WORKING PAPER SERIES

No. 13/2002

MODEL SPECIFICATION AND INFLATION FORECAST UNCERTAINTY

Gunnar Bårdsen
Eilev S. Jansen
Ragnar Nymoen

Department of Economics

Norwegian University of Science and Technology
N-7491 Trondheim, Norway
www.svt.ntnu.no/iso/wp/wp.htm
Model specification and inflation forecast uncertainty*

Gunnar Bårdsen
Norwegian University of Science and Technology and Norges Bank

Eilev S. Jansen
Norges Bank and Norwegian University of Science and Technology

Ragnar Nymoen
University of Oslo and Norges Bank

First Version : 28. April 2000
This version: 29. January 2002

Abstract

Three classes of inflation models are discussed: Standard Phillips curves, New Keynesian Phillips curves and Incomplete Competition models. Their relative merits in explaining and forecasting inflation are investigated theoretically and empirically. We establish that Standard Phillips-curve forecasts are robust to types of structural breaks that harm the Incomplete Competition model forecasts, but exaggerate forecast uncertainty in periods with no breaks. As the potential biases in after-break forecast errors for the Incomplete Competition model can be remedied by intercept corrections, it offers the best prospect of successful inflation forecasting.

Keywords: monetary policy, inflation targeting, wages and prices, model specification, encompassing, model uncertainty, forecasting.


*We would like to thank the participants at the ADRES-CEPREMAP-CREST conference ”The Econometrics of Policy Evaluation” in Paris 10–12 January 2000 for valuable comments to a closely related paper (Bårdsen et al. (1999)). We would also like to thank two referees, Mike Clements, David F. Hendry, Kåre Johansen, Kenneth F. Wallis and the participants at the conference ”Macroeconomic Transmission Mechanisms: Empirical Applications and Econometric Methods” in Copenhagen 18–20 May 2000 for helpful advice and comments. The paper has also been presented at the Project LINK Fall Meeting in Oslo 2–6 October 2000, the Norges Bank workshop ”The conduct of monetary policy in open economies” at the Norwegian Academy of Science 26–27 October 2000, the Norwegian Economists’ Conference in Bø, Telemark 8–9 January 2001 and the Macroeconomic Modelling Seminar at University of Warwick 2–4 July 2001. The views expressed are those of the authors and should not be interpreted as reflecting those of Norges Bank.
1 Introduction

Theoretical research has begun to explore the implications for monetary policy of uncertainty about the inflation process, see e.g. Batini et al. (1999). So far only very specific and limited forms of uncertainty have been considered: For example, the case where the exact specification of the inflation process is known, but the parameters are unknown and have to be estimated. However, uncertainty is quite pervasive in that policy makers face a menu of different models, all claiming to correctly representing the true model of the economy. Also, as emphasized in Svensson (1997), an explicit inflation target implies that the central bank’s conditional forecast 1-2 years ahead becomes the intermediate target of monetary policy. Consequently, there is an unusually strong linkage between forecasting and policy analysis.

The statistical foundation for a conditional forecast as an operational target is that forecasts calculated as the conditional mean are unbiased and no other predictor (conditional on the same information set) has smaller mean-squared forecast error (MSFE), provided the first two moments exist. The practical relevance of the result is reduced by the implicit assumption that the model corresponds to the data generating process (DGP), and that the DGP is constant over the forecast horizon. Credible forecasting methods must take into account that neither condition is likely to be fulfilled in reality.

There are therefore two elements of forecasting particularly relevant for inflation targeting. First, the inflationary process should be captured as correctly as possible. Second, forecasting should take into account that structural changes can occur. As regards the first element, policy makers are faced with several complementary, and sometimes also competing, economic explanations of the inflationary process. Thus, a selection process usually takes place where not only econometric testing but also ‘beliefs’ play a significant role, cf. Granger (1990, 1999). However, the specific inflation models have one important trait in common: they explain inflation—a growth rate—by not only other growth rates but also cointegrating combinations of levels variables. Thus, they are explicitly or implicitly error correction models or, following Hendry (1995), equilibrium correction models, denoted EqCMs. This leads to the second element, namely that EqCMs despite providing good after-the-event explanations of inflation, do not forecast nearly as accurately. Moreover, when forecasting in the presence of structural breaks, there may be a relative advantage of using models specified in differences only—without equilibrium correction terms—so called dVARs, see e.g. Clements and Hendry (1999) and Eitrheim et al. (2002). There is thus a trade-off between the importance of structural modelling for understanding and its cost in terms of losing forecasting robustness relative to dVARs. This paper assesses the importance of this trade-off for inflation forecasting.

Specifically, we consider the two most popular inflation models, namely Phillips curves and wage curve specifications. The standard Phillips curve model (denoted PCM), though formally an EqCM, might come to inherit some of the forecast-robustness of a dVAR. This possibility arises simply because the typical Phillips curve is similar to a dVAR—the only level term included being the output gap or the unemployment rate. The New Keynesian Phillips Curve Model (NPCM), utilizing the staggered contracts framework, has been advocated by Clarida et al. (1999) and Gali and Gertler (1999). It has explicitly forward looking expectations and has
come to dominate the theoretical literature on inflation targeting in particular, as laid out in Svensson (2000). Gali and Gertler (1999) also argues for the inclusion of real unit labour costs instead of an output gap measure, as in the more usual variants—see e.g. Fuhrer (1997). Phillips-curve models, with or without explicitly forward looking terms, therefore continue to hold their ground in both theoretical and empirical models of monetary policy.\footnote{For example, the Bank of England (1999) includes Phillips-curve models in their suite of models for monetary policy. Mervyn King, the Deputy Governor of the Bank of England put it quite explicitly: ‘...the concept of a natural rate of unemployment, and the existence of a vertical long-run Phillips curve, are crucial to the framework of monetary policy’—see King (1998, p.12).}

The wage curve is consistent with a wide range of economic theories, see Blanchard and Katz (1997), but its original impact among European economists was due the explicit treatment of union behaviour and imperfectly competitive product markets, pioneered by Layard and Nickell (1986). Because the modern theory of wage and price setting recognizes the importance of imperfect competition and incomplete information on both product and labour markets, we refer to this class of models as the Imperfect Competition Model—ICM hereafter. Since wage-curve models are EqCM specifications, they are vulnerable to regime shifts, e.g. changes in equilibrium means.

