

# WORKING PAPER SERIES

No. 16/2002

## TESTING STEADY-STATE IMPLICATIONS FOR THE NAIRU

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# Testing steady-state implications for the NAIRU\*

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First version, 25 April, 2000.

This version, 30 April, 2002.

## Abstract

Estimates of the NAIRU are usually derived either from a Phillips curve or from a wage curve. This paper investigates the correspondence between the operational NAIRU-concepts and the steady state of a dynamic wage-price model. We derive the parameter restrictions that secure that correspondence. The full set of restrictions can be tested by econometric analysis of the wage-price system, and this method is demonstrated for Norwegian data. A set of necessary conditions can be tested from estimated wage curves alone. Existing international evidence from empirical wage equations are re-interpreted in light of these conditions.

**Keywords:** *Phillips curve, wage curve, steady state, natural rate, NAIRU, dynamic modelling.*

**JEL classification:** *C51, C52, E24, E31, J30.*

## 1 Introduction

Governments and international organizations customarily refer to NAIRU calculations in their discussions of employment and inflation prospects<sup>1</sup>, and the existence of a NAIRU consistent with a vertical long-run Phillips curve is crucial to the framework of monetary policy.<sup>2</sup> An alternative to a Phillips-curve NAIRU is based on

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\*We would like to thank James H. Stock and two referees for helpful comments on an earlier version. Versions of this paper have been presented at the conference “European Unemployment and Wage Determination” at the European University Institute, Firenze in 1998 and in seminars at the University of Science and Technology, Trondheim, at Sveriges Riksbank, at Statistics Norway and at the University of Oxford. Comments from participants on these occasions are gratefully acknowledged. Discussion with and comments from Øyvind Eitrheim, Steinar Holden, Per Jansson, Kåre Johansen, Dag Kolsrud, Bjørn Naug, Bjarne Strøm and Anders Vredin have been very helpful. All numerical results were produced by PcGive 9.3 and PcFiml 9.3, see Doornik and Hendry (1997, 1999), Please address correspondence to: Ragnar Nymoen, University of Oslo, Department of Economics, P.O. Box 1095 Blindern, N-0317 Oslo, Norway. Phone: + 47 22 85 51 48. Fax + 47 22 50 35 35. Internet: ragnar.nymoen@econ.uio.no

<sup>1</sup>See Elmeskov and MacFarlan (1993), Scarpetta (1996) and OECD (1997, Chapter 1) for examples.

<sup>2</sup>See the discussion in King (1998) for a central banker’s views.

a negative relationship between the level of the real wage and the rate of unemployment, dubbed the wage curve by Blanchflower and Oswald (1994), coupled with firms' price setting schedule. As noted by Blanchard and Katz (1999), the wage curve has become the new consensus framework of the NAIRU in Europe, while a Phillips-curve version, dubbed "the triangle model of inflation" by Gordon (1983), applies to the US, see Gordon (1998) and Staiger et al. (2002) for recent contributions.<sup>3</sup>

In this paper we investigate the steady-state connotations of NAIRUs. Are the Phillips-curve or wage-curve NAIRUs good predictors of the true steady-state values of the rate of unemployment ground out by a dynamic model of the macro economy? To answer this question, we derive in Section 2 the testable conditions for correspondence between a Phillips- or wage-curve NAIRU and the steady-state unemployment rate in a more general system. Section 3 contains an application of the implied testing procedure to Norwegian data. Finally, the framework is used to re-interpret existing evidence from wage equations of a larger number of countries. Section 4 concludes.

## 2 An encompassing framework

The model is an extension of Kolsrud and Nymoene (1998). The variables are, in logarithms, the nominal wage  $w_t$ , the producer price of domestic products  $q_t$ , the consumer price  $p_t$ , the rate of unemployment  $u_t$  and productivity  $pr_t$ .<sup>4</sup> Nominal wages and prices are integrated of order 1, denoted  $I(1)$ . The rate of unemployment may also be non-stationary, e.g., its mean can change, but after removal of deterministic shifts, we assume that  $u_t \sim I(0)$ .

### 2.1 Long-run properties

A cointegrated long-run wage equation, consistent with the bargaining approach and the assumed temporal properties of the data, is thus

$$(1) \quad w_t - q_t - \beta_{pr}pr_t - \beta_{p-q}(p - q)_t = -\beta_u u_t + m_{wt} \sim I(0),$$

The term  $(p - q)_t$  is the *wedge* between the consumer real wage and the producer real wage. The role of the wedge as a source of wage pressure is contested in the literature and the theoretical predictions depend on the specification of the utility functions of unions and employers, see Rødseth (2000, Chapter 8.5) for an exposition. The term  $m_{wt}$  is a catch-all for exogenous institutional and economic factors. In the empirical version of the model we include the replacement ratio in the unemployment insurance system.

