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RAMSEY MODEL OF BARRIERS TO GROWTH AND SKILL-BIASED INCOME DISTRIBUTION IN SOUTH AFRICA

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Ramsey model of barriers to growth and skill-biased income distribution in South Africa*)

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Abstract
The paper integrates two mechanisms of economic growth, barriers to international spillovers and skill-biased effects on the income distribution. South Africa is an interesting case study because of dramatic changes in international barriers over time and policy focus to productivity and distribution. Barriers affect the balance between innovation and adoption in the productivity growth and thereby the skill-bias. The productivity dynamics and the distributional implications are investigated in an intertemporal Ramsey growth model. The model offers a calibrated tariff-equivalence measure of the sanction effect and allows for counterfactual analysis of no-sanctions. Increased openness is shown to reduce barriers to technology adoption leading to skill-biased economic growth and worsened income distribution. The result is consistent with the observation that economic growth under sanctions has been slow and with an increase in the relative wage of unskilled labor. The tradeoff between barriers and skill-bias, foreign spillover driven productivity growth and income distribution, obviously is a challenge for growth policy.

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1. Introduction

The barrier model of economic growth is broadly consistent with the observed income differences between countries and the stability of the world income distribution. Klenow and Rodriguez-Clare (2004) and Parente and Prescott (2004) formulate this model of economic growth based on the importance of international technology spillovers. Recent econometric evidence of the growth experiences of individual countries by Cole et al. (2004) for Latin-America and Harding and Rattsø (2005) for South Africa is in accordance with the barrier model. In this paper we combine the barrier model with an international link to the domestic income distribution. The relationship between growth, openness and inequality is of key concern in the development debate.

The integration of barriers and skill-bias is shown to help understand the recent economic development in South Africa. South Africa is an interesting case study of the dynamics of growth and distribution. The trade regime has been changing over time, and in particular with a long period of international sanctions. The dual economy combines capital-intensive modern manufacturing with large unskilled employment and underemployment. The volatility of growth and the large inequalities are a challenge for research and policy. We capture the essentials of this economic structure by building barriers and skill-bias into an intertemporal general equilibrium model where economic growth is generated by endogenous investment allocation and productivity growth. Foreign trade and capital flows are endogenous and the openness barrier to productivity is influenced by tariffs and sanctions calculated as tariff-equivalent. Income distribution is measured by the relative wages between skilled, semi-skilled and unskilled labor and by separating between rich and poor labor households and capitalist households.

Productivity growth in semi-industrialized economies like South Africa is driven by a combination of innovation and adoption. While innovations are determined by domestic production activity, technology adoption is a foreign spillover. The balance between the domestic and foreign sources of growth is in focus here, as analyzed by Eaton and Kortum (1997). The starting point of the literature is the catching-up advantage of backwardness called the Veblen-Gerschenkron-effect. The mechanism was first formalized by Nelson and Phelps (1966). They assume exogenous growth of a best practice world technology frontier, and productivity growth in the backward country responds to the productivity distance to best practice. All countries can take benefit of the growth of the world technology frontier, albeit in different degrees and speeds, and dependent on the initial conditions. A modern restatement is offered by Parente and Prescott (1994, 2004) introducing the concept barriers to technology adoption. Improvement in productivity is linked to the distance to the exogenous world technology frontier, and investment is needed to benefit from the world technology.

A broad empirical literature has addressed the sources of total factor productivity (TFP) growth. In a study of R&D spillover in 77 developing countries, Coe et al. (1997) conclude that a developing country can boost its productivity by importing a larger variety of intermediate products and capital equipment embodying foreign knowledge. Cameron (1998) has written a helpful survey of studies of the relationship between innovation and growth. Innovations do contribute to growth, and with spillovers between countries, but R&D activity is limited outside the already rich.

Several studies indicate the importance of both openness and domestic factors in the TFP growth in South Africa. The IMF study of Jonsson and Subramanian (2001) is the most enthusiastic about the productivity effect of an open economy. They also find an important role of machinery and equipment investment for TFP growth. Fedderke (2002) offers a richer study and puts more emphasis to domestic factors. He identifies important effects of R&D and the ratio of skilled to unskilled labor in TFP growth. Harding and Rattsø (2005) address the endogeneity problem of openness and concentrate on tariff measures. They identify a shift from domestic to foreign sources of productivity growth after sanctions. Inspired by this literature we study the endogenous formation of productivity growth driven by adoption and innovation. The adoption part is related to the degree of interaction with the rest of the world through international trade, while the innovation part is related to the investment level.

Openness and growth is linked to income distribution. Dollar and Kraay (2004) show the empirical importance for poverty. In the analysis we relate the productivity growth to income distribution by introducing skilled, semi-skilled and unskilled labor and possible skill bias. The specification of technological bias is based on the assumption of an unskilled intensive economy, and is linked to the relative importance of technology adoption and innovation as sources of productivity growth. New technology innovated in skill-intensive developed countries is likely to be skill-biased following from directed technical change (Acemoglu, 1998). Adoption of foreign technology is therefore assumed to generate productivity growth biased towards skilled workers, and the degree of bias increases with the openness of the economy and the availability of foreign technology. Local improvement of technology can be directed based on given factor endowments, which in an unskilled-intensive economy implies technical change biased towards unskilled workers. The more dependent the economy is on adoption of foreign technology, the higher is the degree of skill-bias in technical change. Empirical support is offered by Zhu and Trefler (2003).

To clarify the importance of openness and income distribution for South Africa, we need to place the productivity dynamics in an intertemporal general equilibrium setting. The analysis is embedded in a Ramsey growth model and calibrated to reproduce the main growth path of South Africa during 1960-2003 and projected to 2010. To capture the dual structure of the South African economy, we distinguish between a modern sector using semi-skilled and skilled labor more intensively and a traditional unskilled-intensive sector. On the consumption side, we separate between poor households based on unskilled wage income, rich households based on semi-skilled and skilled wage income, and capitalist households based on profits. The protectionist effect of sanctions is calibrated as a tariff equivalent and with a peak in 1990. This allows the analysis of a counterfactual scenario without sanctions, with consequences for the relationship between adoption and innovation and consequently skill bias. The analysis separates between three time periods: pre-sanctions 1960-74, sanctions 1975-1993, and post-sanctions 1994-2010.

The paper presents the modelling of the productivity dynamics (section 2), the full intertemporal general equilibrium model (section 3), the calibration of South Africa's growth path (section 4), and offers a counterfactual analysis of sanctions (section 5). Section 6 concludes.

2. Productivity dynamics

Productivity growth is generated through technology adoption and own innovations. Technology adoption combines two elements, the distance to the world technology frontier defining the potential productivity level and the role of barriers. We apply the modified Nelson-Phelps specification suggested and empirically documented by Benhabib and Spiegel (2003). The productivity dynamics is consistent with the catching-up hypothesis, where the growth rate increases with the distance to the technological frontier. But compared to the original formulation the relationship between growth and technology gap is linear, and not exponential. This limits the advantage of backwardness and gives possible divergence in cases of high barriers to technology adoption. The barrier may be in the form of human capital as in Nelson and Phelps (1966) and Benhabib and Spiegel (2003) or investment regulations as in Parente and Prescott (1994). We focus on the role of international barriers measured by total trade, as suggested in a broad literature of technology spillovers and formulated by Grossman and Helpman (1991). Innovations are broadly understood as domestic productivity improvements. In the model we assume that the innovation activity is related to the overall investment path. An alternative specification of the productivity dynamics with interaction between trade and human capital as barriers to technology adoption is applied in a Ramsey growth framework by Stokke (2004).

The rate of growth of labor augmenting technical progress is specified as follows (time subscript is omitted):

$$\frac{\dot{A}}{A} = \left(\frac{I}{GDP}\right)^{\theta_1} + \lambda \left(\frac{TRADE}{GDP}\right)^{\theta_2} \left(1 - \frac{A}{T}\right) \tag{1}$$

where A and T represent the domestic and frontier level of productivity, respectively, and A/T is the technology gap. I is total investment, *TRADE* total trade, *GDP* gross domestic product, and λ, θ_1 and θ_2 are constant parameters. Consistent with Benhabib and Spiegel (2003), the first term on the right-hand side is the contribution from innovation activities, while the second term is the technology adoption function. The formulation implies decreasing returns to innovation and adoption with the shares adding up to 1.

Under symmetric growth, the long-run productivity growth is given by the exogenous frontier growth rate g, and the technology gap is constant. The degree of catch-up depends on the level of barriers and the innovative capacity of the economy. The long run equilibrium consequently implies a proportional relationship between A and T:

$$A = \frac{\left(\frac{\mathrm{I}}{\mathrm{GDP}}\right)^{\theta_1} + \lambda \left(\frac{TRADE}{GDP}\right)^{\theta_2} - g}{\lambda \left(\frac{TRADE}{GDP}\right)^{\theta_2}} \cdot T \tag{2}$$

The steady state values of I/GDP and $TRADE/GDP$ are constant, and the relative productivity of the country, A/T, is determined by their values, the frontier growth rate, and the parameters. Changes in the sources of innovation and adoption generate transitional growth to a new technology gap. The dynamics is consistent with the common understanding that differences in income levels are permanent, while differences in growth rates are transitory (Acemoglu and Ventura, 2002).