The existing empirical evidence on the inflationary process is mixed. Although varieties of Phillips curves appear to hold their ground when tested on US data—see Fuhrer (1995), Gordon (1997), Gali and Gertler (1999), and Blanchard and Katz (1999)—studies from Europe usually conclude that ICM models are preferable, see e.g. (Drèze and Bean, 1990, Table 1.4), OECD (1997, Table 1.A.1), Wallis (1993) and Rødseth and Nymoen (1999).

In section 2, we discuss the key differences between both versions of the Phillips-curve and the incomplete competition model. Section 3 presents the empirical results for the three contending models of the inflation process in Norway. In section 4 we discuss the algebra of inflation forecasts based on the competing models. Section 5 evaluate the forecasting properties. Section 6 concludes.

2 Illustrating inflation models

To illustrate the main differences between the alternative specifications, consider the following framework.\footnote{Since detailed derivations of the alternative models are readily available elsewhere, we here focus on the resulting key differences.}

Let \( w \) be wages and \( p \) consumer prices; with \( pr \) as productivity, the wage share (in terms of consumer prices) is given as \( ws = w - p - pr \), or real unit labour costs; \( u \) is the unemployment rate, \( gap \) the output gap and \( pb \) import prices, all measured in logs. We abstract from other forcing variables. A model of the wage-price process general enough for the present purpose then takes the form

\[
\begin{align*}
\Delta w &= \alpha \Delta p^e - \beta ws - \gamma u \\
\Delta p &= \delta \Delta p^e + \zeta \Delta w + \eta ws + \theta gap + \theta \Delta pb
\end{align*}
\]
where $\Delta p^e$ is expected inflation, and the dynamics is to be specified for each model. Although the structure is very simple, the different models drop out as non-nested special cases:

1. The New Phillips Curve Model (NPCM)—Gali and Gertler (1999)—is given as

$$\Delta p_t = \delta_1 \Delta p_{t+1}^e + \eta_1 w s_t + \theta_1 \Delta p b_t,$$

where we have also included import prices, to make it relevant for a small open economy. The expectations term $\Delta p_{t+1}^e$ is assumed to obey rational expectations;


$$\Delta w_t = \alpha_2 \Delta p_t - \gamma_2 u_t$$
$$\Delta p_t = \zeta_2 \Delta w_t + \theta_2 g a p_t + \theta_2 \Delta p b_t,$$

3. The Incomplete Competition Model (ICM) —Layard et al. (1991), Carlin and Soskice (1990), Kolsrud and Nymoen (1998), Bårdsen et al. (1998)—is in its modern form presented as an equilibrium correction model, see Sargan (1964):

$$\Delta w_t = \alpha_3 \Delta p_t - \beta_3 (w s - \gamma_2 u)_{t-1}$$
$$\Delta p_t = \zeta_3 \Delta w_t + \theta_3 (p b + \eta_2 w s)_{t-1} + \vartheta_3 g a p_{t-1},$$

Of course, there exist a host of other, more elaborate, models—a notable omission here being non-linear PCMs. However, the purpose here is to highlight that discrimination between the models is possible through testable restrictions. The difference between the two Phillips curve models is that the NPCM is a reduced form that has explicit forward looking expectations and has real unit labour costs, rather than the output gap of the PCM. The ICM differs mainly from the NPCM in the treatment of expectations and from the PCM in the latter’s exclusion of equilibrium correction mechanisms that are derived from conflict models of inflation, see e.g. Rowthorn (1977), Sargan (1980), Kolsrud and Nymoen (1998). That said, also the PCM can be reformulated as an EqCM. To see this, note first that the PCM assumes a stationary rate of unemployment. Thus, internal consistency requires that the bivariate system describing the PCM is augmented with an equation that links $u_t$ to e.g., the wage share $w s_{t-1}$. Insertion of the equation for $u_t$ in the wage Phillips curve, turns the latter into an explicit EqCM for wages. Thus, the essential difference between the PCM and the ICM is found in the nature of the causal relationships that underlie cointegration, rather than with cointegration as such.

We conclude that the models listed in 1.-3. are identified, in principle, but it is an open question whether data and methodology are able to discriminate between them on a given data set. Also, a highly likely outcome is that the inflationary process contains elements from more than one model, for example by including both cointegration and forward-looking expectations. We therefore test the various identifying restrictions.
When it comes to forecasting, the clearest logical distinction is between the NPCM and the other two models. The PCM and ICM can be formulated as linear difference equations with stable backward solutions. With stochastic disturbances added, they belong to the class of causal or future-independent processes, see Brockwell and Davies (1991, Chapter 3). For both models, the best (in terms of mean square error) forecast corresponds to the conditional mean based on known initial conditions. However, since the conditional means are model dependent, the relative forecast performance of the PCM and ICM merits investigation, see section 4 and 5. In contrast, the NPCM with rational expectations represents non-causal or future-dependent processes. The solution for the rate of inflation in period $T$ is thus a function of the forcing variables in future periods $T + h, h \geq 0$, and requires the application of simulation methods. Thus, there is a premium on prior-to-forecasting econometric testing of the NPCM model, as demonstrated in sections 3.3 and 3.4.

### 3 Empirical inflation models

In this section we develop empirical models of inflation. The wage variable $w_t$ used in the following is average hourly wages in the mainland economy, excluding the North-Sea oil producing sector and international shipping. The productivity variable $pr_t$ is defined accordingly. The price index $p_t$ is measured by the official consumer price index. Import prices $pb_t$ are measured by the official index. The unemployment variable $u_t$ is defined as a “total” unemployment rate, including labour market programmes. The tax-rates $\tau_1 t$ and $\tau_3 t$ are rates of payroll-tax and indirect-tax, respectively.

The output gap variable $gap_t$ is measured as deviations from the trend obtained by the Hodrick-Prescott filter. The other non-modelled variables comprise the length of the working day $\Delta h_t$, which captures wage compensation for reductions in the length of the working day—see Nymoen (1989). In addition, incomes policies and direct price controls have been in operation on several occasions in the sample period, see e.g., Bowitz and Cappelen (2001). The intervention variables $Wdum$ and $Pdum$, and one impulse dummy $i80q2$, are used to capture the impact of these policies. Finally, $i70q1$ is a VAT dummy. This system, where all main variables enter with three lags, is estimated over 1966(4)–1994(4).