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<sup>3</sup>Recently, the Phillips curve has enjoyed a revival in the theory of monetary policy, see Clarida et al. (1999), and it dominates in the theoretical literature on inflation targeting in particular, see e.g. Svensson (2000), which of course lifts its importance in Europe.

<sup>4</sup>We abstract from the payroll tax rate. The rate of unemployment enters linearly in some US studies, see e.g. Fuhrer (1995). However, for most other datasets, a concave transform improves the fit and the stability of the relationship, see e.g. Nickell (1987) and Johansen (1995).

A second cointegration relationship stems from firms' normal-cost pricing<sup>5</sup>

$$(2) \quad q_t - w_t + pr_t = m_{qt} \sim I(0).$$

Identification is achieved through additional restrictions. Section 3.1 gives an example. Other approaches have been used for aggregate data, cf. Bårdsen et al. (1998).

## 2.2 Wage-price dynamics

We specify the following dynamic equations for wage and price growth:

$$(3) \quad \begin{aligned} \Delta w_t &= \gamma_{wq}\Delta q_t + \gamma_{wp}\Delta p_t - \gamma_u u_{t-1} \\ &\quad - \alpha_w [w - q - \beta_{pr}pr - \beta_{p-q}(p - q) + \beta_u u - m_w]_{t-1} + c_w, \\ 0 &\leq \gamma_{wp} + \gamma_{wq} \leq 1, \end{aligned}$$

and

$$(4) \quad \begin{aligned} \Delta q_t &= \gamma_{qw}\Delta w_t + \gamma_{qb}\Delta b_t - \alpha_q [q - w + pr - m_q]_{t-1} + c_q, \\ 0 &\leq \gamma_{qw} + \gamma_{qb} \leq 1, \end{aligned}$$

where  $\Delta$  is the difference operator ( $\Delta x_t = x_t - x_{t-1}$ ) and  $b_t$  denotes import prices.<sup>6</sup> Note that, in 3, either  $\gamma_u$  or  $\alpha_w$  is zero. The consumer price index  $p_t$  is defined by

$$(5) \quad p_t = \phi q_t + (1 - \phi)b_t, \quad 0 < \phi < 1.$$

The model implies a first order system for real wages  $(w - q)_t$  and the real exchange rate  $(b - q)_t$ . Appendix A shows that under mild parameter restrictions, there is a stable solution for an exogenously determined steady-state rate of unemployment,  $u^{ss}$  (e.g., by an independent ARMA process). The model encompasses both the wage curve and the Phillips curve specifications. A Phillips curve requires  $\alpha_w = \alpha_q = 0$ , while a wage curve specification implies  $\gamma_u = \beta_{p-q} = 0$ . Consequently, it should be possible to identify the model specific NAIRUs as special cases of the general steady-state unemployment  $u^{ss}$ .

## 2.3 The wage curve NAIRU

The static wage-curve NAIRU  $\bar{u}^w$  is obtained by imposing  $\beta_{pr} = 1$  and  $\beta_{p-q} = 0$  on (1)-(2):

$$\bar{u}^w = \frac{m_w + m_q}{\beta_u},$$

as shown in Figure 1. It shows the downward sloping wage curve, represented by the restricted (1), while price setting is represented by (2) as the horizontal line. The static wage curve NAIRU is defined by the intersection between the two lines,

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<sup>5</sup>For simplicity we abstract from movements the mark-up over the cycle.

<sup>6</sup>The dynamics is kept deliberately simple for ease of exposition. See Bårdsen and Fisher (1999) for an example with more complex dynamics.

e.g., that there exists a certain level of unemployment  $\bar{u}^w$  at which the conflicting real wage claims are equalized and the rate of inflation is constant.

