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Wild salmon fishing:
Harvesting the old or young?

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Abstract

This paper develops an optimal harvesting model for the wild Atlantic salmon (*Salmo salar*), where various age classes of the population is included. It is shown that the marginal value–fecundity relationship of the spawning population, comprising young and old fish, is crucial for the optimal fishing composition. If the marginal value–fecundity ratio is higher for the old spawning population, this age-class should be harvested more aggressively than the young spawning population, and *vice versa*. It is also shown that changes in prices and interest rate have similar as well as different effects than in the standard fishing biomass model. Changes in the relative price for the harvestable age classes could either increase or reduce the optimal harvest intensity, or have no effect. While a higher interest rate tends to increase the exploitation pressure, there also exist intervals in which the optimal harvest program is not affected by changes in the interest rate.

1. Introduction

For many years, the North Atlantic salmon (*Salmo salar*) has been one of the most important fish species in Norway because of its social, cultural, and economic importance. It was traditionally harvested for food, but today is most important to recreational anglers. Norwegian rivers are the most important spawning rivers for the Atlantic stock, and about 30% of the remaining stocks spawn there. The wild salmon are harvested by commercial and recreational fisheries. The marine harvest is commercial and semi commercial, whereas the harvest in the spawning rivers is recreational (NOU 1999). The amount harvested in marine and river fisheries has been more or less similar over the last few years, but the value of river fishery is much higher because of the higher willingness to pay for sport fishing (NOU 1999, Olaussen and Skonhoft 2008).

However, the abundance of wild salmon stocks has been declining during the last few decades. Stock development has been especially disappointing since the 1990s because of a combination of various factors, such as sea temperature, diseases, and human activity, both in the spawning streams and through the strong growth of salmon sea farming (NASCO 2004). As the wild stock began to decrease during the 1980s, the Norwegian government imposed gear restrictions to limit the marine harvest. Drift net fishing was banned in 1989, and the fishing season of bend net fishing, taking place in the fjords and close to the spawning rivers, has been restricted several times. At the same time, the sport fishing season in the spawning rivers has been subject to various restrictions (NOU 1999). However, despite all these measures taken to secure and rebuild the stock, the abundance of wild salmon seems to be at only half the level experienced in the 1960s and 1970s. It is believed that the rapid expansion of the farmed salmon industry has played the most important role in this decline and today, farmed salmon is regarded as the main threat to the viability of the wild salmon population because of the spread of diseases, escapees, and environment pollution (Hindar et al. 2006). In Liu et al. (2013) possible biological as well as economic effects of farmed salmon escapees are analyzed and numerically illustrated.

Wild salmon fishing has been studied in many papers from an economic perspective. Routledge (2001) studied a mixed stock versus single stock fishery related to Pacific salmon while Laukkanen (2001) analyzed the northern Baltic salmon fishery in a sequential fishing biomass model. Olaussen and Skonhoft (2008) also analyzed a sequential harvesting biomass model, but with recreational fishery in the rivers as its focus. The economics of the Baltic salmon fishery is studied in an age structured dynamic model in Kulmala et al. (2008), comprising migration and seasonal harvest and competing harvesting by commercial and recreational fishermen. Uncertainty is also included, and the model is parameterized and solved numerically for a Finnish river stock. In what follows, an age-structured wild salmon model is analyzed as well, but within a much simpler framework than that of Kulmala et al. The goal is, from a theoretical point of view, to study how the harvesting of different

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age classes influences recruitment and stock abundance, and the main focus is to find the harvest composition that maximizes the economic yield (*MEY*) under various economic conditions. We will think of the fishing related to a river recreational fishery such that the harvesting value is made up of the angler willingness to pay for the catch minus the cost of organizing the fishery and no stock dependent costs are included.

Age-structured models are far more complex than biomass models. On the one hand, it is relatively straightforward to formulate a reasonably good age-structured model and numerically simulate the effects of variations in fishing mortality between age classes and over time. On the other hand, it is notoriously difficult to understand the various biological and economic forces at work in such models. Tahvonen (2009) has analyzed some of these issues in which he finds some results in a dynamic setting, but under quite restrictive assumptions (i.e., only one agent, or one fleet, causing fishing mortality). Early contributions analyzing age-structured models include Reed (1980), who studied the maximum sustainable yield problem. He found that optimal harvesting comprises, at most, two age classes. Further, if two age classes are harvested, the elder is harvested completely. This model is extended in various directions in Skonhoft et al. (2012). Getz and Haight (1988) reviewed various age-structured models, and formulated the solution for the maximum sustainable yield problem as well as the maximum yield problem over a finite planning horizon. The following analysis has similarities with Reed (1980) and Skonhoft et al. (2012), but we study a different biological system in which all the spawning fish, i.e., salmon, die after spawning. This contrasts with the above mentioned works, where the spawning fish (e.g., cod) survive and enter an older age class after spawning. As will be seen, this difference has important implications for the optimal harvesting policy. In addition, just as in Tahvonen (2009), and also the age structured models in Quaas et al. (2013) and Diekert et al. (2010), our analysis is framed in a dynamic setting. While our analysis is directly related to Atlantic salmon, we will find that it fits various Pacific salmon species, such as pink and chum salmon, which also die after spawning (see, e.g., Groot and Margolis 1991).

The paper is organized as follows. In the next section, we describe the population model where two spawning, and hence two harvestable, age classes are included. In section three, we formulate and characterize the maximum economic yield fishing (*MEY*) program under assumption of perfect fishing selectivity. The steady state solution and the dynamics are examined in section four. Section five provides a numerical illustration of the model. In section six, some possible extensions of the model are studied. As perfect selectivity is a questionable assumption, we first take up the selectivity problem and relax this assumption and assume the same fishing mortality for both harvestable age classes. This analysis is also supported by a numerical simulation where differences compared to the perfect selectivity case are illustrated. Next, we relax our assumption of constant marginal willingness

to pay for recreational fishing as implied by the *MEY* program. Section seven concludes the paper where we briefly also discuss the selectivity question.

2. Population model and harvest

Atlantic salmon is an anadromous species that has a complex life cycle with several distinct phases. Freshwater habitat is essential in the early development stages, as this is where it spends the first one to three years from spawning to juvenile rearing before undergoing smoltification and seaward migration. Then, it stays for one to three years in the ocean for feeding and growing and, when mature, returns to the natal, or 'parent' rivers to spawn. After spawning, most salmon die, and less than 10% of the female salmon spawn twice (Mills 1989). The Atlantic salmon is subject to fishing when it migrates back to its parent river in spring - summer. In Norway, the sea fishing takes place in fjords and inlets with wedge-shaped seine and bend nets during a short fishing period (June – July). This fishing is commercial, or semi commercial. In the rivers, salmon are caught by recreational anglers with rods and hand lines, also during a short fishing season (June – August). As indicated, the river recreational fishery is by far the most important from an economic point of view and only this fishery is included here. This means that we abstract from possible complications by considering a sequential fishery; that is, sea fishing taking place first and then river fishing (but see Olaussen and Skonhoft 2008).

In what follows, a specific salmon population (with its native river) is considered in terms of the number of individuals at time t in three young age classes, $N_{1,t}$ (1 yr ≤ age <2 yrs), $N_{2,t}$ (2 yrs ≤ age < 3 yrs) and $N_{3,t}$ (3 yrs ≤ age < 4 yrs), and two adult, spawning classes, $N_{4,t}$ (4 yrs ≤ age < 5 yrs), and $N_{5,t}$ (5 yrs ≤ age <6 yrs). Recruitment is endogenous and density dependent, and the old spawning salmon has higher fertility than the young spawning salmon. Natural mortality is fixed and density independent and, as an approximation, it is assumed that the whole spawning population dies after spawning. It is further assumed that the proportion of the young mature stock that returns to spawn is fixed. This proportion may be influenced by a number of factors, such as the type of river ('small' salmon river vs. 'large' salmon river) and environmental factors (NOU 1999). As fishing takes place when the fish returns back to its native river, only the mature salmon $N_{4,t}$ and $N_{5,t}$ are subject to fishing. Figure 1 illustrates the life cycle of a single cohort of the salmon population considered. A far more detailed description of the life cycle of the Atlantic salmon is found in, e.g., Verspoor et al. (2003).

Figure 1 about here

The dynamics of the fish population is described as follows. We have first:

(1) $\quad N_{a+1,t+1} = s_a N_{a,t}$; $a = 1, 2, 3$

and

(2) $\quad N_{5,t+1} = s_4 N_{4,t}(1-\sigma)$,

where s_a (a = 1, 2, 3, 4) are the age-specific natural survival rates, which are assumed to be density independent and constant over time, and $0 < \sigma < 1$ is the proportion of the young mature stock that returns to spawn, also assumed to be fixed. It is assumed that all mature fish returns back to the native river.

As indicated, only the returning mature stocks are subject to fishing mortality, and when the proportion of the returning fish stock to be harvested in time t is denoted by $0 \le f_{a,t} \le 1$ ($a = 4, 5$), the number of fish harvested in period t becomes $H_{4,t} = \sigma N_{4,t} f_{4,t}$ and $H_{5,t} = N_{5,t} f_{5,t}$. Accordingly, the numbers of spawning fish of the young and old mature populations in the same period are $N_{4,t}^s = \sigma N_{4,t}(1 - f_{4,t})$ and $N_{5,t}^s = N_{5,t}(1 - f_{5,t})$, respectively. With γ_4 and γ_5 as the fecundity parameters, and where the old mature fish is more productive than the young mature fish; that is, $\gamma_5 > \gamma_4$, the size of the spawning population in year t is given as the fecundity weighted sum $B_t = \gamma_4 N_{4,t}^s + \gamma_5 N_{5,t}^s$ (see, e.g., Getz and Haight 1988, Ch. 3), or:

(3) $\quad B_t = \gamma_4 \sigma N_{4,t}(1 - f_{4,t}) + \gamma_5 N_{5,t}(1 - f_{5,t})$.