---

3 With perfect foresight, and $|\delta_1| > 1$, there exist a unique backward solution, see e.g., Gourieroux and Monfort (1997, Chapter 12.4).

4 Ideally, an income tax rate should appear as well. It is omitted from the empirical model, since it is insignificant. This is in accordance with previous studies of aggregate wage formation, see e.g. Calmfors and Nymoen (1990) and Rødseth and Nymoen (1999), where no convincing evidence of important effects from the average income tax rate on wage growth could be found.

5 $Wdum$ and $Pdum$ are defined in the appendix.
3.1 An incomplete competition model

Building on earlier research—Bårdsen et al. (1998)—we estimate the steady-state to be represented by the two cointegrating relationships

\[ w = p + pr - 0.1u + \text{constant}, \]  
\[ p_t = 0.6(w - pr) + 0.4 pb + \tau 3 + \text{constant}. \]  

This is practically the same result as reported by Bårdsen et al. (1998) on a sample that ends in 1993(2).

When estimating a dynamic wage-price system we impose the estimated steady state on a subsystem for \{\Delta w_t, \Delta p_t\} conditional on \{\Delta pr_t, \Delta u_{t-1}, \Delta 1_{t-1}, \Delta 3_{t-1}\} with all variables entering with two additional lags. In addition to gap\_t\_1, we also augment the system with \{\Delta h_t, i80q2, i70q1, Wdum, Pdum\} to capture short-run effects, as described above. The resulting model is given as

\[
\Delta w_t = \Delta p_t - 0.4 \times 0.36 \Delta pb_t - 0.36 \Delta 1_{t-2} - 0.36 \Delta 3_{t-2} - 0.3 \Delta h_t \\
- 0.08 [w_{t-2} - p_{t-2} - pr_{t-1} + 0.1 u_{t-2}] + \text{dummies} \\
\hat{\sigma}_{\Delta w} = 1.02\% \\
\Delta p_t = 0.12 (\Delta w_t + \Delta 1_{t-2}) + 0.05 gapt_{-1} + 0.4 \times 0.07 \Delta pb_t - 0.07 \Delta 3_{t-2} \\
- 0.08 [p_{t-3} - 0.6 (w_{t-1} - pr_{t-1} + 1_{t-1}) - 0.4 pb_{t-1} + \tau 3_{t-3}] + \text{dummies} \\
\hat{\sigma}_{\Delta p} = 0.41\%
\]

The coefficient estimate of \( \Delta pb_t \) is restricted to be 40 per cent of the coefficient estimate of \( \Delta 3_{t-2} \) in both equations and this is denoted by e.g., \( 0.4 \times 0.36 \) in the wage growth equation. These restrictions follow from the underlying theory model, see Bårdsen et al. (1999) for details. The first equation in (3) shows that a one percentage point increase in the rate of inflation raises wage growth by one percent, suggesting an element of real wage rigidity. However, note that this result is conditioned by the inclusion of an indirect tax-rate (\( \Delta 3_{t-1} \)), import price growth (\( \Delta pb_t \)) and the mentioned income policy variables (\( Wdum, \) and \( Pdum, \) not shown). Thus, it is only in periods with no surprises in the exchange rate or in world prices, and when discretionary policies are not in force, that nominal wages adjust quickly to the “normal” or expected consumer price increases as captured by the unit coefficient of \( \Delta p_t \) in the first equation. By the same token, since import price growth (\( \Delta pb_t \)) is likely to be the most important “unexpected” part of price inflation, it is not surprising that \( \Delta pb_t \) is attributed a negative estimated coefficient in the first equation of (3)\(^6\). Finally, the equilibrium-correction term is highly significant in the equation for \( \Delta w_t \), as expected from theory.

Turning to the second equation in (3), price inflation is significantly influenced by wage growth and the output gap, together with effects from import prices and

\(^6\)The coefficients of \( \Delta p_t, \Delta pb_t \) and \( \Delta 1_{t-2} \) in the wage equation are all examples of the overidentifying restrictions whose joint significance is reported in Table 1.
indirect taxes—as predicted by the theoretical model. Deviations from the cointegrating price equation are significant, as is the effect of direct price controls (not shown).

Table 1: Diagnostics for the ICM model (3) and the PCM model (4).

<table>
<thead>
<tr>
<th>Diagnostic tests for the model in (3)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>The sample is 1966(4) to 1994(4), 113 observations.</td>
<td></td>
</tr>
<tr>
<td>$\hat{\sigma}_{\Delta w} = 1.01%$</td>
<td></td>
</tr>
<tr>
<td>$\hat{\sigma}_{\Delta p} = 0.41%$</td>
<td></td>
</tr>
<tr>
<td>Correlation of residuals $= -0.4$</td>
<td></td>
</tr>
<tr>
<td>Overidentification $\chi^2(9) = 9.23[0.42]$</td>
<td></td>
</tr>
<tr>
<td>AR $1 - 5$ $F(20, 176) = 1.02[0.31]$</td>
<td></td>
</tr>
<tr>
<td>Normality $\chi^2(4) = 6.23[0.18]$</td>
<td></td>
</tr>
<tr>
<td>Heteroscedasticity $F(102, 186) = 0.88[0.76]$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Diagnostic tests for the model in (4)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>The sample is 1967(1) to 1994(4), 112 observations.</td>
<td></td>
</tr>
<tr>
<td>$\hat{\sigma}_{\Delta w} = 1.07%$</td>
<td></td>
</tr>
<tr>
<td>$\hat{\sigma}_{\Delta p} = 0.47%$</td>
<td></td>
</tr>
<tr>
<td>Correlation of residuals $= -0.6$</td>
<td></td>
</tr>
<tr>
<td>Overidentification $\chi^2(16) = 25.13[0.07]$</td>
<td></td>
</tr>
<tr>
<td>AR $1 - 5$ $F(20, 176) = 1.02[0.44]$</td>
<td></td>
</tr>
<tr>
<td>Normality $\chi^2(4) = 6.23[0.18]$</td>
<td></td>
</tr>
<tr>
<td>Heteroscedasticity $F(102, 257) = 0.81[0.84]$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Recursive stability tests for the ICM model. The two upper panels show one-step residuals from the wage and the price equations in (3). The lower right panel is recursive N-up Chow-tests for parameter stability (see Chow (1960)), whereas the lower left panel shows recursive tests of the overidentifying restrictions on the estimated model in (3), see Hendry (1971) and Sargan (1988).

