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Is it really worse with a Bird in Hand? A comparison of fiscal rules for resource-rich economies*

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Abstract

This paper produces a normative evaluation of fiscal rules for a resource-rich small open economy. Ad-hoc fiscal rules might be sub-optimal and imply substantial welfare costs: the target is to analyze the magnitude of the costs by evaluating the relative welfare sub-optimality of these rules. I posit a closed-form solution for the infinite horizon maximization problem of the social planner of a small open economy with resource price uncertainty and precautionary saving. The model is subsequently calibrated to provide a welfare-based comparison between the fiscal rule based on the Permanent Income Hypothesis and the *ad-hoc* Bird in Hand rule. The result of the calibration indicates the presence of a *positive* absolute welfare gap and of an approximately *null relative* wealth loss from employing the Bird in Hand rule. This result is shown to be robust under different parameterizations. No, it is not that much worse with a Bird in Hand policy.

JEL Classification: F41, H30, H60, Q32, Q33, Q38

Keywords: oil depletion, fiscal rules, precautionary saving

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1 Introduction

Economies endowed with nonrenewable resource obtain a substantial rent due to scarcity of resources. Normally this rent accrues to central governments through taxation, royalties and direct ownership. Therefore, fiscal policy issues and fiscal rules assume a crucial role for the intertemporal distribution of this rent. Notwithstanding the increased attention for these topics in the literature of economics of resource management, there is yet no consensus on how governments of countries with substantial amounts of exhaustible natural resources should design their policies in order to optimally spend the resource revenues. As argumented by Frankel (2010) in his survey of the resource curse, governments have often overestimated revenues and dangerously relied on the inflated version of their budget constraints (constituted by the overall fiscal surpluses), therefore incurring in sustained budget deficits which could subsequently prove difficult to be reversed once income from resources had started to deplete itself. In order to impede this inconsistency, some resource-rich countries have implemented more prudent *ad-hoc* fiscal rules, so to reduce discretionality of spending rules and the associated macroeconomic risks. The applicability of these rules is of course limited to countries in which not only the domestic political authorities have full control over the resources, but also in which the accountability of such rules would be ensured. In order to be effective, fiscal rules would need to be backed by a strong political will and complemented by administrative reforms. The political economy literature of the resource curse conveys that the requirement for the effectiveness of a rule is that there are costs or penalties of deviating from it, one of these costs being the loss of reputation in case of deviations from the fiscally responsible behaviour.

Spending behaviours in resource-rich economies have been extensively analyzed at the empirical level. Villafuerte and Lopez-Murphy (2010) have documented fiscal policy behaviour in 31 oil-producing countries during the recent oil price cycle 2000-2008. At first, by decomposing the non-oil primary balance of governments into a cyclical and a structural component, they find evidence that fiscal policy has been procyclical during the boom period and contributed to the volatility of the business cycles. The degree of procyclicality has been found to be high for low-income countries and low for high-income countries[1].

[1] In addition, Villafuerte and Lopez-Murphy (2010) conducted a sustainability analysis estimating the effects of a sudden drop of the resource price on the fiscal budgets. Financing these fiscal deficits might constitute a problem for those countries who did not precautionarily accumulate foreign assets and international reserves during the boom period.

This paper produces a normative analyses of fiscal rules in a resource-rich small open economy. The scope is to obtain a welfare-based assessment of fiscal policy rules which could provide fruitful guidance for designing policies in resource-rich countries. Fiscal rules can be either a theoretical derivation or of the *ad-hoc* type. These *ad-hoc* fiscal rules might be sub-optimal and imply substantial welfare costs: the target of this paper is to analyze the magnitude of these costs by evaluating the relative welfare optimality of two different rules. The model builds on the analogy between the consumption/saving maximization problem of an infinitely lived representative consumer who receives an uncertain labour-income, with that of a social planner of a country who receives an uncertain income stream from its exhaustible resource stock. The extensive literature on permanent income states that a consumer who receives such an income stream will simply spend the return of the present discounted value of his entire wealth. Holding the value of the income fixed (*stock*), the actual timing of the income stream (*flow*) becomes irrelevant. This result suggests that only the amount of the resource wealth will actually matter for the government that is behaving as a permanent income consumer.

Bems and Filho (2010) develop a model with resource price uncertainty in order to compute the magnitude of the precautionary savings motive for a large sample of resource-rich economies. Their model is solved numerically and results show the positive significance of the precautionary savings motive. Building on the framework for oil-producing small open economies provided by Engel and Valdes (2000), Maliszewski (2009) has computed numerically the relative welfare gains of different fiscal rules[2]. His results confirm the supposed sub-optimality of the conservative Bird in Hand rule (BIH) with respect to the fiscal rule based on the Permanent Income Hyphotesis (PIH). Another approach is that of Pieschacón (2009) which analyzes the effects of implementing different sustainable fiscal rules in a dynamic stochastic general equilibrium model with a deteriorating oil sector.

The present paper contributes to this existing literature in at least two ways.

Gelb and Grasmann (2010) also confirm the tendency for oil exporters to alternate period of booms with periods of declining GDP, as a consequence of the price cycles. Therefore they introduce in their model two economic constraints, one on the ability of the economy to absorb high levels of public spending in boom periods, and another macroeconomic adjustment constraint in the face of positive and negative demand shocks.

[2] Maliszewski (2009) obtains quantitative comparisons of the rules through Montecarlo simulations. Random realizations of oil price series are used to obtain path for government expenditures under the various fiscal rules considered. The mean of the social welfare functions (all of them implementing different versions of CRRA utility) over the randomized sample is the criteria used to assess the relative performance of the fiscal rules.

