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A DSGE Model with Housing in the Cointegrated VAR Framework*

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Abstract

A vector autoregressive model (VAR) is estimated, and restrictions pertaining to a dynamic stochastic general equilibrium (DSGE) model are imposed in the cointegrated vector autoregressive framework showing that the theoretical restrictions can not be accepted. Comparing impulse response functions from the theoretical model and the restricted empirical model shows that the results from the theoretical model are not found in the data.

Keywords: DSGE, Housing, Cointegrated VAR.
JEL: C32, E32, R21.

1 Introduction

A vector autoregressive model (VAR) is estimated, and steady state properties of a theoretical dynamic stochastic general equilibrium (DSGE) model is tested in the VAR through the cointegrated VAR (CVAR) framework. These properties are not accepted, indicating that the theoretical steady state is not found in the long-run properties of the data. Impulse response functions from the CVAR and the DSGE model are also analyzed, indicating that the data does not seem to be able to recover the results from the theoretical model. The approach of testing relationships in a DSGE model using the CVAR framework is conducted along the lines of Juselius and Franchi (2007).

Specifically, the DSGE model in Iacoviello (2005) is used as a theoretical model. This model includes housing in a new Keynesian model by including nominal debt contracts and collateral constraints tied to housing values. One of the key relationships described by the model in steady state is that there should be a constant relationship between output and housing prices. This is clearly not found in the data for the period used in the estimation in Iacoviello (2005).

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Iacoviello (2005) investigates the impulse response functions for a four-equation VAR model estimated using minimum distance estimation for the impulse response functions, and compare them to impulse response functions an unrestricted VAR using filtered variables. This paper extends the analysis by investigating the impulse response functions of a CVAR model, which will combine the estimated VAR with the theoretical DSGE model in Iacoviello (2005).

I start by estimating an unrestricted VAR using the same variables as in Iacoviello (2005), but instead of using variables that are filtered through the Band Pass filter, I use the variables in logs prior to filtering. This avoids drawbacks of using filtered variables, e.g. that the filter removes information in the data. None of the variables are stationary (tests are shown in table 10), which motivates estimation using cointegration, since steady state relationships often indicate stationary ratios between variables, which is equivalent to stationary linear combinations of variables as indicated by cointegration if logs of variables are used.[1]

The next section gives a brief description of the theoretical DSGE model and how the relationships between the theoretical variables may be represented by the cointegrated VAR model and a moving average representation. Section 3 presents the estimated model and how restrictions from the theoretical model may be imposed on it, while section 4 tests the hypotheses pertaining to the long-run relations implied by the theoretical model. Then, section 5 shows the estimated moving average representation and section 6 presents the structural moving average model and the impulse response functions that follows and compare these to the impulse response functions given by Iacoviello (2005). The final section concludes.

2 Theoretical model

The theoretical model investigated in this paper is, the DSGE model in Iacoviello (2005) which incorporates the financial accelerator by using the housing as an important financial asset. The model is mainly taken from Bernanke et al. (1999), in addition to adding collateral constraints tied to real estate values for firms as in Kiyotaki and Moore (1997) and including nominal debt contracts.

The theoretical steady-state relations implied by the model should be found in the long-run properties of the data, which may be investigated through the cointegrated VAR model. These theoretical relationships can be rearranged in order to get equations that explain a relationship between the observed variables used in the estimated VAR and CVAR model.

2.1 The agents in the model economy

The DSGE model in Iacoviello (2005) contains an economy populated by patient households (their variables are denoted with a prime as superscript), impatient households (denoted with a double prime), entrepreneurs (denoted without any primes), retailers and a central bank.[2]

[1] The log of a ratio is the same as the differences between the log of the variables in the ratio, e.g. $\ln(y/x) = \ln y - \ln x$.

[2] Iacoviello (2005) introduces both a "basic" and a "full" model, where the latter contains an impatient household sector in addition to the patient households as well as allowing for variable capital investment for the entrepreneurs. The full model is considered here.

Patient households maximize a lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln c'_t + j \ln h'_t - (L'_t)^\eta/\eta + \chi \ln(M'_t/P_t)),$$

where $\beta \in (0,1)$ is the discount factor, c'_t is consumption at time t, h'_t is the holding of housing, L'_t are hours work and M'_t/P_t are money balances divided by the price level. j is a preference parameter related to housing, η measures the labor supply aversion and χ measures the preference for holding money, s.t.

$$c'_t + q_t \Delta h'_t + R_{t-1} b'_{t-1}/\pi_t = b'_t + w'_t L'_t + F_t + T'_t - \Delta M'_t/P_t,$$

where $q_t \equiv Q_t/P_t$ denotes the real housing price, $w'_t \equiv W'_t/P_t$ the real wage and $b'_t \equiv B'_t/P_t$ denotes the amount borrowed in real terms where they pay back $R_{t-1}B'_{t-1}/P_t$ where R_{t-1} is the gross nominal interest rate on loans between period $t-1$ and t. $\pi_t = P_t/P_{t-1}$ is the gross inflation rate, F_t are lump-sum profits received from the retailers and T'_t are net transfers from the central bank financed by printing money M'_t.

The impatient households are assumed constrained and maximize

$$E_0 \sum_{t=0}^{\infty} (\beta'')^t (\ln c''_t + j_t \ln h''_t - (L''_t)^\eta/\eta + \chi \ln M''_t/P_t)$$

where $\beta'' < \beta$ which guarantees that they will hit the borrowing constraint in equilibrium. All variables are as for the patient households except that the variables pertaining to the impatient households are denoted by a double instead of a single prime as superscript. This is maximized s.t.

$$c''_t + q_t \Delta h''_t + R_{t-1} b''_{t-1}/\pi_t = b''_t + w''_t L''_t + T''_t - \Delta M''_t/P_t - \xi_{h,t},$$

where $\xi_{h,t} \equiv \phi_h (\Delta h''_t/h''_{t-1})^2 q_t h''_{t-1}/2$ is a housing adjustment cost such that households are not perfectly mobile. The borrowing limit is given as

$$b''_t \leq m'' E_t(q_{t+1} h''_t \pi_{t+1}/R_t),$$

where m'' is the loan-to-value ratio for the impatient households.

Entrepreneurs produce an intermediate good Y_t according to a Cobb-Douglas constant returns-to-scale technology

$$Y_t = A_t K_{t-1}^{\mu} h_{t-1}^{\nu} L_t'^{\alpha(1-\mu-\nu)} L_t''^{(1-\alpha)(1-\mu-\nu)},$$

where A is random technology, K is capital and L' and L'' are the labor from the patient and the impatient households, respectively. They maximize

$$E_0 \sum_{t=0}^{\infty} \gamma^t \log c_t$$

where $\gamma < \beta$, s.t.

$$Y_t/X_t + b_t = c_t + q_t \Delta h_t + R_{t-1} b_{t-1}/\pi_t + w'_t L'_t + w''_t L''_t + I_t + \xi_{e,t} + \xi_{K,t}$$

$$b_t = mE_t(q_{t+1}h_t\pi_{t+1}/R_t)$$

and technology, where $\xi_{e,t} \equiv \phi_e(\Delta h_t/h_{t-1})^2 q_t h_{t-1}/2$ and $\xi_{K,t} \equiv \psi(I_t/K_{t-1} - \delta)^2 K_{t-1}/(2\delta)$ are adjustment costs for housing and capital, respectively.

The retailers buy intermediate goods Y_t from entrepreneurs, differentiate them and sell them. This imposes sticky prices in the model and the New Keynesian Phillips curve in the log-linearized model.

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} - \kappa\hat{X}_t + \hat{u}_t,$$

where $X_t \equiv P_t/P_t^w$ is a markup of final over intermediate goods since P_t^w is the wholesale price and P_t is the final good price index. The hat over a variable implies the percentage deviation from its steady state, i.e. approximated by log deviation, and $\hat{u}_t$ expresses an inflationary shock.

The central bank follows a Taylor-type interest rate rule, which is log-linearly expressed as

$$\hat{R}_t = (1 - r_R)((1 + r_\pi)\hat{\pi}_{t-1} + r_Y\hat{Y}_{t-1}) + r_R\hat{R}_{t-1} + \hat{e}_{R,t},$$

where r_R, r_π and r_Y are coefficients in the Taylor rule pertaining to past interest rate, inflation and output in deviation from steady state, respectively. $\hat{e}_{R,t}$ corresponds to a monetary policy shock.

The first-order condition for debt for the patient households that maximizes their lifetime utility function is

$$\frac{1}{c'_t} = \beta E_t \frac{1}{c'_{t+1}} \frac{R_t}{\pi_{t+1}},$$

which in steady state yields

$$\beta = \frac{\pi}{R}. \tag{1}$$

In steady state, gross inflation, π, is assumed equal to unity, such that we have a zero-inflation steady state, where

$$R = \frac{1}{\beta}, \tag{2}$$

which indicates that the nominal interest rate is stationary in the steady state and the gross nominal interest rate equals $(1/\beta)$.