The spawning population in period t determines the size of stock recruitment, and thus the number of recruits that survives and enters age-class 1 next year:

(4) $\quad N_{1,t+1} = R(B_t) = R(\gamma_4 \sigma N_{4,t}(1 - f_{4,t}) + \gamma_5 N_{5,t}(1 - f_{5,t}))$.

The recruitment function $R(B_t)$ may be a one-peaked value function (i.e., of the Ricker type) or it may be increasing and strictly concave $R' > 0$ and $R'' < 0$ (i.e., of the Beverton - Holt type). In both cases, zero spawning stock means zero recruitment, i.e., $R(0) = 0$. In our analysis and in the numerical simulations we use the Beverton – Holt function.

3. The maximum economic yield harvesting program

The maximum economic yield (MEY) harvesting program is now examined. As already indicated, we are thinking of a recreational fishery where the harvesting value is made up of anglers' willingness to pay for fishing minus the cost of organizing the fishery. Any possible 'quality effect' in the fishery is neglected (cf., e.g., McConnel and Sutinen 1979), implying that a fixed value per fish is assumed and no stock dependent costs are included (but see section six for relaxing this assumption). The

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willingness to pay is higher for the larger and older mature fish than for the younger and smaller fish (see also numerical section five) and the net social benefit is thus higher for the larger fish. With p_a as the per unit net value (NOK/fish), assumed to be fixed over time, we hence have $p_5 > p_4$. The current value (in NOK) of the harvest is thus defined by $Y_t = (p_4 \sigma N_{4,t} f_{4,t} + p_5 N_{5,t} f_{5,t})$, and our

MEY problem is therefore described by $\max\limits_{f_{4,t}, f_{5,t}} \sum\limits_{t=0}^{\infty} \left[p_4 \sigma N_{4,t} f_{4,t} + p_5 N_{5,t} f_{5,t} \right] \rho^t$ subject to the

biological constraints (1), (2) and (4) and the known initial size of the stocks. $\rho = 1/(1+r)$ is the discount factor, where $r > 0$ is the annual interest rate which is assumed to be fixed.

The Lagrangian of this problem may be written as

$$L = \sum_{t=0}^{\infty} \rho^t \{(p_4 \sigma N_{4,t} f_{4,t} + p_5 N_{5,t} f_{5,t}) - \mu_{1,t+1} \rho [N_{1,t+1} - R(\gamma_4 \sigma N_{4,t}(1 - f_{4,t})$$
$$+ \gamma_5 N_{5,t}(1 - f_{5,t}))] - \mu_{2,t+1} \rho(N_{2,t+1} - s_1 N_{1,t}) - \mu_{3,t+1} \rho(N_{3,t+1} - s_2 N_{2,t})$$
$$- \mu_{4,t+1} \rho(N_{4,t+1} - s_3 N_{3,t}) - \mu_{5,t+1} \rho(N_{5,t+1} - s_4 N_{4,t}(1 - \sigma))\}$$

where $\mu_{a,t} > 0$ ($a = 1, ..., 5$) is the current value shadow prices of the fish population in the various age classes. The first order necessary control conditions are:

(5) $\qquad \dfrac{\partial L}{\partial f_{4,t}} = \rho^t \sigma N_{4,t}(p_4 - \gamma_4 R'(B_t) \rho \mu_{1,t+1}) \underset{\leq}{\overset{\geq}{=}} 0$; $0 \leq f_{4,t} \leq 1$, $t = 0, 1, 2...$

and

(6) $\qquad \dfrac{\partial L}{\partial f_{5,t}} = \rho^t N_{5,t}(p_5 - \gamma_5 R'(B_t) \rho \mu_{1,t+1}) \underset{\leq}{\overset{\geq}{=}} 0$; $0 \leq f_{5,t} \leq 1$, $t = 0, 1, 2...$.

Assuming positive stock sizes and $N_{a,t} > 0$ ($a = 1, ..., 5$), we next have the necessary stock, or portfolio conditions, describing the evolvement of the shadow prices, as:

(7) $\qquad \dfrac{\partial L}{\partial N_{1,t}} = \rho^t [-\mu_{1,t} + s_1 \rho \mu_{2,t+1}] = 0$, $t = 1, 2, 3...$,

(8) $\qquad \dfrac{\partial L}{\partial N_{2,t}} = \rho^t [-\mu_{2,t} + s_2 \rho \mu_{3,t+1}] = 0$, $t = 1, 2, 3...$,

(9) $\qquad \dfrac{\partial L}{\partial N_{3,t}} = \rho^t [-\mu_{3,t} + s_3 \rho \mu_{4,t+1}] = 0$, $t = 1, 2, 3...$,

(10) $\qquad \dfrac{\partial L}{\partial N_{4,t}} = \rho^t [p_4 \sigma f_{4,t} + \rho \mu_{1,t+1} R'(B_t) \gamma_4 \sigma(1 - f_{4,t}) - \mu_{4,t} + \rho \mu_{5,t+1} s_4(1 - \sigma)] = 0$, $t = 1, 2, 3...$

and

(11) $\quad \dfrac{\partial L}{\partial N_{5,t}} = \rho^t [p_5 f_{5,t} + \rho\mu_{1,t+1} R'(B_t)\gamma_5 (1-f_{5,t}) - \mu_{5,t}] = 0$, $t = 1, 2, 3...$.

The control conditions (5) and (6) indicate that the marginal harvesting value of each age-class should be equal to, below or above its marginal cost, determined by the discounted shadow value of the reduced number of fish in age-class 1 in the next period. Following the Kuhn-Tucker theorem (when $N_{4,t} > 0$), condition (5) holds as an equation with optimal fishing mortality $0 < f_{4,t} < 1$. On the other hand, if it is not beneficial to harvest at all, this condition reads $(p_4 - \gamma_4 R'(B_t)\rho\mu_{1,t+1}) < 0$ while it states $(p_4 - \gamma_4 R'(B_t)\rho\mu_{1,t+1}) > 0$ when it is profitable to fish the whole young mature age-class. Condition (6) can be given similar interpretation for the harvest of the old mature fish. From these two control conditions it is observed that the price – fecundity ratio p_a / γ_a $(a = 4, 5)$ steers the fishing mortality and the fishing composition. This outcome differs from the Reed (1980) paper and Skonhoft et al. (2012) who found that the marginal fishing value (NOK/fish) together with natural mortality ('biological discounting') determined the optimal fishing composition. As already indicated the reason for this difference is that the fish dies after spawning in our salmon model while it, corrected for natural mortality, enter an older age class after spawning in the Reed and Skonhoft et al. 'cod' model. The portfolio conditions (7) - (9) indicate how the shadow price of the young stocks evolve over time, while (10) and (11) state the shadow price relationship between the harvestable mature age classes and recruitment.

A closer look at the control conditions (5) and (6) reveals that if the price – fecundity ratio, or marginal gain – loss ratio, is higher for the old age-class, i.e., $p_5 / \gamma_5 > p_4 / \gamma_4$, then the harvesting fraction will be higher for the old mature class than for the young class. The optimal harvesting policy can then be further specified as comprising the possibilities i) $f_{5,t} = 1$ and $0 < f_{4,t} < 1$, or ii) $f_{5,t} = 1$ and $f_{4,t} = 0$ and iii) $0 < f_{5,t} < 1$ and $f_{4,t} = 0$. In the opposite case with a higher marginal gain – loss ratio for the young mature age-class, we find that the optimal control conditions can be satisfied as iv) $f_{4,t} = 1$ and $0 < f_{5,t} < 1$, or v) $f_{4,t} = 1$ and $f_{5,t} = 0$, or vi) $0 < f_{4,t} < 1$ and $f_{5,t} = 0$. A possibility may also be that the marginal gain – loss ratio is equal for both age classes; that is, $p_5 / \gamma_5 = p_4 / \gamma_4$. This yields case vii) with $0 \le f_{4,t} \le 1$ and $0 \le f_{5,t} \le 1$. As will be shown below, the optimal harvest in this last case can be arbitrarily allocated among the two age classes.

4. Steady state and dynamics

4.1 Steady state analysis

We have shown that the optimal fishing strategy is steered by the relative marginal value – fecundity ratio of the two mature age-classes. This policy is valid also in a steady state, where the number of fish in each age class, the shadow prices and the fishing mortalities are constant through time. When combining Eqs. (1) - (4) and dropping the time subscript, we find that the steady state should satisfy the following equations:

(12) $N_4 = sR(B)$,

(13) $N_5 = s_4 N_4 (1-\sigma)$

and

(14) $B = \gamma_4 \sigma (1-f_4) N_4 + \gamma_5 N_5 (1-f_5)$,

where $s = s_1 s_2 s_3$ comprises the survival rate of young fish from age 1 to 4. In addition, the control conditions (5) and (6) should hold in steady state as well. From Eqs. (7) - (11) we may also derive an expression for the steady state shadow price for age-class 1:

(15) $\mu_1 = s\rho^3 \{ p_4 \sigma f_4 + R'(B)\gamma_4 \sigma (1-f_4)\rho\mu_1 + s_4 (1-\sigma)\rho[p_5 f_5 + R'(B)\gamma_5 (1-f_5)\rho\mu_1] \}$.

The price – fecundity ratio in the *Salma salar* angler fishery is typically higher for the old mature class than for the young mature class (more details in numerical section five). Therefore, we basically analyze the situation where $p_5 / \gamma_5 > p_4 / \gamma_4$ holds such that the fishing mortalities is described by the above possibilities i) $f_5 = 1$ and $0 < f_4 < 1$, or ii) $f_5 = 1$ and $f_4 = 0$, or iii) $0 < f_5 < 1$ and $f_4 = 0$.

When combining Eqs. (13) and (14), we find $B = [\gamma_4 \sigma (1-f_4) + \gamma_5 (1-f_5)s_4 (1-\sigma)]N_4$.