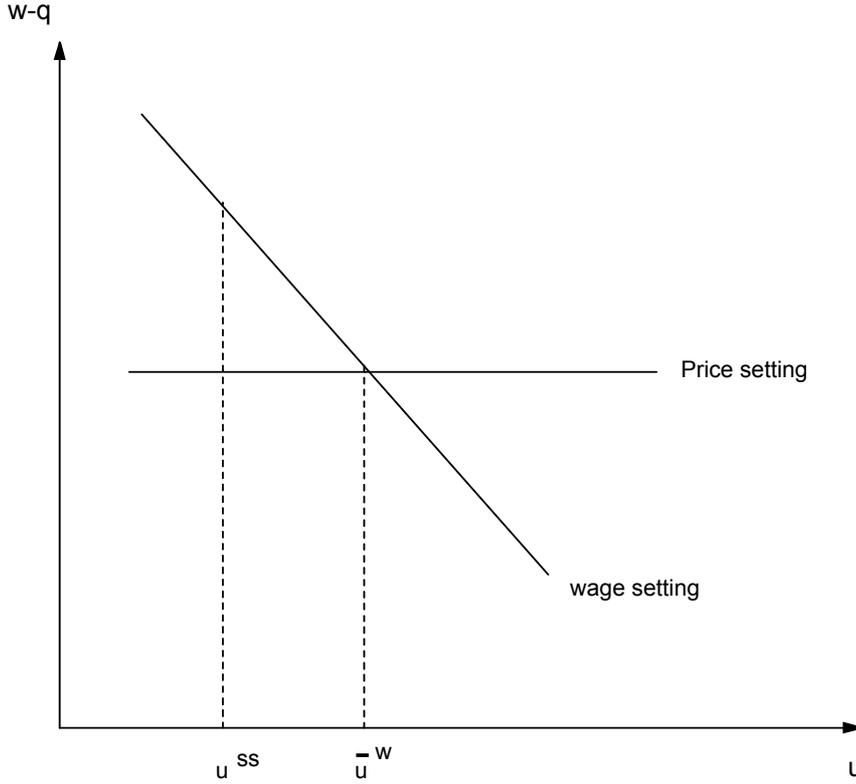


Figure 1: Real wage and unemployment determination in the static wage curve model.

Within our more complete dynamic framework, the wage curve NAIRU  $u^w$  is derived by imposing a set of restrictions on the wage-price system (3)–(4). The following set of restrictions are required:

1. Identification of the unemployment effect on wages:  $\gamma_u = 0$ .
2. Stationarity of the wage share:  $\beta_{pr} = 1$ ; elimination of the wedge in the long-run wage equation:  $\beta_{p-q} = 0$ , but maintaining  $\alpha_w > 0$ , and
3. imposing short-run homogeneity of the particular form  $\gamma_{qw} = \gamma_{wq} = 1$ , and hence  $\gamma_{wp} = \gamma_{qb} = 0$ .

The model can now be expressed in term of two conflicting equations for  $\Delta(w - q)_t$ . The only solution of the battle of mark-ups is that  $u_t \rightarrow u^w$ :

$$(6) \quad u^w = \bar{u}^w + \left( \frac{c_w + c_q}{\alpha_w \beta_u} \right).$$

Proponents of the wage curve argue that  $u_t \rightarrow u^w$  in steady state.<sup>7</sup> Without the restrictions 1.–3., any rate of unemployment, say  $u^{ss}$ , can be fully consistent with a steady-state growth rate of the real wage, a stationary wage share and a constant rate of inflation.

## 2.4 The Phillips curve NAIRU

To derive a NAIRU from a Phillips curve, the following set of restrictions are sufficient:

1. No equilibrium correction  $\alpha_w = \alpha_q = 0$
2. No effect of wage-growth on inflation  $\gamma_{qw} = 0$

Equation (3) then simplifies to the wage Phillips-curve

$$(7) \quad \Delta w_t = c_w + \gamma_{wp} \Delta p_t + \gamma_{wq} \Delta q_t - \gamma_u u_{t-1},$$

and we define

$$(8) \quad u^{phil} = \frac{1}{\gamma_u} [(\gamma_{wp} + \gamma_{wq} - 1) \pi - g_{pr} + c_w]$$

as the Phillips curve NAIRU—where  $\pi$  is steady-state inflation, and  $g_{pr}$  denotes the productivity growth rate.<sup>8</sup>

There is logically no reason why the steady-state real wage should involve  $u^{ss} = u^{phil}$ .<sup>9</sup> In this case neither the real wage nor the real exchange rate are dynamically stable for a given level of unemployment. However,  $u^{phil}$  is a fixed point, from (7), and one can certainly think of a stabilizing mechanism that links  $u_t$  to the lagged real wage share, meaning that  $u_t \rightarrow u^{phil}$  if all shocks are removed from the system.