The entrepreneurs' first-order condition for housing when they maximize their lifetime utility is

$$\frac{q_t}{c_t} = E_t\left(\frac{\gamma}{c_{t+1}}\left(\nu\frac{Y_{t+1}}{X_{t+1}h_t} + q_{t+1}\right) + \lambda_t m\pi_{t+1}q_{t+1}\right),$$

which in steady state becomes

$$\frac{q}{c} = \gamma\frac{q}{c} + \frac{1}{c}\gamma\nu\frac{Y}{X}\frac{1}{h} + \lambda m\pi q. \tag{3}$$

We also have the following first-order condition for debt for the entrepreneurs:

$$\frac{1}{c_t} = E_t\gamma\frac{R_t}{c_{t+1}\pi_{t+1}} + \lambda_t R_t$$

which in steady state is

$$\lambda c = \frac{1}{R} - \frac{\gamma}{\pi}.$$

Inserting this into the steady state of the first-order condition for housing, (3), we get

$$\frac{q}{Y} = \frac{\gamma\nu}{1-\gamma-(\frac{\pi}{R}-\gamma)m}\frac{1}{Xh}. \tag{4}$$

We should also insert for the housing stock, h, which depends on R and π and a combination of various parameters as shown in the technical appendix of Iacoviello (2005) where an expression for h may be found after calculating various first-order conditions for the agents in the economy. Using various equations, we obtain an expression for the steady state of the housing stock. Inserting this yields the following steady state relationship:

$$\begin{aligned}\frac{q}{Y} = \frac{j'}{1-\beta}\Bigg(s' + (1-\beta)\Bigg(m\frac{\gamma\nu}{1-\gamma-(\beta-\gamma)m}\frac{1}{X} \\ +m''\frac{j''}{1-\beta''-m''(\beta-\beta'')}\frac{s''}{1+(1-\beta)m''\frac{j''}{1-\beta''-m''(\beta-\beta'')}}\Bigg)\Bigg) \\ +\frac{j''}{1-\beta''-m''(\beta-\beta'')}\frac{s''}{1+(1-\beta)m''\frac{j''}{1-\beta''-m''(\beta-\beta'')}}+\frac{\gamma\nu}{1-\gamma-(\beta-\gamma)m}\frac{1}{X}\end{aligned} \tag{5}$$

We would like to express a linear relationship between house prices, output, the nominal interest rate and inflation. This may be done by taking logs on each side of (5) and a first-order Taylor approximation such that $\beta = (\pi/R)$ is also expressed linearly. This yields

$$\ln q = \ln Y + a + b(\pi/R), \tag{6}$$

where a and b consists of a combination of a number of different parameters. (6) then suggests a linear combination between log of house prices, log of output and (π/R). Since π is the gross inflation rate and R is the gross nominal interest rate, (π/R) is the inverse of the gross real interest rate, following the Fisher equation (Fisher, 1930).

The steady state relationship expressed in (6) may be related to the statistical concepts of the CVAR model, through the cointegrating relations. If the data for the variables expressed in (6) are non-stationary, a cointegrating relation should explain a stationary relationship between these non-stationary variables.

In the data set used for the estimation, the nominal interest rate and the inflation rate differs from its theoretical counterparts π and R since the theoretical model uses the gross nominal interest rate and inflation, whereas the data set contains net nominal interest rate and net inflation. The steady state relationship (π/R) may be defined as

$$1 + R^{real,net} = \frac{1+R^{net}}{1+\pi^{net}},$$

where $R^{real,net}$, R^{net} and π^{net} are net real interest rate, net nominal interest rate and net inflation, respectively. This may, assuming R^{net} and π^{net} are small, be approximated to

$$R^{real,net} \approx R^{net} - \pi^{net},$$

such that the difference between net nominal interest rate and net inflation also expresses the Fisher relationship.

Furthermore, we may approximate (π/R), assuming R^{net} and π^{net} are small, to $(1+\pi^{net}-R^{net})$[3], such that we may rewrite (6) in order to match the nominal variables (corresponding to the variables in the data set to be defined below) to

$$\ln q = \ln Y + \alpha - b(R^{net} - \pi^{net}), \tag{7}$$

where $\alpha \equiv (a+b)$. This yields an inverse housing demand equation in which housing demand depends positively on aggregate output and negatively on the real interest rate, given $b > 0$. This condition is fulfilled, since $b = 1.28$ assuming $j = j' = j'' = 0.1$ and using the estimated parameter values in Iacoviello (2005).

2.2 Theoretical implications for the cointegrated VAR model

In order to relate (7) to the cointegrating relations in the CVAR model, the choice of rank for the model determines how we should make the connection between the theoretical and the statistical model. The rank determines the number of cointegrating relations, which implies how many stationary relationships that exists in the data.

According to the theoretical model and the relationship in (7), we have a stationary relationship between all four observable variables. If we consider output and housing prices to be non-stationary and driven by the same stochastic trend, and the nominal interest rate and inflation to be stationary, as shown by the theoretical model in (1) and (2), we may set the rank to $r = 3$ since any stationary linear combination is called a cointegrating relation (Lütkepohl and Krätzig, 2004). A relationship between housing prices and output, as well as stationarity of the nominal interest rate and inflation should then be expected as three cointegrating relationships. This corresponds to the following three cointegrating relations:

$$\begin{aligned} &\ln q_t - \ln Y_t \sim I(0) \\ &R_t \sim I(0) \\ &\pi_t \sim I(0), \end{aligned} \tag{8}$$

which implies one common stochastic trend which drives housing prices and output.

If we consider the nominal interest rate and inflation to be non-stationary (which is the case for the observed variables in the sample), we obtain a different interpretation of the theoretical long-run relationship. From (7), we see that if the nominal interest rate and the inflation rate is non-stationary, there should be a long-run relationship between all of the four observable variables that is stationary in the long run, and three common driving trends shuold drive the model. This yields the following cointegrating relation when setting the rank to $r = 1$:

$$\ln q_t - \ln Y_t - \alpha + b(R_t - \pi_t) \sim I(0). \tag{9}$$

If there is a stationary long-run relationship between the nominal interest rate and inflation, this could constitute a separate cointegration vector such that the rank should be set to $r = 2$, yielding

$$\begin{aligned} &\ln q_t - \ln Y_t \sim I(0) \\ &R_t - \pi_t \sim I(0). \end{aligned} \tag{10}$$

[3] $\ln(\frac{1+\pi^{net}}{1+R^{net}}) \approx \pi^{net} - R^{net} \Rightarrow \frac{1+\pi^{net}}{1+R^{net}} = \exp\{\pi^{net} - R^{net}\} \approx 1 + \pi^{net} - R^{net} + \frac{(\pi^{net}-R^{net})^2}{2!} \approx 1 + \pi^{net} - R^{net}$

The hypotheses pertaining to (8), (9) and (10) may be tested statistically through the cointegrating relationships in the estimated CVAR model.

2.3 MA representation

The MA representation of the VAR model is useful for formulating assumptions underlying the DSGE model in Iacoviello (2005) as testable hypotheses within the VAR.

The DSGE model is driven by output, which is driven by productivity A_t, as usually assumed in real business cycle and DSGE models. Furthermore, the productivity is assumed to follow an AR(1) process. The model in Iacoviello (2005) is also extended with AR(1) dynamics in the short-run changes of output, housing prices, inflation and the interest rate.

If we assume that (log of) total factor productivity (TFP), a_t, drives the model, we have the following conceptual MA representation of the DSGE model in Iacoviello (2005):

$$\begin{bmatrix} y_t \\ q_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} d_{11} \\ d_{12} \\ d_{13} \\ d_{14} \end{bmatrix} [a_t] + \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \end{bmatrix}$$

where $v_t = Dv_{t-1} + \xi_t$ and $\xi_t' = [\xi_{1,t}, \xi_{2,t}, \xi_{3,t}, \xi_{4,t}]$ is $IN(0, V)$ and uncorrelated with ε_t which is the error term in the AR(1) process of total factor productivity a_t. The stochastic trend represented by productivity should not affect inflation and the nominal interest rate since they are assumed to be stationary. If a_t is a unit root process, both output and housing prices should be equally affected by the TFP stochastic trend, since output affects housing prices. This is shown in the log-linear housing demand equation below:

$$\hat{q}_t = \gamma_e \hat{q}_{t+1} + (1 - \gamma_e)(\hat{Y}_{t+1} - \hat{X}_{t+1} - \hat{h}_t) - m\beta \hat{rr}_t - (1 - m\beta)\Delta\hat{c}_{t+1} - \phi_e(\Delta\hat{h}_t'' - \beta\Delta\hat{h}_{t+1}'')$$

where $\hat{rr}_t$ denotes the real interest rate, where we see that output (which is driven by TFP) is the only factor driving housing prices.

This implies that we should have $d_{11} = d_{12}$. Furthermore, we should have that $d_{13} = d_{14} = 0$ since inflation and the nominal interest rate is assumed stationary. This implies rank $r = 3$ as in (8), which yields

$$\begin{bmatrix} y_t \\ q_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} d_1 \\ d_1 \\ 0 \\ 0 \end{bmatrix} [a_t] + \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \end{bmatrix}$$

The MA representation implies the three stationary relations outlined in (8) and it is also implied that the cumulated shocks to output is the common driving trend of the model.

