ISSN 1503-299X

WORKING PAPER SERIES

No. 10/2012

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Testing for rational bubbles in the housing market*

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August 31, 2012

Abstract

This paper investigates the presence of a bubble in the US housing market prior to the 2007 subprime mortgage financial crisis. The relationship between housing prices and rental prices, known as the price-rent ratio, is an important measure of a potential deviation between housing prices and its fundamental value. Additionally, the interest rate is taken into account since it is an important factor in determining demand for housing mortgages and thereby influence housing prices. These relationships are then put into a theoretical model framework. The empirical evidence suggests that there was a bubble in the housing market prior to the financial crisis, even when controlling for the decreasing interest rate in the period. Hence, the econometric procedures used in the analysis may be relevant for monitoring the housing market.

JEL: E31, G12, R21, C32.

Keywords: Rational bubbles, Rent-price ratio, House prices, Interest rates, Cointegration, Vector autoregression.

1 Introduction

There seems to be an important relationship between housing prices and the real economy, especially as seen in the turmoil following the subprime mortgage financial crisis which started in the US in 2007. As shown in Reinhart and Rogoff (2008), an investigation of a selection of 18 financial crises from the postwar period in various countries all show that there was a significant run-up in housing prices prior to the financial crisis. Housing prices may therefore be an important macroeconomic variable to investigate concerning the state of the economy. Hence, we should pay close attention to movements in housing prices since large increases in housing prices may indicate an approaching financial crisis.

The increase in housing prices in the US prior to the 2007 financial crisis were particularly large compared to previous housing price increases, as shown in figure 1. For example, the increase in housing prices prior to the savings and loan crisis starting in the 1980s (which

*The author thanks Gunnar Bårdsen, Bent Nielsen and John Leahy for helpful comments and suggestions. I would also like to thank participants at the second workshop in dynamic macroeconometrics at the University of Oslo, and discussant Hans-Martin Straume and other participants at the Norwegian PhD workshop in economics 2012 for helpful comments.

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was one of the crisis investigated by Reinhart and Rogoff (2008)) was a lot smaller than the increase in the 2000s. The savings and loan crisis was considered a substantial crisis since the fiscal costs of cleaning up after the crisis was large (3.2 percent of GDP) compared to other financial crises in other countries (Reinhart and Rogoff, 2008). If the size of the increase in housing prices prior to a crisis indicates the subsequent magnitude of fiscal costs, the 2007 subprime mortgage financial crisis could end up being very costly. We also see from figure 1 that the drop in housing prices after 2007 has been quite large, since the real housing prices in 2011Q1 dropped to a level as low as the level in 1999Q1. This effect is substantial for the households which have lost a large portion of their wealth since the peak of the real housing prices in 2006Q1. This negative wealth effect will dampen aggregate demand and may initiate a recession. An important reason for monitoring housing prices is therefore the substantial negative effects affecting households if the housing prices decreases.

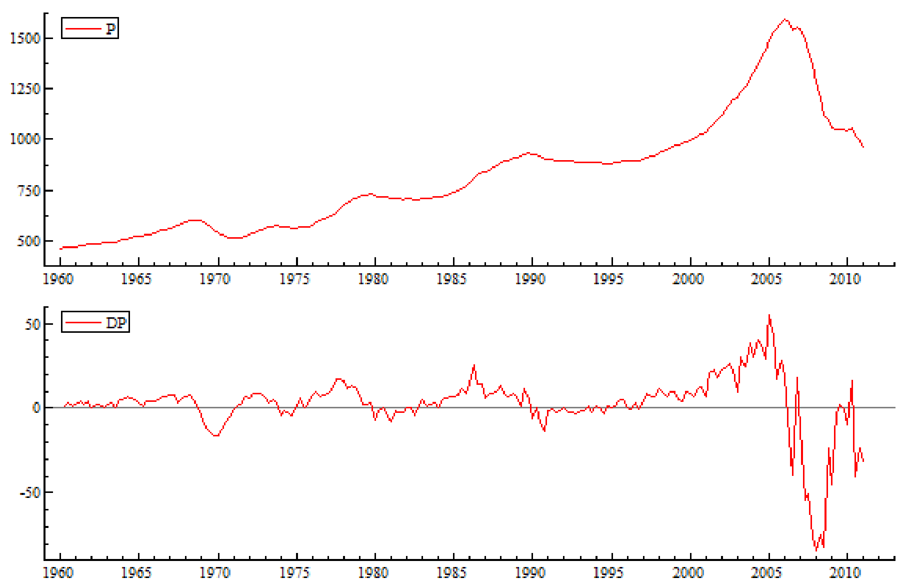

Figure 1: US real house prices (deflated by CPI less shelter), seasonally adjusted in upper panel. Lower panel shows first-differences of the same series. Data source: Davis et al. (2008)

Correspondingly, since increased housing prices may act as a financial accelerator[1], e.g. as shown in Iacoviello (2005), increases in housing prices are important for the real economy. Increased housing prices will cause a positive wealth effect for the households such that increased housing prices may increase aggregate demand. Increased housing prices also increases the collateral value of the borrowers, which will boost borrowing such that there will be increased pressure in demand for housing, as illustrated in Kiyotaki and Moore (1997). Housing prices will then increase, which will further amplify the financial accelerator effect. The effect from the financial accelerator will work in the opposite direction in the case of declining housing prices, since the wealth of the households is decreased and aggregate consumption may decline as a result of this. If decreased housing prices is a result of lower demand for houses in the economy possibly as a result of a recession, the financial accelerator effect will amplify this

[1] The financial accelerator was presented in Bernanke et al. (1999) as a mechanism that boosts the borrowing capacity of debtors when their asset values increase because of increased wealth.