At first, a closed-form analytical solution for the infinite horizon maximization problem of the social planner with resource price uncertainty and precautionary saving is presented. This allows to draw clear theoretical implications by avoiding the black-box effect of numerical analysis (as in Maliszewski (2009)). This result is made possible by the specific assumption of Constant Absolute Risk Aversion (CARA) utility for the representative agent. In addition, the model is calibrated to provide a welfare-based comparison between the fiscal rules based on the PIH and on the *ad-hoc* Bird-in-Hand rule. Comparisons between the welfare implied by the two rules both in the pre and post-depletion eras are also provided.

The present model is built as a partial equilibrium framework in the sense that the government policy decisions do not influence behaviour of private agents in the economy, therefore several macroeconomic variables will automatically be taken as exogenous. The appreciation of the real exchange rate, the diversion of capital and investment resources out from the tradable productive sector into the resource sector and the possibility of rent seeking are all aspects of the resource curse literature which have been deliberately taken away from the point of view of this work. In addition it must be pointed out that the social planning assumption basically requires that the state has full ownership of the economic reward from the utilization of the resources.

The structure of the paper is organized as follows: section 2 introduces the model, section 3 and 4 present the Permament Income Policy and the Bird in Hand Policy, section 5 evaluates the welfare of both the rules whilst section 6 draws the conclusions.

2 The model

I model the intertemporal social planning problem of a representative agent small open economy which receives a stochastic resource windfall. In other words, I look at the optimal consumption/saving problem of the planner of an economy which lasts infinite periods, during which a strictly positive but uncertain exogenous resource income is received. The model is in discrete time. The planner's objective is to choose the optimal level of consumption of the only (public) good in order to maximize the infinite sum of the agent's discounted utility function. Instead of one infinitely-lived agent, the set-up can also be thought of as an infinite sequence of generations of households, each of them living just one period. The government will then try to equalize welfare across generations by optimally allocating consumption using transfers and taking into account the entire stream of oil revenues[3]. In addition, the choice of excluding the possibility of additional government transfers financed by tax revenues from non-oil GDP can be justified by assuming that these non-resource revenues are only transfered within the same generation bearing that specific tax burden.

The motivation for the utility formulation used in this work comes directly from the microeconomics literature about intertemporal consumption, in which Caballero (1990) and Weil (1993) have shown that, for the consumption/saving problem of a consumer with labor-income uncertainty, a CARA instantaneous utility function allows to obtain an analytical closed-form solution with precautionary saving. As previously mentioned, the central role of precautionary saving is also justified by the quantitative results obtained by Bems and Filho (2010). The utility specification of the model is the following:

$$W = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[u(g_t) \right] \right\} \tag{1}$$

$$u(g_t) = -\left(\frac{1}{\alpha}\right) \exp\left(-\alpha g_t\right) \tag{2}$$

W is the social welfare function to be maximized, β represents the intertemporal discount rate parameter, $g_t \, \epsilon \, R_+$ is the government expenditure level at date t (i.e., the consumption of the public good) and $u : R_+ \rightarrow R$ is the CARA instantaneous utility function. In addition to standard assumptions of strictly increasing utility and that $\lim_{g \to 0} u'(g) = \exp\left[-\alpha g\right] = +\infty$, we have that $u'''(g) > 0$, which means strict convexity of marginal utility. In other words, with higher variability of income the planner would choose to save more and consume less. As anticipated above, I assume absence of non-resource income in the economy[4], thus domestic supply-side and investments opportunities are excluded. Moreover, I consider a small open economy on its balanced growth path (which constitute the only realistic option for an infinite-horizon economy since it implies neither growing nor decreasing consumption paths) in which

[3] A similar approach has been adviced by Barnett and Ossowski (2002), although their contribution does not provide a full model of the case with price uncertainty.

[4] Adding a deterministic non-oil GDP series to the framework of this paper would not modify the nature of the results.

An additional observation is that distributing *only* the resource wealth across generations might be motivated by the fact that natural resources, as opposed to domestic non-resource GDP, are indeed an endowment of the whole country's population and not the result of the effort of any specific generation of households.

the domestic interest rate does not deviate from the world interest rate[5]. The planner's infinite horizon constrained optimization problem is then:

$$\underset{\{g_s\}_{s=t}^{\infty}}{Max}\ E_t\left\{\sum_{t=0}^{\infty}\beta^t\left[-\left(\frac{1}{\alpha}\right)\exp\left[-\alpha g_s\right]\right]\right\} \tag{3}$$

$$s.t.\ A_{t+1} = (A_t + Y_t - g_t)R \quad t = 0, 1, 2... \quad A_0 = 0 \tag{4}$$

$$\lim_{t\to\infty} R^{-t}A_{t+1} = 0 \tag{5}$$

Equation [4] represents the flow government's budget constraint. I assume that purchasing of foreign financial assets A_t allows the government to transfer wealth from one period to another. The initial financial wealth endowment of the government is null. By saving a fraction of the resource income revenues, the government starts holding foreign assets. Y_t is the exhaustible resource income, in other words the only income source for the government. Since the private sector does not explicitely appear in the maximization problem, the government does not collect taxes. $R = (1 + r)$ is the constant gross interest rate, and in addition I assume that $\beta R = 1$. In conclusion, a No-Ponzi game condition [5] guarantees that the government is neither borrowing nor lending in the long-run.

The next step is to solve forward the flow budget constraint given in [4] in order to obtain the government's intertemporal lifetime budget constraint, creating a link between the present discounted value of consumption and income:

$$E_t\left[\sum_{s=t}^{\infty}R^{t-s}(g_s)\right] = A_t + E_t\left[\sum_{s=t}^{\infty}R^{t-s}(Y_s)\right] \tag{6}$$

This version of the intertemporal budget constraint states that the expected present discounted value of public consumption has to be at all periods equal to the total current public wealth plus the expected present discounted value of future uncertain resource revenues.