## 3 Testing the NAIRU implications

The analysis of the previous section argued that there is no reason why dynamic stability of real wages and inflation should imply or require a supply side determined steady state rate of unemployment. Moreover, the common practice of estimating such a quantity from a single Phillips curve or wage curve equation implies restrictions on a more general model of wage and price setting. These restrictions can be tested and the outcome can either strengthen or weaken the belief in the NAIRU *qua* model of the stationary rate of the rate of unemployment. We first perform a full system analysis of wages, prices and productivity using Norwegian manufacturing data for the period 1962-1994. Next, we use our framework to re-interpret the evidence in existing studies.

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<sup>7</sup>Thus, “Only if the real wage ( $W/P$ ) desired by wage-setters is the same as that desired by price-setters will inflation be stable. *And the variable that bring about this consistency is the level of unemployment*”, Layard et al. (1991, p. 18).

<sup>8</sup>i.e., we take the expectation of (8) and use that  $E[\Delta w_t - \Delta q_t - \Delta pr_t] = 0$ ,  $E[\Delta q_t] = E[\Delta p_t] = \pi$  and  $E[\Delta pr_t] = g_{pr}$ . With a  $\Delta pr_t$  term in the Phillips curve,  $g_{pr}$  in the expression for  $u^{phil}$  will have a coefficient less than unity. Note that in (6),  $c_q = g_{pr}$  from the property of cointegration.

<sup>9</sup>The real exchange rate is unstable due to the unit root implied by  $\alpha_w = 0$ , however the real exchange rate does not enter into the dynamic equation of the real wage.

### 3.1 Econometric evidence from Norway

We first consider cointegration in a semi-closed system. The modelled variables are all in log scale and are denoted as follows:<sup>10</sup>

$w_t$  = hourly wage cost in manufacturing;

$q_t$  = index of producer prices;

$(p - q)_t$  = the wedge between the consumer and producer real wage;

$pr_t$  = value added labour productivity;

$u_t$  = rate of unemployment;

$rpr_t$  = the replacement ratio.

The wedge variable  $(p - q)_t$  includes a payroll tax-rate and an income tax-rate. The conditioning variables are

$\Delta lmp_t$  is the change in the labour market programmes variable  $lmp$ , defined as  $\ln(1 - \text{the labour market program rate})$ ;

The lagged inflation rate,  $\Delta p_{t-1}$ ;

The change in normal working hours,  $\Delta h_t$ .

In addition we include a constant term (unrestricted) and two institutional dummies.<sup>11</sup> We estimate a system with one lag of each endogenous variable. We use annual data for the period 1964-1994, i.e., the number of observations ( $T$ ) is 32, and the number of coefficients is 12. The main series are shown in Figure 2.

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<sup>10</sup>The data set is available on the internet: <http://folk.uio.no/rnymoen>

<sup>11</sup>The dummy variable  $IP_t$  is designed to capture the effects of the wage-freeze in 1979 and the wage-laws of 1988 and 1989. It is 1 in 1979 and 0.5 in 1980 (low wage drift through 1979), 1 in 1988 (“first wage-law”) and 0.5 in 1989 (“second wage-law”). Similar dummies for incomes policy appear with significant coefficients in earlier studies on both annual and quarterly data (see e.g., Johansen (1995)). The dummy variable  $i67_t$  is a separate dummy which is 1 in 1967 and is zero otherwise. 1967 was a year with large changes in taxes and benefits, in connection with a comprehensive reform of the National Insurance System.