The upper part of Table 1 contains diagnostics for the model (3). Note that the insignificance of Overidentification $\chi^2(9)$ shows that the model in (3) encompasses the unrestricted reduced form, thus encompassing the VAR, see Hendry and Mizon (1993). The reported tests of residual misspecification are all insignificant. Parameter constancy is demonstrated graphically in Figure 1. The two 1-step residuals with their $\pm 2\hat{\sigma}$ estimated residual standard errors, in the graphs are shown in the upper panels, while the lower right panel shows the sequence of recursive forecast Chow-tests together with their one-off 5 per cent critical level. Finally, the lower left panel of Figure 1 shows that the model encompasses of the unrestricted reduced form at every sample size (i.e., the end of the graph corresponds to Overidentification $\chi^2(9)$ in the table).

3.2 A Standard Phillips curve model

When estimating a Standard Phillips curve model, we start out from the same information set as for the ICM, but with more lags in the dynamics, to make sure we end up with a data-congruent specification. The preferred model is reported in (4). Dynamic price homogeneity cannot be rejected in the wage equation, and is therefore imposed. Otherwise it the models shares several of the properties of
(3), despite omitting the equilibrium correction terms. As reported in the lower part of Table 1, the model encompasses its reduced form and shows no sign of misspecification. The estimated equation standard errors, however, are higher than the corresponding ones for its rival.

\[
\begin{align*}
\Delta w_t &= 1.11 \Delta p_t - 0.11 \Delta pb_t - 0.65 \Delta \tau_1 t - 0.41 \Delta \tau_1 t_{-2} - 0.01 \Delta u_{t-3} - 0.006 u_{t-1} \\
&- 0.16 \Delta \tau_3 t_{-1} - 0.34 \Delta \tau_3 t_{-2} - 0.30 \Delta h_t + \text{dummies} \\
\hat{\sigma}_{\Delta w} &= 1.07\% \\
\Delta \hat{p}_t &= 0.14 \Delta w_t + 0.07 \Delta w_{t-3} + 0.17 \Delta p_{t-1} + 0.27 \Delta p_{t-2} + 0.05 \Delta pb_t \\
&- 0.03 \Delta pr_{t-1} + 0.05 gap_{t-1} + \text{dummies} \\
\hat{\sigma}_{\Delta p} &= 0.47\% 
\end{align*}
\]

Parameter constancy of the Phillips curve model is demonstrated graphically in Figure 2. The two 1-step residuals with their ± 2 estimated residual standard errors (±2σ in the graphs) are in the uppermost panels, while the lower right panel shows the a sequence of recursive forecast Chow-tests together with their one-off 5 per cent critical level. The lower left panel shows that the model encompasses of the unrestricted reduced form as the sample size increases (i.e., the end of the graph corresponds to Overidentification $\chi^2(16)$ in the table).
Figure 2: Recursive stability tests for the PCM model. The two upper panels show one-step residuals from the wage and the price equations in (4). The lower right panel is recursive N-up Chow-tests for parameter stability (see Chow (1960)), whereas the lower left panel shows recursive tests of the overidentifying restrictions on the estimated model in (4), see Hendry (1971) and Sargan (1988).

3.3 A New Keynesian Phillips curve model

When estimating a New Keynesian Phillips curve model, we follow the approach of Gali and Gertler (1999), but augment the specification with import price growth and dummies for seasonal effects as well as the special events identified in the previous section. Estimation with GMM produced results very similar to Gali and Gertler (1999):

\[
\Delta p_t = 1.05 \Delta p_{t+1} + 0.04 ws_t - 0.03 \Delta p_{t+1} + \text{dummies} \\
(0.108) (0.025) (0.028)
\]

\[\text{Overid} \chi^2(10) = 10.91 [0.36],\]

where \(ws\) is the wage share and \(\text{Overid} \chi^2\) is the test of the validity of the overidentifying instruments. The instruments used are \(\Delta w_{t-1}, \Delta w_{t-2}, \Delta p_{t-1}, \Delta p_{t-2}, \Delta t1_t, \Delta t1_{t-2}, \Delta u_{t-1}, \Delta u_{t-2}, u_{t-1}, \Delta t3_t, \Delta t3_{t-1}, \Delta t3_{t-2}, \Delta h_t\).

To evaluate the model, we want to investigate the stability of the key parameters of the model as well as investigate the validity of the specification. Since GMM can suffer from small sample problems, we estimate the parameters with rolling regressions, using a fixed window of 85 observations. As Figure 3 shows,
both the coefficients of the wage share and the expected inflation rate exhibit not only instability, but also a trending behaviour over the sample.

![Figure 3: Rolling regression coefficients +/- 2 standard errors of the New Keynesian Phillips curve.](image)

As regards the validity of the specification, the following procedure might be used for testing

\[ \Delta p_t = \delta_1 \Delta p_{t+1}^e + \ldots \]  

as the maintained model of the rate of inflation.\(^7\)

1. Assume that there exists a valid instrument set \( z = [z_1 \ z_2] \), where the sub-
   set \( z_1 \) is sufficient for overidentification of the maintained model.

2. Using \( z_1 \) as instruments, estimate the augmented model \( \Delta p_t = \delta_1 \Delta p_{t+1}^e + \ldots + z_2 \gamma \), under the assumption of rational expectations about \( \Delta p_{t+1}^e \).

3. Under the maintained hypothesis, \( \gamma = 0 \) in the augmented model. Thus, non-
   rejection of the null hypothesis of \( \gamma = 0 \) corroborates the feed-forward Phillips curve. Otherwise, i.e. non-rejection of \( \delta_1 = 0 \) while \( \gamma = 0 \) is rejected statistically, the evidence is against the joint maintained hypothesis of (5) and rational expectation about \( \Delta p_{t+1}^e \).

To apply this test procedure to our data we only need one additional instrument, namely the equilibrium correction term in the inflation equation of the ICM in (3) above

\[ z_2 = \text{ecmp}(t) = p_{t-3} - 0.6(w_{t-1} - pr_{t-1} + \tau 1_{t-1}) - 0.4pb_{t-1} + \tau 3_{t-3}. \]

\(^7\)We thank David F. Hendry for suggesting this test to us.
Table 2: FIML estimated coefficients of 1 and 2 periods leads when introduced in the wage and price equations in ICM (3) and PCM (4). Estimated standard errors in parenthesis.