Examining this equation and Eq. (12), it is straightforward to verify that option i) represents the most aggressive harvesting policy and hence yields the lowest number of fish that returns to spawn as well as the lowest size of the spawning population. On the other hand, if option iii) is optimal, we find the largest spawning population B as well as the largest steady state populations N_4 and N_5. As the combination of Eqs. (13) and (14) is a linear function, whereas Eq. (12) is a strictly concave function

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with Beverton – Holt recruitment (see also numerical section), an internal solution requires that the slope of $B = [\gamma_4 \sigma (1 - f_4) + \gamma_5 (1 - f_5) s_4 (1 - \sigma)] N_4$, or $N_4 = B / [\gamma_4 \sigma (1 - f_4) + \gamma_5 (1 - f_5) s_4 (1 - \sigma)]$, is less steep than $N_4 = sR(B)$ when the size of the spawning population approaches zero.

In case i) control condition (5) holds as $p_4 = \gamma_4 R'(B) \rho \mu_1$, Eq. (14) as $B = \gamma_4 \sigma (1 - f_4) N_4$ and (15) as $\mu_1 = s\rho^3 [p_4 \sigma + s_4 (1 - \sigma) \rho p_5]$. These three equations together with Eq. (12) then determine B, N_4, f_4 and μ_1. All economic as well as biological factors influence stock sizes and harvest in this case such that small shifts in, say, the fishing prices and the interest rate will change the optimal size of the spawning population and harvest of the young mature age class (more details below). Under harvest option ii) with $f_5 = 1$ and $f_4 = 0$, the relationship between the spawning population and the size of the young mature stock is determined simply through Eq. (14) as $B = \gamma_4 \sigma N_4$ and Eq. (12). Hence, the optimal stock sizes are now not directly influenced by small changes in fishing values or interest rate. The age class 1 shadow price is in this case found through Eq. (15) as $\mu_1 = s\rho^3 [R'(B) \gamma_4 \sigma \rho \mu_1 + s_4 (1 - \sigma) \rho p_5]$. Case iii) with $0 < f_5 < 1$ and $f_4 = 0$ implies $p_5 = \gamma_5 R'(B) \rho \mu_1$ from control condition (6), while Eqs. (14) and (15) read $B = \gamma_4 \sigma N_4 + \gamma_5 N_5 (1 - f_5)$ and $\mu_1 = s\rho^4 \{R'(B) \gamma_4 \sigma \mu_1 + s_4 (1 - \sigma) p_5\}$, respectively. Therefore, these three equations together with Eqs. (12) and (13) jointly determine the stock sizes, the spawning population and the age class 1 shadow price and the old mature fishing mortality when harvest option iii) is optimal. All parameters generally now influence the optimal stock size and the harvest of the old mature age class.

When the marginal gain – fecundity ratio is identical for the two mature age-classes, $p_5 / \gamma_5 = p_4 / \gamma_4$, we have case vii) with $0 \leq f_4 \leq 1$ and $0 \leq f_5 \leq 1$. The two extreme situations with $f_4 = f_5 = 0$, indicating no harvest and no income, and $f_4 = f_5 = 1$, which means extinction, can for obvious

reasons not be optimal and are excluded. With these two extreme solutions excluded, the control

conditions (5) and (6) read $\gamma_4 \rho \mu_1 R'(B) = p_4$ and $\gamma_5 \rho \mu_1 R'(B) = p_5$, respectively, which are identical

because $p_5 / \gamma_5 = p_4 / \gamma_4$. With two equations giving the same information, there are multiple optimal

steady state harvest rates. If we define this set of the optimal harvest rates by

$\gamma_4 \sigma(1 - f_4) + \gamma_5 s_4 (1 - \sigma)(1 - f_5) = h$, then h should satisfy condition $\gamma_4 \rho \mu_1 R'(B) = p_4$, or

$\gamma_5 \rho \mu_1 R'(B) = p_5$, where $B = hN_4$ is the optimal spawning population following Eqs. (13) and (14).

The optimal size of the young mature age-class is from Eq. (12), $N_4 = sR(hN_4)$, and the shadow

price of age-class 1 from Eq. (15) is $\mu_1 = s\rho^3[p_4\sigma + s_4(1-\sigma)\rho p_5]$. With h determined, we thus find

that all harvest rates satisfying $\gamma_4 \sigma(1 - f_4) + \gamma_5 s_4 (1 - \sigma)(1 - f_5) = h$, or

$f_5 = \dfrac{\gamma_4 \sigma(1 - f_4) + \gamma_5 s_4 (1 - \sigma) - h}{\gamma_5 s_4 (1 - \sigma)}$, indicating a negative linear relationship between f_4 and f_5, are in

accordance with the steady state maximum economic yield fishing policy. All parameters now

influence the optimal stock sizes and the harvest of the old and young mature age-class.

Finally we have the hypothetical situation (see numerical section five) with a higher price – fecundity

ratio for the young mature age-class and $p_4 / \gamma_4 > p_5 / \gamma_5$, and where the optimal control conditions

can be satisfied as iv) $f_4 = 1$ and $0 < f_5 < 1$, or v) $f_4 = 1$ and $f_5 = 0$, or vi) $0 < f_4 < 1$ and $f_5 = 0$.

How the optimal steady state solution can be determined in each of these cases can be handled in a

similar way as in the opposite situation where $p_5 / \gamma_5 > p_4 / \gamma_4$ and is not discussed further here.

The above analysis has demonstrated the price – fertility ratio difference between the two harvestable

age classes as instrumental in determining the optimal fishing strategy. If $p_5 / \gamma_5 > p_4 / \gamma_4$ holds, then

harvest of the old mature age-class should be prioritized, and the young mature population should be

left unexploited before the old mature age class is harvested completely. In the opposite situation

harvest should start with the young mature age-class, and with $p_5 / \gamma_5 = p_4 / \gamma_4$ both age-classes can

be harvested and there are multiple optimal steady state harvest rates. We have also discussed possible parameter effects in the various cases. We now proceed to look more closely at the impacts of the interest rate and the relative price of the two age-classes on the optimal harvest policy where we basically discuss the situation $p_5 / \gamma_5 > p_4 / \gamma_4$, or $p_5 / p_4 > \gamma_5 / \gamma_4$. The details of the analysis is found in the Appendix.

Figure 2 describes the optimal harvest strategy in the interest rate and relative price space. In the most plausible situation where the price – fecundity ratio is higher for the old mature age-class; that is, the location is above the horizontal line $p_5 / p_4 = \gamma_5 / \gamma_4$, there exists an interest rate denoted by r_2 below which it is optimal to harvest only a proportion of the old mature age-class and nothing of the young age-class; that is, option iii) with $f_4 = 0$ and $0 < f_5 < 1$. The optimal harvest of the old age-class increases as the interest rate increases, but is not affected as the old mature stock becomes more valuable and the relative price p_5 / p_4 increases.

        Figure 2 about here

The interest rate r_2 is defined in such a way that the marginal cost of reducing the spawning population $\rho \mu_1 R'(B)$, evaluated at $f_4 = 0$ and $f_5 = 1$, is equal to the marginal profit of harvesting the old mature population p_5 / γ_5 (again, see the Appendix for more details). When the interest rate equals the critical value r_2, the marginal benefit of increasing the harvest of the young age-class from zero is smaller than the associated marginal opportunity cost if $p_5 / \gamma_5 > p_4 / \gamma_4$. Therefore, the optimal strategy is to harvest all fish in the old mature age-class, but still nothing of the young age-class. This harvest option ii) remains optimal when the interest rate is higher than but sufficiently close to r_2. Given the harvest option ii) with $f_4 = 0$ and $f_5 = 1$, the marginal cost of further reducing the spawning population $\rho \mu_1 R'(B)$ decreases as the interest rate increases and eventually becomes equal to the marginal benefit of harvesting the young mature age class p_4 / γ_4 when the interest rate reaches r_1. With an

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interest rate $r > r_1$ fishing of the young age class also becomes profitable and the fishing intensity for the young mature age-class increases with interest rate and we reach harvest option i). Note that when interest rate equals r_2, a larger relative price p_5 / p_4 implies a greater loss of a marginal increase in the harvest of age-class 4 (from 0). Therefore, the larger p_5 / p_4 is, the larger increase in the interest rate from r_2 is required to make the harvest of the young mature age-class profitable. The line market with r_1 in Figure 2 delimits this interest rate over which it is optimal to harvest a positive proportion of the young mature age-class, in addition to harvesting the entire stock of the old mature age-class.

As also indicated in Figure 2, given an interest rate $r > r_2$, the optimal harvest proportion for the young mature age-class is greater than zero when the relative price p_5 / p_4 is close to the relative fertility γ_5 / γ_4. The harvest of the young age-class decreases as p_5 / p_4 increases, and approaches zero after the relative price has reached a certain level. The relative price increase may follow after a reduction in the price for the young mature age-class or an increase in the price for the old mature age-class. A decrease in the price of the young age-class reduces the marginal benefit of harvesting this age-class, whereas a higher price for the old mature fish increases the marginal cost of harvesting the young mature age-class. Therefore, an increase in the relative price would cause the harvested proportion of the young mature fish to be reduced. Intuitively, when the price of the old mature fish increases relative to that of young mature fish, it is preferable to reduce the harvest of the young age-class so that the spawning population and hence, the sustained yield of the old age-class increases.

The above analysis has shown that the driving economic forces of an optimal steady state harvest policy in our age-structured fishing model are somewhat different from the standard biomass model (e.g., Clark 1990). As in this model, a higher interest rate tends to increase fishing and the exploitation pressure, but it may also keep the optimal harvest policy unchanged. In contrast to the biomass model, however, a more valuable fish may either increase or reduce the fishing pressure, or not influence the

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optimal fishing policy and the size of the standing biomass at all. We find opposite result than in the biomass model as the old mature stock becomes more valuable and the relative price p_5 / p_4 increases when the interest rate is above r_2.