The productivity dynamics enter as part of the production functions. Value added (*X*) is defined as a Cobb-Douglas function of capital (*K*) and total efficient labor use (*L*). Land (*LD*) enters as a sector specific input in the traditional sector. The supply of land is assumed fixed over time, and to have balanced growth we introduce land augmenting technical progress (A_D) growing exogenously at the long-run rate:

$$X_i = K_i^{\alpha_i} L_i^{1-\alpha_i} \qquad i = m, s \tag{3}$$

$$X_a = A_D^{\alpha_{LND}} LD^{\alpha_{LND}} K_a^{\alpha_a} L_a^{1-\alpha_{LND}-\alpha_a} \tag{4}$$

Where the subscripts *a*, *m* and *s* represent traditional sector, modern sector and government services, respectively. Efficient labor is a CES aggregate of unskilled (*Lu*), semi-skilled (*Se*) and skilled (*Ls*) labor:

$$L_i = \left[\gamma_{1,i} A_i^{v-\frac{1}{2}\beta} Lu_i^v + \gamma_{2,i} A_i^v Se_i^v + (1-\gamma_{1,i}-\gamma_{2,i}) A_i^{v+\frac{1}{2}\beta} Ls_i^v \right]^{\frac{1}{v}} \tag{5}$$

In the traditional and modern sector labor augmenting technical progress (*A*) is equal and develops endogenously according to equation (1). The productivity level in government services is assumed to grow exogenously at the frontier rate. Labor and capital are mobile across sectors, but not internationally. γ_1 and γ_2 are the share parameters for unskilled and semi-skilled labor, respectively, and $\sigma = \frac{1}{1-v}$ ($v < 1$) is the elasticity of substitution between different labor types. Marginal productivity of skilled relative to unskilled labor is given as:

$$\frac{\partial X_i / \partial Ls_i}{\partial X_i / \partial Lu_i} = \frac{1-\gamma_{1,i}-\gamma_{2,i}}{\gamma_{1,i}} A_i^{\beta} \left(\frac{Ls_i}{Lu_i} \right)^{v-1} \tag{6}$$

Following from decreasing returns, an increase in the relative use of skilled labor reduces the relative marginal productivity. The direction and degree of technological bias is introduced through the parameter β, which gives the elasticity of the marginal productivity of skilled relative to unskilled labor with respect to labor augmenting technical progress. For β equal to zero, technical change is neutral and does not affect the relative efficiency of the three labor types. With a positive value of β technical change favors skilled workers and to a lesser extent semi-skilled workers (skill-biased technical change), while negative values imply that improvements in technology are biased towards unskilled labor.

To have balanced growth neutral technical change (β = 0) is a necessary long-run condition, but during transition the degree of technological bias is endogenously determined. The common understanding in South Africa is that trade liberalization and skill biased technological change are important to understand the development at the labor market. The specification of technological bias is linked to the relative importance of technology adoption and innovation as sources of productivity growth. The more dependent the economy is on adoption of foreign technology, the higher is the degree of skill-bias in technical change. The reduced form specification of technological bias is assumed to be an increasing and convex function of adoption relative to innovation:

$$\beta = b\left[\left(\frac{TRADE}{I}\right)^2 - 1\right] \tag{7}$$

where b is a constant parameter and *TRADE*/*I* represents the relative contribution of adoption and innovation from equation (1). Given the dimension of the trade and investment level in South Africa, the specification does not need scaling to generate sensible values of technological bias. With adoption as the main source of productivity growth technical change is skill-biased $(\beta > 0)$, while technology improvements driven by own innovations are biased towards unskilled labor $(\beta < 0)$. Equal importance of technology adoption and innovation gives neutral technical change.

3. The intertemporal general equilibrium model

The productivity dynamics is built into a standard intertemporal Ramsey growth model for a small open economy. It follows that capital accumulation and technological growth do not influence world prices and interest rate, which are exogenously given. The model setup of Diao et al. (2002, 2005) is the starting point, but is extended to capture endogenous skill-bias and balance between innovation and adoption in productivity growth, and to analyze income distribution effects. As discussed above, the production structure allows technical change to be biased towards unskilled or skilled labor, and the degree of bias is endogenously determined by the relative importance of adoption versus innovation in productivity improvements. Detailed documentation of the intertemporal general equilibrium model is given in a separate model appendix available from the authors.

Early applied Ramsey models include Goulder and Summers (1989), who study tax policy effects on investment in the US, and Go (1994), who applies the model framework on development issues. Our approach also relates to existing models of growth in dual economies. Stifel and Thorbecke (2003) model the dual character of an archetype African economy that is of relevance here. Irz and Roe (2001) develop a similar Ramsey model to analyze the interaction between agriculture and industry. Love (1997) analyzes industrialization in a dynamic general equilibrium model, also with an emphasis to the role of agriculture.

The Ramsey model describes an economy with macroeconomic stability, full employment of resources, and flexible allocation of resources between sectors according to profitability. The assumptions are certainly heroic, and it is a challenge to develop the model to include political and structural rigidities of the country. At this stage the model should be interpreted as representing the long run market adjustments expected to affect consumption demand and investment behavior, and with labor market adjustments faster than in reality.

The economy is disaggregated into three sectors: traditional, modern and government services. The division is based on skill-intensity, the traditional sector is unskilled-intensive and the modern sector is skill-intensive. The labor market formulation separates between unskilled, semi-skilled and skilled labor, and the relative wages are the key variables describing the income distribution. The model includes three household types according to income level and source of income: A poor household with unskilled wage income, a rich household with semi-skilled and skilled wage income, and a capitalist household with capital income. All savings are done by the capitalist household, which also pays interest on the foreign debt.

Except for government services, which are not traded internationally, we assume imperfect substitution between domestic and foreign goods, and the model then operates with two composite goods (traditional and modern). Imports are endogenously determined through an Armington composite system, while exports are determined through Constant Elasticity of Transformation (CET) functions.

The aggregate capital stock is managed by an independent investor who chooses an investment path to maximize the present value of future profits over an infinite horizon, subject to the capital accumulation constraint. With a waste due to the adjustment costs in investment, net profits as returns to capital go to the capitalist household. Investments can be financed through foreign borrowing, and the decisions about savings and investment can therefore be separated. Domestic savings and investments do not have to be equal in each period, but a long-run restriction on foreign debt exists. Increase in foreign capital inflows (i.e., trade deficits) in the current period, together with interest payments on existing debt, augments foreign debt in the next period.

For each household the consumption of traditional good, modern good and services are constant shares of its total consumption. But aggregate consumption of each good as share of total consumption can change over time. The poor household is assumed to consume relatively more traditional goods, while the rich and the capitalist household spends a relatively higher share of its income on modern goods. While within period consumption patterns differ between the three households, there exists a common intertemporal allocation of total income to consumption and savings to maximize its intertemporal utility. The intertemporal utility function is maximized subject to a budget constraint, which says that discounted value of total consumption cannot exceed discounted value of total income. Assuming intertemporal elasticity of substitution equal to one we have the well-known Euler equation for optimal allocation of total consumption expenditure (*E*) over time:

$$\frac{E_{t+1}}{E_t} = \frac{1+r}{1+\rho} \qquad (8)$$

where r is the world market interest rate and ρ the positive rate of time preference. The growth in consumption depends on the interest rate, the time preference rate, and the price path. Higher interest rate or lower time preference rate motivate more savings and thereby higher consumption spending in the future.

4. Productivity growth and income distribution in South Africa

South Africa achieved remarkable high growth from 1960 to the mid-1970s, here called the pre-sanctions period, with an annual average of above 6%. The implication was that the whites enjoyed a living standard at the level of the richest countries of the world, but the majority lived in poverty. According to our model this can be understood as transition growth generated by reduced barriers. Then the economic growth shifted down in the mid-1970s with the liberalization struggle and the international isolation. Many developing and developed economies experienced economic stagnation because of the oil crisis. The growth process in South Africa also was affected by local economic and political factors. It is a common understanding that Apartheid labor policies came to be a constraint on growth in South Africa. While initially the discrimination of blacks may have stimulated growth by cheap labor, now shortages of skilled labor are building up. When the sanctions were tightened, at the same time political unrest and labor strikes affected the economic development. The higher barriers contribute to the explanation of the economic stagnation. In the post-sanctions period the economic performance has improved, but the growth has been erratic and low on average. Lewis (2001) and Gelb (2004) offer a nice record of the recent economic history.

The early growth episode followed by stagnation is clearly described by the relative performance of South Africa. GDP per capita relative to the US was about 0.21 in 1960 and reached a peak of 0.25 in 1974. By 1994 relative GDP per capita has declined to 0.14, and the domestic level of real GDP per capita is lower than in 1970. The relative position to the US is further reduced to 0.13 in 2003. Overall the income gap to the frontier, here defined as the US, is steadily rising since 1974. Dijk (2002) documents a similar pattern of manufacturing labor productivity relative to the US, decreasing from 32% in 1970 to 20% in 1999. Domestic level of real GDP per capita is rising in the post-sanction period and reaches about the 1970 level in 2003. Table 1 presents some comparing statistics for the three periods.