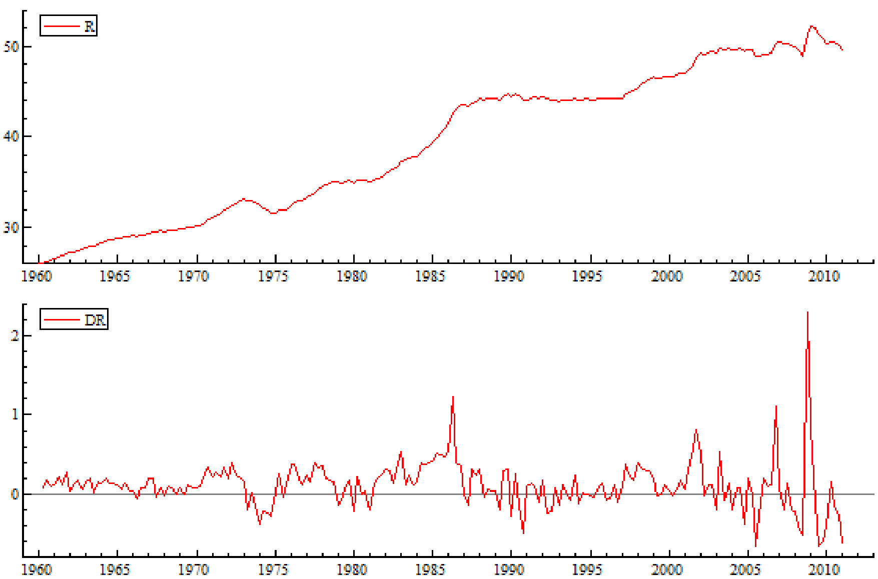

Figure 2: US real rental prices (deflated by CPI less shelter), seasonally adjusted in upper panel. Lower panel shows first-differences of the same series. Data source: Davis et al. (2008)

effect.

The presence of a bubble in the housing market will amplify this mechanism, such that a potential burst of the bubble will have a large negative impact on the real economy. A potential bubble therefore needs to be identified before it bursts such that precautionary means may be taken in order to dampen the negative accelerating effects that may influence the real economy. Stiglitz (1990) defines bubbles as a high price being high only because investors believe that the selling price is high tomorrow when fundamental factors do not seem to justify the high price. This kind of psychology is then the main reason behind price increases, and economic fundamental values play a smaller role as a driving force.

The ratio between housing prices and the rental price (the price-rent (PR) ratio) have been studied in order to investigate whether there may exist a bubble in the housing market, see e.g. Himmelberg et al. (2005). An important fundamental value behind house prices is the rental price, such that investigating the PR ratio is important for detecting bubbles. A large long-term deviation between housing prices and the rental price may indicate a bubble in the housing market. If we compare figures 1 and 2, we see that the high increase in the housing prices prior to the 2007 financial crisis is not followed by a similar high increase in the rental price. Some other factors than the rental price must therefore exist in order to explain the increase in housing prices. Since the alternative to purchasing a house is to rent it, the rental price and the housing price should move together. People will be interested in one or the other depending on the price difference, such that the demand for the two alternatives will always adjust back to a value coherent with the fundamental value. If housing prices increase, demand for rented dwellings will increase which yields an increase in the rental price. Housing prices are then no longer relatively higher than rental prices such that the spread between the two is reduced. A bubble which is present because of psychological reasons will distort this equilibrium relationship such that a non-constant PR ratio could be the sign of a bubble in the housing market.

If a bubble exists, it may be either irrational or rational regarding the psychological factors driving the price. Irrational bubbles may result from investors being driven by irrationally optimistic expectations, fashion, or fads (Schiller, 2000), while rational bubbles occur when asset prices continue to rise because investors believe that they will be able to sell the overvalued asset at a higher price in the future (Flood and Hodrick, 1990). In the housing market, an irrational bubble may result from increased demand for houses because households have visions of improving their life when buying an expensive house or a trend that drives the market towards owning rather than renting houses. This trend may be because of expected increases in housing prices in the future such that the households may sell their home with a profit, which will indicate a rational bubble. However, such behavior is more relevant from an investors point of view, since an investor seeks to buy a house today and sell it later with a profit. If a household should behave as a profit maximizing investor, it needs to alternate between owning an renting which is not that likely to be the case for the majority of households. When a household sells its house, it is more likely that they buy a new house. They will then sell and buy in the same market such that increases in the value of their homes is used for buying the new house which also have increased in value since the time of purchasing their first house. If this behavior is more likely in the housing market, a potential bubble should be regarded as an irrational bubble.

When a bubble is identified, we should take precautionary moves in order to dampen the explosive behavior such that the price increase may be dampened, thus reducing the possibility of a burst of the bubble. The price can then move towards its fundamental value over time rather than dropping rapidly after a burst of the bubble. Using monetary policy by increasing interest rates such that the demand for housing loans decreases, or using macro prudential tools such as stricter requirements on the debt burden of households, may work as means to dampen the increase in housing prices.

Another way to view a house, is as an asset. The payoff from the asset will then be the rental price which is received by the owner (investor) when he lets out the house. This will also result in an equilibrium between the housing price and the rental price, since an increase in one of the prices will lead to an increase in the other price through changes in demand such that the PR ratio is held constant in the long run. The relationship between housing prices and rental prices will then be an analogue to the relationship between asset prices and dividends. Since the fundamental value of an asset will be the present value of all the future cash flows of the asset, a potential divergence of the actual price of the asset from its fundamental value is a speculative bubble (Brooks et al., 2001). The fundamental value will be the rental price in this case, such that a deviation between the rental price and the housing price will indicate a bubble. Various tests for bubbles in asset markets found in the literature may therefore be of relevance when testing for bubbles in the housing market. I will use the econometric procedures presented in Engsted and Nielsen (2012), who tests for rational bubbles in the US stock market for the period 1974-2000, in order to investigate how these methods are able to test for a housing bubble.