When the inflation and the nominal interest rate are assumed to be non-stationary, and we have one stationary relationship between all of the four variables as in (9), we have three common driving trends. This implies that not only total factor productivity drives the model, but also two other factors. The theoretical model includes four exogenous shocks, related to productivity, prices (cost-push shock), housing preferences and a shock process for the central bank's interest rate rule. The shock to the nominal interest rate is assumed to be of mean zero, while the three remaining shocks are expressed as stochastic AR(1) processes in Iacoviello (2005). If we assume that the three AR(1) processes may contain a unit root, the MA representation can be expressed as

$$\begin{bmatrix} y_t \\ q_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \\ d_{14} & d_{24} & d_{34} \end{bmatrix} \begin{bmatrix} \sum u_{1,i} \\ \sum u_{2,i} \\ \sum u_{3,i} \end{bmatrix} + \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \end{bmatrix} \tag{11}$$

where the first stochastic trend is assumed to be the shocks to total factor productivity $\sum u_{1,i} = a_t$. For $q_t - y_t$ to be stationary, we need to have $d_{11} = d_{12}$, $d_{21} = d_{22}$ and $d_{31} = d_{32}$ such that the stochastic trends feeds into y_t and q_t equally. Imposing the restriction for the Fisher relationship to hold, the stochastic trends should feed equally into π_t and r_t, such that we need $d_{13} = d_{14}$, $d_{23} = d_{24}$ and $d_{33} = d_{34}$.

The third interpretation of the theoretical model is given by (10). This suggests two common driving trends and the following MA representation:

$$\begin{bmatrix} y_t \\ q_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} d_{11} & d_{21} \\ d_{12} & d_{22} \\ d_{13} & d_{23} \\ d_{14} & d_{24} \end{bmatrix} \begin{bmatrix} \sum u_{1,i} \\ \sum u_{2,i} \end{bmatrix} + \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \end{bmatrix},$$

where $d_{11} = d_{12}$, $d_{21} = d_{22}$, $d_{13} = d_{14}$ and $d_{23} = d_{24}$ in order for long-run homogeneity between output and house prices and the Fisher relationship to hold. Shocks to TFP and inflation may be the two driving trends of this model.

3 Statistical model

In order to make this paper closely linked to the model in Iacoviello (2005), I will use the same data set over the same sample (1974Q1 to 2003Q2) as he does. The only exception is that I instead of using real house prices and real GDP filtered via a band-pass filter removing frequencies above 32 quarters, will use the log of the variables without filtering. I will also not multiply inflation and the interest rate by 100. The variables (except the interest rate) are seasonally adjusted.

The VAR model estimated in Iacoviello (2005) also includes one lag of the log of the Commodity Research Bureau (CRB) price index, a constant, a time trend, and a shift dummy that is zero until 1979Q4 and unity thereafter. I will also include these in the estimated CVAR model.

3.1 Specification of the VAR model

The common trends (or MA) representation e.g. in (2.3) was specified for a VAR(1) model. However, two lags are needed in order to in order to get an estimated model without auto-correlation (tests will be shown below). The VAR(2) model may then be expressed in error correction form (ECM) as[4]

[4]Note the change of ordering of the variables in x_t in the estimated model compared to the ordering in the MA representation of the theoretical model. This is done to use similar ordering as in the estimated VAR model in Iacoviello (2005).

$$\Delta x_t = \omega \Delta W_t + (\Gamma_{11} - \omega \Gamma_{21}) \Delta z_{t-1} + \alpha_1 \left[\beta', \beta_0, \beta_1\right] \tilde{z}_{t-1} + \Phi D_t + \gamma_0 + \gamma_1 t + \tilde{\varepsilon}_t \qquad (12)$$

where $x_t' = [R_t, \pi_t, q_t, y_t]$, $\tilde{z}_{t-1}' = [z_{t-1}, 1, t]'$, $z_t' = [x_t, W_t]$, $\Gamma_1' = [\Gamma_{11}, \Gamma_{21}]$, $\alpha' = [\alpha_1, \alpha_2]$, $\tilde{\varepsilon}_t = \varepsilon_{1t} - \omega \varepsilon_{2t}$, $\varepsilon_t' = [\varepsilon_{1t}, \varepsilon_{2t}]$, $W_t = \ln CRB_{t-1}$ and $\varepsilon_t \sim IN(0, \Omega)$ for $t = 1, \ldots, T$ and x_{-1}, x_0 is given. This implies that the log of the CRB spot price index is included as a weakly exogenous variable.

The variables are y_t which is log of real GDP (nominal GDP deflated by the GDP deflator), q_t which is log of real house prices (Conventional Mortgage Home Price Index - CMHPI by Freddie Mac deflated by the GDP deflator), π_t which is log difference of the GDP deflator, R_t which is the Fed Funds rate calculated as the average value in the first month of each quarter as well as log of the CRP spot price index. The sample is chosen to be similar to the sample used in the estimation in Iacoviello (2005), i.e. 1974Q1 to 2003Q2. D_t is a vector of dummy variables, μ_0 is a constant and μ_1 is a coefficient for the trend t.

In order to prevent quadratic trends in (12), the trend component needs to be restricted to the cointegration relations such that $\beta_1 \neq 0$ and $\gamma_1 = 0$. This enables the cointegration space to include time as a trend-stationary variable such that unknown exogenous growth (e.g. technical progress) may be accounted for (Harris and Sollis, 2003, p. 133). The constant term needs to be unrestricted in order to allow for a constant term in the equations describing the slope of the linear trends in the data (Juselius, 2006). Including the trend is in line with the estimated VAR in Iacoviello (2005), although the theoretical model as shown in the MA representation in e.g. (2.3) suggests that the trend should not be included in the cointegrating relations since the trend of y_t and q_t should cancel. We may therefore expect to be able to exclude the trend after estimating the unrestricted model.

3.2 Institutional events

By using no variables representing institutional events (which may be results of interventions, reforms, etc.) except the shift dummy for 1979Q4 and thereafter, the model is misspecified. This is explicitly shown by the non-normal residuals in the VAR model. By including dummy variables for the periods with large residuals (which should correspond to periods with institutional events), we shuold be able to have a well-specified VAR model, even though the theoretical model does not take these events into account.

Multivariate tests (p-values underneath test values)	
Residual autocorrelation LM(1)	$\chi^2(16) = 33.315$ (0.007)
Residual autocorrelation LM(2)	$\chi^2(16) = 55.280$ (0.000)
Test for normality	$\chi^2(8) = 177.267$ (0.000)
Test for ARCH LM(1)	$\chi^2(100) = 257.700$ (0.000)

Table 1: Misspecification tests for the full period.

In order to get a well-specified model which includes no autocorrelation and normality[5] we need to add dummy variables for 19 periods according to Autometrics' (Doornik, 2006)

[5] Although the model contains autoregressional conditional heteroskedasticity (ARCH), the VAR results should be robust to moderate ARCH effects (Rahbek et al., 2002).

large outlier detection, which should account for institutional events. These are listed in table 2. Table 3 shows that the model is well specified when these dummy variables are included.

Dummy variable	F(4,83)	p-value	Dummy variable	F(4,83)	p-value
Dp1974Q3	16.86	0.00	Dp1980Q3	16.05	0.00
Dp1975Q1	2.14	0.00	Dp1980Q4	11.37	0.00
Dp1975Q2	1.57	0.19	Dp1981Q1	30.86	0.00
Dtr1975Q3	6.79	0.00	Dp1981Q3	13.50	0.00
Dp1976Q1	4.97	0.00	Dp1982Q2	10.87	0.00
Dp1976Q3	3.59	0.01	Dp1982Q3	3.27	0.02
Dp1977Q2	8.55	0.00	Dp1983Q1	1.83	0.13
Dp1977Q4	4.93	0.00	Dp2000Q1	1.73	0.15
Dp1978Q2	10.10	0.00	Dp2000Q2	2.18	0.08
Dp1980Q2	21.85	0.00			

Table 2: F-tests on the dummy variables. "Dp" indicates a permanent innovation and "Dtr" a transitory innovation for the mentioned and the preceding period.

Multivariate tests (p-values underneath test values)	
Residual autocorrelation AR 1-5	$F(80, 250) = \underset{(0.080)}{1.278}$
Test for normality	$\chi^2(8) = \underset{(0.699)}{5.536}$
Test for ARCH	$F(84, 290) = \underset{(0.006)}{1.523}$

Table 3: Misspecification tests for the full period after adding dummy variables

Even though the estimated VAR is now well-specified as shown in table 3, the number of dummy variables is quite large. We also see that the vast majority of the dummy variables belong to the first part of the sample, since almost half of the periods in the sample prior to 1983 needs to contain a dummy variable. All of the dummy variables may be motivated by institutional events such as oil crises and large recessions, but it may also be a sign of missing observable variables. Using the same observables as in Iacoviello (2005) requires adding the dummy variables in table 2, which is not in line with the theoretical model where the institutional events are not taken into account.

Following Juselius and Franchi (2007), I split the sample in one period going until 1979Q4 (prior to the second oil crisis) and a second period going from 1981Q2 until the end of the sample, as well as omitting the period 1980Q1-1981Q1 in order to avoid the outlier observations corresponding to these periods. Iacoviello (2005) splits the sample in his VAR estimation with a shift dummy in 1979Q4 such that the sample split done here is also in line with what Iacoviello (2005) does. However, since the sample split period starting from the beginning of the sample (1974Q1) going until 1979Q4 is quite short (24 observations), I choose to only do the estimation for the second split of the sample. Hence, the sample in the estimated CVAR model in this paper is 1981Q2-2003Q2.