[5] In a model with endogenous non-resource income in which the returns from domestic projects and foreign assets are allowed to be different, domestic capital investments would provide an alternative diversification channel for the social planner in addition to purchasing of foreign assets.

It has to be mentioned as well that the assumption of $r = r^*$ rules out essential features of developing economies, in which capital scarcity might determine the rate of return on domestic spending to be higher than the returns from saving abroad.

A further diversification channel considered in the literature is that of hedging on financial markets (i.e. over-the-counter markets). However, Bems and Filho (2010) document that the total volume of exchange on those markets was estimated to be in 2009 of only 0.18% of proven oil reserves.

2.1 Modeling price uncertainty and income

Bems and Filho (2010) have shown that exhaustible resource prices happen to be substantially more volatile than extraction quantities[6]. This empirical evidence motivates the following approach which abstracts from resource extraction decisions and considers the resource price volatility as the one and only source of uncertainty. For simplicity, I assume oil income for the small open economy to be given in each period by the quantity of oil sold X_t, evaluated at real spot market prices:

$$Y_t = P_t X_t \tag{7}$$

As far as the stock of reserves is concerned, I assume that the peak of oil production has already been met, and no further discoveries of new fields are going to replace the depleting stock. Thus the stock of oil is inevitably depleting until it vanishes. At that point in time, the model will become exclusively deterministic since the only source of uncertainty will disappear. I formalize the depletion dynamics of the stock in the following way, where the depletion rate is represented by the exogenous parameter δ[7]:

$$X_t = (1 - \delta) X_{t-1} \qquad \delta > 0 \tag{8}$$

Hence the crucial role in the present model is played by the volatile price component. An important work in the literature of resource prices is the study by Pindyck (1998) who builds a model in which oil prices are mean-reverting to a quadratic trend which is fluctuating over time. The economic intuition behind the mean-reversion property is that, building on the assumption that the resources are sold in a competitive market, their price will sooner or later revert to the long-run marginal cost. However, after his estimations Pindyck (1998) concludes that, in case oil prices would rise substantially in the subsequent decade (which we know has indeed happened), the multivariate stochastic process model proposed in his work would have not provided any better predic-

[6] For a large sample of oil-producing economies, price volatility has been 2-3 times higher then extraction volatility over the period from 1980 to 2007. In addition, they show that oil production had limited responses to the changes in the price of oil all over the period 1999-2008, therefore demonstrating a very small price elasticity of supply. An explanation for this can be that extraction capacity is costly and time-consuming, making extraction plans not to respond rapidly to short-run changes in prices.

[7] The parameter δ governing the speed and the path of resource depletion is not a control variable for the planner of the economy. Instead, the amount of resource income which is spent or saved will be endogenous.

tions than a simple model with mean-reversion to a fixed linear trend[8]. Hamilton (2009) has conducted a comprehensive statistical investigation of the properties of oil prices. He shows that changes in oil prices have always tended to be permanent, difficult to predict and governed by different stochastic regimes in different epochs. In conclusion he claims that, although forecasts might likely turn out to be far from actual future values, it is *actual* current values which happen to be the best available forecasts. Thus it seems both reasonable and practical to assume in this model that the price of oil follows a random walk without drift of the following kind:

$$P_t = P_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{9}$$

I formulate the income process as a combination of an *exogenously-decreasing trend component* represented by the depleting resource stock and of a *random walk deviation* from this trend represented by the price of oil:

$$Y_t = (P_{t-1} + \varepsilon_t)\left[(1-\delta)\, X_{t-1}\right] \tag{10}$$

3 The Permanent Income Policy

Let us proceed to derive the fiscal spending rule based on the PIH, in other words the optimal consumption function of the maximization problem presented in section 2. This spending rule will be consistent with maintaining the stock of wealth constant over the long-run. This means that the *forward-looking* government does not simply spend out of current resource and financial income, but instead spends out of permanent income or total wealth. In other words, the government optimally chooses a combination of consumption and savings which allows to equalize the welfare of the agent over his/her entire lifetime horizon. The value equation for the problem is given by:

$$V(A_t) = \max_{\{g_t\}} \left\{ u(g_t) + \beta E_t V(A_{t+1}) \right\} \tag{11}$$

Standard solving procedure with the help of the envelope theorem gives the

[8] More recently, Rogoff and Dvir (2009) argue that a very long-run perspective is necessary to understand the true stochastic process lying behind oil prices, because of the structurally different statistical behaviour of prices in different epochs.

classic Euler equation for the marginal utilities of consumption:

$$u'(g_t) = \beta R E_t \left[u^{'}(g_{t+1}) \right] \tag{12}$$

Let us now observe how the introduction of income uncertainty (as a consequence of the resource price uncertainty $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$) triggers the presence of precautionary motives in the optimal consumption rule. The CARA utility specification implies that equation [12] becomes:

$$\exp(-\alpha g_t) = \beta R E_t \exp[-\alpha g_{t+1}] \tag{13}$$

As a consequence of the fact that the income process has normally distributed innovations, I am guessing (and verifying, see A1 in Appendix) that the consumption process will obey the following dynamics:

$$g_{t+1} = g_t + \log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2 + \varepsilon_{t+1} \tag{14}$$

Based on this expected dynamics, we need to specify how future consumption levels are predicted. Conditioning the future unknown level of consumption on the current information gives (see A2 in Appendix):

$$E_t(g_{t+1}) \quad = \quad g_t + \log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2 \tag{15}$$

$$\Rightarrow \quad E_t(g_s) = g_t + (s-t)\left[\log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2\right] \tag{16}$$

This result shows that a more volatile resource income will trigger higher expected consumption growth, in other words a steeper optimal consumption path. This is a consequence of precautionary motives which determine higher current savings to offset possible future adversities. As a result of this, the government's consumption will be expected to grow faster from one period to another.