<table>
<thead>
<tr>
<th></th>
<th>ICM</th>
<th>PCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ(w_t)-equation</td>
<td>Δ(p_t)-equation</td>
<td>Δ(w_t)-equation</td>
</tr>
<tr>
<td>Δ(p_{t+1})</td>
<td>0.24</td>
<td>(0.52)</td>
</tr>
<tr>
<td>Δ(p_{t+2})</td>
<td>0.21</td>
<td>(0.58)</td>
</tr>
<tr>
<td>Δ(w_{t+1})</td>
<td></td>
<td>-0.19</td>
</tr>
<tr>
<td>Δ(w_{t+2})</td>
<td></td>
<td>-0.31</td>
</tr>
</tbody>
</table>

The results, using GMM, are

\[ \hat{\Delta p_t} = 0.06 \Delta p_{t+1} - 0.10 w_{t+1} + 0.04 \Delta p_{t+2} - 0.12 ecmp(t) + \text{dummies} \]

Overident\(\chi^2(10) = 8.19 [0.61] \), establishing that the NPCM is not a valid representation of the inflationary process in Norway.

3.4 Elements of NPCM: Testing for forward-looking expectations

Summing up so far, the NPCM appears to be too stylized and specific to act as a good model of the inflationary process in Norway. The PCM and the ICM are better candidates. However, alternative expectations hypotheses of explicit forward looking terms in the two latter specifications merit further investigation by themselves, see Moghadam and Wren-Lewis (1994).

Both the ICM and PCM are simultaneous equations models in Δ\(w_t\) and Δ\(p_t\) and thus estimation by FIML implies that the models already have a (rational) expectations interpretation in terms of the current dated wage and price growth. Care must be taken when period \(t + 1\) and \(t + 2\) expectation terms of the same two variables are included in the models, since identification problems occur. In the calculations underlying table 2 we have tackled this problem by using a restricted reduced form to predict e.g., Δ\(w_{t+1}\) rather than the unrestricted reduced form, see Blake (1991). In choosing the restrictions we have kept an eye on the estimated residual standard errors of the affected structural equations—they typically become markedly larger than in (3) and (4) if the expectations formation is not sufficiently restricted. However, we avoided that the Overidentification Chi-square test became significant, since that would entail a too restrictive expectations formation. Table 2 shows that in a majority of cases the forward terms are statistically insignificant. The most significant term is Δ\(p_{t+1}\) in the PCM wage growth equation, however this is also where the identification problems are most pronounced.

Part of the explanation for these results may be that there is a second expectations interpretation of the current dated variables, namely that e.g., Δ\(w_t\) in the consumer price equation is by itself a predictor of Δ\(w_{t+1}\). The implied forecasting
mechanism is quite simple, i.e. $\Delta \Delta w_{t+1} = 0$, which is consistent with what students of decision making tell us: that agents often resort to rules of thumb or “routines” when faced by complex uncertainty, see Simon (1965), Nelson and Winter (1982) and Shleifer (2000). Fundamental uncertainty is indeed a valid characteristic of economic time series as they are influenced by unit-root and deterministic shifts. Comparison of forecasting rules confirm that $\Delta \Delta w_{t+1} = 0$ is a robust forecasting tool in that the effects of deterministic shifts are corrected, see Clements and Hendry (1999), Eitrheim et al. (1999).

4 Forecast errors of stylized inflation models

On the basis of above evidence, there is no need to shoulder the impracticalities of forecasting inflation with the rational expectations NPCM. Thus, the remaining sections are devoted to the forecast properties of the two backward looking models. For that purpose, we formulate a simple DGP to investigate the theoretical forecasting capabilities of the ICM and the PCM estimated in the previous section, thus providing a background for the interpretation of the actual forecast errors in section 5.

In order to obtain an analytically tractable distillation of the gist of the empirical models, we introduce several simplifying assumptions. For example, we retain only one cointegrating relationship, the “wage-curve”, and we also abstract from productivity. Thus (6) is a simplified version of the equation in the first line of (3):

$$
\Delta (w - p)_t = \kappa - \pi_w [(w - p)_{t-1} + \lambda u_{t-1} - \mu] + \epsilon_{w,t}, \pi_w > 0, \lambda > 0.
$$ (6)

The wage-curve is the term in square brackets. The parameter $\mu$ denotes the mean of the long run relationship for real wages, i.e. $\mathbb{E} [(w - p)_{t-1} - \lambda u_{t-1} - \mu] = 0$. Since we abstract from the cointegration relationship for consumer prices, the simultaneous equation representation of the inflation equation is simply that $\Delta p_t$ is a linear function of $\Delta pb_t$ and $\Delta w_t$, and the reduced form equation for $\Delta p_t$ is

$$
\Delta p_t = \phi_p + \varphi_{pb} \Delta pb_t - \pi_p [(w - p)_{t-1} + \lambda u_{t-1} - \mu] + \epsilon_{p,t}, \varphi_{pb} \geq 0, \pi_p \geq 0.
$$ (7)

Multi-step (dynamic) forecasts of the rate of inflation entails forecasts of import price growth and the rate of unemployment. In order to simplify as much as possible, we let $\Delta pb_t$ and $u_t$ follow exogenous stationary processes:

$$
\Delta pb_t = \phi_{pb} + \epsilon_{pb,t},
$$

$$
\Delta u_t = \phi_u - \pi_u u_{t-1} + \epsilon_{u,t}, \pi_u > 0.
$$ (8)

$I_T$ denotes the information set available in period $T$. The four disturbances ($\epsilon_{w,t}, \epsilon_{p,t}, \epsilon_{pb,t}, \epsilon_{u,t}$) are innovations relative to $I_T$, with contemporaneous covariance matrix $\Omega$. Thus, the system (6)-(9) represents a simple data generation process (DGP) for inflation, the real wage, import price growth and the rate of unemployment. The forecasting rule

$$
\hat{\Delta p}_{T+h} = \mathbb{E} [\Delta p_{T+h} | I_T] = a_0 + a_1 \Delta pb_T + a_2 \mathbb{E} [(w - p)_{T+h-1} | I_T] + a_3 \mathbb{E} [u_{T+h-1} | I_T],
$$

$$
h = 1, 2, ..., H.
$$ (10)
with coefficients
\[ a_0 = \phi_p + \pi_p \mu, \]
\[ a_1 = \varphi pb, \]
\[ a_2 = -\pi_p \]
\[ a_3 = -\pi_p \lambda \]
is the minimum mean squared forecast error (MSFE) predictor of \( \Delta p_{T+h} \), by virtue of being the condition expectation.