In order to correct for large outliers pertaining to the residuals for the nominal interest rate, we need to impose dummies for the periods 1981Q4 and 1982Q2 and no other periods which is in line with the Volker disinflation policy period. This gives a well-specified model with normality, no autocorrelation and no ARCH. The VAR is then not as dependent on dummy variables, which provides a closer link between the estimated VAR and the theoretical model.

Multivariate tests (p-values underneath test values)	
Residual autocorrelation LM(1)	$\chi^2(16) = 24.746$ (0.074)
Residual autocorrelation LM(2)	$\chi^2(16) = 34.466$ (0.005)
Test for normality	$\chi^2(8) = 12.691$ (0.123)
Test for ARCH LM(1)	$\chi^2(100) = 85.430$ (0.850)

Table 4: Misspecification tests for the split sample

3.3 Exclusion

The first hypothesis related to the theoretical model that we would like to test is whether the trend is excludable from the long run relations. If it is found to be excludable, the model may be estimated without a trend since the trend will not provide any useful information to the long-run structure (Juselius, 2006, p. 351). Since all the linear trends in the levels should cancel, the trend should be excludable from the long-run relations.

r	DGF	5% C.V.	R	π	q	y	$\ln(CRB)$	TREND
1	1	3.841	26.755 [0.000]	27.637 [0.000]	13.920 [0.000]	0.013 [0.910]	5.079 [0.024]	0.814 [0.367]
2	2	5.991	32.182 [0.000]	48.120 [0.000]	17.215 [0.000]	23.684 [0.000]	8.198 [0.017]	22.929 [0.000]
3	3	7.815	45.489 [0.000]	65.439 [0.000]	17.226 [0.000]	35.986 [0.000]	9.985 [0.019]	29.744 [0.000]

Table 5: Test of long-run exclusion from the VAR for various choices of the rank

The results in table 5 (p-values in brackets below test values) show that the trend and output may be excludable for $r = 1$ while it is significant for $r = 2$ and $r = 3$. This implies that the trend and output is not needed in the first cointegrating relation but is needed for the other cointegrating relations, which is in line with the indications from the rank test after estimating the model without a trend shown in table 8, suggesting that the output and the linear trend constitutes the second cointegrating relationship. Also, if a variable is found to be excludable according to this test, it may be a sign of strong multicollinearity with other variables and it may suggest that some relevant variables are missing from the cointegration analysis (Dennis et al., 2006, p. 72). The estimated model without a trend shows, according to the exclusion test shown in table 6, that output may not be excluded from the long-run relationships. This suggests that there is strong multicollinearity between the linear trend and the log of real output. According to the theoretical long-run relations, output should be included and the trend should be excluded, so I choose to keep output when estimating the long-run relations but restricting the trend to be excluded.

r	DGF	5% C.V.	R	π	q	y	$\ln(CRB)$
1	1	3.841	32.288 [0.000]	46.738 [0.000]	15.516 [0.000]	11.491 [0.001]	4.357 [0.037]
2	2	5.991	40.610 [0.000]	54.614 [0.000]	15.938 [0.000]	15.117 [0.001]	7.756 [0.021]
3	3	7.815	48.573 [0.000]	67.169 [0.000]	19.248 [0.000]	20.864 [0.000]	10.399 [0.015]

Table 6: Test of long-run exclusion in model without trend for various choices of the rank

3.4 Rank

The theoretical models in (8), (9) and (10) correspond to different choices of the rank ($r = 3$, $r = 1$ and $r = 2$ respectively). I will use statistical tests in order to determine the rank together with these theoretical interpretations. This will pick out one of the three models pertaining to the three different rank options, which will be used as the relevant model for the further analysis in the paper.

r	$p-r$	λ_i	p	p^*	ρ_{max}
0	4	0.565	0.000	0.000	0.603
1	3	0.402	0.000	0.003	0.620
2	2	0.215	0.239	0.568	0.891
3	1	0.036	0.947	0.960	0.861

Table 7: Determining the rank based on the unrestricted VAR model

The trace test suggests a rank of $r = 2$ (the row marked $r = i$ tests $H_0 : r \leq i$) for a 5% significant level, both using the Bartlett corrected test (the column marked p^* in table 7 shows the p-value) and the standard trace test (the column marked p in table 7 shows the p-value). The (modulus of) the largest unrestricted characteristic root for all choices of the rank suggests a rank of $r = 1$. Setting $r = 3$ would leave a root of 0.861 in the model (see the column marked ρ_{max} in table 7), which is quite large. Choosing $r = 2$ would give the larges root equal to 0.891, and if rank $r = 1$ the modulus of the largest unrestricted root is as small as 0.62. The largest unrestricted root should be significantly different from unity in order to do the various stationarity tests conducted later in the paper, but since we do not know the standard deviation of the roots, we do not know exactly whether the reported roots suggest a rank of $r = 1$. Still, there is a quite small increase in the largest root when going from rank $r = 3$ to $r = 2$ such that we should set the rank to $r = 1$ if looking at the roots. The column marked λ_i shows the estimated eigenvalues, which should be low.

The theoretical model suggest a rank of $r = 3$ for the scenario in (8), rank $r = 2$ for (10) and rank $r = 1$ for the scenario in (9). This implies that the choice of rank based on the statistical tests will influence which theoretical scenario we will pursue in the analysis that follows.

3.4.1 Estimating the model without the trend

Since the theoretical implication suggests that there should be no deterministic trend in the model, we should try to test for rank using an estimated model where we do not include the trend. The estimated cointegrated VAR model below will also show that the trend may be excludable in the long-run relationships, which also indicates that an estimated model without a trend could be a good representation of the data. However, the estimated VAR model in Iacoviello (2005) includes a trend, so I will include the trend in the estimated model in the analysis below. In order to do a sensitivity analysis regarding finding the rank I will perform a rank test where the trend is not included in the model. The trace test and the modulus of the largest unrestricted roots then yields

r	$p-r$	λ_i	p	p^*	ρ_{max}
0	4	0.561	0.000	0.000	0.603
1	3	0.229	0.392	0.612	0.616
2	2	0.151	0.798	0.923	0.856
3	1	0.014	0.999	0.999	0.901

Table 8: Determining the rank in model with no trend

The model estimated without a trend suggests a rank of $r = 1$ both when looking at the trace test and the modulus of the largest unrestricted root. This may be a sign that the second cointegrating relationship found by the trace test for the model which included a trend was a relationship between the trend and output yielding a trend-stationary output gap. The tests of exclusion shown in table 5 and table 6 also supports this proposition.

3.5 Stationarity

r	DGF	5% C.V.	R	π	q	y
1	3	7.815	48.470 [0.000]	41.265 [0.000]	62.473 [0.000]	31.118 [0.000]
2	2	5.991	24.099 [0.000]	24.879 [0.000]	35.281 [0.000]	3.426 [0.180]
3	1	3.841	0.452 [0.502]	1.318 [0.251]	13.774 [0.000]	1.556 [0.212]

Table 9: Test of stationarity

r	DGF	5% C.V.	R	π	q	y
1	3	9.488	59.271 [0.000]	52.740 [0.000]	62.648 [0.000]	59.950 [0.000]
2	2	7.815	32.699 [0.000]	31.792 [0.000]	35.331 [0.000]	32.363 [0.000]
3	1	5.991	12.545 [0.002]	11.487 [0.003]	19.386 [0.000]	20.222 [0.000]

Table 10: Test of stationarity in model with excluded trend

The stationarity test in table 9 is a test for trend-stationarity since the trend is included in the cointegrating relations, while the test in table 10 is for stationarity where the trend is excluded from the cointegrating relations, conditional on the choice of rank. Since none of the variables is shown to be stationary, the assumption of stationary inflation and nominal interest rate for rank $r = 3$ according to (8) does not seem to hold. The test for trend-stationarity shows that output, the nominal interest rate and inflation are trend-stationary when rank is $r = 3$. Since housing prices are not trend-stationary while the other variables are, different trends may drive the financial markets and the real economy. This is evidence against long-run homogeneity between housing prices and output.

4 Testing the long-run relations

The test and choice of rank above determined the number of unique cointegration vectors spanning the cointegration space. However, any linear combination of the stationary vectors is a stationary vector, such that the estimates for any of the columns in the β matrix are not unique. It is therefore necessary to impose restrictions in order to test if the columns of β are identified (Harris and Sollis, 2003, p. 143). Since the rank test suggested a rank of $r = 1$, I will use the theoretical scenario pertaining to (9) and disregard the other scenarios. This implies one stationary relationship and three common driving trends in the model.

For rank $r = 1$ in (9), we only have one cointegration vector, which should include the four endogenous variables, but not the exogenous variable log of the CRB price index or the trend. There should also be long-run homogeneity between housing prices and output and between the nominal interest rate and inflation (corresponding to the definition of the real interest rate according to the Fisher relationship). Jointly imposing these restrictions yields the results given in table 11.