Table 1 about here.

The growth model described above is calibrated to reproduce the main elements of the economic development during the three periods. The first step of the analysis is to calibrate a growth path that is close to the growth experienced in South Africa during 1960-2003 and projected to 2010. The model allows for a new measure of the protectionist effect of international sanctions. The empirical literature addressing foreign trade and trade policy faces the problem that sanctions cannot be measured directly. We calibrate a tariff-equivalent level that reproduces the actual development of the trade. Figure 1 reports the reproduction of the trade path. While tariffs are kept low (at 3%) during 1960-74, the slow growth of foreign trade during sanctions require a gradual increase of the tariff-equivalent after 1975, and with a peak in 1990 of about 78%. Interestingly, this tariff-equivalent measure of openness is consistent with the openness indicator for South Africa calculated by Aron and Muellbauer (2002) based on econometric estimation. The tariff-equivalent serves as the source of the barrier to international spillover.

Figure 1 about here.

The economic growth of the period under study is of transitional character, but is consistent with a long run growth path. Changing barriers lead to transitional growth with a long run equilibrium determined by a constant gap to the world technology frontier. Figure 2 shows how we track the declining, but erratic, actual growth rate as a steady decline in the model growth rate. The long-run equilibrium growth rate is assumed to be 3 percent (1 percent technological progress rate and 2 percent labor growth). The parameters supporting the long-run equilibrium path are discussed in the appendix. The calibration assumes long run balanced growth, i.e. the savings-investment balance can support a sustainable growth path, the structure of the economy is stable, and the trade surplus with interest payments balances the projected development of foreign debt.

Starting from the base year 1998, we calibrate backward a growth path that is close to the observed real GDP growth for the previous four decades and then allow this to project the post-sanctions growth through 2010. To reproduce the actual GDP of 1960, the initial level of the capital stock is reduced to about 10 percent of the base year level. Supply of skilled, semi-skilled and unskilled labor are also scaled down, and the skill-ratio (defined as skilled and semi-skilled relative to unskilled) is calibrated to increase from 0.62 in 1960 to about 0.8 at the end of the period studied (broadly consistent with data in Fedderke et al., 2003). The share of unskilled labor in total labor force declines from 0.62 to 0.56 during five decades, with a corresponding increase in the skilled labor share from 0.06 to 0.12. Sectoral TFPs are reduced according to the long run growth rate and foreign debt is adjusted to reproduce the initial year. The scaling back serves as an exogenous shock that takes the economy outside the equilibrium long run path in 1960. The initial capital stock is below the long run path and economic growth is driven by endogenous adjustment back to equilibrium growth. The calibrated economic growth rate during the pre-sanctions period 1961-74 is 5.6% on average, while the growth rate during sanctions (1975-1993) averages 3.5%. The post sanctions period has an increasing model growth rate with an average of about 3.5%.

Figure 2 about here.

The pre-sanctions period broadly observed the prediction of the model with high, but declining, growth. The understanding is that the reduced barriers generated profit opportunities that encouraged high investment. In standard fashion the marginal return to capital consequently was reduced over time. This is the core of the neoclassical convergence mechanism. In the beginning of the growth period studied the low level of the capital stock gives high marginal return to investment with consequent high investment growth and capital accumulation (Figure 3). Part of the investment must be imported from abroad with imperfect substitution between foreign and domestic goods. Technology spillovers embodied in foreign capital goods stimulate productivity growth, and contribute (together with domestic improvements of technology) to the increasing productivity growth path and catching-up relative to the frontier (see Figure 4). The capital and GDP growth rates decline over time due to decreasing returns to investment.

During the sanctions period the negative growth trend is strengthened. The international isolation represented by an increasing tariff-equivalent affects productivity growth directly by increasing the barriers to technology adoption and limiting the transfer of foreign spillovers. A possible scenario for South Africa would be to compensate the reduced openness with higher domestic investments. As seen from Table 1, this did not happen. Our understanding is that the cost of investment increases as imports of capital goods became more expensive with sanctions, and lower productivity growth further reduces the profitability of investments. The fall in capital growth strengthens the negative effect on productivity growth by reducing the growth in total imports and holding back domestic innovations, and the technology gap relative to the frontier increases over time. The growth path of the model is consistent with the low level of investment and the declining growth rate of productivity during the sanctions period.

Figure 3 and 4 about here.

While economic sanctions have negative effects on economic growth, the income distribution improves. Driven by increasing skill-ratio, the relative wage between unskilled and skilled labor increases in the pre-sanction period. Figure 5 shows how this positive distributive effect is strengthened during sanctions. Our understanding of the increased relative wage for unskilled labor is related to the development of technological bias. Increased tariffs have a negative effect on both technology adoption and innovation through higher barriers and lower capital accumulation, respectively. In our simulations the first effect dominates, and the relative importance of technology adoption decreases during the sanction period. The economy is forced to rely more on own improvements of technology, and the degree of skill-bias in technical change declines from 0.35 to 0.3. As explained in section 2, the degree of skill-bias is the elasticity of the marginal productivity of skilled relative to unskilled labor with respect to labor augmenting technical progress. Positive values imply bias towards skilled labor. Since technical change is relatively less skill-biased under sanctions, the relative marginal product of unskilled labor increases. The relative demand for unskilled workers is stimulated, and the relative wage gradually increases to meet the higher demand.

The change in income distribution generates shifts in the consumption pattern that strengthen the positive effect on the relative wage. Relative higher income for the poor household increases relative demand for traditional goods, which further increases the demand for unskilled labor (since the traditional sector uses unskilled labor relatively more intensively). The relative unskilled to skilled wage rate is below 0.16 in 1975, but increases to about 0.18 during the sanction period. Declining skill-bias improves the income distribution, but the increase in the relative wage is held back due to a shortage of skilled labor. Larger expansion of the skill-ratio would keep skilled wages down and contribute to the reduction of the wage gap between skilled and unskilled labor.

In the post-sanction period trade liberalization reduces the barriers to technology adoption, and the degree of skill bias increases gradually from 0.3 to 0.36. The increase in the skill-ratio is not sufficient to meet the higher skill demand and the wage gap widens over time. The relative wage between semi-skilled and skilled labor follows a similar pattern, increasing from 0.31 in 1960 to about 0.38 at the end of the period studied. According to Fedderke et al. (2003), the relative wage for semi-skilled labor increases from 0.32 in the 1970s, via 0.34 in the 80s, to about 0.37 in the 90s. Similar figures for the unskilled wage rate are 0.10, 0.16 and 0.25, respectively. The relative wage paths generated by the model are broadly consistent with this observed pattern.

Economic research in South Africa has addressed the relationship between wage inequality and skill bias. Edwards (2001) argues that skill bias has contributed to increased skill employment in South Africa. Abdi and Edwards (2002) address the puzzle that relative wages of unskilled has gone up, while unskilled employment has gone down since the mid-1970s. Since this is hard to explain in a standard labor market model, appeal to political and institutional factors to understand this is common, including increased union power. In our setting we emphasize a different channel of effects. The degree of skill-bias is reduced with sanctions and the higher demand for unskilled labor increases the relative wage of unskilled. Institutional factors are not built into our analysis and are hard to handle in this context.

Figure 5 about here.

The post-sanctions period shows increasing growth rate with our assumptions. The elimination of sanctions reduces the costs of imported investment goods and opens the economy to more technology adoption. Again the investment and productivity effects strengthen each other, but now in a positive direction. The increasing growth rate is closely related to the increased openness and assumes that reduction of protectionism continues steadily. Also the projection is the result of favorable conditions for investment allocation to take advantage of the improved profitability. Finally, the higher growth rate is driven by technology adoption, in practice associated with foreign direct investment. The actual growth has increased according to Table 1, but not fully at the potential indicated by the model projections. This can be due to macroeconomic disturbances excluded from the model. But it is more realistic to assume that the structural conditions of the economy are different from the flexible adjustments assumed in the model. The limited foreign direct investment observed may indicate that technology adoption has been below the projection shown.

5. Counterfactual analysis of sanctions

South Africa allows an interesting counterfactual analysis of the role of international sanctions and thereby the effect of barriers. As explained above, we have calibrated a tariff-equivalent growing from 1974 and with a peak in 1990 to reproduce the actual trade and growth path. Eliminating this rise in the tariff-equivalent during the sanctions period, we can simulate the economic development in an open economy without sanctions. In the experiment, the import tariff-equivalent is kept at a constant low level (3%) for the entire period studied. The new GDP growth path is shown in Figure 6 below. The main message is that South Africa could have avoided some of the decline in the growth rate. Sanctions have contributed to more costly investment goods and less technology adoption and consequently held back economic growth. The growth effect adds up to a rather large permanent income gap between the two scenarios. Without sanctions the 1998 level of real GDP would have been about 10 percent higher than its actual level in that year.

Figure 6 about here.