Now define $\kappa = \log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2$ so that equation [16] becomes $E_t(g_s) = g_t + (s-t)\kappa$. In order to proceed with the derivation of the optimal consumption function, we need to obtain the present discounted values to be inserted in the intertemporal budget constraint given by [6]:

$$E_t\left[\sum_{s=t}^{\infty} R^{t-s}(g_s)\right] = g_t \sum_{s=t}^{\infty} (1+r)^{t-s} + \kappa \sum_{s=t}^{\infty} (1+r)^{t-s}(s-t) \tag{17}$$

$$= g_t\left(\frac{1+r}{r}\right) + \kappa\left(\frac{1+r}{r^2}\right) \tag{18}$$

Let us now turn the attention to the income process. Given the income process we described in equation [10] we have that (see A3 in Appendix):

$$E_t\left(Y_s\right) = (1-\delta)^{s-t} P_t X_t \tag{19}$$

Computing now the present discounted value of income gives:

$$E_t\left[\sum_{s=t}^{\infty} R^{t-s}(Y_s)\right] = P_t X_t \sum_{s=t}^{\infty} (1+r)^{t-s} (1-\delta)^{s-t} \tag{20}$$

$$= P_t X_t \left(\frac{1+r}{r+\delta}\right) \tag{21}$$

We can now substitute these results into equation [6] and solve for the optimal consumption function of the government:

$$g_t\left(\frac{1+r}{r}\right) = A_t + P_t X_t\left(\frac{1+r}{r+\delta}\right) - \kappa\left(\frac{1+r}{r^2}\right) \tag{22}$$

$$g^*_{t,PIH} = \left(\frac{r}{r+\delta}\right) P_t X_t + \left(\frac{r}{1+r}\right) A_t - \left(\frac{1}{2r}\right)\alpha\sigma^2_{\varepsilon} \tag{23}$$

The first term on the right-hand side reflects the government's direct consumption of the resource income. The propensity to consume directly out of the resource revenues is lower than unity since part of the resource revenues is invested by purchasing foreign assets. The second term represents the annuity value of the financial wealth, in other words the government consumes the interest income of its previously accumulated financial wealth. Finally, the last term (obtained by remembering that we assumed that $\beta R = 1$) indicates that uncertain future resource income prospects makes it desirable for the government to precautionarily consume less and save some of its current total wealth. The result implies that, after resources have been depleted, both the direct consumption term and the precautionary motives term will disappear from the optimal consumption function, thus the model becomes deterministic and the PIH-based fiscal rule will simply turn out to be given by $g^*_{t,PIH} = \left(\frac{r}{1+r}\right) A_t$.

In other words, when oil resources are depleted the government will finance its expenditure exclusively by relying on the returns from past savings from resources.

However, the formulation in [23] is not yet closed-form. In fact, the value of the net foreign assets is determined endogenously in the current model and must therefore depend only on model's initial conditions as well as income shocks. By setting $\phi = \frac{\kappa}{r}$ and inserting the spending rule [23] in the dynamics of the budget constraint given in [4] implies:

$$A_{t+1}^{pih} = (1+r)\left[A_t^{pih} + Y_t - \left(\frac{r}{r+\delta}\right)Y_t - \left(\frac{r}{1+r}\right)A_t^{pih} + \phi\right] \quad (24)$$

$$A_{t+1}^{pih} = A_t^{pih} + \left[\frac{\delta(1+r)}{r+\delta}\right]Y_t + (1+r)\phi \quad (25)$$

Solving this difference equation gives back:

$$A_t^{pih} = A_0^{pih} + \left[\frac{\delta(1+r)}{r+\delta}\right]\sum_{s=0}^{t-1} Y_s + t(1+r)\phi \quad (26)$$

Inserting back into equation [23] allows finally to obtain a reduced-form PIH spending rule, as a function only of exogenous terms:

$$g_{t,PIH}^{*} = \left(\frac{r}{1+r}\right)A_0^{pih} + \left(\frac{r}{r+\delta}\right)\left[Y_t + \delta\sum_{s=0}^{t-1} Y_s\right] + (rt-1)\phi \quad (27)$$

4 The Bird in Hand Policy

As opposed to the theoretical and *forward-looking* spending rule based on the PIH, a few countries have recently adopted *ad-hoc* fiscal rules to govern the use of their resource incomes. These rules might help to reduce the procyclicality of fiscal policy and to direct the use of the resource revenues towards long-term sustainability objectives. An example of an *ad-hoc* fiscal rule with high degree of *fiscal conservatorism* is that of Norway[9]. In order to spread the gains from hydrocarbon revenues to future generations of citizens, Norwegian authorities

[9] Another interesting example is that of Chile, as mentioned in Frankel (2010). Chile managed to have a countercyclical fiscal policy due to a structural balance rule which allowed the government to run deficits larger than the target only in case of deep recessions and price of resource (copper) being lower than expected. The structural balance rule factors out the cyclical and random effects of GDP and of the copper price. The cyclical adjustment to the copper is based on the gap between the actual export price and an estimated long-term moving average reference price.