	R	π	q	y	$\ln(crb)$	Trend
β_1	29.30 (2.93)	-29.30 (-2.93)	1	-1	-	-

Test of restricted model: $\chi^2(4) = 56.887[0.000]$
Test of restricted model, Bartlett corrected: $\chi^2(4) = 44.896[0.000]$

Table 11: Testing imposed theoretical restrictions

The model after imposing rank $r = 1$ and the restrictions pertaining to the hypotheses in (9) yields the long-run relationship for $\beta' x_{t-1}$

$$\ln q_t = \ln Y_t - 29.30(R_t - \pi_t),$$

which expresses an inverse demand function for housing that is the only stationary relation among the variables. However, the hypotheses are rejected when imposed on the cointegration vector of the estimated model. This is shown in table 11 and figure 1, where we observe a non-stationary relationship. The upper panel of figure 1 shows the residuals from the estimated long-run cointegrating relation and the lower panel of figure 1 shows the residuals from the estimated long-run cointegrating relation corrected for short-run effects.

The only hypothesis not rejected is excluding the trend from the long-run relationship, which also corresponds to the results from the general test for exclusion above.

The long-run relationship from $\beta' x_{t-1}$ is then given as (with absolute values of t-ratios in parentheses below the coefficients)

$$\ln q_t = \underset{(14.975)}{0.693} \ln Y_t \underset{(7.112)}{-10.638} R_t + \underset{(11.177)}{45.166} \pi_t,$$

which also shows an inverse demand function for housing, but not with long-run homogeneity between housing prices and output and the Fisher relationship. Both the GDP deflator and log of the CRB price index is significant and included as price measures. This restriction is not rejected, with the test values and p-values in brackets $\chi^2(2) = 5.171[0.075]$ and for the Bartlett corrected case $\chi^2(2) = 3.988[0.136]$. This suggests that there may exist a long-run relation pertaining to housing demand, but long-run homogeneity between house prices and output and between the nominal interest rate and inflation does not exist. The cointegrating relation is shown in figure 2, where there seems to be a stationary relationship.

The financial accelerator (which is one of the key motivations behind the theoretical model) suggests that increased housing prices should increase aggregate demand which again should increase housing prices, and so on. This indicates that an increase in housing prices should have a self-reinforcing effect such that housing prices might increase more than output, assuming that output growth is limited by increased technology and the housing supply lags housing demand. This would eventually lead to a demand surplus in the housing market following the amplifying effects given by increased housing prices. The long-run homogeneity between housing prices and output may therefore not be consistent with the financial accelerator.

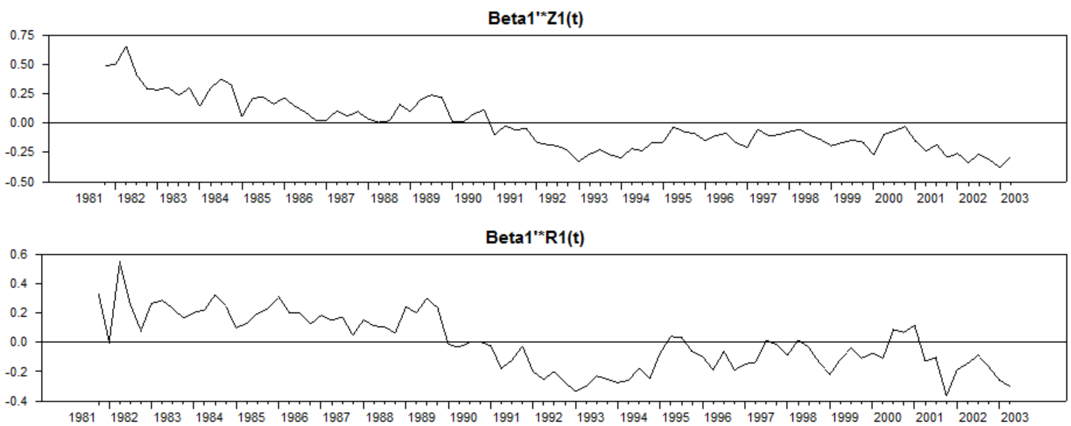

Figure 1: Cointegrating relationship, with restrictions on β

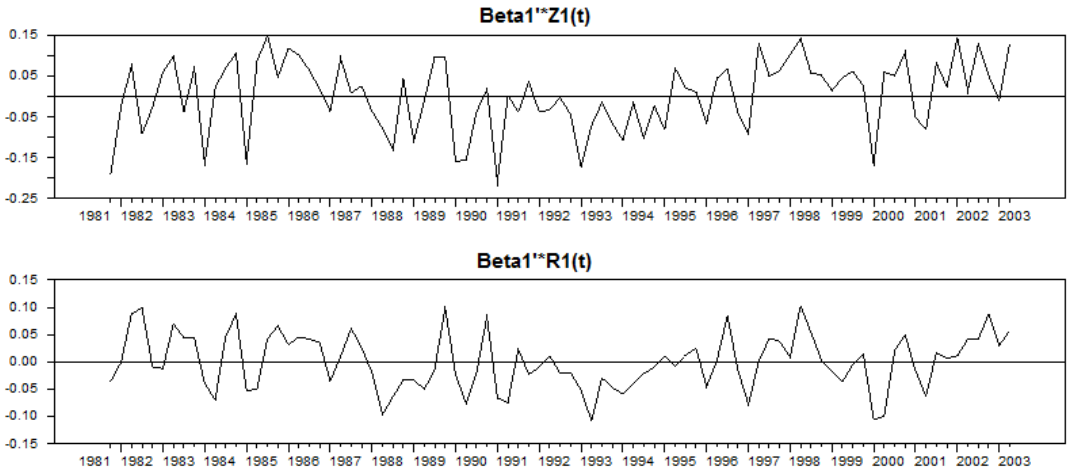

Figure 2: Cointegrating relationship, with excluded trend and $\ln CRB_{t-1}$

5 The moving average (MA) representation

The vector error correction model shown in (12) may also be represented by a moving average structure through the Granger representation theorem (Engle and Granger, 1987) as shown in theorem 4.2 in Johansen (1995). This results in

$$x_t = C \sum_{i=1}^{t} (\varepsilon_i + \Psi D_i) + C^*(L)(\varepsilon_t + \mu_0 + \alpha\beta_1 t + \Phi D_t) + A, \tag{13}$$

where

$$C = \beta_\perp(\alpha'_\perp(I - \Gamma_1 - \Gamma_2)\beta_\perp)^{-1}\alpha'_\perp = \tilde{\beta}_\perp\alpha'_\perp,$$

where the values of the parameters in the matrices $\beta_\perp$ and $\alpha_\perp$ can be calculated from the estimates of α, β and Γ. The matrix A depends on the initial values and satisfies $\beta' A = 0$. This moving average representation may be used to analyze the structure of the common stochastic trends, which represents the pushing forces that create the non-stationary properties of the data. The unrestricted or restricted estimates of α and β can therefore be used in order to find the common stochastic trends and their weights. The cointegration analysis will be covered through restrictions on the β vectors, while the estimate of C and $\alpha_\perp$ will give information of the estimated common trends. Since the rank is set to $r = 1$, we have $p - r = 3$ common stochastic trends.

The estimated CVAR model with the theoretical restrictions given by (9) imposed, may be transformed into a common trends representation using the estimated parameters pertaining to the matrices α, β and Γ. This yields the results for the parameters in (13), given in table 12 which shows the matrices which is relevant for the common trends representation with t-values in braces below the parameters.

	$\hat{\varepsilon}_R$	$\hat{\varepsilon}_\pi$	$\hat{\varepsilon}_q$	$\hat{\varepsilon}_y$
$\hat{\sigma}_{\varepsilon_i}$	0.0017	0.0016	0.0122	0.0108
$\hat{\alpha}^c_{\perp,1}$	0.23 [0.54]	1.00	0.00	0.00
$\hat{\alpha}^c_{\perp,2}$	−0.04 [−0.04]	0.00	0.00	1.00
$\hat{\alpha}^c_{\perp,3}$	−1.61 [−1.32]	0.00	1.00	0.00
	R	π	q	y
$\hat{\beta}^c_{\perp,1}$	0.77 [4.25]	0.80 [4.78]	1.83 [1.39]	1.01 [0.86]
$\hat{\beta}^c_{\perp,2}$	0.22 [3.14]	0.13 [1.97]	−0.69 [−1.35]	2.03 [4.49]
$\hat{\beta}^c_{\perp,3}$	0.05 [0.73]	0.10 [1.66]	2.13 [4.33]	0.55 [1.27]
The C matrix				
	$\hat{\varepsilon}_R$	$\hat{\varepsilon}_\pi$	$\hat{\varepsilon}_q$	$\hat{\varepsilon}_y$
R	0.09 [0.33]	0.77 [4.25]	0.05 [0.74]	0.22 [3.14]
π	0.02 [0.06]	0.80 [4.78]	0.10 [1.66]	0.13 [1.97]
q	−2.97 [−1.47]	1.83 [1.39]	2.13 [4.33]	−0.69 [−1.35]
y	−0.74 [−0.41]	1.01 [0.86]	0.55 [1.27]	2.03 [4.49]