When $p_5 / p_4 = \gamma_5 / \gamma_4$ and we have harvest option vii), it does not matter which of the two mature age-classes is harvested as long as the fishing mortalities are governed by

$$f_5 = \frac{\gamma_4 \sigma (1 - f_4) + \gamma_5 s_4 (1 - \sigma) - h}{\gamma_5 s_4 (1 - \sigma)}$$ (see above). However, we find that the size of the optimal

spawning population decreases monotonously as the interest rate increases; that is, $\partial B / \partial r < 0$, implying that the proportion of the returning fish population to be harvested increases with interest rate. On the other hand, changes in prices would not affect the optimal harvesting intensity as long as $p_5 / p_4 = \gamma_5 / \gamma_4$ is not invalidated.

To complete the analysis, Figure 2 also includes the situation where the price-fecundity ratio for the young mature age-class is higher than for the old mature fish, and the location is hence below the line $p_5 / p_4 = \gamma_5 / \gamma_4$. The optimal harvest strategy in this situation is parallel to what was found with the opposite price-fecundity ratio, except that harvest of the young mature age-class is prioritized over the old mature population. Specifically, harvest option vi) with fishing of a fraction of the young age-class and nothing in the old age-class is optimal when the interest rate is low. When the interest rate exceeds a certain level r_3, all returning fish in the young age-class should be harvested, while the optimal harvest fraction of the old mature fish becomes either zero or strictly positive. Which of the harvest options iv) or v) is optimal here depends on the difference between p_5 / p_4 and γ_5 / γ_4, i.e., whether the price-fecundity ratio for the young mature age-class is significantly higher or close to the old mature age-class.

4.2 Dynamic analysis

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Above some properties of possible steady states with a constant number of fish through time was analyzed. As the profit function is linear in the controls, economic theory suggests that fishing should be adjusted to lead the population to steady state as fast as possible; that is, the Most Rapid Approach Path (*MRAP*) dynamics. However, the *MRAP* is not a regular one in our age-structured fish population because control of fish population is realized indirectly through control of the spawning population and because the optimal fishing policy in the steady state, except in the situation where the price-fecundity ratio is identical for the two mature age- classes, is a corner solution. The age structure implies that the population could be above that of the optimal steady state level for one age-class and at the same time lower than the optimal steady state level for the other age-class. Since fishing is confined to the two mature age-classes, the *MRAP* may imply a large harvest in one period and zero harvest in the next.

Secondly, harvest of the two mature age-classes need to be considered jointly before the optimal steady state is achieved, because the population in each mature age-class affects, through its impact on the breeding population (see Eq. 3), the optimal harvest rate for the other age-class (see the first order necessary control conditions 5 and 6). Consider the typical situation where the price - fecundity ratio is highest for the old mature age-class. The proportion of the old mature age-class to be harvested will then depend on the number of young mature fish. If the number of fish in this age-class is small, it may be necessary to reduce the harvest of old fish to obtain the desirable size of the spawning population. On the contrary, if the number of fish in the young mature age-class is large, it may be necessary to harvest all or some of the old mature fish even if the number of fish in this age-class is lower than the optimal steady state level.

A third feature of the *MRAP* in our age-structured model is that if the population returning to the native river is above that of the optimal steady state spawning population, then it might be optimal to harvest such that spawning population is kept slightly under, or over, its optimal steady state level. The reason is that the optimal steady state marginal harvest benefit typically differs from the marginal

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cost, determined by the discounted shadow value of the reduced number of fish in age-class 1 in the next period (see section three), because of corner solutions. For instance, consider the optimal harvest policy option ii) with $f_4 = 0$ and $f_5 = 1$. Suppose that the stock of the young mature fish $N_{4,t}$ is considerably smaller than its optimal steady state level N_4 while the old mature stock $N_{5,t}$ exceeds its steady state level N_5. We then have $\gamma_4 \sigma N_{4,t} + \gamma_5 N_{5,t} > B$, where $B = \gamma_4 \sigma N_4$ is the optimal size of the steady state spawning population. In this case it would be optimal to harvest the old mature age-class to such an extent that $\gamma_4 \sigma N_{4,t} + \gamma_5 N_{5,t}(1 - f_{5,t}) < B$ because at the optimal steady state spawning population the marginal benefit of harvest the old mature age-class is larger than its marginal cost. If $N_{4,t}$ is larger than its optimal steady state level, we may conclude with the same reasoning that it is optimal to harvest all the fish in the old mature age-class and some of the young fish such that $\gamma_4 \sigma N_{4,t}(1 - f_{4,t}) > B$.

5. Numerical illustration

5.1 Data

The above theoretical reasoning will now be illustrated numerically. Hansen et al. (1996) estimated a salmon recruitment function for a small/medium sized river in Norway (the Imsa River in southern Norway) based on the Shepherd recruitment function, which includes three parameters. In our generic model, we choose a simpler approach and use the strictly concave Beverton–Holt function. This function is specified as $R(B_t) = s_0 a B_t / (1 + B_t / K)$ where $0 < s_0 < 1$ is the fraction of the recruits that survive and enter age class 1. $a > 0$ is the intrinsic growth rate, or maximum number of recruits per (fertility adjusted) spawning salmon, and $K > 0$ as the stock level for which density-dependent mortality equals density-independent mortality. The size of aK yields the maximum number of recruits and scales the system ('size of the river'), which is assumed to be 40,000 (number of recruits). The value of a indicates the 'quality' of the river, and we choose $a = 160$ (number of recruits per spawning salmon). The migration parameter σ depends on type of river ('small' or 'large' salmon river), sea temperature and other factors, and is assumed to be 0.5. See Table 1 where the natural survival rates and the other parameters are shown.

Following McGinnity et al. (2003) weight (kg/fish) is a questionable fertility (fecundity) approximation for wild Atlantic salmon. Instead they postulate fertility to be described by a strictly concave function of weight (and age), indicating that the weight – fertility ratio increases with weight; that is, $w_5 / \gamma_5 > w_4 / \gamma_4$. When normalizing the fertility parameter for the young to one, $\gamma_4 = 1$, and using the fertility–weight function in McGinnity et al. (2003) we find $\gamma_5 = 2.4$ under the assumption of (average) fishing weights of $w_4 = 2.0$ and $w_5 = 5.5$ (kg/fish). These weights fit a typical medium-sized Norwegian salmon river (NOU 1999). The fishing prices are related to recreational fishery, which, as indicated, is far more important economically than the marine fishery. We assume that the fishing permit price in a reasonably good river is about 200 (NOK/day) (Olaussen and Liu 2011). Based on average catch success, this permit price may translate into fishing prices in the range of 100–400 (NOK/kg). We assume the same price for both age-classes, and use $\tilde{p}_4 = \tilde{p}_5 = 150$ (NOK/kg). With these weights and prices, we find the fixed prices as $p_4 = 300$ and $p_5 = 825$ (NOK/fish) such that $p_5 / \gamma_5 = 825 / 2.4 = 344 > p_4 / \gamma_4 = 300 / 1 = 300$ in the baseline calculations.

Table 1 about here

5.2 Results[1]

We first present some dynamic results. Assuming that the population and the harvest rates stabilize within a limited time of T years, we solved the optimization model for an infinite time horizon. This was accomplished by calculating the present value of the net harvest revenues from year T and onwards based on the revenue in year T. The value of T was determined by trial and error such that (1) the optimal solution converges to a steady state before T, and (2) further increase in T does not affect the optimal solution. The optimization results for the first 50 years are presented in Table 2. To illustrate the irregularity in the optimal harvest path, we choose an initial state with a large number of recruits, a large number of fishes in age classes 2 and 4, and a small number of fishes in age classes 1, 3 and 5. See Table 2. As already indicated, because the profit function is linear in the controls, *MRAP* dynamics, but necessarily exactly a *MRAP*, is supposed to describe the optimal transitional dynamics. Table 2 seems partly to confirm this where the age-class distribution of the fish population and harvest rates approach the steady state quite fast. With the baseline parameter values, the optimal steady state comprises 500 fishes in the young mature age-class N_4 and 125 fishes in the old mature age-class N_5. The initial age-class distribution implies that the fish number fluctuates around the steady state level for both mature age-classes in the first few years, and therefore the optimal harvest fluctuates as well. The optimal harvest options shift between ii) with $f_5 = 1$ and $f_4 = 0$ (years 1 and 3) and i) with $f_5 = 1$

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[1] The model is solved by using EXCEL. The program is available from the authors upon request.

and $0 < f_4 < 1$ (years 0, 2 and 4). From year 5 and onwards, however, the fishing prevails with option ii).

      Table 2 about here

It should be emphasized that at the optimal steady state with $f_5 = 1$ and $f_4 = 0$, the benefit of the last fish harvested in the old mature age-class exceeds its cost of reduction in spawning population. On the other hand, the benefit of harvesting the first fish in the young mature age class is lower than its marginal cost (Eqs. 5 and 6), cf. also section 4.2. This explains why the harvest of the old mature age-class is not sufficiently reduced to maintain the optimal steady state spawning population when the young mature age-class is understocked (see, e.g., years 1 and 3). Similarly, when the young mature age class is overstocked it is not optimal to harvest the young mature age-class to the optimal steady state spawning population (see, e.g., years 0, 2 and 4).

Table 3 presents the detailed optimal steady state results where the optimal harvest option ii) with $f_5 = 1$ and $f_4 = 0$ under the baseline parameter scenario in shown in row one. Increasing the interest rate to 5% ($r = 0.05$), changes the solution to the more aggressive case i) with $f_5 = 1$ and $0 < f_4 < 1$. This yields a smaller fish population, but also a slightly reduction in the harvest of the old mature fish population from 125 to 122 fishes. In contrast to this, a reduction of the interest rate to 1% changes the optimal solution to option iii) with $0 < f_5 < 1$ and $f_4 = 0$. The lower harvest rate leads to a larger steady state fish population, but does not affect the quantity of the harvest which still counts 125 fishes.