have established in 1996 a sovereign wealth fund in which all resource revenues are placed. According to the annexed spending rule, only 4% per annum of the fund wealth can be used for consumption (although there are exceptions to this spending level in case of recessions)[10]. In other words, the Bird in Hand policy allows consumption only of the resource revenues which have already been liquidated (i.e. it has a *backward-looking* nature). Why should this rule be preferred over the policy based on the PIH? A possible disadvantage of the PIH rule is that it ignores future expenditure committments related to population dynamics and ageing. In addition, is it really that the BIH rule qualifies as a more prudent spending rule? The rule is supposed to limit the macroeconomic impact of the resource revenues by smoothing the spending prospect of these revenues. However, it has not be taken for given that this rule will allow to accomodate unexpected fiscal committments in the long-run. Harding and Van der Ploeg (2009) have looked specifically at the Norwegian economy and have argued that neither the PIH spending rule nor the more conservative BIH rule will determine a level of foreign assets accumulation high enough to face the increasing future burden coming from population ageing and related rising pension commitments[11]. The BIH rule is defined as:

$$g^*_{t,BIH} = \left(\frac{r}{1+r}\right) A_t^{bih} \tag{28}$$

Because of the fact that the BIH rule prescribes that the entire resource income be stored in a sovereign fund, no intertemporal consumption/saving problem arises for the government. When this rule is adopted, the stochastic process of the oil price becomes a negligible variable because the spending rule will not *directly* react to it anymore. Government consumption will be affected only indirectly through changes in accumulated financial assets rather than through variations in the present value of resource revenues. What does the rule imply for the dynamics of the budget constraint? Let us go back to

[10] It has to be pointed out that the specific target of the norwegian spending rule is to smooth the combination of both domestic and resource income, whilst in the current framework I abstracts from the former as explained in section 2. This implies that the Bird in Hand policy formulation of this paper is a somehow stylized form of the more detailed norwegian spending rule, although the rationale behind the rule remains the same, i.e. trying to reduce the uncertainty from the resource depletion and income volatility in order to spread the benefits equally through generations and face future unexpected fiscal committments.

[11] Jafarov and Leigh (2007) have also analyzed the long-run sustainability of Norway's public finances under different fiscal rules. Their conclusion is that no rule dominates the others, and that under any reasonable rule Norway's oil wealth will unlikely be enough to cover the projected increase in future spending commitments.

equation [4] and substitute in the BIH rule to get:

$$A_{t+1}^{bih} = (1+r)\left[A_t^{bih} + Y_t - \left(\frac{r}{1+r}\right)A_t^{bih}\right] \quad (29)$$

$$A_{t+1}^{bih} = A_t^{bih} + (1+r)Y_t \quad (30)$$

This tells us that after resource depletion, the amount of wealth which is saved in the fund stays at a constant level, since the oil income entirely depleted itself: $A_{t+1}^{bih} = A_t^{bih}$ (the same will occur for the PIH rule since both rules become identical after depletion, net of the difference in accumulated assets). Solving the difference equation obtained in [30] gives:

$$A_t^{bih} = A_0^{bih} + (1+r)\sum_{s=0}^{t-1} Y_s \quad (31)$$

In turn this allows to express the BIH spending rule as a function only of exogenous terms and initial values:

$$g_{t,BIH}^{*} = \frac{r}{1+r}\left[A_0^{bih} + (1+r)\sum_{s=0}^{t-1} Y_s\right] \quad (32)$$

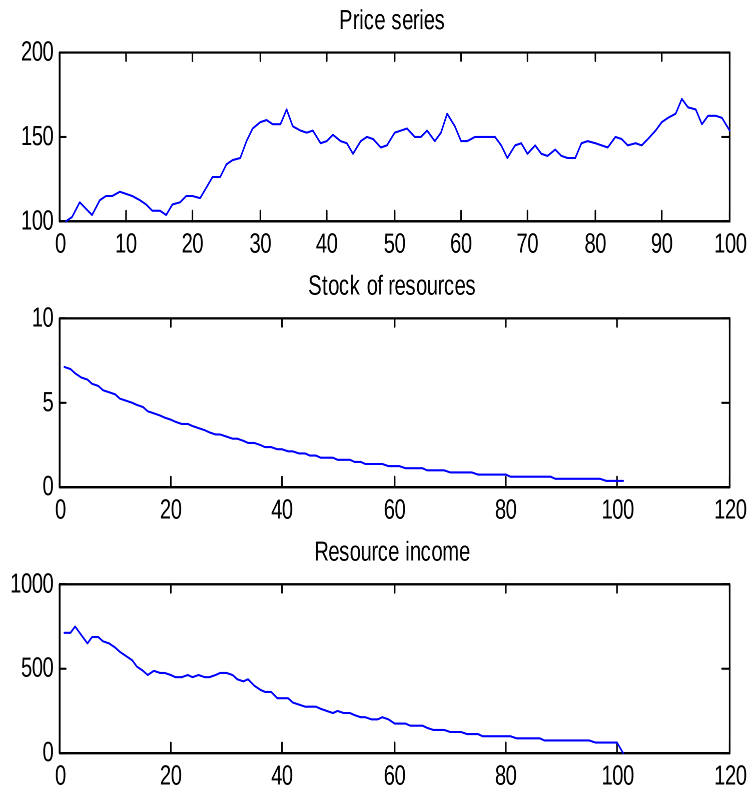
5 Welfare based evaluation of fiscal rules

Let us proceed with the welfare-based analysis. The simplest approach for a welfare comparison between the two fiscal rules is to investigate the magnitude of the compensation parameter which would make the representative agent at least as well off as under a fiscal rule rather than the other. Lucas (2003) used a similar approach to identify the welfare gain from fully eliminating income uncertainty for a risk-averse consumer. When it comes to comparing rules, a fiscal rule would be logically preferred over the other if its contingent plan for consumption and asset accumulation yields a higher level of expected conditional welfare, both before and after depletion. Since the PIH benchmark represents the optimal rule under the assumptions of the previous section's optimization problem, the compensation parameter which measures the welfare cost of switching from the optimal rule to the BIH rule will have to be positive. Let us begin by assuming that:

$$W'_{BIH} \equiv E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[-\frac{\exp(-\alpha g^*_{t,BIH}(1+\lambda))}{\alpha}\right]\right\} = \quad (33)$$
$$= E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[-\frac{\exp(-\alpha g^*_{t,PIH})}{\alpha}\right]\right\} \equiv W_{PIH}$$