Table 12: Parameters for the MA representation

Using the estimated values, we get the following estimated common trends representation:

$$\begin{bmatrix} R_t \\ \pi_t \\ q_t \\ y_t \end{bmatrix} = \begin{bmatrix} \underset{[4.25]}{0.77} & \underset{[3.14]}{0.22} & \underset{[0.73]}{0.05} \\ \underset{[4.78]}{0.80} & \underset{[1.97]}{0.13} & \underset{[1.66]}{0.10} \\ \underset{[1.39]}{1.83} & \underset{[-1.35]}{-0.69} & \underset{[4.33]}{2.13} \\ \underset{[0.86]}{1.01} & \underset{[4.49]}{2.03} & \underset{[1.27]}{0.55} \end{bmatrix} \begin{bmatrix} \sum u_{1,i} \\ \sum u_{2,i} \\ \sum u_{3,i} \end{bmatrix} + \text{stationary and det. components}$$

The stochastic trends may be interpreted as (with t-ratios in brackets below the coefficients)

$$\sum u_{1,i} = \sum \varepsilon_\pi + \underset{[0.54]}{0.23} \sum \varepsilon_R$$

$$\sum u_{2,i} = \sum \varepsilon_y - \underset{[0.03]}{0.04} \sum \varepsilon_R$$

$$\sum u_{3,i} = \sum \varepsilon_q - \underset{[1.32]}{1.61} \sum \varepsilon_R$$

Disregarding the insignificant variables, we see that the first stochastic trend can be interpreted as cumulated shocks to inflation, the second as cumulated shocks to output and the third as cumulated shocks to housing prices, as the theoretical model may suggest. We may therefore write the MA representation as

$$\begin{bmatrix} R_t \\ \pi_t \\ q_t \\ y_t \end{bmatrix} = \begin{bmatrix} \underset{[4.25]}{0.77} & \underset{[3.14]}{0.22} & \underset{[0.73]}{0.05} \\ \underset{[4.78]}{0.80} & \underset{[1.97]}{0.13} & \underset{[1.66]}{0.10} \\ \underset{[1.39]}{1.83} & \underset{[-1.35]}{-0.69} & \underset{[4.33]}{2.13} \\ \underset{[0.86]}{1.00} & \underset{[4.49]}{2.03} & \underset{[1.27]}{0.55} \end{bmatrix} \begin{bmatrix} \sum \varepsilon_\pi \\ \sum \varepsilon_y \\ \sum \varepsilon_q \end{bmatrix} + \text{stationary and det. components}$$

The nominal interest rate is mainly influenced by cumulated shocks to output and inflation while inflation is mainly influenced by cumulated shocks to inflation only. This may indicate non-stationarity of the real interest rate, which rejects the hypothesis pertaining to the second relation in (10), i.e. the Fisher equation. Housing prices are mainly influenced by cumulated shocks to itself, and output is influenced by cumulated shocks to itself. This may indicate evidence against a stationary relationship between output and housing prices, since two separate trends are driving them. We also see that there are few parameter estimates corresponding to the expected relationships from (11) ($d_{11} = d_{12}$, $d_{21} = d_{22}$, $d_{31} = d_{32}$, $d_{13} = d_{14}$, $d_{23} = d_{24}$ and $d_{33} = d_{34}$).

The estimated C-matrix in table 12 suggests that cumulated empirical shocks to the nominal interest rate only have transitory effects to the variables of the system since all the parameters in the first column of the C-matrix are insignificant, which is also suggested by the theoretical model. It also suggests that cumulated shocks to inflation have permanent effects on the nominal interest rate and inflation, cumulated shocks to housing prices have only permanent effects on housing prices and cumulated shocks to GDP have permanent effects on the nominal interest rate and GDP. The normalization of $\alpha'_\perp$ as shown in table 12 and the interpretation of the three stochastic trends is in line with this.

6 Structural MA model and impulse response functions

We can impose the orthogonality condition on the residuals while separating between r transitory shocks, u^T, and $p - r$ permanent shocks, u^P, and impose exclusion restrictions on the parameters of the model, in order to identify the structural shocks $u_t = (u_t^T, u_t^P)$ (Juselius, 2006, p. 275) and (Dennis et al., 2006). These structural shocks can then be given an economic interpretation in order to extend the analysis of the model with impulse response functions and relate it to the theoretical scenarios.

The MA representation is given by (13). We may relate the VAR residuals ε_t to the structural shocks u_t by a matrix B such that

$$u_t = B\varepsilon_t \Leftrightarrow \varepsilon_t = B^{-1}u_t,$$

where the error terms u_t are *iid* $N_p(0, I_p)$ (Dennis et al., 2006). By inserting for ε_t in (13) using matrix B, we get[6]

[6]The weakly exogenous variable $\ln CRB_{t-1}$ is excluded here to ease notation.

$$
\begin{aligned}
x_t &= CB^{-1}\sum_{i=1}^{t}(u_i) + C\sum_{i=1}^{t}\Phi D_i + C^*(L)(B^{-1}u_t + \mu_0 + \Phi D_t) + A \\
&= \tilde{C}\left(\sum_{i=1}^{t} u_i + \sum_{i=1}^{t} B\Phi D_i\right) + \sum_{i=0}^{\infty}\tilde{C}_i^*\left(u_{t-i} + B\Phi D_{t-i}\right) + A
\end{aligned}
$$

where $\tilde{C} = CB^{-1}$ and $\tilde{C}^*(L) = C^*(L)B^{-1}$. I choose to normalize the transitory shock on the nominal interest rate, the first permanent shock on inflation, the second permanent shock on output and the third permanent shock on housing prices. This is in line with the theoretical model which assumes AR(1) processes for the shocks pertaining to the three latter variables, while shocks to the nominal interest rate is assumed to be white noise. Results from the estimated $\alpha'_\perp$ in the MA representation as shown in table 12 also supports this. Additionally, I restrict the second permanent shock (output) to have no long-run effect on inflation and the third permanent shock (housing prices) to have no long-run effect on inflation and the nominal interest rate as shown in (14). This is in line with the theoretical model that suggests no long-run impact on inflation and the nominal interest rate which is assumed stationary in the long run.

The (normalized) rotation matrix B determines how the orthogonalized permanent and transitory shocks are associated with the estimated VAR residuals, and is given by

$$
B = \begin{bmatrix} 1.00 & 0.06 & 0.07 & -0.04 \\ 0.02 & 1.00 & 0.13 & 0.16 \\ 0.84 & -0.49 & -0.62 & 1.00 \\ -1.83 & -0.84 & 1.00 & 0.54 \end{bmatrix}
$$

The structural long-run impact matrix is given by $\tilde{C} = CB^{-1}$

$$
\tilde{C} = \begin{bmatrix} 0.00 & 1.02 & 0.08 & \mathbf{0.00} \\ 0.00 & 1.00 & \mathbf{0.00} & \mathbf{0.00} \\ 0.00 & 2.36 & -1.33 & 1.00 \\ 0.00 & 2.94 & 1.00 & 1.00 \end{bmatrix}, \tag{14}
$$

where the assumed exclusion restrictions are in bold face and show that shocks to output should not have long-run effects on inflation and shocks to house prices should not have any long-run effects on inflation and the nominal interest rate.

The impulse response functions from the structural MA with imposed restrictions on β in line with (9) are shown in figure 3 together with impulse response functions from the estimated VAR in Iacoviello (2005). The residuals are restricted to be orthogonal.

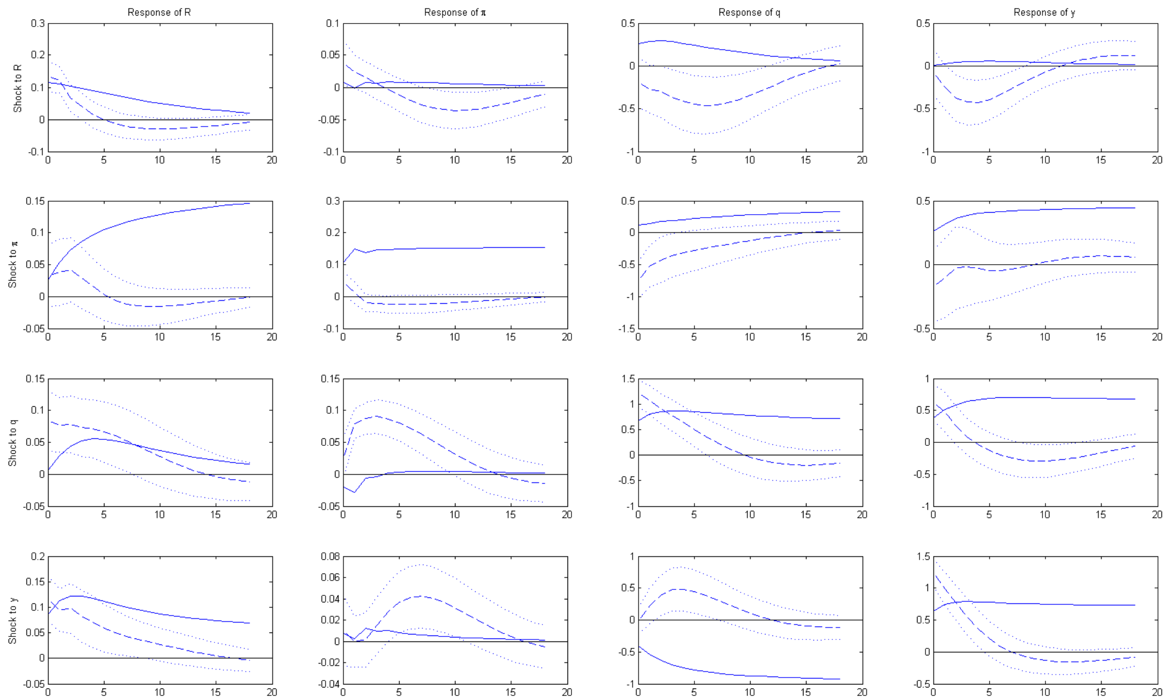

Figure 3: Impulse response functions from the structural MA model (solid lines) and from Iacoviello (2005) (dashed lines) with 90% confidence bands (dotted lines).

