

# LARS ONSAGER

Chemistry, 1968

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Early in the morning of 30 October 1968, reporters got in touch with Margarethe Onsager to tell her that her husband had just won the Nobel Prize. Her reaction surprised them: "In physics or chemistry?" Onsager happened to be in California but a reporter from *Svenska Dagbladet* got hold of him at about half past five that morning and congratulated him on winning the "Nobel Prize for the Onsager theory". Onsager's reaction was in the same mould: "The Onsager theory? That could be all sorts of things, which one?" 2

Actually Onsager won the Nobel Prize in his work on what goes under the name of reciprocal relations, the foundation of the theory of irreversible processes. But the couple's response was not totally inappropriate, on the contrary. Onsager had been nominated both in physics and in chemistry, and for different scientific achievements. As we will explain below, there are "Onsager theories" in several different areas. Professor Robert H. Cole said once that Onsager could have won the prize for half a dozen different achievements.<sup>3</sup>

Who was this man whose scientific scope and depth made him one of the greatest scientists of the past century?

## THE PATH TO THE PRIZE-WINNING WORK

Lars Onsager was born on 27 November 1903 in Kristiania (now Oslo), the eldest of three boys, to Erling and Astrid Kirkeby Onsager. The surname "Onsager" means "Odin's field" and is the name of a farm in Hole in Ringsaker where Lars's paternal grandfather – Lars Onsager – was born and raised; he moved subsequently to Drammen. Erling Onsager was a lawyer and successful businessman. Lars's mother was a trained teacher. Lars grew up in a stimulating home environment. His parents were interested in fine art – Erling admired Edvard Munch and owned several of his paintings. The family also enjoyed outdoor activities. Lars

was attracted to botany from an early age – at the age of three he was counting stamens – and botany remained a lifelong interest.<sup>5</sup>

His early schooling was unorthodox: he was taught by a private tutor and his mother, and went to a private school.<sup>6</sup> It was obvious he was an intelligent boy; he solved the cubic equation when he was twelve and his academic performance at Frogner school was so good that he skipped a year.<sup>7</sup>

Hardly seventeen, he started studying chemical engineering at the Norwegian Institute of Technology (NTH – Norges Tekniske Høiskole), taking to it like a fish to water. "As a school it was very good," he told an interviewer later in life. "And there were lots of good students." One of them introduced him to the work of the Swedish physicist Carl Wilhelm Oseen. Lars read Oseen's lecture notes on hydrodynamics with great interest. He developed his formidable mathematical skills by reading classics like Whittaker and Watson's *Modern Analysis*, solving the immensely difficult exercises one by one. Learning to think analytically turned him into a mathematical virtuoso: his solution of the two-dimensional Ising model is his crowning achievement in that respect. Fellow students nicknamed him "the mathematician". 10

Chemistry professors Oscar Collenberg and Claus N. Riiber, both excellent teachers, were clearly major influences on Onsager during his studies in Trondheim. More so, in fact: Onsager was intrigued by Riiber's precise experiments on the reaction kinetics of sugars, by how two or three different forms (tautomers) of a substance can transmute into each other. 11 This was something Onsager would pursue in his later work on coupled processes - for which he won the Nobel Prize. But Johan Holtsmark, a 29-year-old, recently appointed physics professor, may have been an even stronger influence on Onsager towards the end of his Trondheim studies. Holtsmark's wide-ranging interests included theoretical and experimental physics, particularly acoustics and nuclear physics. He and Onsager got on very well, and Holtsmark supervised his master's thesis. Onsager had been reading scientific journals for some time, and in his second year he came across two fascinating articles in Holtsmark's possession, 12 in which Debye and Hückel presented their new theory of electrolytes. 13 Electrolytes contain positive and negative ions and can conduct an electric current. Two well-known examples are hydrochloric acid and a solution of ordinary salt in water. The electrical forces between the ions are long-range, and every ion therefore interacts with many others, making theoretical computations complicated. Onsager worked through the mathematics and physics in Debye and Hückel's papers, and discovered to his surprise that their results on the conductivity of electrolytes failed to obey certain symmetry requirements (for example, if an ion moves at a given velocity relative to another ion, the latter will move at the same velocity relative to the first, but



Onsager as a student in 1920.

with the opposite sign). He found a way of correcting the error, and the result was a revised theory which corresponded very neatly with experimental observations. Debye and Hückel's expression for electrical conductivity in electrolytic solutions contained an undetermined constant which they had to adjust to fit the experimental measurements. Onsager, however, equipped with his new theory, managed to *calculate* the constant. <sup>14</sup> That a student, halfway through his studies, could contribute to frontline research in this way is, obviously, very impressive. More importantly, though, this early experience had a formative impact on his career. Electrolyte theory became a lifelong love.