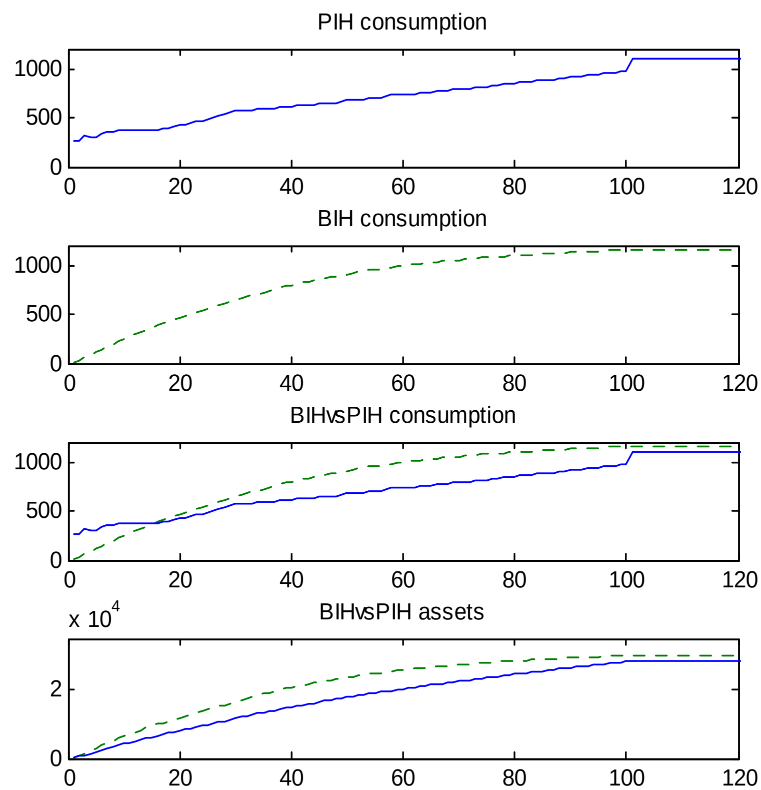
where W'_{BIH} is the total welfare implied by the BIH rule augmented by the compensation parameter.

The scope is to calibrate the model using parameters and initial values for one generic oil-exporting country, for example Norway. One period is set as to be equivalent to one year, the net interest rate is $r = 0.04$ (thus subjective discount rate becomes $\beta = 1/1.04$), the coefficient of absolute risk aversion is standard as in the literature and given by $\alpha = 0.5$. The initial net foreign asset position shows that Norway had a remarkable NFA of 79% of GDP at the end of 2009 (source: Statistics Norway). The amount of exhaustible oil reserves for Norway was estimated to be of 7.1 thousand million barrels at the end of 2009 (source: BP Statistical Review of World Energy 2010). The depletion rate is $\delta = 0.03$, and the lifetime of oil reserves has been arbitrarily set to 100 years (figure 1 of the Appendix shows instead the real dynamics of depletion of oil reserves in Norway). Initial value for the real price of oil has been set up at the price of Brent at the end of 2010, which was approximately of $100 (source: BP Statistical Review of World Energy 2010), in the figure 2 of the Appendix is plotted the annual crude real and nominal oil price series with range 1968-2011, (source EIA). In addition, the variance of the errors for the price series has been calibrated to $\sigma^2_\varepsilon = 30$ according to author's calculations. The next figure shows an example (one random realization) of the dynamics of the uncertain resource revenues:



The simulated series for the resource price, stock and revenues subsequently determine the consumption and asset accumulation prospects under the two fiscal rules. As explained before, PIH-based rule implies that a fraction of the resource revenues are accumulated in a fund and subsequently their capital income is consumed whenever resource stock is depleted, in order to sustain the consumption of the representative agent over its infinite future. Therefore the consumption series for this rule will look substantially flat, with only limited variation due to price volatility in the pre-depletion era.

The BIH rule calls instead for a faster initial asset accumulation because it prescribes that all the resource income be initially used to buy foreign assets. The amounts of the assets accumulated in the fund turns out to be constantly higher for the case of the BIH rule. The faster pace of foreign asset accumulation determines an initial lower consumption level with respect to that of the PIH. However, due to the higher amount of financial assets accumulated up until depletion, this rule allows after depletion a sustained *higher* level of public consumption. Let us visually compare the consumption series implied by both fiscal rules up until the depletion year (after depletion the two rules coincide and the consumption gap stays constant):



We can observe that up until approximately the 20th year/period, PIH-based rule dominates BIH-based rule, because of the above mentioned conservatorism of the latter. However, the faster asset accumulation and the decreasing revenues due to the depleting stock of reserves determine that from that period onwards the BIH rule allows a higher consumption level.