The impulse response functions given by the restricted CVAR model represented in a structural MA form may be compared to the impulse response functions given by the estimated VAR model in Iacoviello (2005). However, all the variables are assumed non-stationary in the CVAR model such that some shocks need to have permanent effects in order to "drive" these variables. The first row in figure 3 shows the transitory shock, which may be interpreted as a shock to the nominal interest rate. The remaining rows represent the $p-r=3$ permanent shocks. Ordered from top to bottom we may interpret these as shocks to inflation, housing prices and output. The two different IRFs in figure 3 are comparable with regards to the interpretation of shocks since I have ordered the IRFs following the structural MA model as in Iacoviello (2005)[7].

6.1 Analyzing the impulse response functions

In order to separate between the impulse response functions in Iacoviello (2005) and the impulse response functions from the structural MA model following the restricted cointegrated VAR model estimated here, I will calculate two additional sets of impulse response functions, shown in appendix B in addition to the direct comparison in figure 3.

IRFs using filtered variables instead of unfiltered when estimating the structural MA model following the CVAR model are shown in figure B.1, which may indicate differences between the IRFs in figure 3 due to using unfiltered variables. The IRFs using the structural MA model without imposing restrictions (homogeneity between housing prices and output

[7]Note that the impulse responses in figure 3 are shown from one period after the impact in order to match the IRFs shown in Iacoviello (2005)

and the Fisher equation) on β are shown in figure B.2, and may identify differences in IRFs due to the imposed theoretical restrictions.

The first line of figure 3 should be interpreted as a transitory contractionary monetary shock, through an increase in the nominal interest rate R. The immediate effect on the interest rate itself and on inflation is positive, while housing prices and output reacts with opposite signs in the VAR in Iacoviello (2005) and in the structural MA model. A reason for this may be that housing prices and output to a larger extent reacts to the real interest rate than the nominal such that the size of the effect on inflation following the monetary policy shock determines whether housing prices and output increase or decrease. When not imposing the long-run restrictions, both housing prices and output decrease as shown in figure B.2. The imposed theoretical long-run relations therefore seems to yield effects on housing prices and output opposite of what we would expect from increasing the interest rate, which may indicate different effects on the real interest rate.

The shock in line 2 in figure 3 may be interpreted as a positive shock to inflation[8]. This yields permanently increased interest rate, housing prices and output in the structural MA model and a temporary increase in the interest rate together with decreased housing prices and output in the VAR in Iacoviello (2005). Both in the structural MA without long-run restrictions and in the structural MA using unfiltered variables, housing prices and output increase as shown in figures B.1 and B.2. The difference (i.e. opposite effects on housing prices and output) may therefore be because the nominal interest rate increase more than inflation such that the real interest rate decreases, causing increased output and housing prices. All of the IRFs in the structural MA models shows a large increase in the nominal interest rate following the inflationary shock.

Increased housing prices as shown in the third row cause a permanent increase in output, a transitory increase in the nominal interest rate and a transitory decrease in inflation in the structural MA model as shown in figure 3. This indicates the presence of the financial accelerator since increased housing prices cause a permanent increase in output. Inflation decreases as shown in IRFs from the structural MA models (all versions of it), while the VAR in Iacoviello (2005) shows increased inflation following increased housing prices. This may therefore be caused by the identification of the shocks in the structural MA model, especially since shocks to house prices are restricted not to have any long-run effects on inflation.

The fourth row can be interpreted as shocks to output, i.e. a positive demand shock. In the unrestricted VAR in Iacoviello (2005), this causes increased housing prices, interest rate, and inflation. The decrease in housing prices following a permanent positive demand shock in the structural MA model (all versions of it) is the main difference when compared to the VAR in Iacoviello (2005). This is opposite of what we would expect from the financial accelerator. Since the nominal interest rate increases permanently and inflation decreases temporarily in the structural MA model, the real interest rate will increase permanently. If the negative effect on housing prices following an increased real interest rate is larger than the positive effect of increased output, housing prices may decrease. The identification of the shocks in the structural MA model which imposes no permanent effect on inflation from the demand shock may be the cause of this. Additionally, the use of filtered variables in the structural MA model yields increased housing prices in the long run as shown in figure B.2, such that the use of filtered variables may also affect the results.

The results from the impulse response functions following the structural MA model may

[8] permanent in the structural MA model and transitory in the VAR in Iacoviello (2005)

often be hard to explain, so the interpretations here are not necessarily correct. Especially, since the imposed long-run restrictions are not accepted when imposing them on the CVAR model, the 'economic labels' imposed on the shocks here may not be correctly connected to the empirical shocks (Juselius, 2006, p. 286). Imposing that the second permanent shock (labeled as a positive shock to housing prices and shown in the third row in the figures) should have no long-run effect on inflation or the nominal interest rate is also perhaps a strong restriction, since the importance of housing prices which is the label for this shock is then reduced. We also see that using filtered variables and imposing restrictions on the long-run relations affects the IRFs. In addition, it seems that the change in the real interest rate following a shock has a strong effect on output and housing prices, in some cases larger than the effect of the initial shock.

7 Conclusion

The framework applied in this paper combines the theoretical model and the estimated VAR in Iacoviello (2005) by imposing restrictions from the theoretical model on the estimated VAR by using the cointegrated VAR model. Even though there is found evidence for the financial accelerator in the estimated VAR and the impulse response functions following the minimum-distance estimation of the theoretical model in Iacoviello (2005), the results from impulse responses of the theoretically restricted structural VAR estimated here does not show evidence for this.

Imposing a constant relationship between housing prices and output according to one of the steady states of the theoretical model implies that an increase in housing prices should lead to a similar increase in output and vice versa. Looking at the time series for housing prices and output indicates that this hypothesis does not hold since there has been a much larger growth in housing prices than output for a large period of time. The theoretical model also assumes that inflation and the nominal interest rate should separately be stationary, even though the empirical evidence suggests that these are non-stationary for the sample period the model is being estimated over. Stationarity between the nominal interest rate and inflation, pertaining to the Fisher equation, is therefore imposed on the long-run relations here in order to restrict the real interest rate to be stationary. This seems to be non-stationary as well.

The results following the structural MA model suggests some puzzling results which should be analyzed carefully. The imposed long-run homogeneity between housing prices and output and the Fisher equation should impact how the variables respond to various shocks, and since these imposed restrictions are not accepted, the interpretation of the results becomes less realistic. Additionally, by using the Band-pass filter on output and real housing prices separately, the potential difference in the trends driving these which is not in line with the long-run homogeneity between housing prices and output may be removed. This is also indicated by the rejection of trend-stationarity of housing prices but acceptance for all the other variables. However, the effect of the real interest rate should also be considered, and the results following the structural MA model may indicate that the real interest rate has a large impact on the economy such that changes in the real interest rate following a shock may alter the initial effect of the shock.

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A The data series

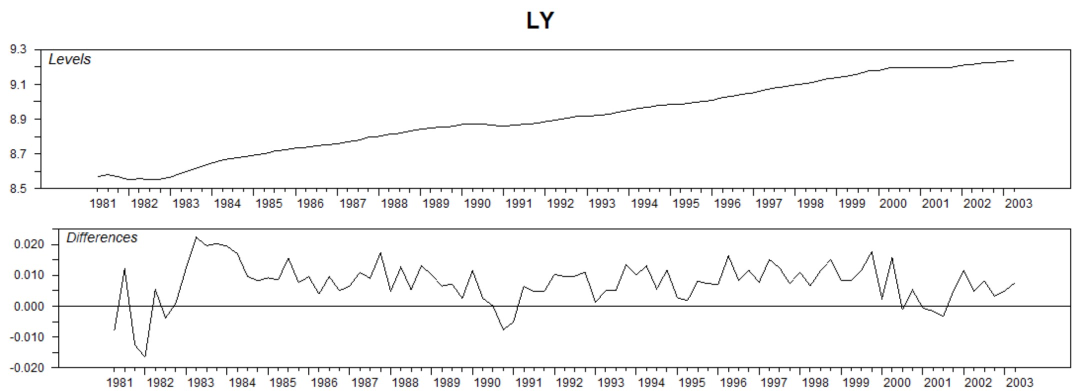

Figure A.1: Log of Real GDP (nominal GPD deflated by the GDP deflator).

