_h = \frac{\rho \gamma}{\varphi_h(1 - \alpha)}\), which implies the efficient resource tax for Home
\[
\dot{\gamma}_h = \frac{\varphi_h (1 - \alpha) - \rho}{\rho}. \tag{B.46}
\]
The optimal resource tax in Foreign $\tilde{\tau}_f$ then follows from (B.40). Since $1 + \tilde{\tau}_h = 1 - \bar{\sigma}_h^d$ by (B.46), the only way to satisfy $\tilde{\theta} = \tilde{\bar{\theta}}$ in equations (A.28) and (B.40) is to set $1 + \tilde{\tau}_f = 1 - \bar{\sigma}_h^f = \frac{1}{\bar{p}} \varphi_h (1 - \alpha)$, which proves (30). It can be easily verified that, residually, (28)-(30) imply $\bar{\sigma}_f^r = \bar{\sigma}_f^c$. 
## Arellano-Bond dynamic panel-data estimation

Endogenous variable: shareoim

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Table 1: Estimation results for income shares of oil-importing countries. Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$. 

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