In order to abstract from estimation uncertainty, we identify the parameters of the ICM with the probability limits of the corresponding estimated coefficients. The dynamic ICM forecasts errors have the following means and variances:

\[
E[\Delta p_{T+h} - \hat{\Delta} p_{T+h}, ICM | \mathcal{I}_T] = 0, \tag{11}
\]
\[
\text{Var}[\Delta p_{T+h} - \hat{\Delta} p_{T+h}, ICM | \mathcal{I}_T] = \sigma_p^2 + \sigma_{pb}^2 \tag{12}
\]
\[
+ a_2^2 \sum_{i=1}^{h-1} (1 - \pi_w)^{2(h-1-i)} \sigma_w^2
\]
\[
+ a_2^2 (\pi_w \lambda)^2 \sum_{i=1}^{h-1} (1 - \pi_w)^{2(h-1-i)} \sum_{j=1}^{i} (1 - \pi_u)^{2(i-j)} \sigma_u^2
\]
\[
+ a_3^2 \sum_{i=1}^{h-1} (1 - \pi_u)^{2(h-1-i)} \sigma_u^2
\]
The first two terms on the right hand side of (12) are due to \( \epsilon_{p,T+h} \) and \( \epsilon_{pb,T+h} \). The other terms on the right hand side of (12) are only relevant for \( h = 2, 3, 4...H \). The third and fourth terms stem from \( (w - p)_{T+h-1} \) — it is a composite of both wage and unemployment innovation variances. The last line contains the direct effect of \( \text{Var}[u_{T+h-1}] \) on the variance of the inflation forecast. In addition, off-diagonal terms in \( \Omega \) might enter.

We next the consider the case where a forecaster imposes the PCM restriction \( \pi_w = 0 \) (implying \( \pi_p = 0 \) as well). The “Phillips-curve” inflation equation is then given by

\[
\Delta p_t = \tilde{a}_0 + \tilde{a}_1 \Delta pb_t + \tilde{a}_3 u_{t-1} + \tilde{\varepsilon}_{p,t}, \text{ with} \tag{13}
\]
\[
\tilde{a}_0 = a_0 + a_2 \lambda E[u_{t-1}] + a_2 \mu, \text{ and}
\]
\[
\tilde{\varepsilon}_{p,t} = \epsilon_{p,t} + a_2 [(w - p)_{t-1} - \lambda u_{t-1} - \mu].
\]
This definition ensures a zero-mean disturbance \( E[\varepsilon_{p,t} | \mathcal{I}_T] = 0 \). Note also that \( \text{Var}[\varepsilon_{p,t} | \mathcal{I}_{T-1}] = \sigma_p^2 \), i.e., the same innovation variance as in the ICM-case. The PCM forecast rule becomes

\[
\hat{\Delta} p_{T+h,PCM} = E[\Delta p_{T+h,PCM} | \mathcal{I}_T] = \tilde{a}_0 + \tilde{a}_1 \delta pb + \tilde{a}_4 \hat{u}_{T+h-1}.
\]
The mean and variance of the 1-step forecast-error are

\[
E[\Delta p_{T+1} - \hat{\Delta} p_{T+1,PCM} | \mathcal{I}_T] = \{a_1 - \tilde{a}_1\} \delta pb + \sigma_T (a_2) u_T + a_2\{((w - p)_{T} - \lambda E[u_{t}] - \mu), \]
\[
\text{Var}[\Delta p_{T+1} - \hat{\Delta} p_{T+1,PCM} | \mathcal{I}_T] = \sigma_p^2 + \sigma_{pb}^2.
\]
The 1-step ahead prediction error variance conditional on $I_T$ is identical to the ICM-case. However, there is a bias in the 1-step PCM forecast arising from two sources: First omitted variables bias imply that $a_1 \neq \tilde{a}_1$ and/or $a_3 \neq \tilde{a}_3$, in general. Second, 

$$(w - p)_T - \lambda E[u_t] - \mu \neq 0$$

unless $(w - p)_T = E[(w - p)_t]$, i.e., the initial real wage is equal to the long-run mean of the real-wage process.

For dynamic $h$ period ahead forecasts, the PCM prediction error becomes

$$\Delta p_{T+h} - \Delta p_{T+h,PCM} = (a_1 - \tilde{a}_1)\delta pb + (a_3 - \tilde{a}_3)\mu T_{T-1} + a_3 \sum_{i=1}^{h-1} (1 - \pi_u)^{h-1-i} u_{T+i} + \epsilon_{pb,T+h} + \epsilon_{p,T+h} + a_2(w - p)_{T+h-1} - a_2(\lambda E[u_t] - \mu)$$

Taking expectation and variance of this expression gives

$$E[\Delta p_{T+h} - \Delta p_{T+h,PCM} | I_T] = (a_1 - \tilde{a}_1)\delta pb + (a_4 - \tilde{a}_4)\mu T_{T-1},$$

$$Var[\Delta p_{T+h} - \Delta p_{T+h,PCM} | I_T] = \text{Var}[\Delta p_{T+h} - \Delta p_{T+h,ECM} | I_T].$$

Hence systematic forecast error is again due to omitted variables bias and the fact that the conditional mean of real wages $h - 1$ periods ahead, departs from its (unconditional) long-run mean. However, for long forecast horizons, large $H$, the bias expression can be simplified to become

$$E[\Delta p_{T+h} - \Delta p_{T+h,PCM} | I_T] \approx (a_1 - \tilde{a}_1)\delta pb + (a_4 - \tilde{a}_4)\mu T_{T-1}$$

since the conditional forecast of the real wage and of the of the rate of unemployment approach their respective long run means.