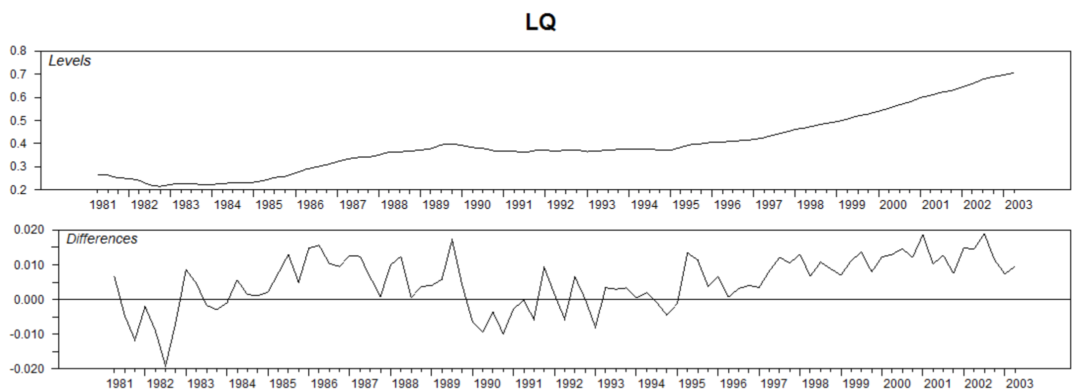

Figure A.2: Log of real house prices (deflated by the GPD deflator) from the Conventional Mortgage Home Price Index (CMHPI) by Freddie Mac.

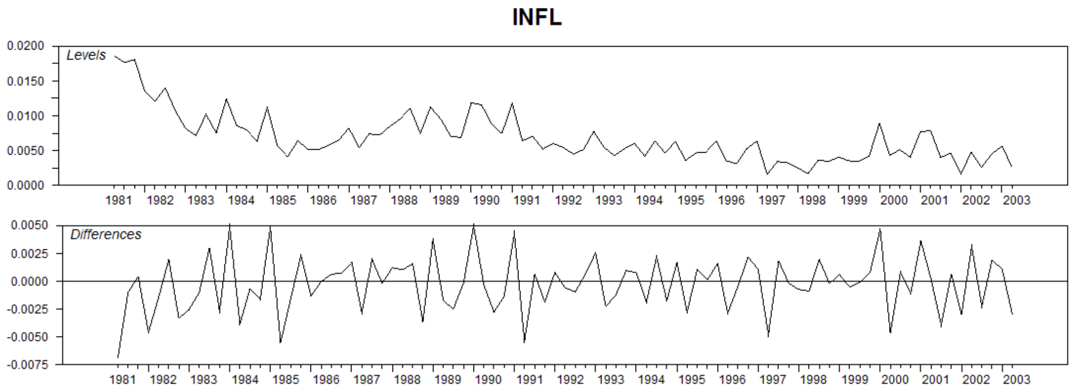

Figure A.3: Log difference of the GDP deflator, measuring inflation.

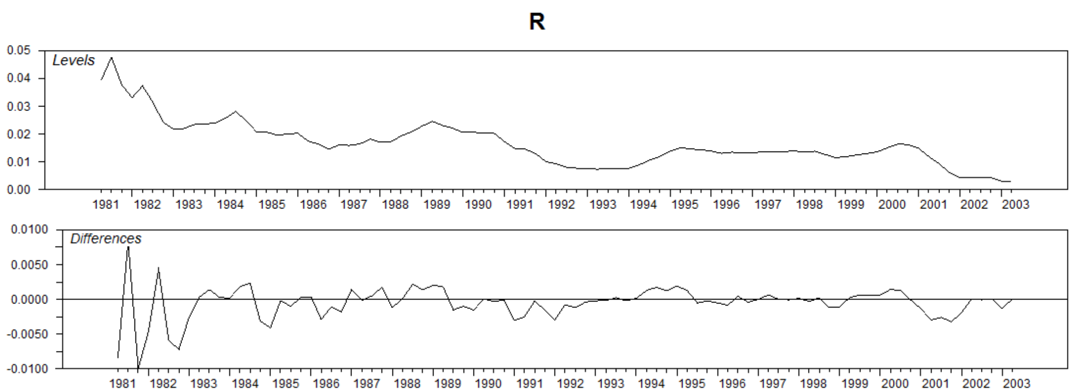

Figure A.4: The Fed Funds rate, calculated as the average value in the first month of each quarter.

B Impulse response functions

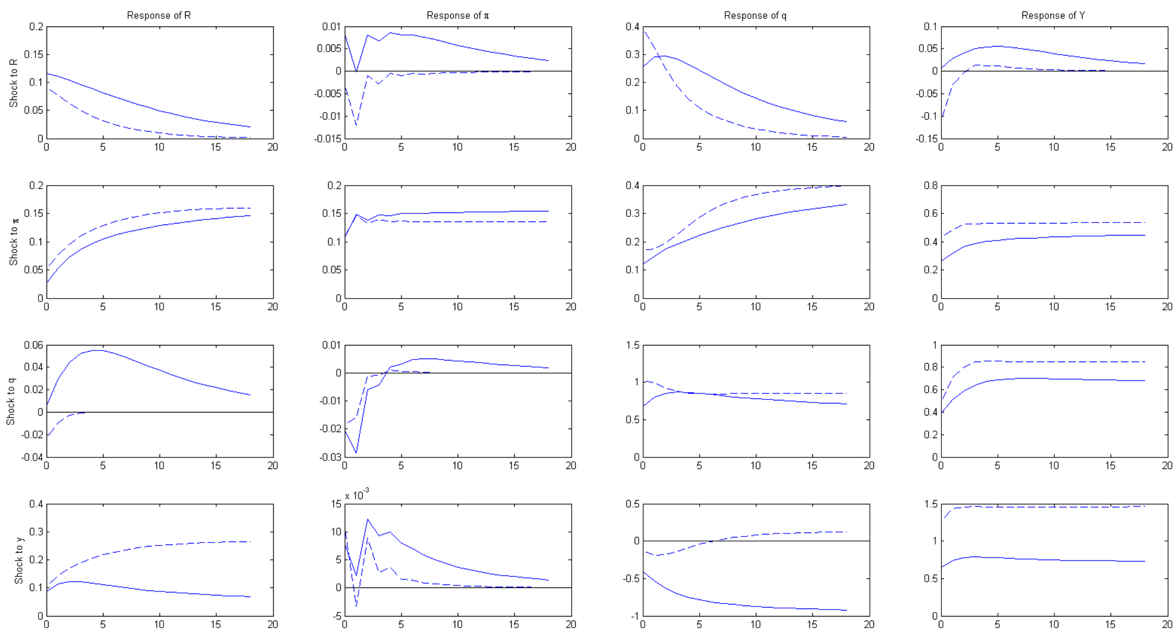

Figure B.1: Impulse responses from the structural MA model (solid lines) and from the estimated structural MA model using filtered variables and long-run restrictions (dashed lines). Size of impacts are scaled in order to ease comparisons.

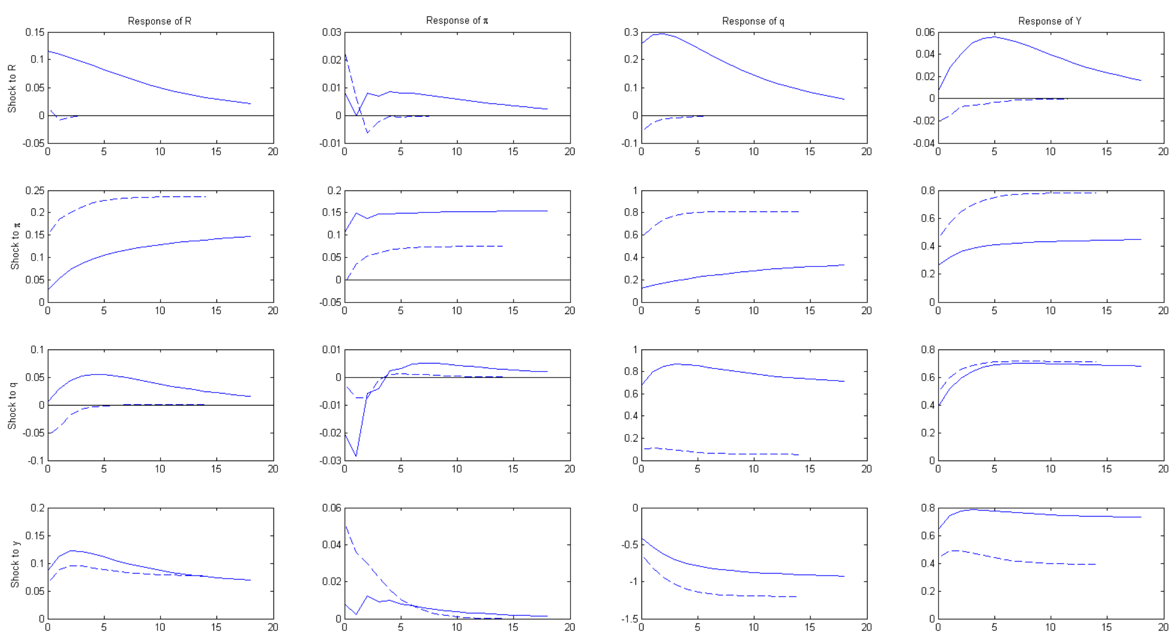

Figure B.2: Impulse responses from the structural MA model (solid lines) and from the estimated structural MA model using unfiltered variables and no long-run restrictions (dashed lines). Size of impacts are scaled in order to ease comparisons.