      Table 3 about here

Sensitivity analysis also demonstrates that an increase in the price of the old mature age-class does not change the optimal fishing from the baseline situation (row four). On the other hand, 10% reduction in the price of the old mature age class, from 825 to 742.5 (NOK/fish), makes it profitable to harvest some of the young mature fish in addition to harvest all the old fish. That is, we reach harvest option i) with $f_5 = 1$ and $0 < f_4 < 1$. In row six in Table 3 the price of the old mature age class is reduced further to 720 (NOK/fish) such that the price-fecundity ratio becomes similar for both age-classes, i.e., $p_5 / \gamma_5 = 720 / 2.4 = 300 = p_4 / \gamma_4 = 300 / 1 = 300$. Harvest option vii) with multiple steady states is then reached, and all harvest rates satisfying $f_5 = 1.02 - 0.83 f_4$ and $0 \le f_i \le 1$ ($a = 4,5$) are optimal

(see section 4.1). After a further price reduction of the old mature age-class, exemplified by $p_5 = 660$ (NOK/fish), we reach harvest option v) with $f_5 = 0$ and $f_4 = 1$ (last row).

The above sensitivity analysis is consistent with the theoretical analysis presented in section 4.1 and Figure 2 (see also the Appendix). For the given biological parameter values (Table 1), the critical interest rate r_2 is slightly above 2 %. Accordingly, when the price-fecundity ratio is highest for the old mature age-class, the optimal fishing mortality is governed by option iii) with $0 < f_5 < 1$ and $f_4 = 0$ as long as the interest rate is lower than about 2 %. When the interest rate exceeds this critical level, the fishing mortality for the old mature age-class becomes fixed at its maximum level $f_5 = 1$, whereas the fishing rate for the young mature age-class depends on the interest rate and the relative price of the two age-classes.

For a further illustration of the effect of the interest rate and the relative price on the fishing mortality, we calculate the optimal spawning population associated with two different interest rates and a variety of prices for the old mature age-class. Only prices higher than $p_5 = 720$ (NOK/fish) is considered such that the old mature age-class always has the highest price-fecundity ratio. We then find that the optimal steady state spawning population increases for the fixed interest rate 3 % because the young mature age-class fishing mortality decreases. See Figure 3. When p_5 reaches a certain level, the fishing mortality of the young mature age-class reduces to zero and harvest option ii) is reached, and the optimal spawning population is not affected by further price increase anymore. When the relative price is close to the relative fecundity for the two age classes, a higher interest rate exemplified by 5%, leads to a lower spawning population, implying that the fishing mortality for the young mature age class increases with the interest rate. When the relative price is sufficiently high, however, the optimal spawning population is not affected by the interest rate also in this case. These results confirm the behavior of the optimal steady state solution described in Figure 2 within the region where the interest rate is higher than the critical value r_2 and the price fecundity ratio is highest for the old mature age-class.

Figure 3 about here.

6. Some extensions of the model

In the above analysis we have assumed perfect fishing selectivity. We now relax this assumption, and consider non-selective fishing pattern with the same fishing mortality for both the young and old harvestable age classes; that is, $f_{4,t} = f_{5,t} = f_t$. The Lagrangian of this new *MEY* problem reads

$$L = \sum_{t=0}^{\infty} \rho^t \{ (p_4 \sigma N_{4,t} + p_5 N_{5,t}) f_t - \mu_{1,t+1} \rho[N_{1,t+1} - R((\gamma_4 \sigma N_{4,t}$$
$$+ \gamma_5 N_{5,t})(1-f_t))] - \mu_{2,t+1} \rho(N_{2,t+1} - s_1 N_{1,t}) - \mu_{3,t+1} \rho(N_{3,t+1} - s_2 N_{2,t})$$
$$- \mu_{4,t+1} \rho(N_{4,t+1} - s_3 N_{3,t}) - \mu_{5,t+1} \rho(N_{5,t+1} - s_4 N_{4,t}(1-\sigma)) \}$$

The first order necessary control condition is now:

(16) $\dfrac{\partial L}{\partial f_t} = \rho^t [(p_4 \sigma N_{4,t} + p_5 N_{5,t}) - R'(B_t) \rho \mu_{1,t+1} (\gamma_4 \sigma N_{4,t} + \gamma_5 N_{5,t})] \overset{\geq}{\underset{\leq}{}} 0 \; ; \; 0 \leq f_t \leq 1$,

$t = 0,1,2...$.

The necessary stock, or portfolio conditions, are Eqs. (7), (8) and (9) while

(17) $\dfrac{\partial L}{\partial N_{4,t}} = \rho^t [p_4 \sigma f_t + \rho \mu_{1,t+1} R'(B_t) \gamma_4 \sigma(1-f_t) - \mu_{4,t} + \rho \mu_{5,t+1} s_4(1-\sigma)] = 0$, $t = 1,2,3...$

and

(18) $\dfrac{\partial L}{\partial N_{5,t}} = \rho^t [p_5 f_t + \rho \mu_{1,t+1} R'(B_t) \gamma_5 (1-f_t) - \mu_{5,t}] = 0$, $t = 1,2,3...$

replace Eqs. (10) and (11).

The control condition (16) now indicate that the stock weighted marginal harvesting value should be equal to, below or above its marginal cost, now determined by the stock weighted discounted shadow value of the reduced number of fish in age-class 1 in the next period. Again, following the Kuhn-Tucker theorem, this condition holds as an equation with optimal fishing mortality $0 < f_t < 1$ while the corner solutions now indicate no fishing at all, or fishing down the whole spawning population. The last option will lead to extinction, and can be optimal only when the interest rate is 'high'. Notice that our formulation of the non-selective fishing decision problem differs from the perfectly selective case only because of the restriction on the permissible fishing mortalities. Therefore, *MRAP* dynamics still remain optimal, and thus the fishing mortality should be determined such that the population approaches the optimal steady state as fast as possible. Notice also this outcome differs from what is found in the literature on age structured harvesting models with imperfect harvesting selectivity where cycles, or 'pulse harvesting', seem to be intrinsic feature (e.g., Tahvonen 2009). Unlike Tahvonen, and others, however, stock variables are not included in our control conditions. See also Naevdal et al. (2012).

The optimal steady state will generally differ from that of the perfect selectivity case, depending on the relative price-fecundity ratios for the two mature age classes. Considering steady state and ignoring $f = 0$ which not can be optimal, the control condition must hold as an equation. Condition (16) reads then:

(19) $(p_4 \sigma N_4 + p_5 N_5) = R'(B) \rho \mu_1 (\gamma_4 \sigma N_4 + \gamma_5 N_5)$,

and where the optimal spawning population is:

(20) $B = [\gamma_4 \sigma N_4 + \gamma_5 N_5](1 - f)$,

and the shadow price for age-class 1 derived from Eqs. (7) – (9), (17) and (18) reads:

(21) $\mu_1 = s\rho^3 \{ p_4 \sigma f + R'(B) \gamma_4 \sigma (1 - f) \rho \mu_1 + s_4 (1 - \sigma) \rho [p_5 f + R'(B) \gamma_5 (1 - f) \rho \mu_1] \}$.

Eqs. (19) - (21) together with Eqs. (12) and (13) enable us to determine the optimal steady state with non-selective fishing.

If the price-fecundity ratio for the two mature age-classes are equal, $p_5 / \gamma_5 = p_4 / \gamma_4$, it is easy to prove that the optimal steady state population with non-selective fishing is the same as in the perfectly selective fishing case. The optimal harvest rate becomes then

$f = 1 - \dfrac{\gamma_4 \sigma (1 - f_4) + \gamma_5 s_4 (1 - \sigma)(1 - f_5)}{\gamma_4 \sigma + \gamma_5 s_4 (1 - \sigma)}$ where f_4 and f_5 are the optimal age specific steady state

harvest rates under perfectly selective fishing.   In this case, as explained above (section 4), it does not matter which age-class is harvested as long as the total harvest is kept at the optimal level. Therefore, the constraint that the harvest rates for the two mature age-classes should be equal implied by non-selective fishing, will not affect the optimal solution.

On the other hand, when the price-fecundity ratio is different, i.e., when $p_5 / \gamma_5 \neq p_4 / \gamma_4$, the optimal harvest rate differs between the two mature age-classes under perfectly selective fishing. The uniform harvesting pattern in non-selective fishing then obviously gives an economic loss compared to the selective harvesting scheme. We may also suspect that the harvesting pressure increases and the stock sizes and the spawning biomass reduce compared to the selective harvesting scheme in steady state. However, analytical comparisons of the optimal steady state associated with the two fishing schemes are difficult. Instead, we made the comparison numerically using the example described in numerical Section 5. The results (see Table 4) show that with different price-fecundity ratio, non-selective fishing leads to a lower optimal steady spawning population. The difference seems more sensitive to shifts in the interest rate than in price changes. With respect to the present value harvest benefits, we

find that the gain from selective fishing increases as the discount rate decreases and when the difference in price-fecundity ratio between the two mature age-classes increases.

Table 4 about here

Our assumption of constant marginal willingness to pay for fishing may also be questioned. With still no 'quality effect' present, but assuming decreasing marginal willingness to pay as indicated by the concave function $U(Y_t)$, the current utility may be written as

$U_t = U(Y_t) = U(p_4 \sigma N_{4,t} f_{4,t} + p_5 N_{5,t} f_{5,t})$ when again having perfect fishing selectivity. The control conditions now read:

(22) $\quad \dfrac{\partial L}{\partial f_{4,t}} = \rho^t \sigma N_{4,t} [U'(.)p_4 - \gamma_4 R'(B_t) \rho \mu_{1,t+1}] \overset{\geq}{\underset{\leq}{}} 0$; $0 \leq f_{4,t} \leq 1$, $t = 0,1,2...$

and

(23) $\quad \dfrac{\partial L}{\partial f_{5,t}} = \rho^t N_{5,t} (U'(.)p_5 - \gamma_5 R'(B_t) \rho \mu_{1,t+1}) \overset{\geq}{\underset{\leq}{}} 0$; $0 \leq f_{5,t} \leq 1$, $t = 0,1,2...$.