Thus far we have considered a constant parameter framework: The parameters of the model in equations (6)-(9) remain constant not only in the sample period $(t = 1, ..., T)$ but also in the forecast period $(t = T + 1, ..., T + h)$. However, a primary source of forecast failure is structural breaks, especially shifts in the long-run means of cointegrating relationships and in parameters of steady-state trend growth, see e.g. Doornik and Hendry (1997) and Clements and Hendry (1999, Chapter 3). Moreover, given the occurrence of deterministic shifts, it is no longer true that the “best” econometric model over the sample period also gives rise to the minimum MSFE. Instead, the model forecasts can be beaten in a forecast contest by non-causal forecasting rules based on differencing, i.e. dVARs, because such rules are robust to regime shifts that have occurred prior to the forecast period, see e.g., Clements and Hendry (1999, Chapter 5), Eitrheim et al. (1999) and Eitrheim et al. (2002).

This trade-off between modelling of structure versus robustness in forecasting is illustrated by the following example: Assume that the long-run mean $\mu$ of the
wage-equation changes from its initial level to a new level, i.e. \( \mu \to \mu^* \), before the forecast is made in period \( T \), but that the change is undetected by the forecaster. There is now a bias in the (1-step) ICM real-wage forecast:

\[
E[(w - p)_{T+1} - (\hat{w} - p)_{T+1,ICM} \mid I_T] = -\pi_w[\mu - \mu^*],
\]

which in turn produces a non-zero mean in the period 2 inflation forecast error:

\[
E[\Delta p_{T+2} - \hat{\Delta} p_{T+2,ICM} \mid I_T] = -a_2 \pi_w[\mu - \mu^*].
\]

The PCM-forecast on the other hand, is insulated from the parameter change in wage formation, since \( (w - p)_{T+h-1} \) does not enter the predictor—the forecast error is unchanged from the constant parameter case. Consequently, both set of forecasts for \( \Delta p_{T+2+h} \) are biased in the situation with a shift in \( \mu \), and there is no logical reason why the PCM forecast could not outperform the ICM forecast on a comparison of biases. In terms of forecast properties, the PCM, despite the inclusion of the rate of unemployment, behaves as if it was a dVAR, since there is no feed-back from wages and inflation to the rate of unemployment in the example DGP.

Finally, consider the consequences of using estimated parameters in the two forecast rules. This does not change the results about the forecast biases. However, the conclusion about the equality of forecast error variances of the ICM and PCM is changed. Specifically, with estimated parameters, the two models do not share the same underlying innovation errors. In order to see this, consider again the case where the ICM corresponds to the DGP. Then a user of a PCM does not know the true composition of the disturbance \( \hat{\epsilon}_{p,t} \) in (13), and the estimated PCM will have an estimated residual variance that is larger than its ICM counterpart, since it is influenced by the omitted wage-curve term. In turn, the PCM prediction errors will overstate the degree of uncertainty in inflation forecasting. We may write this as

\[
\text{Var}[\hat{\epsilon}_{p,t} \mid I_T, PCM] > \text{Var}[\epsilon_{p,t} \mid I_T, ICM]
\]

to make explicit that the conditioning is with respect to the two models (the DGP being unknown). From equation (13) it is seen that the size of the difference between the two models' residual variances depend on i) the strength of equilibrium correction \( (a_2) \) and ii) the variance of the long-run wage curve.

The main results of this section can be summarized in three points

1. With constant parameters in the DGP, PCM will bias the forecasts and overstate the degree of uncertainty, if it involves invalid restrictions.

2. PCM forecasts are however robust to changes in means of (omitted) long-run relationships.

3. Thus PCM share some of the robustness of dVARs, but also some of its drawbacks (excess inflation uncertainty).

In sum, the outcome of a forecast comparison is not a given thing, since in practice we must allow for the possibility that both forecasting models are misspecified relative to the generating mechanism that prevails in the period we are trying to
forecast. *A priori* we cannot tell which of the two models will forecast best. Hence, there is a case for comparing the two models’ forecasts directly, even though the econometric evidence of section 3 favoured the ICM as the better model over the sample period.

5 Forecasting inflation

Both models condition upon the rate of unemployment $u_t$, average labour productivity $pr_t$, import prices $pb_t$, and GDP mainland output $y_t$. In order to investigate the dynamic forecasting properties we enlarge both models with the same relationships for these four variables. All of these variables are potentially affected by interest rates and are therefore potential channels for monetary instruments to influence inflation. Also, none of these variables are likely to be strongly exogenous. For example, import prices depend by definition on the nominal exchange rate. Below we report a model that links the exchange rate to the lagged real exchange rate, which in turn depend on the domestic price level. The details of the additional relationships are given in Bårdsen et al. (1999), but the qualitative properties can summarized as

$$\Delta v_t = f\left( \text{rex}_{t-1}, \text{oilprice}_t, \Delta RS_t, \right)$$

$$\Delta y_t = f\left( EqCM\ y_{t-1}, \Delta y_{t-1}, \Delta cr_{t-1} \right)$$

$$\Delta u_t = f\left( \Delta y_t, \Delta u_{t-1}, st\ u_{t-1}, \Delta (w - p)_{t-1}, lmp_t \right)$$

$$\Delta pr_t = f\left( \Delta 3pr_{t-1}, \Delta u_{t-1} \right)$$

where $\text{rex}_t$ is the log of the real exchange rate, $RS_t$ is the money market interest rate, $EqCM\ y_t$ is an equilibrium correction term for an aggregate demand relationship, and $cr_t$ is a function of credit demand—see Bårdsen and Klovland (2000). Furthermore, $stu_t$ denotes non-linear effects in unemployment adjustment, while $lmp_t$ measures the effect of labour market programmes.

Figure 4 illustrates how the ICM-based model forecast some important variables over the period from 1995(1) to 1996(4). The model parameters are estimated on a sample that ends in 1994(4). These dynamic forecast are conditional on the actual values of the non-modelled variables (ex post forecasts). The quarterly inflation rate $\Delta p_t$ only has one significant bias, in 1996(1). In that quarter there was a reduction in the excises on cars that explains around 40 per cent of this particular overprediction. In the graphs of the annual rate of inflation $\Delta 4p_t$ this effect is naturally somewhat mitigated. The quarterly change in the wage rate $\Delta w_t$ is very accurately forecasted, so the only forecast error of any importance for the change in real wages $\Delta (w - p)_t$ also occurs in 1996(1). The forecasts for the rate of unemployment are very accurate for the first 5 quarters, but the reduction in unemployment in the last 3 quarters does not appear to be predictable with the aid of this model.
Figure 4: Dynamic forecasts of the ICM model for the period 1995(1)–1996(4), with 95% prediction bands.