As argued e.g. in Leamer (2002), a high ratio between prices and earnings of an asset may be justified if other assets also are highly priced, for example if bond yields and mortgage rates are low. We should therefore correct for changes in the interest rate when doing this analysis. For fundamental values in the housing market, Poterba (1992) calculates an imputed rental value that takes into account the interest rate, various tax rates, maintenance costs and inflation as important factors that affect the annual costs after buying a house. This is also taken into account in the Gordon growth model, which is analyzed regarding a housing bubble e.g. in Campbell et al. (2009). I will therefore also take into account the interest rate, since it is an important factor for the cost of a housing loan, and it is clearly not constant for the period analyzed below. The interest rate will influence the cost of borrowing, and hence influence demand for houses and housing prices. An increase in housing prices could therefore be a result of decreasing interest rates instead of the presence of a speculative bubble.

2 Modeling house prices

The stream of future rental income, R, received by a household letting a house or another type of dwelling to someone, is represented by $R_{t+1}, R_{t+2}, R_{t+3}$, etc. These future payments may be replicated by a portfolio of bonds with different maturities. We consider buying a set of bonds with different maturities, such as a one-period bond with face value R_{t+1}, a two-period bond with face value R_{t+2}, a three-period bond with face value R_{t+3}, and so on for an infinite number of future periods. This portfolio will then pay off R_{t+1} at time $t+1$, R_{t+2} at time $t+2$ and so on. If we let i_t be the yield to maturity per period on the bond, the price at time t for the j-th period bond has price

$$Q_{t+j} = \frac{R_{t+j}}{(1+i_t)^j}. \tag{1}$$

The price of a house needs to be equal to the total cost of this portfolio of bonds such that

$$P_t = \sum_{j}^{\infty} Q_{t+j}. \tag{2}$$

If the equality in (2) does not hold (i.e. the house price is less or more than the portfolio of bonds), investors will invest in the most profitable such that prices adjusts until the equilibrium is re-obtained. Using (1), we can write (2) as

$$P_t = \sum_{j=1}^{\infty} \frac{R_{t+j}}{(1+i_{t+j})^j}. \tag{3}$$

This says that the price of a house should equal the present value of all future rental payments the house can provide if it is rented out. Hence, (3) should hold when we consider a house as an investment object. This gives a model that relates housing prices and rental prices, but in a more formal setting than simply looking at the PR-ratio.

Next, we may move (3) one period forward such that we get an expression for P_{t+1}:

$$P_{t+1} = \sum_{j=2}^{\infty} \frac{R_{t+j}}{(1+i_{t+j})^j},$$

which we may insert on the right-hand side of (3), such that we get

$$P_t = \frac{1}{1+i_t} E_t(P_{t+1} + R_{t+1}). \tag{4}$$

This indicates that the housing price today should reflect the value of next period's payouts (rents) and possible increase in housing prices. This is in line with the reasoning that the alternative to owning a house is to rent the house. Therefore, the owner of a house (assuming he uses the house as his primary residence himself) saves the rent he alternatively had to pay in order to live in the house if he did not own it (Himmelberg et al., 2005).

If we write this out for the one-period gross return and assume that the discount factor (i.e. the interest rate) is constant, we get one-period gross real return to housing, as shown in Campbell et al. (2009):

$$(1+i) = E_t\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right). \tag{5}$$

A constant discount factor is common in the empirical bubble literature (Engsted and Nielsen, 2012). We may additionally formulate (4) (assuming a constant discount factor) as

$$M_t = P_t + R_t - (1+i)P_{t-1}, \tag{6}$$

where $E_{t-1}M_t = 0$ is a martingale difference, corresponding to the efficient market hypothesis as outlined e.g. in LeRoy (1989). The efficient market hypothesis implies that an expected value does not depend on its prehistory and that all public information is reflected in the price as soon as it is known. This corresponds to M_t being a martingale difference.

Additionally, the "spread" (as defined in Campbell & Shiller (1987, 1988)),

$$S_t \equiv P_t - R_t/i, \tag{7}$$

expresses the relationship between housing prices and the rental price, and corrects for i (which is assumed constant over time here). We then have

$$M_t = (1+i)\Delta_1 P_t - i \cdot S_t, \tag{8}$$

which will be used later to test the efficient market hypothesis.

We also see that (3) is equivalent to the dividend valuation model for stock prices, see e.g. Mishkin (2009). The relationship between housing prices and rent should therefore be similar to the relationship between stock prices and dividends. Methods for analyzing bubbles in the stock market using the dividend valuation model may therefore also be applied to housing prices in order to test for bubbles in the housing market, simply by replacing asset prices with housing prices and dividends with rental prices. I will use the framework outlined by Engsted and Nielsen (2012), who perform tests for bubbles in the US stock market, in order to investigate whether there was a bubble in the housing market prior to the financial crisis that started in 2007, and to what extent this framework is able to identify such bubbles.

2.1 Rational bubbles in the housing market

As shown e.g. in Gilles and LeRoy (1992), every continuous dynamic price system can be divided into two parts; a fundamental and a bubble component. We should therefore distinguish these two apart to see if there is evidence of a bubble in the data. If such bubbles are present, P_t will be determined as

$$P_t = \sum_{j=1}^{\infty}\left(\frac{1}{1+i}\right)^j E_t R_{t+j} + bB_t, \tag{9}$$

where $B_t = (1+i)^{-1}E_t B_{t+1}$, i.e. $B_{t+1} = (1+i)B_t + \xi_{t+1}$ where ξ_{t+1} is a rational forecast error. Additionally, we know that $i > 0$ such that B_t is explosive and an explosive component is added to P_t if $b \neq 0$.