It is easily recognized that these conditions can be given more or less the same interpretation as conditions (5) and (6). Therefore, when again assuming higher marginal gain – loss ratio for the old age-class, $p_5 / \gamma_5 > p_4 / \gamma_4$, we find that the harvesting of the old age-class also now should be prioritized with fishing mortalities described by the above possibilities i), ii) and iii) (section three). However, because of the decreasing marginal benefit, we may suspect that the optimal harvesting policy may be less aggressive than in the linear benefit situation.

Alternatively, we may assume that the utility of the age classes is additive such that the current benefit may be written as $U_t = U_4(p_4 \sigma N_{4,t} f_{4,t}) + U_5(p_5 N_{5,t} f_{5,t})$ with $U_a(.)$, $a = 4,5$, as concave functions. Under this benefit assumption, we find that the ratio $U'_5(.)p_5 / \gamma_5$ versus $U'_4(.)p_4 / \gamma_4$ steers the optimal fishing priority. Therefore, the stock sizes now influence whether fishing of the old or young mature class should be prioritized. To demonstrate analytically how these stock effects work are not considered further here.

7. Concluding remarks

In this paper, we have studied the maximum economic yield management of an age-structured wild Atlantic salmon (*Salmo salar*) population with two spawning and harvestable age classes. Under our basic assumption of perfect fishing selectivity and constant marginal willingness to pay for

recreational fishing, the main finding is that the price – fertility ratio difference between the two harvestable age-classes is instrumental in determining the optimal fishing strategy. This outcome differs from Reed (1980) and Skonhoft et al. (2012) who found that price, or weight, together with natural mortality were the decisive factors. Thus, both economy and biology play a role also in these studies as well, but the importance of the biological factors is different. The reason for this discrepancy is the different biological characteristics of the fish stocks, as the mature salmon die after spawning in our model, whereas a fixed fraction of the spawning fish survive and enter older age classes in the Reed and Skonhoft et al. 'cod' model. Our analysis and findings are based on the Atlantic salmon, but the results will also apply to, e.g., the various Pacific salmon stocks, which also die after spawning.

Given that the weight -fecundity ratio increases with weight (and age) (McGinnity et al. 2003), and also that the price -fecundity increases with age, we find that the optimal fishing mortality will be highest for the old mature age-class. We further find that three possible optimal harvest options are possible, and where the various steady state options are related to the size of the interest rate and the relative harvesting price. While higher interest rate leads to either unchanged or increased exploitation pressure, we find that higher price of the old mature age-class actually may reduce the harvesting pressure and hence increase the size of the spawning population. This finding is different from the standard biomass model (e.g., Clark 1990). However, notice that no stock dependent harvesting costs are included in our model. In our model, it is the relative price between the two harvestable age-classes that is of importance. The model is illustrated numerically where some irregularities in the optimal transitional harvesting pattern during the first few years are observed when we start with a skewed initial age distribution of the salmon population. After this transitional period the age-class distribution of the fish population and harvest rates approach the steady state quite fast.

We have assumed perfect fishing selectivity in the basic version of the model, but in real life there is no such thing. We have therefore considered the non-selective, or uniform, fishing pattern as well. The optimal fishing policy is characterized, and the economic loss compared to the perfect selectivity situation is illustrated numerically. We find that the economic loss ranges from 0 to about 14 %, depending on the discount rate and the relative price-fecundity ratio of the two mature age-classes. While it is difficult to select between harvesting old and young mature fish, it can in a river recreational fishery be influenced by several factors. It includes regulation of the fishing gear, but possibly more effectively through seasonal regulation. Catch and release policy with properly designed size limits may also be an option. Seasonal regulation will affect fishing selectivity because young mature and old mature salmon to some extent migrate back to the parent river sequentially, with the broad pattern being that the young mature return before the old (see, e.g., NOU 1999). Hence,

seasonal regulations combined with e.g., age-class targeted catch and release policy, are possible ways to more efficiently target the Atlantic salmon fish stock.

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Appendix

The effects of prices and interest rate

In section 4.1 the impacts of interest rate and the relative price on the optimal steady-state population and harvest is discussed. In what follows, a formal analysis of these impacts is presented. For this purpose we need an explicit expression of the shadow price of the fish stock in age-class 1. Solving Eq. (15) for μ_1 and using the definition of $\rho = 1/(1+r)$, yields:

(A1) $$\mu_1 = \frac{s[(1+r)\sigma f_4 p_4 + s_4(1-\sigma)p_5 f_5]}{(1+r)^4 - sR'(B)[\sigma\gamma_4(1-f_4) + (1-\sigma)\gamma_5 s_4(1-f_5)/(1+r)]} .$$

Situation 1: $p_5/\gamma_5 > p_4/\gamma_4$ and harvest of the old mature age-class is more profitable

We first consider harvest option ii) with $f_5 = 1$ and $f_4 = 0$. The associated steady state population is then described by $B = \gamma_4\sigma N_4 \equiv \bar{B}$ and $N_4 = sR(\bar{B}) \equiv \bar{N}_4$. Substitution of $f_5 = 1$ and $f_4 = 0$ into (A1)

yields $\mu_1 = \dfrac{ss_4(1-\sigma)p_5}{(1+r)^4 - s\sigma\gamma_4 R'(\bar{B})}$. The partial derivatives of the Lagrangian $\partial L / \partial f_4$ and $\partial L / \partial f_5$,

evaluated at $f_5 = 1$ and $f_4 = 0$, are:

(A2)
$$\frac{\partial L}{\partial f_4} = \sigma\bar{N}_4[p_4 - \gamma_4 R'(\bar{B})\rho\frac{ss_4(1-\sigma)p_5}{(1+r)^4 - s\sigma\gamma_4 R'(\bar{B})}]$$
$$= \frac{\sigma\bar{N}_4[s\sigma\gamma_4(1+r)p_4 + \gamma_4 ss_4(1-\sigma)p_5]}{(1+r)^5 - s\sigma\gamma_4 R'(\bar{B})(1+r)}[\frac{p_4(1+r)^5}{s\sigma\gamma_4(1+r)p_4 + \gamma_4 ss_4(1-\sigma)p_5} - R'(\bar{B})]$$

and

(A3)
$$\frac{\partial L}{\partial f_5} = s_4(1-\sigma)\bar{N}_4[p_5 - \gamma_5\rho R'(\bar{B})\frac{ss_4(1-\sigma)p_5}{(1+r)^4 - s\sigma\gamma_4 R'(\bar{B})}]$$
$$= \frac{s_4(1-\sigma)\bar{N}_4 p_5[s\sigma(1+r)\gamma_4 + ss_4(1-\sigma)\gamma_5]}{(1+r)^5 - s\sigma\gamma_4 R'(\bar{B})(1+r)}[\frac{(1+r)^5}{s\sigma(1+r)\gamma_4 + ss_4(1-\sigma)\gamma_5} - R'(\bar{B})]$$

The conditions for harvest option $f_5 = 1$ and $f_4 = 0$ to be optimal are $\partial L / \partial f_4 \leq 0$ and $\partial L / \partial f_5 \geq 0$.

Under our assumption that $R(B)$ is increasing and concave (Beverton – Holt recruitment function),

we have $\bar{N}_4 > sR'(\bar{B})\bar{B}$, which means that $sR'(\bar{B})\gamma_4\sigma < 1$ and hence

$(1+r)^5 - s\sigma\gamma_4 R'(\bar{B})(1+r) > 0$. Therefore, the optimality conditions $\partial L / \partial f_4 \leq 0$ and $\partial L / \partial f_5 \geq 0$

are equivalent to:

(A4) $\quad R'(\bar{B}) \geq \dfrac{(1+r)^5}{s\sigma(1+r)\gamma_4 + ss_4(1-\sigma)\gamma_4(p_5 / p_4)}$

and

(A5) $\quad R'(\bar{B}) \leq \dfrac{(1+r)^5}{s\sigma(1+r)\gamma_4 + ss_4(1-\sigma)\gamma_5}$.

Let r_1 be the interest rate where $\partial L / \partial f_4 = 0$ (evaluated at $f_5 = 1$ and $f_4 = 0$); that is,

(A6) $\quad \dfrac{(1+r_1)^5}{s\sigma(1+r_1)\gamma_4 + ss_4(1-\sigma)\gamma_4(p_5 / p_4)} = R'(\bar{B})$.

Let r_2 be the interest rate where $\partial L / \partial f_5 = 0$ (evaluated at $f_5 = 1$ and $f_4 = 0$), i.e.,

(A7) $\quad \dfrac{(1+r_2)^5}{s\sigma(1+r_2)\gamma_4 + ss_4(1-\sigma)\gamma_5} = R'(\bar{B})$.

From (A6) and (A7) we know that $r_1 = r_2$ when $\gamma_4(p_5 / p_4) = \gamma_5$, or $p_5 / \gamma_5 = p_4 / \gamma_4$. Furthermore, r_2

is determined solely by the biological parameters, whereas r_1 depends on the relative price of the two

mature age-classes. If we define $\alpha = p_5 / p_4$ and differentiate both sides of equation (A6), we find

$\dfrac{\partial r_1}{\partial \alpha} = \dfrac{(1+r_1)s_4(1-\sigma)}{5s_4(1-\sigma)\alpha + 4\sigma(1+r_1)} > 0$. Thus, r_1 increase as p_5 / p_4 increases from γ_5 / γ_4. Note that the

RHS of inequalities (A4) and (A5) increase with r. Therefore, conditions (A4) and (A5) are satisfied

when $r_2 \le r \le r_1$.