In 1925, just twenty-one years old, Onsager graduated from NTH in Trond-heim. His intellectual tool chest was formidable, and tailor-made for his later scientific work. Not only was he extremely knowledgeable in the fields of mathematics, hydrodynamics and chemistry, his electrolyte theory was already an achievement. He had also adopted Holtsmark's basic attitude: it is the duty of a theorist to examine the experimental consequences of his theories. It was, therefore, natural for the theoretically brilliant chemistry student to choose a purely experimental problem for his thesis work, with Holtsmark as his supervisor. 15

The first thing the newly graduated chemical engineer did was pay an

unannounced visit to Peter Debye in Zurich. Erich Hückel, Debye's assistant, recounts the event in his autobiography:

On a spring morning in 1925, while we were sitting and talking in Debye's office in Zurich, there was a knock on the door. On the word "enter", in strode a very young, big, fair-haired fellow. To Debye's enquiry, "What can I do for you?" came the frank reply, "Your theory is wrong." 16

Debye did not throw the impudent young fellow out; he offered him a cigar, listened to the main features of his work, then said generously: "But that makes the whole thing much more beautiful!"

Impressed by the young Norwegian, Debye offered him a job as an assistant. Onsager accepted and started over the new year, returning in the meantime to Trondheim to write a revised version of the Debye and Hückel theory for *Physikalische Zeitschrift*. It was his first published work. <sup>17</sup> He spent two years with Debye. The professor of physics and future Nobel laureate in chemistry considered Onsager a genius and presented his ideas at a 1927 scientific meeting in Cambridge. <sup>18</sup> Onsager's stay in Zurich was also a pleasant one: his brother Per was studying there, too. They both had a tall, athletic build and enjoyed rowing on the Zürichsee.

Europe's economic climate in the mid-1920s was difficult, but there was optimism in the US, and American universities were expanding rapidly. When Onsager was offered a well-paid job at Johns Hopkins University in Baltimore, he accepted. He was not the only Norwegian engineer to cross the Atlantic; in fact, NTH was known as the "Norwegian America Line", after the famous shipping line, with as many as 200 Norwegian engineers emigrating in a year. 20

It was a busy time at Johns Hopkins, where he had to prepare lectures in elementary chemistry. Preparations notwithstanding, he was a pedagogical disaster, simply incapable of understanding that others – students or colleagues – might not share his prodigious intellect. It was a barrier he never overcame. It was not arrogance; he just assumed others knew and understood things he took for granted. Reliable sources say that when people mustered the courage to ask for an explanation, Onsager would patiently adjust the level of difficulty down one step at a time.<sup>21</sup>

However, his contract with Johns Hopkins was not extended, doubtless to the general relief of the students.

Luckily for Onsager, the head of the chemistry department at Brown University, Professor Charles A. Kraus, had heard Onsager present a paper to the American Chemical Society, and was immensely impressed. He offered him a job at Brown; Onsager accepted and worked there for the next five years, producing the work on irreversible thermodynamics which later won him the Nobel Prize.



The triumphant four with cox; Onsager is second from left. Lake Zurich, 1926.

Irreversible thermodynamics is about currents caused by certain "forces". Wellknown examples are as follows: a difference in temperature causes a flow of heat, a concentration difference causes a stream of particles and an electric potential difference (voltage) causes an electric current. But there are also cross-effects: as a temperature difference causes a flow of heat, it can, in addition, cause a weak electrical current (known as the Seebeck effect). Conversely, an electrical current can produce a weak heat flow. These flows are assumed to be proportional (the processes are linear) to the inducing forces, the temperature or voltage differences, and the magnitude of the flows is determined by proportionality coefficients. An example of such a coefficient is the thermal conductivity, which expresses the quantitative relation between temperature difference and heat flow. Now, the intriguing fact is that coefficients for cross-effects in a pair, such as the two thermoelectric phenomena mentioned already, are not independent. On the contrary, if we write the driving forces in a certain way, the two cross-coefficients will be of identical size. This identity is known today as the Onsager symmetry. Certain examples of this symmetry were known as experimental facts as early as the mid-1800s, but no convincing explanation had been offered. William Thomson suggested a quasi-thermodynamic explanation for the above-mentioned thermoelectric effects, but it was recognised as flawed.22

Onsager was aware of symmetries like this from his work on electrolytes, where currents are transported by different types of ions in the electrolytic solution. Whether these symmetries are examples of a general phenomenon was a question he had struggled with back in his Zurich days. He suspected that the symmetries, which occur experimentally on the everyday macro scale, are based on fundamental, micro-scale molecular properties. Eventually, he became confident that the symmetry in the transport coefficients mirrors the symmetry between past and future in atomic and molecular dynamics. In our everyday, macroscopic world, we accept without much thought time's inability to run backwards. When a film is shown backwards, we are amused because that's not how reality works. But with a hypothetical film showing atoms and molecules in motion, we would not be able to decide whether the film was being run forwards or backwards. 23 At the atomic level future and past are completely symmetrical. And with such a basic factor as the crux of the explanation of the symmetry of transport coefficients, the result must be universal: all cross-effects in irreversible thermodynamics show the "Onsager symmetry".