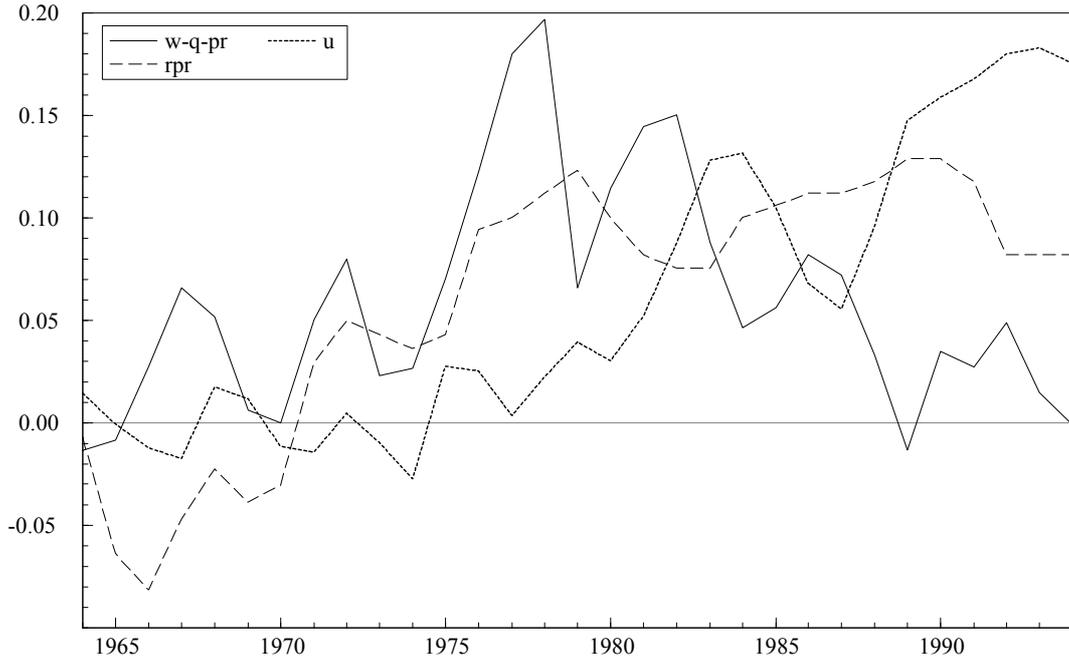


Figure 2: Logs of real unit labour costs, unemployment rate and replacement rate. Scales and means are adjusted.

Table 1: Cointegration analysis

$r$	1	2	3	4	5	6
eigenvalue	0.91	0.68	0.47	0.31	0.22	0.17
$Max$	59.8**	29.1	15.9	9.5	6.2	4.9*
$Tr$	125.4**	65.7	63.5	20.7	11.1	4.9*

The cointegration analysis is shown in table 1, in terms of the 6 eigenvalues and the maximum eigenvalue ( $Max$ ) and trace ( $Tr$ ) statistics.<sup>12</sup> The tests are corrected for degrees of freedom, see Doornik and Hendry (1997). The analysis confirms the presence of a single steady-state relationship given as

$$w = 0.9q + 1.06pr - 0.07u + 0.31(p - q) + 0.24rpr$$

We next want to test for long-run homogeneity and exogeneity. The joint test statistic of long-run homogeneity and weak exogeneity of  $q_t$ ,  $pr$ ,  $u$  and  $rpr$  is  $\chi^2(7) = 6.81[0.45]$ . The incremental test of weak exogeneity of  $u_t$  yields  $\chi^2(1) = 0.99[0.32]$  so the long run relationship is *not* an attractor for the rate of unemployment. Next, it is relevant to test whether either  $(p - q)$  or  $rpr$  can be omitted from the long-run relationship. To obtain the significance of each restriction we calculate  $\chi^2(1) = 1.65[0.20]$  for the no-wedge restriction, and  $\chi^2(1) = 5.61[0.02]$  for the zero restriction

<sup>12</sup>For evidence on the residual properties of the VAR, see Bårdsen and Nymoen (2000).

on the replacement ratio. Hence, hence  $\beta_{p-q} = 0$  is a valid restriction, while  $rpr$  appears to be significant. The final relationship is thus

$$(9) \quad w - q - pr = \underset{(0.013)}{-0.12}u + \underset{(0.06)}{0.3} rpr.$$

The corresponding equilibrium correction coefficients is  $-0.32$  (0.02) for the real wage and  $-0.22$ (0.06) for the wedge.

The strong equilibrium-correction of wages with respect to the long-run relationship is evidence against the Phillips-curve concept  $u^{Phil}$ . However, the finding that the wedge drops out is consistent with the wage curve NAIRU ( $u^w$ ). To reach a verdict about its empirical status, we need to investigate in which form (if any) dynamic homogeneity can be imposed.