If homeowners are willing to pay inflated prices for houses today because they expect unrealistically high housing appreciation in the future, there exists a housing bubble (Case and Shiller, 2004). This is indicated by $B_t > 0$ in (9) which will increase house prices in period t through this psychological factor driving housing prices to grow explosively.

There may be an explosive root in housing prices in addition to a unit root because of speculative bubbles as shown in Engsted (2006). This may be investigated by looking at the roots in a vector autoregressive (VAR) model and a cointegrated VAR model where housing prices and rental prices are included.

2.2 A non-constant interest rate

The interest rate used in the dividend valuation model above is assumed constant as is common in much of the empirical literature on bubbles in the asset market. However, as shown in figure 3, interest rates varied a lot during the sample period. Assuming a constant interest rate is therefore a quite strict assumption and may cause a loss of valuable information in our model. As previously mentioned, the interest rate is important for the fundamental value of housing since low interest rates may boost demand for house mortgages and increase housing prices. If we view housing as a potential investment, the alternative to investing in housing is to invest in bonds, such that a low rate on bonds leads to higher demand for housing and increased housing prices. The increase in housing prices in the period before the crisis may therefore be partially explained by the decrease in interest rates.

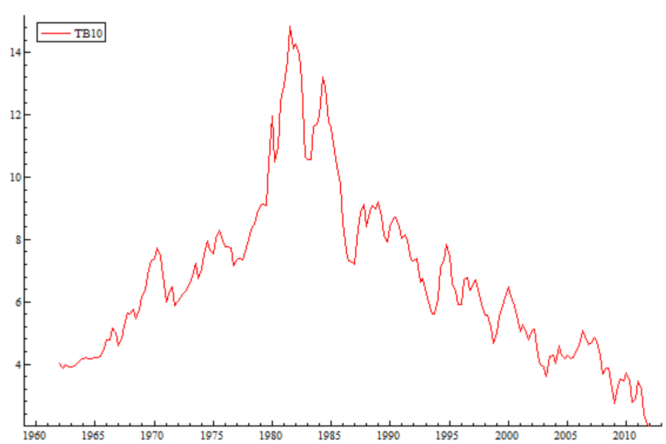

Figure 3: Nominal interest rate on 10 year US Treasury Bill.

Poterba (1992) calculates an "imputed rental value", which in a simplified form as shown by André (2010) is expressed as

$$R^{imp} = P(i^a + \tau + f - \pi) \tag{10}$$

where i^a is the after-tax nominal mortgage interest rate, τ is the property tax rate on owner-occupied houses, f denotes the recurring costs and π symbolizes the expected capital gains on houses. The variables included in the parenthesis should therefore be considered when we investigate the PR ratio. We are particularly interested in correcting for the declining interest rate prior to the subprime mortgage crisis, such that I include the interest rate in the estimated model. However, instead of calculating the imputed rental price by using the housing price and the interest rate, I use the actual rental price and the interest rate. This allows preserving the information and the dynamics for the rental price in the estimation. I will therefore calculate the imputed rental price by dividing the actual rental price by the gross interest rate;

$$R_t^{imp} = \frac{R_t}{1 + i_t^{TB10}}, \tag{11}$$

where i_t^{TB10} is the interest rate on a 10-year government bond. This is an ad hoc way of incorporating the interest rate in the model, but it is in line with the ways of incorporating the interest rate used in Poterba (1992) and Campbell et al. (2009) except for using the gross interest rate here and not taking other factors as done in Poterba (1992) shown in (10) into consideration. It is also in line with correcting the rental price for the interest rate as done in the spread relationship in (7). Using the imputed rental value will correct the rental price for changes in the interest rate on mortgages which influences demand for mortgages and hence housing prices. A decreasing interest rate will then increase the imputed rental price such that renting is relatively more expensive compared to buying a house (which is less expensive due to lower costs on mortgages). This will then give the following model for house prices:

$$P_t = \sum_{j=1}^{\infty} \frac{R_{t+j}/(1 + i_{t+j}^{TB10})}{(1 + i_{t+j})^j}, \tag{12}$$

where i_{t+j} is assumed constant. The calculated imputed rent will then take into consideration the real cost of a mortgage when we compare it to the housing price. This allows using only two observable variables when estimating the model since the interest rate will be incorporated into the rental price. The alternative would be to use as a third variable in the estimation by allowing a non-constant value of the interest rate in (5) which would complicate the coexplosive VAR model.

Even though it may be relevant to use the mortgage rate as the interest rate in (11), I have chosen to use the 10-year government bond yield, since the difference between the interest rate on 10-year Treasury bonds and the annual appreciation on housing measures the real cost of a mortgage (Leamer, 2002). As shown in André (2010), the PR-ratio for the US when using imputed rental value is quite similar when using the mortgage rate or the government bond rate, except for the period after the subprime mortgage crisis which nevertheless will not be analyzed in this paper. The time horizon is also realistic since the average homeowner lives in the house financed by a mortgage for about ten years on average.

In the following section, I will first investigate if there is a bubble in the housing market using the actual rental price, and secondly I will use the 'imputed rent' such that the declining interest rate in the period before the crisis is taken into consideration. This may enable us to see whether declining interest rates was an important driver for the housing prices or if the evidence of a speculative bubble in the housing market changes when correcting for decreasing interest rates.