Let us now evaluate welfare of the two fiscal rules. We estimate the compensation parameter λ which appeared in [33] by computing the average of the total difference in discounted utility between the two series of consumption over the entire representative agent's horizon. To obtain this measure, I simulated several thousands times the revenue series and the correspondent consumption and discounted utility series , and then computed the average of these results in order to obtain a robust estimate for the compensation parameter, which turns to be indeed based on the assumptions of the model but at the same time not dependent on a single realization of the resource price and revenue series. This is the formulation of λ:

$$\lambda(A_t) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[-\frac{\exp(-\alpha g^*_{t,PIH})}{\alpha} \right] - \sum_{t=0}^{\infty} \beta^t \left[-\frac{\exp(-\alpha g^*_{t,BIH})}{\alpha} \right] \right\} \quad (34)$$

This calibration exercise gives the expected result: the PIH-based rule provides higher absolute total welfare for the representative agent of our economy. As seen above, this welfare gap turns to be entirely built in the beginning of the pre-depletion era in which the PIH-based rule temporarily enjoys a higher welfare. The following results summarize the simulation findings:

Table 1	*PIH - BIH*
λ *before depletion*	0.4209
λ *after depletion*	-0.0002
λ *total*	0.4207
$(\lambda \ total)/\overline{g}^*_{PIH}$	0.00003

However, the interesting observation is that in case the total compensation parameter λ is expressed in terms of the country's average consumption level provided by the benchmark PIH fiscal rule ($\overline{g}^*_{PIH}$), we obtain that $\frac{\lambda}{\overline{g}^*_{PIH}} \approx 0$. In other words, the *relative* welfare loss in terms of average PIH consumption suffered from switching from the PIH rule to the *ad-hoc* BIH rule is *approximately null*[12].

In addition, let us observe whether these results are robust with respect to some variations in the parameters included in the analysis. I will allow variation for one parameter at a time. The interest rate will now be allowed to vary between $0.03 < r < 0.06$, the coefficient of absolute risk aversion would be moving within $0.03 < \alpha < 0.08$, and different values of the variance of the resource price will be considered. The relative value of λ in terms of average

[12] This result is robust to the following slightly different way of estimating the compensation parameter: $\lambda(A_t) = \frac{E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[-\frac{\exp(-\alpha g^*_{t,PIH})}{\alpha} \right] \right\}}{E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[-\frac{\exp(-\alpha g^*_{t,BIH})}{\alpha} \right] \right\}}$

consumption of the PIH rule is entered in brackets:

Table 2	*PIH - BIH*	*PIH - BIH*	*PIH - BIH*	*PIH - BIH*
	$r = 0.3$	$r = 0.4$	$r = 0.5$	$r = 0.6$
$\lambda(\lambda/\overline{g}^*_{PIH})$	$0.6145(\approx 0)$	$0.4207(\approx 0)$	$0.2904(\approx 0)$	$0.2017(\approx 0)$
	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.8$
$\lambda(\lambda/\overline{g}^*_{PIH})$	$1.2884(\approx 0)$	$0.7130(\approx 0)$	$0.4207(\approx 0)$	$0.1057(\approx 0)$
	$\sigma^2_\varepsilon = 30$	$\sigma^2_\varepsilon = 40$	$\sigma^2_\varepsilon = 50$	$\sigma^2_\varepsilon = 60$
$\lambda(\lambda/\overline{g}^*_{PIH})$	$0.4207(\approx 0)$	$0.4207(\approx 0)$	$0.4207(\approx 0)$	$-0.1352(\approx 0)$

At first let us notice that from the specific formulation of the model used above, no straightforward conclusions could be exerted as regards the effects of a higher interest rate r on the welfare gap. In fact, an increase in the interest rate increases *ceteris paribus* the relative convenience from investing the resource income into financial assets, thus determining a higher post-depletion welfare gain for the BIH rule over the PIH rule, hence *reducing* the overall welfare gap. However, as it can be seen from equation [27] a higher net interest rate translates as well into higher *direct* consumption of the resource income and *generally* higher level of consumption under the PIH rule, hence *increasing* again the welfare gap between the two rules. The results in table 2 show that the former effect plays a bigger role, since the absolute welfare gap steadily decreases with higher levels of r.

As far as the variations in the compensation parameter with respect to the coefficient of absolute risk aversion α are concerned, the results predict again a clear diminishing tendency for the overall welfare gap with higher α. This intuitively means that for higher degrees of risk aversion stronger precautionary motives will determine a more prudent PIH rule and a *decreasing* overall absolute welfare gap.

As concerning the variance of the resource revenues, the structure of the model would predict that higher volatility of income translates into higher amount of precautionary savings and subsequently a lower average consumption level under the PIH rule. On the other hand, as it has been mentioned in section 4, the properties of the *ad-hoc* BIH policy determine that this rule does not react directly to changes in income and thus to changes in his volatility (although increased volatility does indeed determine higher volatility of the consumption levels under this rule as well). However, this prediction of steadily decreasing welfare gap is not clearly confirmed by the simulation's results, which shows that the absolute welfare gap does *not* react to increasing volatility up until $\sigma_{\varepsilon}^{2} = 50$, whilst it decreases exponentially and becomes even negative for $\sigma_{\varepsilon}^{2} \geqslant 60$.

More importantly, the result for the *relative* welfare gap is robust to all these changes in the parameters of the model, confirming the result that under the assumptions of the current model the BIH is not at all a "worse" option as regards the welfare of the representative agent.

6 Concluding remarks

In this paper I constructed a model of a small open economy endowed with a stochastic income from exhaustible resources. The stylized features of the model do not allow to provide straightforward policy recommendations (country-specific parameters often play a crucial role in determining the design of spending policies), however they provide a clear understanding of the features and properties of the two alternative fiscal rules under observation. I assumed that the planner of the economy could decide to spend the income from the resource according to two different fiscal rules, one rule being derived from his intertemporal consumption/saving problem whilst the other being an *ad-hoc* rule. The purpose of the paper was to quantitatively evaluate the relative welfare-based optimality of these two fiscal rules.

After presenting the closed-form expressions of both the maximization-based rule and the alternative *ad-hoc* rule, the model was calibrated in order to simulate the resource price and subsequently obtain the income dynamics and the consumption series. In addition, applying the CARA utility function allowed to evaluate their relative welfare-based optimality. The result shows indeed the presence of an absolute welfare loss suffered from switching from the PIH rule to the *ad-hoc* BIH rule, whilst the welfare loss relative to one period of average consumption under the PIH rule turned out to be null. In addition, sensitivity tests have proven the robustness of this result under different parameterizations.