Having established  $\beta_{p-q} = 0$  and the exogeneity of  $pr$ ,  $u$  and  $rpr$ , we can proceed with a single equation model of wage growth. The final equation is reported below:

$$(10) \quad \Delta(\widehat{w_t - q})_t = \underset{(0.05)}{-0.59}\Delta(q_t - p_{t-1}) - \underset{(0.13)}{0.55}\Delta h_t - \underset{(0.006)}{0.02}\Delta u_t \\ - \underset{(0.02)}{0.33}(w - q - pr + 0.12u - 0.3rpr)_{t-1} \\ + \underset{(0.008)}{0.04}i67_t, - \underset{(0.006)}{0.06}IP_t + \underset{(0.006)}{0.11} \\ \text{Method: OLS } T = 31 [1964 - 1994], \quad R^2 = 0.97, \quad \hat{\sigma} = 0.8\% \\ \chi_{NOR}^2(2) = 0.88 \quad F_{AR}(2, 22) = 2.03 \\ F_{HET}(11, 12) = 0.58 \quad F_{ARCH}(1, 22) = 0.25$$

Dynamic price homogeneity was found to be acceptable and is imposed in equation (10). Note that although the change in producer prices is highly significant in this specification, its estimated elasticity is also significantly different from unity, i.e., strong rejection of the  $u^w$  theory.<sup>13</sup>

The hours-variable  $\Delta h_t$  picks up the direct wage compensation in connection with the reductions in the length of the working day in 1964, 1968, 1969, 1976 and 1987. The two policy variables  $IP_t$  and  $i67_t$  are explained in footnote 11.

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<sup>13</sup>Instrumental variables estimation produced very similar results.

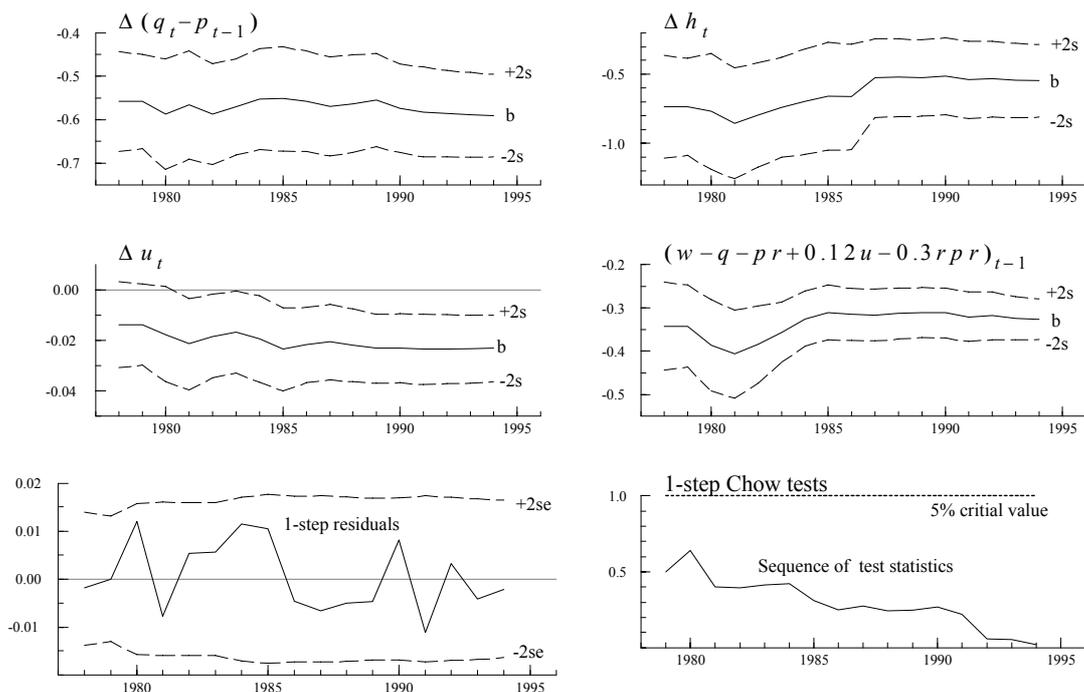


Figure 3: Recursive estimation of wage equation (23).

The statistics reported below the equation are a Chi-square test of residual normality and the F-forms of the test of 2. order residual autocorrelation, of heteroscedasticity due to squares of the regressors and of ARCH effects.

Figure 3 shows the stability of equation (10) over the period 1978-94. The four first graphs show the recursively estimated elasticities in (10), with  $\pm 2$  estimated coefficient standard errors, denoted  $\beta$  and  $\pm 2\sigma$  in the graphs. The last two panels show the 1-step residuals with  $\pm 2$  residual standard errors,  $\pm 2se$  in the graph, and finally the sequence of 1-step Chow statistics scaled with their 5% critical levels. All graphs show a high degree of stability.

These results are consistent with earlier work on Norwegian manufacturing wages, see e.g. Nymoen (1989). Johansen (1995) in particular find that the wedge can be dropped from the wage curve. The quarterly model of total economy wages by Bårdsen et al. (1998) also provides corroboration.