More openness reduces the cost of adopting foreign technology by limiting the barriers to technology transfer, and productivity growth increases over time (Figure 4). While the productivity growth in the reference path is declining after the mid-1970s, the productivity growth now is increasing with steadily higher trade. During the period of study we observe a weak degree of catch-up with relative productivity increasing from 0.36 to 0.38. The growth rate effect of higher trade is decreasing over time since the magnitude of the spillover effect and the return to own innovations gradually decline. In accordance with the catching-up hypothesis the learning potential from technology adoption declines as the technology gap decreases. The profitability of capital accumulation is stimulated by less expensive foreign capital goods and higher productivity growth. Decreasing returns to investment is counteracted, and capital growth is kept high over time (Figure 3). Increased capital accumulation generates domestic innovations and implies more imports, generating further technology spillovers from abroad. This productivity-investment interaction stimulates growth and contributes to the large growth differential between the two scenarios during transition. In the early pre-sanction period (1961-74) both capital and GDP growth are slightly higher along the calibrated South Africa path compared to the counterfactual path. This follows from intertemporal adjustment with perfect foresight, since expected higher tariffs (more expensive capital goods) in the future gives an incentive to increase current capital accumulation.

Given our model specification, there is a trade-off between economic growth and income distribution. While the aggregate economy benefits from a more open economy, the difference between poor and rich households increases. With lower tariffs the cost of technology transfer is kept low, and the economy takes advantage of foreign technology. Falling capital growth rate reduces the ability to generate local improvements of technology, and the relative importance of technology adoption increases over time. The new technology favors skilled workers, and the degree of skill-bias in technical change increases gradually from 0.33 in 1960 to 0.44 at the end of the period studied. This generates an increase in the relative demand for skilled labor, which counteracts the increasing skill supply and gives about constant relative wage over time. Changes in the consumption pattern following relative larger wage gap strengthen the negative effect on the income distribution. The rich household with semi-skilled and skilled wage income consumes relatively more modern goods, which uses skilled labor more intensively. This increases the demand for skilled labor and widens the wage gap even more. The economy is stuck in a vicious circle, where skill-biased technical change and demand-side effects of changing consumption pattern work together to worsen the income distribution. On average the unskilled wage, both relative to semi-skilled and skilled wage, drops about 2 percentage points compared to the sanction scenario (see Figure 5). But even though the relative unskilled wage rate is lower, the absolute income level for the poor household is eventually higher than along the calibrated path due to higher growth.

6. Concluding remarks

The analysis addresses the role of barriers for economic growth and income distribution in South Africa. The barriers to productivity growth are integrated in a standard intertemporal Ramsey growth model. Barriers to international technology spillovers influence both productivity growth and skill-bias. Reduced barriers stimulate transitional productivity growth and leads to more skill-intensive technology. The model is disaggregated to capture interactions between traditional and modern industrial sectors and adjustments at the labor markets for skilled, semi-skilled and unskilled labor. South Africa is an interesting case study of changing openness with consequences for technology adoption and skill-bias and thereby productivity growth and income distribution.

The model reproduces the declining growth rate since 1960 and separates between the pre-sanctions, sanctions and post-sanctions periods. The high and declining growth during pre-sanction 1961-1974 is consistent with reduced barriers and neoclassical convergence, the exploitation of profit opportunities with declining return. To understand the low growth during sanctions, 1975-1993, the importance of barriers to international spillovers should be recognized. The isolation of the economy implies higher costs of investment and reduced technology adoption. Interestingly, this period shows increase in the relative wage of unskilled labor. The protected economy has less skill-bias in technology. The model projects an increasing growth rate in the post-sanctions period, driven by cheaper investment goods and technology adoption with reduced barriers. The actual growth is somewhat below this projection, probably reflecting domestic barriers to competition and spillover.

The analysis reveals a trade off between economic growth and income distribution. Openness stimulates growth (spillovers, less expensive capital goods and productivity-investment interaction), but worsens the income distribution because foreign technology is skill-biased. The development of relative wages depends on the sources of productivity growth. While adoption of foreign technology generates skill-biased technical change, local improvement of technology through innovation can be directed towards unskilled labor.

The relationship between barriers and income distribution works through both supply-side effects (higher degree of skill-bias in technical change increases the demand for skilled labor) and demand-side effects (changes in the consumption pattern). Since the poor household consumes relatively more traditional goods, a worsening of the income distribution shifts consumption away from unskilled-intensive goods and reduces the demand for unskilled labor. The general equilibrium model puts this demand story in a broader context.

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Figure 1. Total trade: Calibrated path of model versus actual path (given in Billions of 1995 Rand)

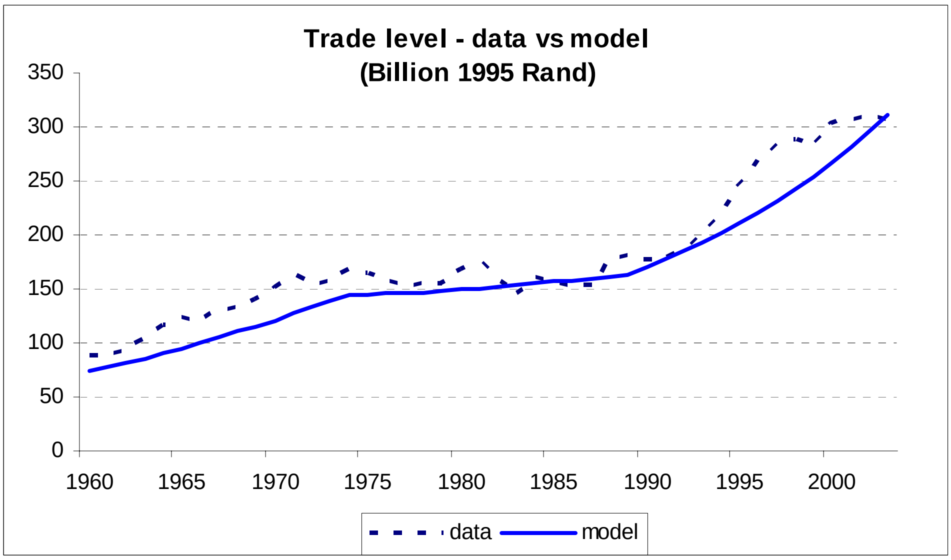

Figure 2. Real GDP growth rate: Calibrated path of model versus actual growth (measured as 5-year moving average)

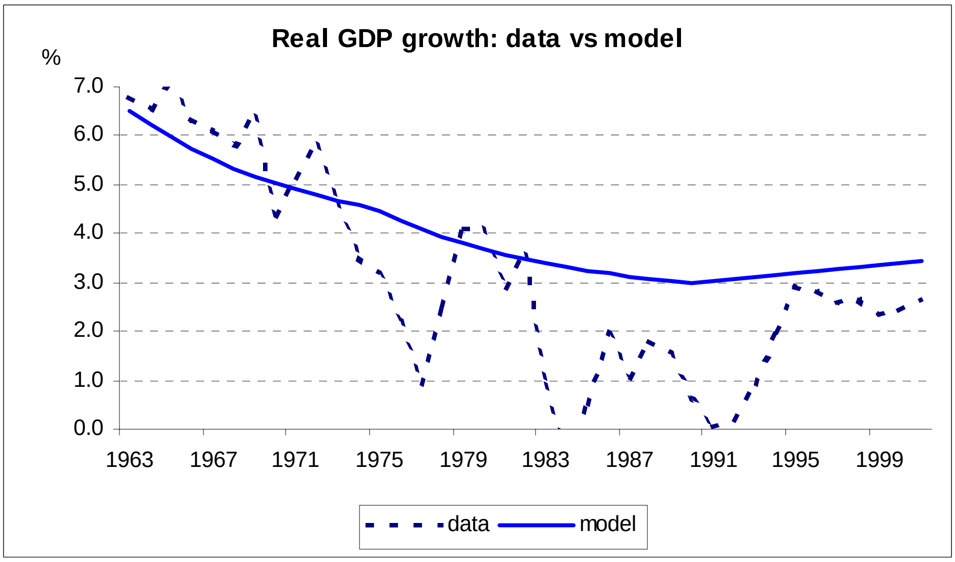

Figure 3. Growth rate of capital: calibrated path versus counterfactual path

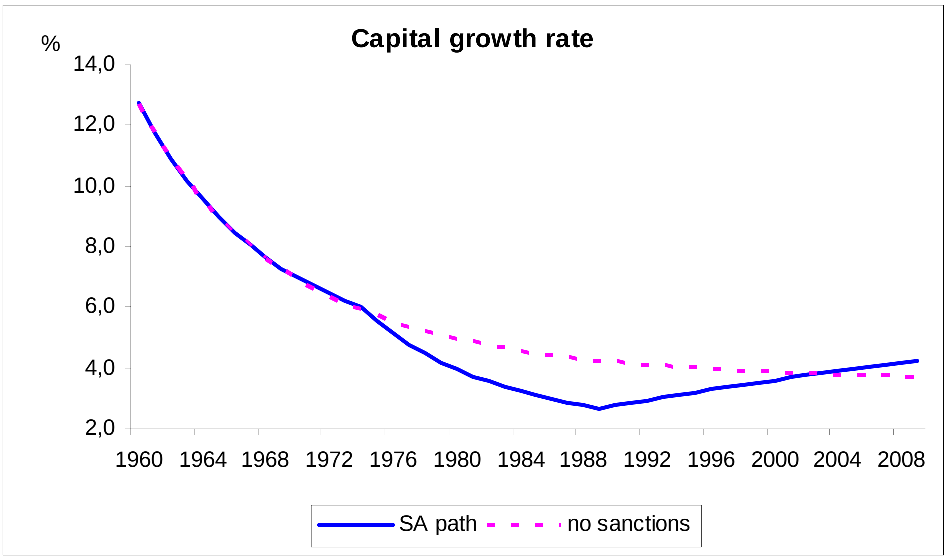

Figure 4. Labor augmenting technical progress: calibrated path versus counterfactual path