Figure 4 also contains the 95% prediction intervals in the form of ±2 standard errors, as a direct measure of the uncertainty of the forecasts. The prediction intervals for the annual rate of inflation are far from negligible and are growing with the length of the forecast horizon.

Next, Figure 5 illustrates how the model based on the Phillips curve forecast the same variables over the same period from 1995(1) to 1996(4). For most variables the differences are negligible. For the quarterly inflation rate $\Delta p_t$ in particular, the Phillips curve specification seems to be no worse than the ICM as regards the point forecasts, although the prediction intervals are somewhat wider, due to the larger residual variances in wage and price setting.

However, in the graphs of the annual rate of inflation $\Delta_4 p_T$ the result is after all a difference between the predictions on this one-off comparison. Since $\Delta_4 \hat{p}_{T+h,\text{mod}}$ is a 4 quarter moving average of the quarterly rates, the same is true for the prediction errors, thus

$$\Delta_4 p_{T+h} - \Delta_4 \hat{p}_{T+h,\text{mod}} = \sum_{i=0}^{3} (\Delta p_{T+h-i} - \Delta \hat{p}_{T+h-i,\text{mod}}), \text{ mod } = ICM, PCM. \quad (19)$$

Until 1995(4) there is zero bias in $\Delta_4 \hat{p}_{T+h,\text{PCM}}$ because all the preceding quarterly forecasts are so accurate. However, $\Delta_4 \hat{p}_{T+h,\text{PCM}}$ becomes biased from 1996(1) and onwards because, after the overprediction of the quarterly rate in 1996(1), there is no compensating underprediction later in 1996. The ICM forecasts on the other hand achieve exactly that correction, and do not systematically overpredict inflation.
Figure 5: Dynamic forecasts of the Phillips curve model for the period 1995(1)–1996(4), with 95% prediction bands.

For the annualized inflation rate the uncertainty increases quite rapidly for both models, but markedly more so for the Phillips curve forecast. Indeed, by the end of the two year period, the forecast uncertainty of the Phillips curve is about twice as big as the dynamic ICM model. This effect is clearly seen when the annual inflation forecasts from the two models are put together in the same graph. The dotted lines denote the point forecasts and the 95% prediction bands of the dynamic ICM, while the solid lines depict the corresponding results from the forecasts of the Phillips curve specification. At each point of the forecast the uncertainty of the Phillips curve is bigger than for the ICM. Indeed, while the ICM has a standard error of 0.9 percentage points 4-periods ahead, and 1.2 percentage points 8-periods ahead, the Phillips curve standard errors are 1.6 and 2 percentage points, respectively. Considering equation (19) it transpires that the explanation is not only that each $\text{Var}[\Delta p_{T+h} - \hat{\Delta}p_{T+h,PCM}] > \text{Var}[\Delta p_{T+h} - \hat{\Delta}p_{T+h,ICM}]$, but also that the PCM quarterly prediction errors are more strongly positively autocorrelated than the ICM counterparts.
Figure 6: Comparing the annual inflation forecasts of the two models. The thin line is actual annual inflation in Norway. The dotted lines denote the point forecasts and the 95% prediction error bands of the ICM model in (3), while the solid lines depict the corresponding results from the forecasts of the standard Phillips curve specification (PCM in (4)).
6 Conclusions

The strong linkage between forecasting and policy analysis makes the role of econometric models more important than ever. Policy makers face a menu of different models and an explicit inflation target implies that the central bank’s conditional forecast 1-2 years ahead becomes the operational target of monetary policy. The presence of non-stationary data and frequent structural breaks makes inevitable a trade-off between the gain and importance of correct structural modelling and their cost in terms of forecasting robustness. We have explored the importance of this trade-off for inflation forecasting.

Specifically, we have considered the two most popular inflation models, namely Phillips curves and wage curve specifications. We establish that Phillips-curve forecasts are robust to types of structural breaks that harm the wage-curve forecasts, but exaggerate forecast uncertainty in periods with no breaks. Moreover, omitted relevant equilibrium correction terms induces omitted variables bias in the usual way. Conversely, for the wage curve model, the potential biases in after-break forecast errors can be remedied by intercept corrections. As a conclusion using a well-specified model of wage-price dynamics offers the best prospect of successful inflation forecasting.
References


A Data definitions

A.1 Notes
1. Unless another source is given, all data are taken from RIMINI, the quarterly macroeconometric model used in Norges Bank (The Central Bank of Norway).

2. For each RIMINI-variable, the corresponding name in the RIMINI-database is given by an entry [RIMINI: variable name] at the end of the description. (The RIMINI identifier is from Rikmodnotat 140, Norges Bank, Research department, 19th April 1999)

3. Several of the variables refer to the mainland economy, defined as total economy minus oil and gas production and international shipping.

4. In the main text, impulse dummies are denoted $iyyq_x$, where $yy$ gives the year with two digits and $x$ contains the quarter (1,2,3). Hence $i80q2$ is 1 in the second quarter of 1980, and is 0 in all other quarters.

A.2 Definitions

gap Output gap defined as log mainland GDP (log of the variable $Y$ as defined below) deviations from trend, where the trend is estimated by the HP-filter using $\lambda = 1600$. Fixed baseyear (1991) prices. Mill. NOK.

$H$ Normal working hours per week. [RIMINI: NH]

$P$ Consumer price index. 1991=1. [RIMINI: CPI].

$PI$ Deflator of total imports. 1991=1. [RIMINI: PB].


$PR$ Mainland economy value added per man hour at factor costs, fixed baseyear (1991) prices. Mill. NOK. [RIMINI: ZYF].

$RS$ 3 month Euro-krone interest rate. [RIMINI: RS].

$\tau 1$ Employers tax rate. $\tau 1 = WCF/WF - 1$.

$\tau 3$ Indirect tax rate. [RIMINI: T3].

$U$ Rate of unemployment. Registered unemployed plus persons on active labour market programmes as a percentage of the labour force, calculated as employed wage earners plus unemployment. [RIMINI: UTOT].

$W$ Nominal mainland hourly wages. Constructed from Rimini-database series as:

$$W = WIBA \times TWIBA + WOTVJ \times (TWTV + TWO + TWJ))/TWF$$