In terms of the theoretical model, the empirical results correspond to

- $\alpha_w > 0$
- $\beta_{pr} = 1$
- $\beta_{p-q} = 0$
- $\gamma_{wp} + \gamma_{wq} = 1$
- $\gamma_{wp} < 1$ .

Thus this test rejects the Phillips curve, and does not support the wage curve NAIRU. The upshot is that one should be wary of basing any policy analysis on the presumption that the long term rate of unemployment is pinned down by the mainstream models. A wider setting and more modelling is needed, which is however beyond the scope of this paper.

### 3.2 Interpreting existing evidence from other countries

Following the impact of Layard and Nickell (1986) there is a range of studies that estimate dynamic real-wage equations and that can be re-interpreted in the light of our framework. While not claiming to be complete, this section aims to summarize the evidence found in several econometric studies of wage formation.<sup>14</sup>

Empirical models of Nordic manufacturing wage formation are reviewed and updated in Rødseth and Nymoen (1999). Their results for Denmark, Finland, Norway and Sweden strongly reject the Phillips-curve specification. The evidence against the Phillips curve hypothesis,  $\alpha_w = 0$ , is not confined to the Nordic countries, see e.g., Grubb (1986) and Drèze and Bean (1990a) who analyze manufacturing wages for a number of European economies.

Turning to the wage-curve model: If there is no wedge term in the wage equation, the NAIRU is independent of the real exchange rate. However, the above analysis shows that only subject to specific restrictions do the wage curve NAIRU correspond to the steady-state of the system. The Nordic study by Rødseth and Nymoen, while supporting that  $\beta_{p-q} = 0$ , imply strong rejection of the NAIRU restrictions on the dynamics. Results for other European countries give the same impression: For example, six out of ten country-studies surveyed by Drèze and Bean (1990a) do not imply a wage curve NAIRU, since they are not genuine *product* real wage equations: Either there is a wedge effect in the levels part of the equation ( $\beta_{p-q} > 0$ ), or the authors fail to impose  $\gamma_{wq} = 1$ ,  $\gamma_{wp} = 0$ .<sup>15</sup>

For the United Kingdom, there are several individual studies to choose from, some of which include a significant wedge effect, i.e.,  $\beta_{p-q} > 0$ , see for example Carruth and Oswald (1989) and Cromb (1993). In a comprehensive econometric study of U.K. inflation, Rowlatt (1992) is able to impose dynamic homogeneity,  $\gamma_{wp} + \gamma_{wq} = 1$  in wage formation, but the NAIRU restriction  $\gamma_{wq} = 1$  is not supported by the data.<sup>16</sup> The work of Davies and Schøtt-Jensen (1994) contains similar evidence for several EU-countries. For the majority of the data sets, consumer price growth is found to be important alongside producer prices, and as we have showed this is sufficient to question the logical validity of the claims made in the same study,

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<sup>14</sup>See also Holden and Nymoen (2002) for empirical evaluation of the NAIRU concept used by the OECD.

<sup>15</sup>From (Drèze and Bean, 1990b, Table 1.4), and the country papers in Drèze and Bean (1990a) we extract that the equations for Austria, Britain and (at least for practical purposes) Germany are “true” product real-wage equations. The equation for France is of the Phillips-curve type. For the other countries we have, using our own notation: Belgium and the Netherlands: Consumer real-wage equations, i.e.  $\gamma_{wp} = 1$ ,  $\gamma_{wq} = 0$  and  $\beta_{p-q} = 1$ . Denmark:  $\beta_{p-q} = 1$ ,  $\gamma_{wp} = 0.24$ ,  $\gamma_{wq} = 0.76$ . Italy:  $\beta_{p-q} = 0$ ,  $\gamma_{wp} = 0.2(1 - \phi)$ ,  $\gamma_{wq} = 0.8(1 - \phi)$ . United-States  $\beta_{p-q} = 0.45(1 - \phi)$ ,  $\gamma_{wq} = 1$ ,  $\gamma_{wp} = 0$ . Spain:  $\beta_{p-q} = 0.85 \cdot 0.15$ ,  $\alpha_w = 1$ ,  $\gamma_{wp} = \beta_{p-q}$ ,  $\gamma_{wq} = 1 - \beta_{p-q}$  (The equation is static).

<sup>16</sup>See (Rowlatt, 1992, Chapter 3.6).

namely that a steady state unemployment equilibrium is implied by the estimated real-wage equations.