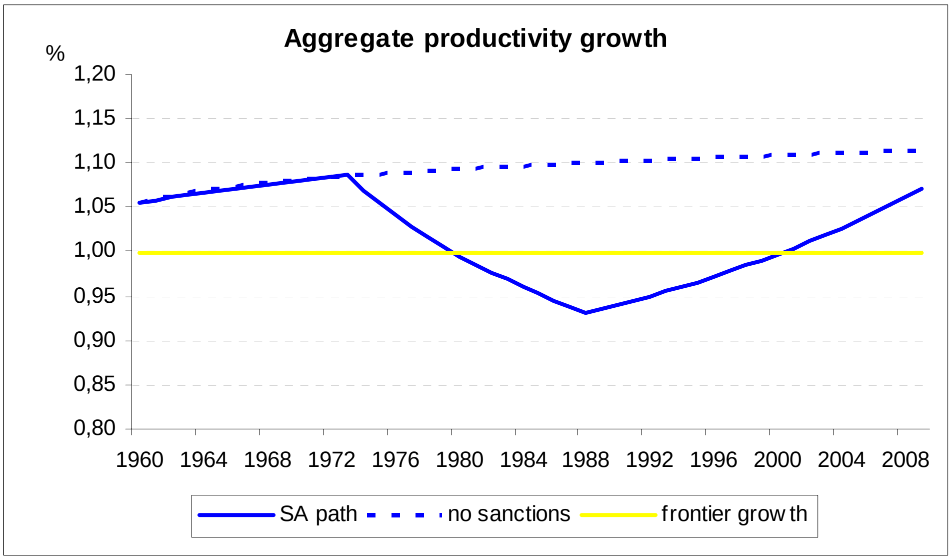

Figure 5. Unskilled wage rate relative to skilled wage rate: Calibrated path versus counterfactual path

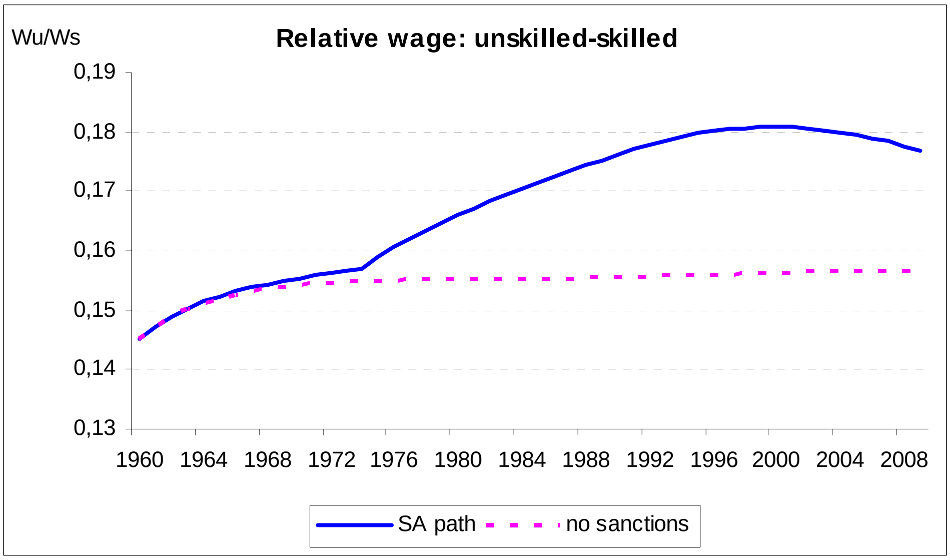

Figure 6. Real GDP growth: Calibrated path versus counterfactual path

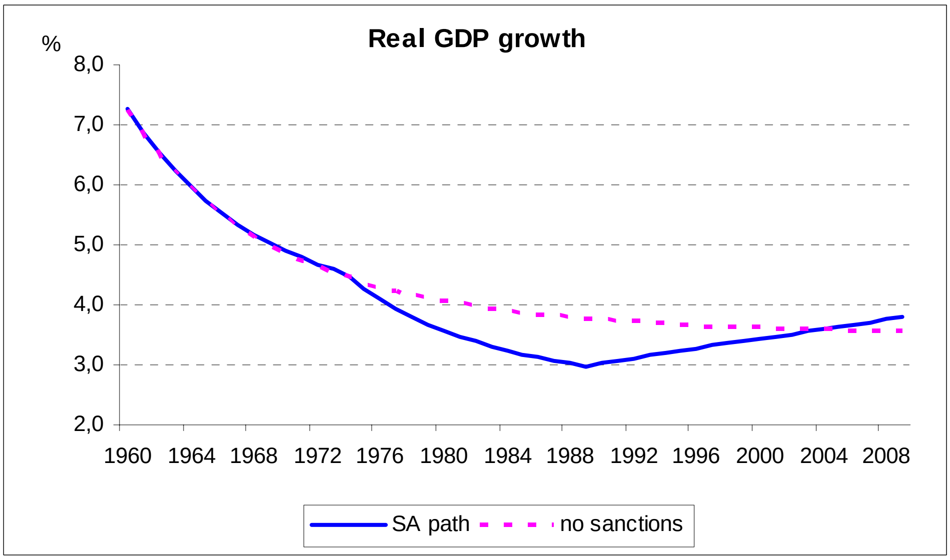

Table 1. South Africa growth experience 1961 – 2003

	1961 – 74 Pre-sanctions	**1975 – 93 Sanctions**	**1994-2003 Post-sanctions**
GDP growth rate	6.1 %	1.3 %	2.7 %
Growth in total trade	4.9 %	1.0 %	4.6 %
Growth in gross fixed capital formation	8.1 %	-0.2 %	3.9 %

Source: World Bank Development Indicators 2004

Appendix: Calibration

The parameters in the production, demand, and trade functions are set according to the method adopted in most static computable general equilibrium models and are based on the 1998 social accounting matrix (SAM) documented in a separate model appendix available from the authors. The long run growth path calibrated as supply side response to sectoral investment and productivity adjustments must be made consistent with the macroeconomic equilibrium as represented by the Euler equation: $r = (1+\rho)(1+g+n)-1$, where $g+n$ is the exogenous long-run growth rate. With a world market interest rate of 12.5 percent and long-run growth rate of 3 percent, the time preference rate is equal to 9.2 percent. Then, with the long run assumptions, most parameters of the intertemporal part of the model can be calibrated from the SAM. Given marginal product of capital, the initial capital stock is calculated based on capital income. Investment is calibrated from the long-run constraint on capital accumulation, for given values of depreciation rate and long run growth rate. The shadow price of capital equals the firm value relative to the capital stock, and follows when we know the interest rate. The initial level of foreign debt is set by the long-run constraint on debt accumulation, given data about trade deficit/surplus together with the long-run growth rate and interest rate. The θ values in the productivity growth function allocate the effects of the two sources of productivity growth, and θ_1 is set to 0.3 and θ_2 to 0.7. Based on the long run technological progress rate, initial values of the adoption and innovation variables, and the relative level of productivity, the parameter λ follows as a residual. To have balanced growth the skill-bias variable (β) is set equal to 0 in the calibration. The elasticity of substitution in both the Armington and CET functions are assumed to be 2, in accordance with national and international estimates as documented by Gibson (2003). These elasticities represent substitution possibilities between domestic and foreign goods (Armington), and between sales to domestic markets versus export markets (CET). The elasticity of substitution between different labor categories is important for the adjustment of relative wages, and is set equal to 2, which implies that unskilled, semi-skilled and skilled labor are substitutes.

Jørn Rattsø and Hildegunn E. Stokke,

Ramsey model of barriers to growth and skill-biased income distribution in South Africa

Appendix: Documentation of model and calibration

Date: February 7, 2005

1998 Social Accounting Matrix

The original SAM is developed by Thurlow and van Seventer (2002)[1] and includes 43 sectors, 14 household types and three labor categories: unskilled, semi-skilled and skilled. We aggregate this micro-SAM into a three-sector framework with three household types. To capture the dual structure of the South African economy, we distinguish between a modern sector using semi-skilled and skilled labor intensively and a traditional unskilled-intensive sector. In addition, government services are treated as a separate sector.