2.3 The vector autoregressive model

The economic model outlined in (4) may be estimated through a vector autoregressive (VAR) model of order k where $X_t = (P_t, R_t)'$ (Alternatively, R_t^{imp} may be used in place of R_t in order to allow for a varying interest rate as shown in (11).)

$$X_t = \sum_{j=1}^{k} A_j X_{t-j} + \mu + \varepsilon_t. \tag{13}$$

In equilibrium correction form, a reformulation of this yields

$$\Delta_1 X_t = \Pi X_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta_1 X_{t-j} + \mu + \varepsilon_t, \tag{14}$$

where $\Delta_1 X_t = X_t - X_{t-1}$, $\Gamma_j = -(I - A_1 - \cdots - A_{k-1})$ and $\Pi = -(I - A_1 - \cdots - A_k)$.

This may be restricted in order to obtain the coexplosive model outlined in Nielsen (2010). A reformulation of (14) yields (given $k \geq 2$)

$$\Delta_1 \Delta_\rho X_t = \Pi_1 \Delta_\rho X_{t-1} + \Pi_\rho \Delta_1 X_{t-1} + \sum_{j=1}^{k-2} \Phi_j \Delta_1 \Delta_\rho X_{t-j} + \mu + \varepsilon_t, \tag{15}$$

where $\Delta_\rho X_t = X_t - \rho X_{t-1}$, $\Pi_1 = \frac{\Pi}{1-\rho}$, $\Pi_\rho = -\rho(I_p + \Pi_1 - \sum_{j=1}^{k-1} \rho^{-j})$ and $\Phi_j = \sum_{l=j+1}^{k-1} \rho^{j-l} \Gamma_l$. Additionally, it is assumed that X_t has one unit root and one explosive root through reduced rank restrictions. The explosive root, $\rho > 1$, is a freely varying parameter obtained from the estimated characteristic polynomial. The long-run restrictions for the coexplosive model are that the rank is set to $r = 1$, called H_1, such that we get

$$\Delta_1 \Delta_\rho X_t = \alpha_1 \beta_1' \Delta_\rho X_{t-1} + \alpha_\rho \beta_\rho' \Delta_1 X_{t-1} + \sum_{j=1}^{k-2} \Phi_j \Delta_1 \Delta_\rho X_{t-j} + \mu + \varepsilon_t, \tag{16}$$

where a constant is also included in the cointegrating space.

Furthermore, $\beta_1' \Delta_\rho X_t$, $\beta_\rho' \Delta_1 X_t$ and $\Delta_1 \Delta_\rho X_t$ can be given a stationary distribution such that β_1 is the cointegrating vector and β_ρ is the coexplosive vector. The cointegration rank is determined through the likelihood test procedure by Johansen (1996).

The rental price is assumed to be integrated of order one such that it is non-explosive. This can be tested through the hypothesis on the coexplosive vector that

$$H_R : \beta_\rho = (0, 1)' \tag{17}$$

The equation for the coexplosive model is then reduced to

$$\Delta_1 \Delta_\rho X_t = \alpha_1 \beta_1' \Delta_\rho X_{t-1} + \alpha_\rho \Delta_1 R_{t-1} + \sum_{j=1}^{k-2} \Phi_j \Delta_1 \Delta_\rho X_{t-j} + \mu + \varepsilon_t. \tag{18}$$

For a given value of ρ, the likelihood is maximized by reduced rank regression of $\Delta_1 \Delta_\rho X_t$ on $\Delta_\rho X_{t-1}$ correcting for lagged rental price growth $\Delta_1 R_{t-1}$ and lagged differences $\Delta_1 \Delta_\rho X_{t-j}$. This restricted model is for forthcoming references labeled M_{1R}.

By imposing two additional restrictions on the model in (18), we get the bubble model. These two restrictions are that the "spread" $S_t = P_t - R_t/i$ is a cointegrating relation so that the coefficient i which represents the expected one-period return is linked to the explosive root, ρ, through $\rho = 1 + i$. This gives the hypothesis

$$H_S : \beta_1 = (1, -1/i)'. \tag{19}$$

In addition, the martingale restriction outlined in (6) should be imposed as a restriction. This implies that (8) is imposed, such that (18) may be re-written as

$$\Delta_1\Delta_\rho X_t = \iota'\alpha_1\Delta_\rho S_{t-1} + \iota'\alpha_\rho\Delta_1 R_{t-1} + \iota'\sum_{j=1}^{k-2}\Phi_j\Delta_1\Delta_\rho X_{t-j} + \iota'\alpha_1\zeta_1 + \iota'\varepsilon_t, \qquad (20)$$

where $\iota' = (1,1)$ such that the equation reduces to $M_t = \iota'\varepsilon_t$ denoted ε_M if the following hypothesis is accepted

$$H_B : \iota'\alpha_1 = -1,\ \iota'\alpha_\rho = -(1+i)^2/i,\ \iota'\Phi_j = 0,\ \zeta_1 = 0. \qquad (21)$$

The model M_{1R} restricted by H_S and H_B is denoted M_{1RSB}. This model may be reparameterized. First, ω should denote the population regression coefficient of $\varepsilon_{R,t} = (0,1)\varepsilon_t$ on $\varepsilon_{M,t} = (1,1)\varepsilon_t$ and rewrite the model in terms of the marginal equation for M_t and the conditional equation for $\Delta_1 D_t$ as shown in Engsted and Nielsen (2012). This yields the model M_{1RSB}:

$$M_t = \varepsilon_{M,t} \qquad (22)$$

$$\Delta_1 R_t = \alpha_{1,R}\Delta_\rho S_{t-1} + (\alpha_{\rho,R}+\rho)\Delta_1 R_{t-1} + \sum_{j=1}^{k-2}\Phi_{j,R}\Delta_1\Delta_\rho X_{t-j} + \omega M_t + \varepsilon_{R\cdot M,t}, \qquad (23)$$

where $\varepsilon_{R\cdot M,t} = \varepsilon_{R,t} - \omega\varepsilon_{M,t}$ is uncorrelated with $\varepsilon_{M,t}$. For a known value of i, (22) and (23) are unrelated such that the likelihood is maximized by maximizing over i using a profile argument. This implies that i should be set to the value that maximizes the likelihood function. This also affects ρ such that the likelihood profile for various choices of the discount factor must be investigated when estimating the parameter values which yields the maximum likelihood. This applies for all of the estimated restricted models outlined above. This is also explained in section 3.