When $r < r_2$, $\dfrac{(1+r)^5}{s\sigma(1+r)\gamma_4 + ss_4(1-\sigma)\gamma_5} < \dfrac{(1+r_2)^5}{s\sigma(1+r_2)\gamma_4 + ss_4(1-\sigma)\gamma_5} = R'(\bar{B})$. From (A3) we

know that $\partial L / \partial f_5 < 0$ at $f_5 = 1$ and $f_4 = 0$ when $r < r_2$. Accordingly, the optimal harvest option is iii)

with $0 < f_5 < 1$ and $f_4 = 0$ when $r < r_2$. The optimal value of f_5 should then satisfy

$p_5 = \gamma_5 R'(B)\rho\mu_1$ (control condition 6) , which together with $f_4 = 0$ imply

$\mu_1 = s\rho^3 \{ R'(B)\gamma_4\sigma\rho\mu_1 + s_4(1-\sigma)\rho[p_5 f_5 + R'(B)\gamma_5(1-f_5)\rho\mu_1] \}$
$\quad = s\rho^3 \{ \gamma_4\sigma p_5 / \gamma_5 + s_4(1-\sigma)\rho p_5 \}$.

Substitution into $\partial L / \partial f_5 = 0$ yields then:

(A8) $\quad R'(\gamma_4\sigma N_4 + \gamma_5 s_4(1-\sigma)N_4(1-f_5)) = \dfrac{1}{s\rho^4 \{ \gamma_4\sigma + s_4(1-\sigma)\rho\gamma_5 \}}$

where

(A9) $\quad N_4 = sR(\gamma_4\sigma N_4 + \gamma_5 s_4(1-\sigma)N_4(1-f_5))$.

From these two equations we know that changing prices have no effect on the optimal harvest

decision. Moreover, we have

$$\frac{\partial f_5}{\partial r} = \frac{4s\gamma_4\sigma(1+r)^5 + 5(1+r)^4 ss_4(1-\sigma)\gamma_5}{[s\gamma_4\sigma(1+r) + ss_4(1-\sigma)\gamma_5]^2} \frac{sR'(B)(\gamma_4\sigma + \gamma_5 s_4(1-\sigma)(1-f_5)) - 1}{R''(B)N_4\gamma_5 s_4(1-\sigma)} > 0.$$

Therefore, when $p_5/\gamma_5 > p_4/\gamma_4$ and $r < r_2$, the optimal harvest rate for the old mature age-class increases when the interest rate increases.

When $r > r_1$, $\dfrac{(1+r)^5}{s\sigma(1+r)\gamma_4 + ss_4(1-\sigma)\gamma_4(p_5/p_4)} > \dfrac{(1+r_1)^5}{s\sigma(1+r_1)\gamma_4 + ss_4(1-\sigma)\gamma_4(p_5/p_4)} = R'(\bar{B})$.

From (A2) we also know that with $r > r_1$, $\partial L/\partial f_4 > 0$ at $f_5 = 1$ and $f_4 = 0$. Accordingly, the optimal fishing rates are option i) with $f_5 = 1$ and $0 < f_4 < 1$ when $r > r_1$. The optimal solution are determined by $p_4 = \gamma_4 R'(B)\rho\mu_1$ (control condition 5), $B = \gamma_4\sigma(1-f_4)N_4$, $N_4 = sR(B)$ and

$\mu_1 = s\rho^3[p_4\sigma + s_4(1-\sigma)\rho p_5]$. From these conditions we find that

$$\frac{\partial f_4}{\partial \alpha} = \frac{(1+r)^5 ss_4(1-\sigma)}{\{s\sigma(1+r)\gamma_4 + ss_4(1-\sigma)\alpha\gamma_4\}^2} \frac{1 - sR'(B)[\gamma_4\sigma(1-f_4)]}{R''(B)\sigma N_4} < 0 \text{ and}$$

$$\frac{\partial f_4}{\partial r} = \frac{(1+r)^4\{4s\sigma(1+r) + 5ss_4(1-\sigma)\alpha\}}{\{s\sigma(1+r)\gamma_4 + ss_4(1-\sigma)\alpha\gamma_4\}^2} \frac{sR'(B)[\gamma_4\sigma(1-f_4)] - 1}{R''(B)\sigma N_4} > 0 \text{ must hold.}$$

Accordingly, when $p_5/\gamma_5 > p_4/\gamma_4$ and $r > r_1$, the optimal harvest rate for the old mature age-class is fixed as $f_5 = 1$ while the optimal harvest rate for the young fish decreases with p_5 and increases with r.

Situation 2: $p_5/\gamma_5 = p_4/\gamma_4$ harvest in the old and young mature age-class is equally profitable

In this situation, there are multiple optimal steady state harvest rates defined by

$\gamma_4\sigma(1-f_4) + \gamma_5 s_4(1-\sigma)(1-f_5) = h$ (section 4.1), or:

(A10) $\quad \gamma_4\sigma f_4 + \gamma_5 s_4(1-\sigma)f_5 = \gamma_4\sigma + \gamma_5 s_4(1-\sigma) - h$

where h is the solution of the equations

(A11) $\quad R'(hN_4) = \dfrac{(1+r)^5}{s\sigma(1+r)\gamma_4 + ss_4(1-\sigma)\gamma_5}$

and

(A12) $\quad N_4 = sR(hN_4)$

(see section 4.1). From Eqs. (A11) and (A12) we find

$\dfrac{\partial h}{\partial r} = \dfrac{(1-sR'(hN_4)h)(1+r)^4[4s\sigma(1+r)\gamma_4 + 5ss_4(1-\sigma)\gamma_5]}{R''(hN_4)[s\sigma(1+r)\gamma_4 + ss_4(1-\sigma)\gamma_5]^2} < 0$. This inequality holds under our

assumption that $R(B)$ is increasing and concave which implies $1-sR'(hN_4)h > 0$ and $R''(hN_4) < 0$.

Therefore, from (A10) we see that the weighted sum of the optimal harvest rates

$\gamma_4\sigma f_4 + \gamma_5 s_4(1-\sigma)f_5$ increases as the interest rate increases.

Situation 3: $p_5/\gamma_5 < p_4/\gamma_4$ and harvest of young mature age-class is more profitable

This situation is analyzed in a parallel manner to the above situation 1, and this part of the Appendix is

available from the authors upon request.

Table 1. Biological and economic baseline parameter values

PARAMETER	DESCRIPTION	VALUE	SOURCE
s_0	Natural survival rate of recruits	0.31	NOU (1999)
s_1	Natural survival rate of age class 1	0.40	NOU (1999)
s_2	Natural survival rate of age class 2	0.40	NOU (1999)
s_3	Natural survival rate of age class 3	0.50	NOU (1999)
s_4	Natural survival rate young adult mature age class	0.50	NOU (1999)
a	Intrinsic growth rate recruitment function	160 (# of recruits/ fertility adjusted spawner)	Scaled
K	Scaling parameter recruitment function	250 (# of spawners)	Scaled
σ	Migration parameter	0.5	Assumption
γ_4	Fecundity parameter young adult	1.0	McGinnity et al. (2003)
γ_5	Fecundity parameter old adult	2.4	McGinnity et al. (2003)
p_4	Fish price young adult	300 (NOK/ fish)	Olaussen and Liu (2011)
p_5	Fish price old adult	825 (NOK/fish)	Olaussen and Liu (2011)
r	Interest rate	3%	Assumption

Table 2. Dynamics of the fish population (in # of fish) and optimal harvest (in # of fish) with baseline parameter values (Table 1). Numbers in bold indicate the initial state

Year	$N_{1,t}$	$N_{2,t}$	$N_{3,t}$	$N_{4,t}$	$N_{5,t}$	$N^s_{4,t}$	$N^s_{5,t}$	B_t	$H_{4,t}$	$H_{5,t}$
			Number of fish in each age class				Spawning pop.			Harvest
0	**5000**	**3000**	**800**	**600**	**100**	**255**	**50**	**375**	49	100
1	7500	2000	1200	400	150	251	0	251	0	139
2	6258	3000	800	600	100	200	11	227	47	100
3	5945	2503	1200	400	150	253	0	253	0	139
4	6282	2378	1001	600	100	200	11	227	47	100
5	5946	2513	951	501	150	253	0	253	0	150
6	6283	2378	1005	476	125	250	0	250	0	125
7	6254	2513	951	503	119	238	0	238	0	119
8	6094	2502	1005	476	126	251	0	251	0	126
9	6266	2437	1001	503	119	238	0	238	0	119
10	6094	2506	975	500	126	251	0	251	0	126
11	6266	2438	1003	487	125	250	0	250	0	125
12	6252	2507	975	501	122	244	0	244	0	122
13	6171	2501	1003	488	125	251	0	251	0	125
14	6258	2468	1000	501	122	244	0	244	0	122
15	6171	2503	987	500	125	251	0	251	0	125
16	6258	2468	1001	494	125	250	0	250	0	125
17	6251	2503	987	501	123	247	0	247	0	123
18	6210	2500	1001	494	125	250	0	250	0	125
19	6254	2484	1000	501	123	247	0	247	0	123
20	6210	2502	994	500	125	250	0	250	0	125
21	6254	2484	1001	497	125	250	0	250	0	125
22	6250	2502	994	500	124	248	0	248	0	124
23	6230	2500	1001	497	125	250	0	250	0	125
24	6252	2492	1000	500	124	248	0	248	0	124
25	6230	2501	997	500	125	250	0	250	0	125
26	6252	2492	1000	498	125	250	0	250	0	125
27	6250	2501	997	500	125	249	0	249	0	125
28	6240	2500	1000	498	125	250	0	250	0	125
29	6251	2496	1000	500	125	249	0	249	0	125
30	6240	2500	998	500	125	250	0	250	0	125