OECD (1997, Table 1.A.1) contains detailed wage equation results for 21 countries. For 14 countries the reported specification is of the wage-curve type but the necessary restrictions derived above on the short run dynamics are rejected. Phillips curve specifications are reported for the other seven countries, notably for the United States which corroborates evidence in other studies, see Blanchard and Katz (1997) for a discussion.

## 4 Summary and conclusion

We have derived restrictions to identify steady-state unemployment as models of the NAIRU. On the basis of the evidence, we are unable to confirm that the open economies in Europe possess a strong automatic stabilization towards a level of unemployment consistent with mainstream theories.

The main finding is that there is in general no correspondence between the steady-state rate of unemployment and the NAIRUs emanating from either the Phillips curve or the wage-bargaining model.

A defining feature of the mainstream approaches is that they advise an empirical strategy where wage and prices are modelled, but not the rate of unemployment itself. Given the apparent failure of this approach for many countries, it seems worthwhile to explore an alternative methodology that has been successful in other areas of empirical economics, see e.g., Bårdsen and Fisher (1999), and to model unemployment jointly with wages and prices. Thus, in general, the determination of the joint steady state of real wages, the real exchange rate and the rate of unemployment, requires a full dynamic model rather than wage and price setting equations alone.

## A Solving the model

### A.1 Stability

The above model can be solved for the real wage  $(w - q)_t$  and the real exchange rate  $(b - q)_t$ . One set of sufficient conditions for stable roots (adapted from Kolsrud and Nymoene (1998)) is:

$$(11) \quad \alpha_w > 0, \text{ and } \alpha_q > 0, \text{ and } \beta_{p-q} > 0, \text{ and } \gamma_{qw} < 1.$$

The first two conditions represent equilibrium correction of wages and prices with respect to deviations from the wage curve and the long run price setting schedule. The third condition states that there is a long run wedge effect in wage setting. Finally, a particular form of dynamic homogeneity is precluded by the fourth condition: a one point increase in the rate of wage growth, must lead to less than one point increase in the rate of price growth. Note that the fourth condition is much more restrictive than dynamic homogeneity in general which would be  $\gamma_{qw} + \gamma_{qb} = 1$  and  $\gamma_{wp} + \gamma_{wq} = 1$ . Dynamic homogeneity, in this usual sense is fully consistent with a stable steady state.

## A.2 General steady state

If the sufficient condition in (11) hold, we obtain a dynamic equilibrium—the “tug of war” between workers and firms reaches a stalemate. The system is stable in the sense that if all shocks are switched off,  $(w - q)_t \rightarrow (w - q)^{ss}(t)$  and  $(b - q)_t \rightarrow (b - q)^{ss}(t)$ , where  $(w - q)^{ss}(t)$  and  $(b - q)^{ss}(t)$  denote the deterministic steady state growth paths of the product real wage and the real exchange rate. They are independent of the initial conditions, but depend on steady state growth rate of import prices  $gb$ , of the mean of  $u_t$  denoted  $u^{ss}$ , and of the expected time path of productivity:

$$(12) \quad (w - q)^{ss}(t) = -\delta^0 + \xi^0 g_b + g_{pr}(t - 1) + pr_0,$$

$$(13) \quad (b - q)^{ss}(t) = -d^0 + e^0 g_b + n^0 u_{ss} + \frac{1 - \beta_{pr}}{\beta_{p-q}(1 - \phi)} [g_{pr}(t - 1) + pr_0].$$

where  $g_{pr}$  and  $pr_0$  denote the drift parameter and the initial value of productivity, respectively. Obviously, the rate of inflation in steady state is constant, and is given by  $g_b$ .

The coefficients of the two steady state expressions are given by (14):

$$(14) \quad \begin{aligned} -\delta^0 &= (c_q + \alpha_q m_q) / \alpha_q, \\ \xi^0 &= (1 - \gamma_{qw} - \gamma_{qb}) / \alpha_q, \\ -d^0 &= [\alpha_q(c_w + \alpha_w m_w) + \alpha_w(c_q + \alpha_q m_q)] / \alpha_w \alpha_q \beta_{p-q}(1 - \phi), \\ e^0 &= [\alpha_q(1 - \gamma_{wq} - \gamma_{wp}) + \alpha_w(1 - \gamma_{qw} - \gamma_{qb})] / \alpha_w \alpha_q \beta_{p-q}(1 - \phi) \\ n^0 &= \beta_u / \beta_{p-q}(1 - \phi). \end{aligned}$$

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