Traditional sector	Modern sector
Agriculture, forestry and fishing	Beverages & Tobacco
Coal mining	Paper and paper products
Gold and uranium ore mining	Printing, publishing and recorded media
Other mining	Coke and refined petroleum products
Food	Basic chemicals
Textiles	Other chemicals and man-made fibers
Wearing apparel	Rubber products
Leather and leather products	Non-metallic minerals
Footwear	Basic non-ferrous metals
Wood and wood products	Machinery and equipment
Plastic products	Electrical machinery and apparatus
Glass and glass products	Television, radio and communication equipment
Basic iron and steel	Professional and scientific equipment
Metal products excluding machinery	Motor vehicles, parts and accessories
Furniture	Other transport equipment
Building construction & civil engineering	Other manufacturing
Wholesale and retail trade	Electricity, gas and steam
	Water supply
	Catering and accommodation services
	Transport and storage
	Communication
	Finance and insurance
	Business services
	Medical, dental, veterinary & other services
	Other products

[1] Thurlow, J. and D. E. van Seventer (2002), A standard computable general equilibrium model for South Africa, TMD Discussion paper no. 100, IFPRI.

Based on average relative wages for the manufacturing sector during the 1990s given by Fedderke et al. (2003)[2], the relative wage between unskilled and skilled labor is assumed to equal 0.25, and the relative wage between semi-skilled and skilled labor is set to 0.37. With the labor income data from the SAM this means that the total labor force consists of 52% unskilled workers, 41% semi-skilled and 7% skilled workers. Both the traditional and the modern sector employ 34% of the labor force, while the remaining 32% works in government services. The original SAM does not include land as a production factor, and we therefore assume that 1/3 of the capital stock in traditional sector represents the input of land. Appendix table 2 gives further characteristics of the three sectors.

The model includes three household types according to income level and source of income: A poor household with unskilled wage income, a rich household with semi-skilled and skilled wage income, and a capitalist household with income from capital and land. All savings are done by the capitalist household, which also pays interest on the foreign debt. Income from sales taxes and import tariffs are transferred to the household sector lump sum. The distribution between the three household types is made according to income shares. The poor and the rich households do not save, and all income is used on consumption goods. The poor household is assumed to consume relatively more traditional goods, while the rich and the capitalist households spend a relatively higher share of its income on modern goods. This is consistent with the consumption pattern in the original SAM. The consumption share of government services is assumed to be relatively lower in the capitalist household. The income gap between the two wage-earning households (measured as the income of the poor household relative to the income of the rich household) equals 0.57. According to household wage income data in the original SAM, the poor household corresponds to more than the seven lowest income deciles (the 70 percent poorest of the population). Appendix table 3 gives further characteristics of the three households.

Except for import tariffs (which is an important parameter in the modeling of economic sanctions), we ignore transfers between the rest of the world and domestic agents. Capital and wage income going abroad are included as income to the capitalist and the rich household, respectively. We do not adjust total export and import, and the current account therefore differs from the original SAM. The adjustments give negative foreign savings (trade surplus). In intertemporal models the SAM is assumed to represent long-run balanced growth, and a trade surplus is consistent with growing foreign debt (as opposed to growing assets in the case of long-run trade deficit).

[2] Fedderke, J., Y. Shin and P. Vaze (2003), Trade, technology and wage inequality in the South African manufacturing sectors, mimeo, University of Witwatersrand.

Appendix Table 1: 1998 SAM SOUTH AFRICA (3 sectors, 3 households, 3 labor categories)

(Measured in Millions of Rand)

	ACT_A	ACT_M	ACT_S	COMD_A	COMD_M	COMD_S
ACT_A				446 541		
ACT_M					683 580	
ACT_S						197 431
COMD_A	182 706	125 330	5 106			
COMD_M	164 071	294 073	26 404			
COMD_S		167	10 494			
UNSK	56 896	25 291	52 590			
SEMI-SK	37 699	77 551	42 057			
SKILLED	22 276	47 517	9 886			
LND	30 000					
CAPITAL	61 193	185 206	50 186			
POOR HH						
RICH HH						
CAP HH						
MTAX				2 234	4 408	
ATAX	2 830	7 479	708			
SAV-INV						
RoW				41 159	140 441	
TOTAL	557 671	762 614	197 431	489 934	828 429	197 431

Appendix Table 1 continues:

	UNSK	SEMI-SK	SKILLED	LND	CAPITAL	POOR HH	RICH HH
ACT_A							
ACT_M							
ACT_S							
COMD_A						71 829	21 628
COMD_M						13 813	130 100
COMD_S						52 490	91 262
UNSK							
SEMI-SK							
SKILLED							
LND							
CAPITAL							
POOR HH	134 777						
RICH HH		157 307	79 679				
CAP HH				30 000	296 585		
MTAX							
ATAX							
SAV-INV							
RoW							
TOTAL	134 777	157 307	79 679	30 000	296 585	138 132	242 990

Note: The table continues on the next page.

Appendix Table 1 continues:

	CAP HH	MTAX	ATAX	SAV-INV	RoW	TOTAL
ACT_A					111 130	557 671
ACT_M					79 034	762 614
ACT_S						197 431
COMD_A	53 070			30 265		489 934
COMD_M	116 753			83215		828 429
COMD_S	42 450			568		197 431
UNSK						134 777
SEMI-SK						157 307
SKILLED						79 679
LND						30 000
CAPITAL						296 585
POOR HH		1 262	2 093			138 132
RICH HH		2 258	3 746			242 990
CAP HH		3 122	5 178		-8564	326 321
MTAX						6 642
ATAX						11 017
SAV-INV	114 048					114 048
RoW						181 600
TOTAL	326 321	6 642	11 017	114 048	181 600	

Note: ACT_A = Traditional activity, ACT_M = Modern activity, ACT_S = Government service activity, COMD_A = Traditional commodity, COMD_M = Modern commodity, COMD_S = Government service commodity, UNSK = Unskilled labor, SEMI-SK = Semi-skilled labor, SKILLED = Skilled labor, LND = land, POOR HH = Poor household, RICH HH = Rich household, CAP HH = Capitalist household, MTAX = Import tariffs, ATAX = Sales taxes, SAV-INV = Savings/Investments, RoW = Rest of world.

Appendix Table 2: Sector characteristics (based on the SAM in Appendix Table 1)

	Traditional	Modern	Govm. service
Value added share	0.30	0.48	0.22
Within sector distribution of labor:			
Unskilled	0.65	0.28	0.63
Semi-skilled	0.29	0.59	0.34
Skilled	0.06	0.13	0.03
Between sectors distribution of labor:			
Unskilled	0.42	0.19	0.39
Semi-skilled	0.24	0.49	0.27
Skilled	0.28	0.60	0.12
Total	0.34	0.34	0.32
Capital/Total capital	0.21	0.62	0.17
Export/Output	0.20	0.10	0.00
Export/Total export	0.58	0.42	0.00
Import/Supply	0.08	0.17	0.00
Import/Total import	0.23	0.77	0.00

Note: Within sector labor shares are calculated based on the assumption that the relative wage between skilled and unskilled equals 0.25 and the relative wage between skilled and semi-skilled equals 0.37.

Appendix Table 3: Consumption pattern (based on the SAM in Appendix Table 1)

	Poor household	Rich household	Capitalist household
Share traditional good	0.52	0.08	0.25
Share modern good	0.10	0.54	0.55
Share govm services	0.38	0.38	0.20
Income share	0.20	0.34	0.46
Income share (without capitalist household)	0.36	0.64	

The mathematical documentation of the model

1. Equations

The following equations are the detailed description of the model. The numerical model is solved by the General Algebraic Modeling System (GAMS).

The consumer's decision

We separate between three kind of households, which differ with respect to their within period consumption patterns. But since we apply a representative agent model, there is a common intertemporal consumption decision based on utility maximization:

$$Max\;\; U_1 = \sum_{t=1}^{T}(1+\rho)^{-t}\ln(Q_t) + \ln(Q_T)\frac{(1+\rho)^{1-T}}{\rho}$$

$$s.t. \sum_{\mathrm{i}} P_{i,t}\cdot TC_{i,t} = Y_t - SAV_t$$

U_1 is the value of the intertemporal utility evaluated at time period 1's price, and aggregate consumption, Q_t, for each time period is defined as:

$$Q_t = \sum_i TC_{i,t}$$

where $i = a, m, s$ representing traditional, modern and government services, respectively, and $TC_{i,t}$ is total consumption for each good. Y_t is consumer income for each period and is defined as the sum of the incomes of the three household types:

$$Y_t = \sum_h Y_{h,t}$$

where $h = poor, rich, cap$ representing poor, rich and capitalist household, respectively. The poor household receives income from unskilled wages, the rich household from semi-skilled and skilled wages, and finally the capitalist household from capital and land income. All savings are assumed done by the capitalist household, which also covers interest payments on the foreign debt. Income from sales taxes and import tariffs are transferred lump-sum to the household sector. The distribution between the three households is made according to income shares.

$$Y_{poor} = Wu_t Lu_t + ay1\cdot\sum_i atr_i\cdot PX_{i,t}\cdot X_{i,t} + by1\cdot\sum_j mtr_j\cdot PWM_j\cdot M_{j,t}$$

$$Y_{rich} = Ws_t Ls_t + Wse_t Se_t + ay2\cdot\sum_i atr_i\cdot PX_{i,t}\cdot X_{i,t} + by2\cdot\sum_j mtr_j\cdot PWM_j\cdot M_{j,t}$$

$$Y_{cap} = Rk_t K_t + Wd\cdot LD + ay3\cdot\sum_i atr_i\cdot PX_{i,t}\cdot X_{i,t} + by3\cdot\sum_j mtr_j\cdot PWM_j\cdot M_{j,t} + FSAV_t$$

where $j = a, m$ represents traditional and modern good, respectively, which are traded internationally.