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7 Appendix

Here are presented in more details all the steps which were omitted in the article.

A1) From equation [13] to [14]. The "guess" on the consumption dynamics formulation is:

$$g_{t+1} = g_t + \log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2 + \varepsilon_{t+1}$$

Let us verify whether this process specification works by inserting in [13]:

$$\begin{aligned} 1 &= \beta R E_t \exp[-\alpha(\log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2 + \varepsilon_{t+1} + g_t - g_t)] \\ 1 &= \beta R \exp[-\alpha(\frac{1}{\alpha})\log(\beta R)] \exp(-\frac{\alpha^2}{2}\sigma_\varepsilon^2) E_t \exp(-\alpha\varepsilon_{t+1}) \end{aligned}$$

We know from the properties of the log-normal distribution function that, whenever $X \sim N(\mu, \sigma^2)$, then $E\exp(X) = \exp(\mu + \frac{1}{2}\sigma^2)$. Thus we can apply this result to the normally distributed innovations in order to obtain $E\exp(-\alpha\varepsilon_{t+1}) = \exp[E_t(-\alpha\varepsilon_{t+1}) + \frac{1}{2}Var(-\alpha\varepsilon_{t+1})] = \exp(\frac{\alpha^2\sigma_\varepsilon^2}{2})$ and simplify as follows:

$$\begin{aligned} 1 &= R\beta \exp[-\alpha(\frac{1}{\alpha})\log(R\beta)] \exp(-\frac{\alpha^2\sigma_\varepsilon^2}{2}) \exp(\frac{\alpha^2\sigma_\varepsilon^2}{2}) \\ 1 &= R\beta \exp[\log(R\beta)^{-1}] \\ 1 &= 1 \end{aligned}$$

A2) From equation [15] to [16]:

$$\begin{aligned} E_t g_{t+1} &= g_t + \log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2 \\ E_t g_{t+2} &= E_t(E_{t+1} g_{t+2}) \\ &= E_t(g_{t+1} + \log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2) \\ &= g_t + 2\log(\beta R)^{\frac{1}{\alpha}} + \alpha\sigma_\varepsilon^2 \\ &\Rightarrow E_t g_s = g_t + (s-t)\left[\log(\beta R)^{\frac{1}{\alpha}} + \frac{\alpha}{2}\sigma_\varepsilon^2\right] \end{aligned}$$

A3) From equation [10] to [19]:

$$
\begin{aligned}
E_t\left(Y_{t+1} \mid Y_t\right) &= E_t\left[\left(P_t+\varepsilon_{t+1}\right)(1-\delta) X_t\right] \\
&= E_t P_t(1-\delta) E_t X_t = P_t(1-\delta) X_t \\
E_t\left(Y_{t+2} \mid Y_t\right) &= E_t\left[E_{t+1} Y_{t+2}\right] \\
&= E_t\left[P_{t+1}(1-\delta)^2 X_t\right] = (1-\delta)^2 P_t X_t \\
E_t\left(Y_s \mid Y_t\right) &= (1-\delta)^{s-t} P_t X_t
\end{aligned}
$$

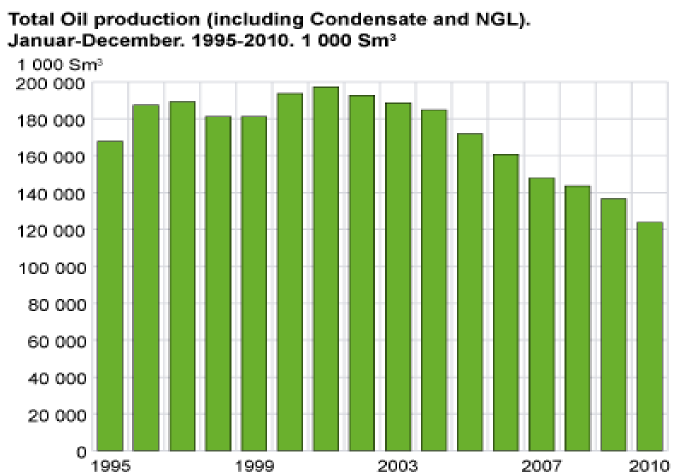

Figure 1: Total oil production in Norway. Source: SSB

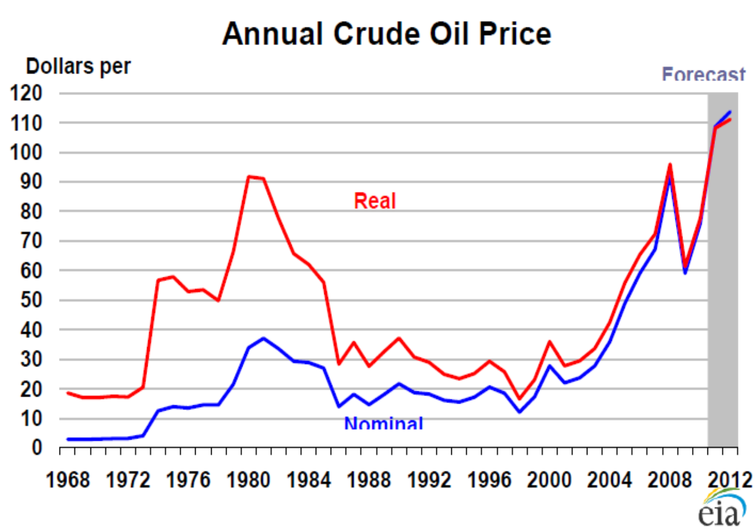

Figure 2: Annual Crude Oil price. Source: EIA Energy Outlook.