3 Estimation

The two data series used for housing prices and rents are real house prices and real rental prices for the U.S. The data are collected from Davis et al. (2008), where quarterly time series from 1960Q1 are still being updated. These series are estimated rents and house prices for the aggregate stock of housing in the U.S, and I choose to use the S&P Case-Shiller price index for the house prices after 2000, since it includes all home sales, in contrast to the FHFA index which only includes conforming home mortgages. The rental price is taken from the quarterly index for the rent of primary residence published by the Bureau of Labor Statistics (BLS), and it is used to interpolate average net rent from the decennial census of housing and extrapolated beyond 2000. Since the rental price is measured as a total cost per year for each quarter, I have divided it by four in order to obtain quarterly rental costs as a measure for R_t.

I have seasonally adjusted the data using the U.S. Census Bureau's X12 seasonal adjustment program in order to remove the cyclical seasonal movements from the series prior to the estimation. In order to obtain the real house price and the real rental price, I have deflated the nominal price series (after seasonally adjusting them) by the national consumer price index excluding shelter.

I choose to use the sub-sample 1986Q1-2005Q1. The beginning of the sample coincides with the tax reform act of 1986 which stimulated investing in owner-occupied housing after the real estate boom pertaining to the first half of the 1980s, and starting in 1986Q1 avoids the presence of multiple housing price bubbles in the sample since we exclude the increasing housing prices before the savings and loans crisis. The sample ends in 2005Q1, since the growth in house prices declines after this period as shown in figure 1. This is needed since the sample has to end before the potential bubble bursts, and also illustrates that this method is useful for identifying bubbles before they burst.

First, the unrestricted VAR will be estimated, and the model needs to be well specified in order for the various tests carried out below to be valid. As shown by the misspecification tests in table 1, the model does not contain residual autocorrelation or non-normality. The test for autocorrelation is proven to be valid when the model contains explosiveness (Nielsen, 2006), and the tests for normality and autoregressional conditional heteroskedasticity (ARCH) are believed to be valid (Engsted and Nielsen, 2012). The residuals may contain ARCH, but the rank test should be robust to moderate ARCH effects (Rahbek et al., 2002) and statistical inference is quite robust to residual heteroskedasticity (Juselius, 2006, p. 47). However, it is not proven that the same holds when explosiveness is present in the model. The tests related to the theoretical model are nevertheless carried out below, and it is assumed that the model is well-specified based on the results from the misspecification tests even though all of them are not proven to be valid.

Additionally, the proper lag length for the VAR is set to two, since this is the smallest lag length which still provides no residual autocorrelation. The largest root needs to be larger than one $\rho > 1$, such that the co-explosive model may be formulated. The rank should be $r = 1$, and the largest root should still be larger than unity after imposing this. Next, the various hypotheses related to the housing price model and the bubble model are tested in the coexplosive VAR framework. These hypotheses were outlined in section 2.3, and they are summarized in table 2.

Table 1: Multivariate misspecification tests

Residual autocorrelation AR 1-5	$F(20, 118) = \underset{(0.147)}{1.379}$
Test for normality	$\chi^2(4) = \underset{(0.039)}{10.086}$
Test for ARCH LM(1)	$F(24, 186) = \underset{(0.000)}{3.544}$

Table 2: Restricted models and their maintained hypotheses

Model	Hypothesis	Description
M_1	$H_1, r = 1$	Impose rank $r = 1$
M_{1R}	H_1, H_R	Rental price is non-explosive
M_{1RS}	H_1, H_R, H_S	$S_t = P_t - R_t/i$ is a cointegrating relation
M_{1RSB}	H_1, H_R, H_S, H_B	Efficient market hypothesis

First, H_R is tested given $r = 1$ and $\rho > 1$. Then H_S is tested with $i = \rho - 1$, and finally H_B is tested.

First, we may estimate the ECM from (14) labeled model $\mathbf{M}$:

$$\Delta_1 X_t = \Pi X_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta_1 X_{t-j} + \mu + \varepsilon_t,$$

which given rank $r = 1$ may be reparameterized to the model $\mathbf{M_1}$ from (15):

$$\Delta_1 \Delta_\rho X_t = \alpha_1 \beta_1' \Delta_\rho X_{t-1} + \alpha_\rho \beta_\rho' \Delta_1 X_{t-1} + \sum_{j=1}^{k-2} \Phi_j \Delta_1 \Delta_\rho X_{t-j} + \mu + \varepsilon_t,$$

where $\Delta_\rho X_t = X_t - \rho X_{t-1}$ and $\Delta_1 X_t = X_t - X_{t-1}$, $\Pi_1 = \frac{\Pi}{1-\rho}$, $\Pi_\rho = -\rho(I_p + \Pi_1 - \sum_{j=1}^{k-1} \rho^{-j})$ and $\Phi_j = \sum_{l=j+1}^{k-1} \rho^{j-l} \Gamma_l$. The explosive root $\rho > 1$ is a freely varying parameter obtained from the estimated characteristic polynomial.