The Euler equation for the consumer problem is:

$$\frac{E_{t+1}}{E_t} = \frac{1+r}{1+\rho}$$

$$E_t = \sum_i P_{i,t} TC_{i,t}$$

Total demand for each commodity is aggregated over the three households:

$$TC_{i,t} = \sum_h C_{i,h,t}$$

For each household the consumption of good *i* is a constant share of household income (minus savings for the capitalist household). The poor household is assumed to consume relatively more traditional goods compared to the rich household.

$$P_{i,t} C_{i,h,t} = \mu_{i,h} \cdot Y_{h,t} \qquad \text{for } h = poor, rich$$

$$P_{i,t} C_{i,h,t} = \mu_{i,h} \cdot (Y_{h,t} - SAV_t) \qquad \text{for } h = cap$$

Endogenous productivity

The rate of labor augmenting technical progress is endogenously driven by technology adoption and own improvements of technology. While adoption of foreign technology depends on the degree of international barriers (measured by the trade share) and the technology gap, the innovative activity is approximated by the investment level. Labor augmenting technical progress in traditional and modern sector is specified as:

$$\frac{\dot{A}}{A} = \left(\frac{I}{GDP}\right)^{\theta_1} + \lambda \left(\frac{TRADE}{GDP}\right)^{\theta_2} \left(1 - \frac{A}{T}\right)$$

The productivity level in government services is assumed to grow exogenously. In the long run productivity grows at the frontier rate (*g*) and the technology gap is constant.

Production decision and technological bias

Value added is defined as a Cobb-Douglas function of capital and total efficient labor use. Land (*LD*) enters as a sector specific input in the traditional sector. The supply of land is assumed fixed over time, and to have balanced growth we introduce land augmenting technical progress (A_D) growing exogenously at the long-run rate:

$$X_{i,t} = K_{i,t}^{\alpha_i} L_{i,t}^{1-\alpha_i} \qquad i = m, s$$

$$X_a = A_D^{\alpha_{LND}} LD^{\alpha_{LND}} K_a^{\alpha_a} L_a^{1-\alpha_{LND}-\alpha_a}$$

where *L* is efficient labor use, which is a CES aggregate of unskilled (*Lu*), semi-skilled (*Se*) and skilled (*Ls*) labor:

$$L_{i,t} = \left[\gamma_{1,i} A_{i,t}^{v-\frac{1}{2}\beta_t} Lu_{i,t}^v + \gamma_{2,i} A_{i,t}^v Se_{i,t}^v + (1-\gamma_{1,i}-\gamma_{2,i}) A_{i,t}^{v+\frac{1}{2}\beta_t} Ls_{i,t}^v \right]^{\frac{1}{v}}$$

where $\sigma = \frac{1}{1-v}$ is the elasticity of substitution between different labor types.

β is a variable representing the degree and direction of bias in technical change, and is assumed to depend on the relative importance of technology adoption and innovation as sources of productivity growth:

$$\beta_t = b\left[\left(\frac{TRADE_t}{I_t}\right)^2 - 1\right]$$

where *TRADE/I* represents the relative contribution of adoption and innovation.

First order conditions are:

$$\alpha_i PV_{i,t} X_{i,t} = Rk_t \cdot K_{i,t}$$
$$\alpha_{LND} PV_{a,t} X_{a,t} = Wd_t \cdot LD$$
$$(1-\alpha_i-\alpha_{LND}) PV_{i,t} X_{i,t} \gamma_{1,i} A_{i,t}^{v-\frac{1}{2}\beta_t} Lu_{i,t}^{v-1} = Wu_t \cdot \left[\gamma_{1,i} A_{i,t}^{v-\frac{1}{2}\beta_t} Lu_{i,t}^{v} + \gamma_{2,i} A_{i,t}^{v} Se_{i,t}^{v} + (1-\gamma_{1,i}-\gamma_{2,i}) A_{i,t}^{v+\frac{1}{2}\beta_t} Ls_{i,t}^{v}\right]$$
$$(1-\alpha_i-\alpha_{LND}) PV_{i,t} X_{i,t} (1-\gamma_{1,i}-\gamma_{2,i}) A_{i,t}^{v+\frac{1}{2}\beta_t} Ls_{i,t}^{v-1} = Ws_t \cdot \left[\gamma_{1,i} A_{i,t}^{v-\frac{1}{2}\beta_t} Lu_{i,t}^{v} + \gamma_{2,i} A_{i,t}^{v} Se_{i,t}^{v} + (1-\gamma_{1,i}-\gamma_{2,i}) A_{i,t}^{v+\frac{1}{2}\beta_t} Ls_{i,t}^{v}\right]$$
$$(1-\alpha_i-\alpha_{LND}) PV_{i,t} X_{i,t} \gamma_{2,i} A_{i,t}^{v} Se_{i,t}^{v-1} = Wse_t \cdot \left[\gamma_{1,i} A_{i,t}^{v-\frac{1}{2}\beta_t} Lu_{i,t}^{v} + \gamma_{2,i} A_{i,t}^{v} Se_{i,t}^{v} + (1-\gamma_{1,i}-\gamma_{2,i}) A_{i,t}^{v+\frac{1}{2}\beta_t} Ls_{i,t}^{v}\right]$$

where α_{LND} is the land share, which is zero in modern sector and government services.

Value-added price for each sector:

$$PV_{i,t} = PX_{i,t}(1-atr_i) - \sum_j P_{j,i,t} IO_{j,i}$$

Intermediate goods are employed according to the fixed coefficient:

$$INT_{i,t} = \sum_j IO_{ij} \cdot X_{j,t}$$

GDP at factor price:

$$GDP_t = \sum_i PV_{i,t} \cdot X_{i,t}$$

Investment decision

Investment decision is made according to intertemporal profit maximization, subject to the accumulation of the capital stock over time:

$$\underset{I,K}{Max} \sum_{t=1}^{\infty} (1+r)^{-t} \left[Rk_t \cdot K_t - (PI_t \cdot I_t + ADJ_t)\right]$$

$$s.t. \;\; K_{t+1} = K_t \cdot (1-\delta) + I_t$$

where

$$I_t = AK \cdot \prod_i IVD_{i,t}^{iels_i}$$

The adjustment costs in real terms, *ADJ*, consume the modern good and are specified as:

$$ADJ_t = a \cdot P_{m,t} \cdot \frac{I_t^2}{K_t}$$

where a is constant, P_m is the price of the modern good, I_t investment in real term, and K_t is the capital stock.

Differentiating the intertemporal profit function of the investor with respect to I_t gives:

$$q_t = PI_t + 2 \cdot P_{m,t} \cdot a \cdot \frac{I_t}{K_t}$$

where PI is unit cost of investment net adjustment costs. This relationship says that the investor equilibrates the marginal cost of investment, which is given on the right hand side, and the shadow price of capital, q. Differentiating the same function with respect to K_t gives us the well-known no-arbitrage condition:

$$r \cdot q_{t-1} = Rk_t + a \cdot P_{m,t} \cdot \left(\frac{I_t}{K_t}\right)^2 - \delta \cdot q_t + \dot{q}_t$$

which states that marginal return to capital has to equal the interest payments on a perfectly substitutable asset of size q_{t-1}. The first term on right hand side, Rk_t, is the capital rental rate, while the second term is the derivative of capital in the adjustment cost function. The marginal return to capital also has to be adjusted by the depreciation rate, δ, and capital gain or loss, $\dot{q}$.

Investment demand:

$$IVD_{i,t} = \frac{iels_i \cdot PI_t \cdot I_t}{P_{i,t}}$$

Total investment demand for the modern good includes the adjustment cost:

$$TIVD_{m,t} = \frac{iels_m \cdot PI_t \cdot I_t}{P_{m,t}} + a \cdot \frac{I_t^2}{K_t}$$

Exports and Imports

Except for government services, which are not traded internationally, we assume imperfect substitution between domestic and foreign goods, and the model operates with two composite goods (traditional and modern). Imports and domestic demand are endogenously determined through an Armington composite system. The demand functions are derived from minimizing current expenditure, subject to the Armington function:

$$Min \;\; PM_j \cdot M_{j,t} + PD_{j,t} \cdot D_{j,t}$$
$$s.t. \;\; CC_{j,t} = aa_j [ma_j \cdot M_{j,t}^{-exa} + (1 - ma_j) D_{j,t}^{-exa}]^{-1/exa}$$

where the subscript j represents traditional (a) and modern (m) good.

$PM_j = PWM_j \cdot er(1 + mtr_j)$ is the price of import goods.