31	6251	2496	1000	499	125	250	0	250	0	125
32	6250	2500	998	500	125	250	0	250	0	125
33	6245	2500	1000	499	125	250	0	250	0	125
34	6251	2498	1000	500	125	250	0	250	0	125
35	6245	2500	999	500	125	250	0	250	0	125
36	6251	2498	1000	500	125	250	0	250	0	125
37	6250	2500	999	500	125	250	0	250	0	125
38	6248	2500	1000	500	125	250	0	250	0	125
39	6250	2499	1000	500	125	250	0	250	0	125
40	6248	2500	1000	500	125	250	0	250	0	125
41	6250	2499	1000	500	125	250	0	250	0	125
42	6250	2500	1000	500	125	250	0	250	0	125
43	6249	2500	1000	500	125	250	0	250	0	125
44	6250	2500	1000	500	125	250	0	250	0	125
45	6249	2500	1000	500	125	250	0	250	0	125
46	6250	2500	1000	500	125	250	0	250	0	125
47	6250	2500	1000	500	125	250	0	250	0	125
48	6250	2500	1000	500	125	250	0	250	0	125
49	6250	2500	1000	500	125	250	0	250	0	125
50	6250	2500	1000	500	125	250	0	250	0	125

Table 3. Optimal steady state sensitivity results. Fishing mortalities, population sizes and spawning population (in # of fish) and harvest (in # of fish)

	f_4	f_5	N_4	N_5	B	H_4	H_5
Baseline values	0	1	500	125	250	0	125
2 %-point increase interest rate ($r = 0.05$)	0,0253	1	487	122	237	6	122
2%-point reduction interest rate ($r = 0.01$)	0	0,9789	512	128	262	0	125
20% increase price old age-class ($p_5 = 990$)	0	1	500	125	250	0	125
10% reduction price old age-class ($p_5 = 742.5$)	0,0111	1	494	124	244	3	124
$p_5 = 720$	$f_5 = 1.02 - 0.83 f_4$		490	123	240		
20% reduction price old age-class ($p_5 = 660$)	1	0	583	146	350	291	0

Table 4. Non-selective and selective fishing. Optimal present value economic yield and optimal steady state spawning population

	Present value economic yield (in 1000 NOK)		Size spawning population B (in # of fish)	
	Non-selective fishing	Selective fishing[a]	Non-selective fishing	Selective fishing
Baseline values	3334	3527 (5.8 %)	232	250
2 %-point increase of interest rate ($r = 0.05$)	2069	2175 (5.1 %)	205	237
2%-point reduction of interest rate ($r = 0.01$)	9776	10738 (9.8 %)	260	262
20% increase price old age-class ($p_5 = 990$)	3720	4224 (13.6 %)	232	250
10% reduction price old age-class ($p_5 = 742.5$)	3141	3181 (1.3 %)	232	244
$p_5 = 720$	3088	3088 (0 %)	240	240
20% reduction price old age-class ($p_5 = 660$)	2948	2991 (1.5 %)	232	350

a. Percentage gain selective fishing in parentheses.

Figure 1. Schematic representation of the life cycle of a wild Atlantic salmon for a single cohort (the time index is omitted). See main text for definition of symbols.
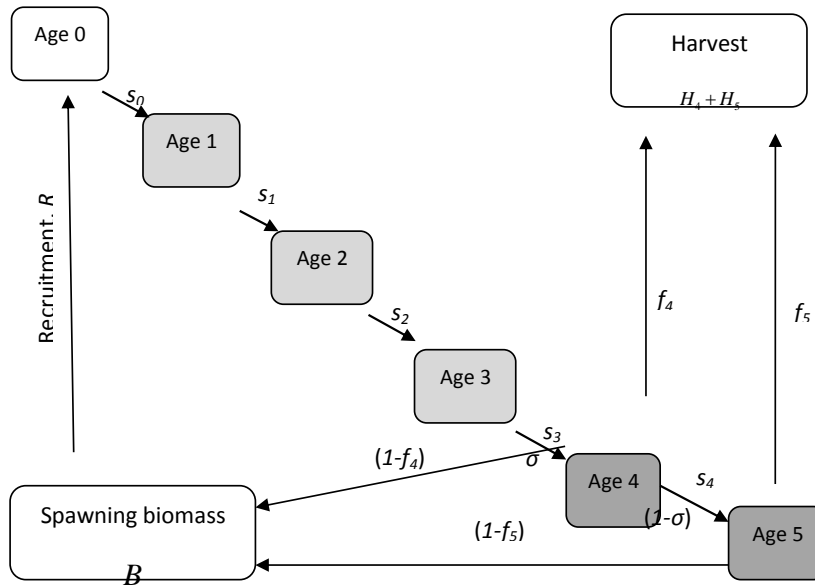
Figure 2. Effects of the relative price ($\alpha = p_5 / p_4$) and interest rate on the optimal steady-state solution. r_1 is the interest rate at which $\partial L / \partial f_4 = 0$ and r_2 is the interest rate at which $\partial L / \partial f_5 = 0$ when evaluated at $f_4 = 0$ and $f_5 = 1$. r_3 is the interest rate at which $\partial L / \partial f_4 = 0$ and r_4 is the interest rates at which $\partial L / \partial f_5 = 0$, when evaluated at $f_4 = 1$ and $f_5 = 0$. On the horizontal line $p_5 / p_4 = \gamma_5 / \gamma_4$, there exist multiple optimal solutions at each interest rate, and the weighted sum of the optimal harvest rates $\gamma_4 \sigma f_4 + \gamma_5 s_4 (1 - \sigma) f_5$ increases as the interest rate increases.
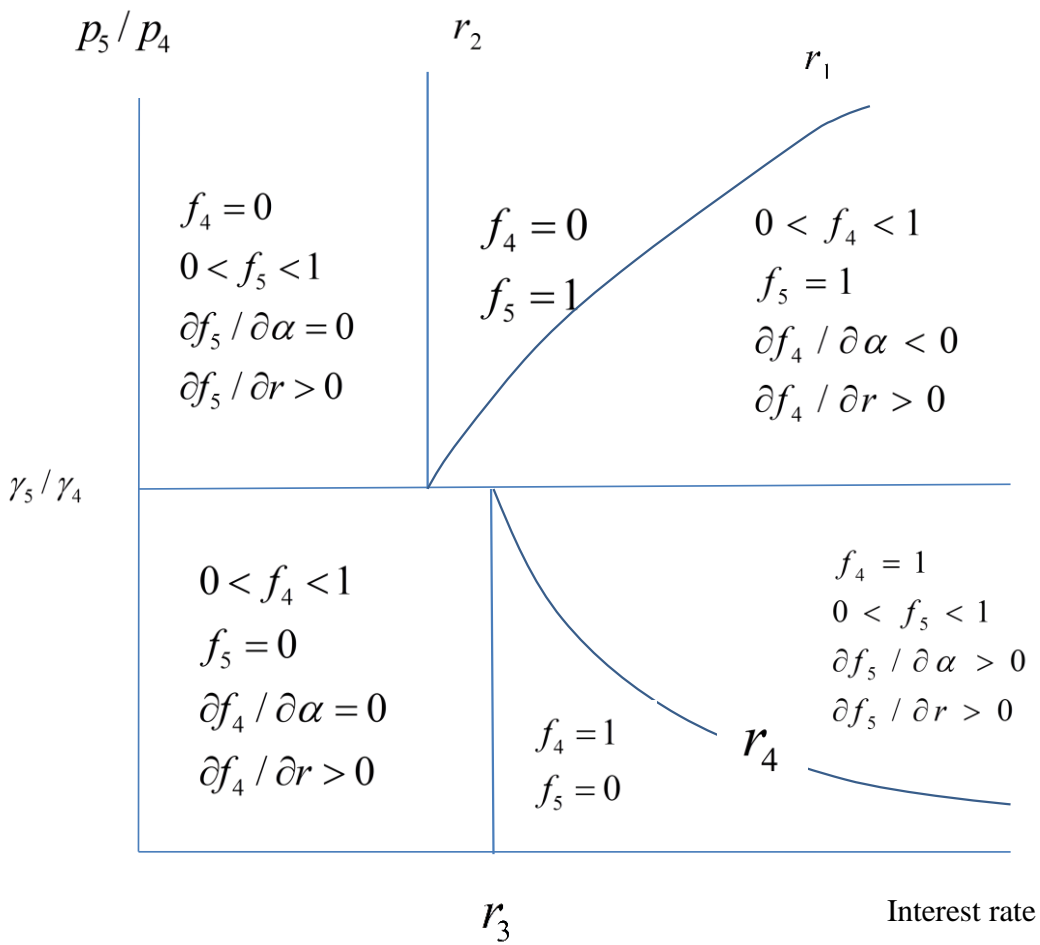
p_5 / p_4 r_2 r_1

$$f_4 = 0$$
$$0 < f_5 < 1$$
$$\partial f_5 / \partial \alpha = 0$$
$$\partial f_5 / \partial r > 0$$

$$f_4 = 0$$
$$f_5 = 1$$

$$0 < f_4 < 1$$
$$f_5 = 1$$
$$\partial f_4 / \partial \alpha < 0$$
$$\partial f_4 / \partial r > 0$$

γ_5 / γ_4

$$0 < f_4 < 1$$
$$f_5 = 0$$
$$\partial f_4 / \partial \alpha = 0$$
$$\partial f_4 / \partial r > 0$$

$$f_4 = 1$$
$$f_5 = 0$$

$$f_4 = 1$$
$$0 < f_5 < 1$$
$$\partial f_5 / \partial \alpha > 0$$
$$\partial f_5 / \partial r > 0$$

r_4

r_3

Interest rate

35

Figure 3. The optimal steady state spawning population B (fecundity weighted sum in # of fish) associated with different interest rates r (%) and prices of the old mature age-class p_5 (NOK/fish). Baseline value (Table 1) all other parameters