Next, we test the hypothesis that R_t is non-explosive by imposing $\beta_\rho = (0, 1)'$. This gives the model $\mathbf{M_{1R}}$:

$$\Delta_1 \Delta_\rho X_t = \alpha_1 \beta_1' \Delta_\rho X_{t-1} + \alpha_\rho \Delta_1 R_{t-1} + \sum_{j=1}^{k-2} \Phi_j \Delta_1 \Delta_\rho X_{t-j} + \mu + \varepsilon_t$$

For a given value of ρ, the likelihood is maximized by reduced rank regression of $\Delta_1 \Delta_\rho X_t$ on $\Delta_\rho X_{t-1}$ correcting for $\Delta_1 R_{t-1}$, lags of $\Delta_1 \Delta_\rho X_{t-j}$ and a constant. The likelihood is then maximized by a grid search over ρ.

The hypothesis pertaining to the "spread", H_S, may be tested by imposing $\beta_1 = (1, \frac{-1}{i})$, where $i = \rho - 1$ such that we get the model $\mathbf{M_{1RS}}$:

$$\Delta_1 \Delta_\rho X_t = \alpha_1 \Delta_\rho S_{t-1} + \alpha_\rho \Delta_1 R_{t-1} + \sum_{j=1}^{k-2} \Phi_j \Delta_1 \Delta_\rho X_{t-j} + \mu + \varepsilon_t$$

where $S_t = P_t - \frac{1}{i} R_t$. The likelihood is again maximized over ρ.

Finally, the Martingale difference restrictions may be tested by imposing H_B:

$$(1, 1)\alpha_1 = 1, (1, 1)\alpha_\rho = -\frac{(1+i)^2}{i}, (1, 1)\Phi_j = 0, (1, 1)\mu = 0$$

which yields the model $\mathbf{M_{1RSB}}$:

$$\begin{aligned} M_t &= \varepsilon_{M,t} \\ \Delta_1 R_t &= \alpha_{1,R} \Delta_\rho S_{t-1} + \alpha_{\rho,R} \Delta_1 R_{t-1} \\ &\quad + \sum_{j=1}^{k-2} \Phi_{j,R} \Delta_1 \Delta_\rho X_{t-j} + \mu_R + \omega M_t + \varepsilon_{R \cdot M,t} \end{aligned}$$

where $\varepsilon_{M,t}$ and $\varepsilon_{R \cdot M,t} = \varepsilon_{R,t} - \omega \varepsilon_{M,t}$ are uncorrelated as shown in Engsted and Nielsen (2012). This may be estimated by constrained optimization.

3.1 Results

First, by looking at Figures 1 and 2, we clearly see signs of explosiveness in the housing price for the period before the subprime mortgage crisis while the rental price shows no such signs. Section 3.1.1 shows the results from the tests when the interest rate is not taken into account when estimating the model, while section 3.1.2 shows the results when the declining interest rate during the sample period is taken into account.

3.1.1 Assuming a constant interest rate

The initial model is a VAR with two lags (which is enough lags needed to avoid autocorrelated residuals). The characteristic roots are 1.104, 1.037, 0.613 and 0.136, which indicates an explosive root in the system. The cointegration rank test is shown in table 3, and indicates a rank of $r = 1$, although we may reject the hypothesis of $r = 1$ if we have a significance level smaller than 2%.

Table 3: Rank test

Hypothesis	Likelihood	Test statistic	p-value
$r \leq 2$	-93.035		
$r \leq 1$	-95.715	$LR(r \leq 1 \| M) = 5.36$	0.26
$r = 0$	-104.4375	$LR(r = 0 \| M) = 22.81$	0.02

If we impose a unit root, i.e. set the rank to $r = 1$, the largest root is still high at 1.113 and the second to largest is reduced to 1.000. This points to the anticipated result after investigating the data graphically, namely that there is one explosive root and one unit root.

Table 4: Tests of the rational bubble restrictions

Model	Hypothesis	log-likelihood	Test statistic		d.f.	p-value
M_1	$H_1, r = 1$	-95.7151631				
M_{1R}	H_1, H_R	-96.2405288	$LR(M_{1R} \| M_1)$	1.05	1	0.31
M_{1RS}	H_1, H_R, H_S	-97.5908743	$LR(M_{1RS} \| M_{1R})$	2.70	1	0.10
			$LR(M_{1RS} \| M_1)$	3.75	2	0.15
M_{1RSB}	H_1, H_R, H_S, H_B	-147.124282	$LR(M_{1RSB} \| M_{1RS})$	99.07	3	0.00
			$LR(M_{1RSB} \| M_{1R})$	101.77	4	0.00
			$LR(M_{1RSB} \| M_1)$	102.82	5	0.00

The results in table 4 report whether the various hypotheses pertaining to the restrictions on the VAR model may be rejected or not. We see that the hypothesis of a non-explosive rental price cannot be rejected such that the explosive component of the estimated VAR model belongs to housing prices through the p-value pertaining to H_R. The hypothesis H_S gives p-values of 0.10 and 0.15 when tested against M_{1R} and M_1, respectively, such that the hypothesis of a stationary spread between house prices and the rental price cannot be rejected either.

When testing the hypothesis H_S, the imposed β vector is $\beta_1 = (1, \frac{-1}{i})$, and since $i = \rho - 1$, where $\rho = 1.091$ is found by a grid search, we have that $i = 0.091$. The unrestricted β vector is $\beta_1 = (1, -94.25)$ such that the unrestricted estimate of i is $\hat{i} = 0.011$. Since H_S is not rejected even with this large deviation between the restricted and the actual value, there is little information in the data on this parameter which measures the expected return. Additionally, the imposed value of $i = 0.091$ is quite high, since this implies an expected growth in real housing prices at 9.1% per quarter.

The hypothesis pertaining to the bubble, H_B is rejected. This indicates that the standard house price model with a constant discount rate and a rational bubble are not supported and excess returns do not behave as a martingale difference. This may indicate that the rental price is not fully able to explain the movements in house prices even when correcting for explosiveness. Additionally, this may be evidence of an "irrational bubble", since there clearly is an explosive component in housing prices but the efficient market hypothesis does not hold when we correct for this.