The first order conditions:

$$\frac{M_{j,t}}{CC_{j,t}} = aa_j^{\frac{-exa}{exa+1}} \cdot \left(ma_j \frac{P_{j,t}}{PM_j} \right)^{\frac{1}{exa+1}}$$

$$\frac{D_{j,t}}{CC_{j,t}} = aa_j^{\frac{-exa}{exa+1}} \cdot \left((1-ma_j) \cdot \frac{P_{j,t}}{PD_{j,t}} \right)^{\frac{1}{exa+1}}$$

where $exa = \frac{1}{\sigma_m} - 1$.

Sales to export market versus domestic market are endogenously determined through a CET function, and domestic and export goods are imperfect substitutes. The supply functions are derived from maximizing current sales income, subject to the CET function:

$$Max \quad PD_{j,t} \cdot D_{j,t} + PE_j \cdot EX_{j,t}$$

$$s.t. \quad X_{j,t} = ac_j [mc_j \cdot EX_{j,t}^{exc} + (1-mc_j) D_{j,t}^{exc}]^{1/exc}$$

$PE_j = PWE_j \cdot er$ is the export price.

The first order conditions:

$$\frac{D_{j,t}}{X_{j,t}} = ac_j^{\frac{exc}{1-exc}} \cdot \left((1-mc_j) \cdot \frac{PX_{j,t}}{PD_{j,t}} \right)^{\frac{1}{1-exc}}$$

$$\frac{EX_{j,t}}{X_{j,t}} = ac_j^{\frac{exc}{1-exc}} \cdot \left(mc_j \cdot \frac{PX_{j,t}}{PE_j} \right)^{\frac{1}{1-exc}}$$

where $exc = \frac{1}{\sigma_e} + 1$.

Foreign borrowing and foreign debt

$$FSAV_t = \sum_j (PWM_j \cdot M_{j,t} - PWE_j \cdot EX_{j,t})$$

$$DEBT_{t+1} = DEBT_t \cdot (1+r) + FSAV_t$$

Foreign debt is accumulated over time from trade deficits and interest payments on outstanding debt.

Factor market equilibrium

$$Lu_t = \sum_i Lu_{i,t}$$

$$Se_t = \sum_i Se_{i,t}$$
$$Ls_t = \sum_i Ls_{i,t}$$
$$K_t = \sum_i K_{i,t}$$

From these equations we determine wage rates and marginal product of capital.

Commodity market equilibrium

$CC_{j,t} = INT_{j,t} + TC_{j,t} + TIVD_{j,t}$ for traditional and modern good

$X_{s,t} = INT_{s,t} + TC_{s,t} + TIVD_{s,t}$ for government services

These equations determine the equilibrium price, $P_{i,t}$.

Terminal conditions (long run constraints)

The terminal conditions are imposed in the model, such that when the time is beyond T, which is the last period in the model, all endogenous variables have to approach approximately to their long run situation.

$$I_T = (\delta + g + n)K_T$$
$$FSAV_T = (g + n - r)DEBT_T$$
$$Rk_T + a \cdot P_{m,T}\left(\frac{I_T}{K_T}\right)^2 = (r + \delta)q_T$$

These conditions state that foreign debt and capital stocks grow at a constant rate given by $g + n$, and that marginal return to capital becomes constant. With positive foreign debt in the long run, the country has to run trade surplus as $r > g + n$ from the Euler equation.

2. Glossary

Parameters

α_i	share parameter for capital in value added function sector *i*
α_{LND}	share parameter for land in traditional value added function
v	exponent in CES function of skilled and unskilled labor
σ	elasticity of substitution between skilled and unskilled labor
$\gamma_{1,i}$	share parameter in CES function for unskilled labor sector *i*
$\gamma_{2,i}$	share parameter in CES function for semi-skilled labor sector *i*
IO_{ij}	input-output coefficient for commodity i used in sector j
exa	exponent in Armington functions
σ_m	elasticity of substitution between imported and domestic goods
ma_i	distribution parameter in Armington function for commodity i
aa_i	shift parameter in Armington function for commodity i
exc	exponent in CET functions
σ_e	elasticity of substitution between domestic goods and exports
mc_i	distribution parameter in CET function for commodity i
ac_i	shift parameter in CET function for commodity i
$cles_i$	share of consumer's demand for commodity i
cs	shift parameter in total consumption function
AK	shift parameter in total investment function
$iels_i$	share of investment demand for commodity i
a	coefficient in adjustment cost function
ρ	rate of consumer's time preference
δ	capital depreciation rate
b	parameter in technological bias function
θ_1	elasticity of innovation-driven productivity growth wrt the investment rate
θ_2	elasticity of technology adoption wrt the trade share
λ	coefficient in labor augmenting technical progress function
ay1	share of sales taxes going to the poor household
ay2	share of sales taxes going to the rich household
ay3	share of sales taxes going to the capitalist household
by1	share of import tariffs going to the poor household
by2	share of import tariffs going to the rich household
by3	share of import tariffs going to the capitalist household
$\mu_{i,h}$	share of total consumption by household *h* going to good *i*

Exogenous variables

PWM_i	world import price for commodity i
PWE_i	world export price for commodity i
T	productivity level at the frontier
LD	land supply
A_D	land augmenting technical progress

atr_i	sales tax rate for commodity i
mtr_i	tariff rate for commodity i
er	nominal exchange rate
r	world market interest rate
n	exogenous growth rate for unskilled and skilled labor supply
g	exogenous productivity growth
Lu_t	unskilled labor supply
Se_t	semi-skilled labor supply
Ls_t	skilled labor supply

Endogenous variables

$X_{i,t}$	output of commodity i
$K_{i,t}$	sector's capital demand
$L_{i,t}$	efficient labor use
$Lu_{i,t}$	sector's unskilled labor demand
$Se_{i,t}$	sector's semi-skilled labor demand
$Ls_{i,t}$	sector's skilled labor demand
$D_{i,t}$	good i produced and consumed domestically
$M_{i,t}$	imports of commodity i
$CC_{i,t}$	total absorption of composite good i
$EX_{i,t}$	exports of commodity i
$TRADE_t$	total trade
$TC_{i,t}$	total consumer demand for good i
$C_{i,h,t}$	demand for good i from household h
E_t	total consumption expenditure
$INT_{i,t}$	intermediate demand for good i
$IVD_{i,t}$	investment demand for good i
$TIVD_{m,t}$	total investment demand for industrial good (including adjustment cost)
I_t	investment in quantity
K_t	capital stock
ADJ_t	adjustment costs
Q_t	aggregate consumption
Y_t	total consumer income
$Y_{h,t}$	income household h
SAV_t	total savings
GDP_t	GDP
$FSAV_t$	trade deficit
$DEBT_t$	foreign debt

PV_{it}	value added price for commodity i
Wu_t	unskilled wage rate
Wse_t	semi-skilled wage rate
Ws_t	skilled wage rate
Wd_t	rate of return to land
Rk_t	rate of return to capital
$PX_{i,t}$	producer price for commodity i
$P_{i,t}$	Armington composite price for commodity i
$PD_{i,t}$	price for D_i
PM_i	import price for commodity i
PE_i	export price for commodity i
PI_t	unit cost of investment that builds up capital equipment
q_t	shadow price of capital
β_t	degree of bias in technical change
$A_{i,t}$	labor augmenting technical progress

Values of selected parameters and initial values of endogenous variables

Definition	Symbol in the model	Value
Parameters		
Share of capital in value added for traditional sector	α_a	0.29
Share of capital in value added for modern sector	α_m	0.55
Share of capital in value added for government services	α_s	0.32
Share of land in value added for traditional sector	α_{LND}	0.14
Elasticity of substitution between different labor types	σ	2
Distribution parameter in CES labor function, traditional sector	$\gamma_{1,a}$	0.31
Distribution parameter in CES labor function, modern sector	$\gamma_{1,m}$	0.17
Distribution parameter in CES labor function, services	$\gamma_{1,s}$	0.34
Distribution parameter in CES labor function, traditional sector	$\gamma_{2,a}$	0.31
Distribution parameter in CES labor function, modern sector	$\gamma_{2,m}$	0.36
Distribution parameter in CES labor function, services	$\gamma_{2,s}$	0.37
Parameter in technological bias function	b	0.05
Elasticity of innovation-driven growth w.r.t. investment	θ_1	0.3
Elasticity of technology adoption w.r.t. trade	θ_2	0.7
Parameter in technical progress function	λ	1.07
Distribution parameter CET function traditional sector	mc_a	0.67
Distribution parameter CET function modern sector	mc_m	0.75
Distribution parameter Armington function traditional sector	ma_a	0.24
Distribution parameter Armington function modern sector	ma_m	0.32
Elasticity in Armington function	σ_{m}	2
Elasticity in CET function	σ_{e}	2
Coefficient in adjustment cost function	a	2.61
Time preference rate	ρ	0.092
Depreciation rate	δ	0.04
Endogenous variables		
Marginal returns to capital	$Rk + a \cdot P_m \left(I/K \right)^2$	0.19
Marginal product of capital	Rk	0.18
Derivative of adjustment cost w.r.t capital	$-a \cdot P_m \left(I/K \right)^2$	-0.01
Shadow price of capital	q	1.17
Adjustment cost per unit of investment	$a \cdot P_m\, I/K$	0.18
Unskilled wage rate	Wu	0.25
Semi-skilled wage rate	Wse	0.37
Skilled wage rate	Ws	1