Assuming that all individuals do not have different comparative information in acquiring information, is perhaps a strong assumption for the housing market. It is more likely that this is the case in the stock market than the housing market, since the stock market to a larger extent consists of professional investors and the housing market have a large share of private households buying and selling homes. Individual preferences and different abilities to obtain information about the market may be an important factor in the housing market, and will act as evidence against the efficient market hypothesis.

Table 5: Tests of the rational bubble restrictions when the interest rate is included in the estimation

Model	Hypothesis	log-likelihood	Test statistic		d.f.	p-value
M_1	$H_1, r = 1$	-101.670502				
M_{1R}	H_1, H_R	-102.276898	$LR(M_{1R}\|M_1)$	1.21	1	0.27
M_{1RS}	H_1, H_R, H_S	-103.372442	$LR(M_{1RS}\|M_{1R})$	2.19	1	0.14
			$LR(M_{1RS}\|M_1)$	3.40	2	0.18
M_{1RSB}	H_1, H_R, H_S, H_B	-152.555888	$LR(M_{1RSB}\|M_{1RS})$	98.37	3	0.00
			$LR(M_{1RSB}\|M_{1R})$	100.56	4	0.00
			$LR(M_{1RSB}\|M_1)$	101.77	5	0.00

3.1.2 Correcting for a varying interest rate

When using the 'imputed rent' as shown in (11) instead of the actual rental price when testing the restrictions on the model, we get the results as shown in table 5 for a VAR model with two lags. Also in this case, the model is econometrically well-specified and the largest root is larger than unity. The characteristic roots are 1.10, 1.04, 0.60 and 0.14, so there still seems to be an explosive root in the system which is only slightly smaller than in the model without the interest rate.

The same pattern emerges in this case, namely that there is a rank of $r = 1$ and there is an explosive component that belong to house prices. The largest root is 1.11 when imposing rank $r = 1$, and the second to largest root is 1.000 such that there seems to be an explosive root and a unit root. H_R and H_S are not rejected, such that the explosive component belongs to housing prices and there is cointegration between housing prices and rental prices (which now also includes the interest rate). The estimate of β is not changed much from the model without the interest rate, such that the estimation still provides little information on the expected return. Furthermore, the bubble model is rejected, which indicates that the efficient market hypothesis does not hold even when we correct for the non-constant interest rate.

The inclusion of the interest rate in order to use the information in its dynamics over the sample is therefore not helpful in supporting the efficient market hypothesis. Correcting for explosiveness, cointegration and the interest rate is therefore not sufficient to obtain a result that supports the efficient market hypothesis. The difference between the stock market and the housing market therefore still seems to be evident.

However, the perhaps most interesting finding when correcting for the interest rate is that we still find evidence of a bubble. This indicates that the explosive growth in housing prices prior to the subprime mortgage crisis was not a result of lower interest rate and a possibly higher credit demand boosting housing demand. At least this was not enough to explain the explosive behavior. Psychological factors therefore seems to be the driver behind housing prices when we correct for rental prices and the interest rate, such that a bubble is found in the data.

The lack of evidence supporting the efficient market hypothesis may also be a result of the difference between rational bubbles and irrational bubbles. Since we reject the efficient market hypothesis, this may indicate that the bubble is an irrational bubble. As previously mentioned, one of the most important differences between stock markets and housing markets is the participants in the markets. Since the housing market to a larger extent consists of unprofessional investors, it is more likely that incorrect judgments are made in the housing market since households may not obtain all available information about the market. Professional investors that to a large extent populates the stock market should be expected to be more rational. The difference between the results here and the results in Engsted and Nielsen (2012) therefore seems to support this, since the efficient market hypothesis holds when the bubble model is tested on the stock market but not on the housing market. However, as argued in Dale et al. (2005), the distinction between rational and irrational bubbles is not very clear since the misjudgment of the investors in the market needs to be investigated in order to make the distinction. Further analysis of the behavior of the agents in the housing market therefore needs to be done if one should find proper evidence for whether the bubble is rational or irrational. Another important difference between the cost of owning versus renting, is that renting is paid by after-tax income while e.g. the cost of a housing mortgage is tax deductible. However, this advantage of owning is constant for the estimated period such that the PR-ratio should not been affected by this.

4 Conclusion

The empirical evidence presented in this paper suggests that there is an explosive root in housing prices, while the rental price does not contain explosive elements. This also holds in the case where the 'imputed' rental price is used. This means that the declining interest rate in the period before the subprime financial crisis is not a strong enough effect to account for the large increase in the housing price that exceeds the increase in the rental price. This therefore indicates that a speculative bubble was an important driving force behind the increase in housing prices when we look at data prior to the crisis. Additionally, the bubble seems to be irrational rather than rational, which indicates that there is a difference between how the housing market behaves compared to the stock market.

Since the econometric methods used here were able to find evidence for a bubble in the early 2000s, it may be an important instrument for monitoring the housing market. The relevance of the method is particularly important since it does not take into consideration the timing regarding the starting point of the bubble or try to pin point the time period when the bubble bursts. If housing prices deviate from their fundamental value and there is an explosive root in a VAR model estimated on the basis of these two variables, this framework should be able to find evidence for a bubble through the various tested hypotheses.

This framework was applied to the US housing market for the sample 1986Q1-2005Q1.

Correspondingly, this implies that the estimation and test of hypotheses could be conducted the first quarter of 2005, which was before the subprime financial crisis. A bubble could then be identified such that precautionary moves, such as using macro prudential or monetary policy tools, could be implemented in order to dampen the demand for housing. The explosive behavior of housing prices would then be dampened such that a smoother adjustment towards fundamental values could be made instead of a burst of the bubble in order to make the same correction.

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