He was not able to get all the details right at first. According to Onsager, the final piece fell into place in Boulogne-sur-Mer, in northern France. He was waiting for the boat to America and had ample time to think.

Getting the theory published was not an urgent concern to Onsager. He turned the arguments inside out to find the simplest and most convincing formulation. He published the theory first at a meeting of Scandinavian scientists in Copenhagen<sup>24</sup> in mid-1929, and then as a ten-minute presentation at the American Physical Society's annual meeting, spring 1930. The response was disappointing. Nobel laureate Niels Bohr and the highly respected professor in physical chemistry, Niels J. Bjerrum, 25 took part in the discussions at the Copenhagen meeting, but Onsager and his audience failed to see eye to eye. 26 Things improved at a summer school in theoretical physics arranged in late summer of 1930, organised by George Uhlenbeck and Samuel Goudsmit in Ann Arbor, Michigan. Onsager briefed a small group of interested physicists, including Paul Ehrenfest and Uhlenbeck. Ehrenfest thought it was a brilliant piece of work, and suggested ways Onsager could improve his presentation. He also suggested the title, Reciprocal Relations in Irreversible Processes, and Onsager used it. The two articles were published by Physical Reviewand, many years later, earned him the Nobel Prize. 27 Like the fundamental equations of classical thermodynamics, Onsager's results were elegant, simple and universally applicable, 28 and are frequently called the fourth law of thermodynamics.

Onsager was fully aware of the importance of his work and one could reasonably have assumed that the solution of a long-standing problem would attract

attention. But the response of the scientific community was lukewarm, to put it mildly. Onsager ordered a thousand reprints of the articles, but received not a single reprint request!<sup>29</sup>

A decade passed before reference was made to Onsager's reciprocal relations.<sup>30</sup> After the Second World War, however, appreciation of his work grew rapidly and it was finally recognised as a fundamental contribution to irreversible thermodynamics.

At Brown University Professor Kraus was keen to get Onsager to do experimental work. Onsager therefore suggested an experiment on isotope separation under thermal diffusion. Thermal diffusion is one of the cross-processes reciprocal relations are all about: a gas mixture of two isotopes is fed into a tube. Heat is applied to one end. As a result, concentration differences emerge along the tube, allowing the removal of the concentrated substance. This was an excellent idea, Kraus thought, until he found out that the tube would have to be of platinum and extend from the basement to the third floor. Given the worldwide economic slump, there was no way a university could afford such a platinum tube, and the experiment was put on indefinite hold. Ten years later the method was used to separate uranium isotopes (in gaseous form as uranium hexafluoride) in connection with the top-secret Manhattan project leading to the atom bomb. (Onsager was not directly involved in developing the atom bomb, although he did work on isotope separation with William Watson at Yale, who was.) So Onsager never had an opportunity to show Kraus how useful irreversible thermal diffusion could be. On the other hand, Kraus stopped pestering him to do experimental work.

Although Onsager, before the war, was not recognised for his work on the reciprocal relations, he was widely respected as an expert on the theory of electrolytes. He continued to do research in the field, working together with a brilliant young Brown student called Raymond Fuoss. Together, they published a major paper in 1932, which soon became a standard work on irreversible processes in electrolytes.<sup>31</sup>

#### TO YALE

As autumn progressed into winter in 1932, the economic depression hit rock bottom and Brown University foundered. A despairing Kraus had to let some of his young staff go, including Onsager. Herbert Harned, the leading physical chemist at Yale University, saw a golden opportunity to snap up Onsager. He was offered a Sterling Fellowship beginning in autumn 1933, and Onsager remained at Yale un-

til he retired. Harned later said that "capturing" Onsager was the most important decision of his career. <sup>32</sup> The new job proved beneficial from Onsager's point of view over and above the science. Visiting the Austrian chemist Hans Falkenhagen in Europe in mid-1933, Onsager was introduced to Falkenhagen's 21-year-old sister-in-law, Margarethe Arledter, and fell in love. They were engaged within the week, married before the end of the summer and never looked back. They had four children, Erling, Inger, Hans and Christian.

A year later Yale promoted Onsager to "assistant professor". However, local bureaucrats were astonished to discover that he had no PhD, and Onsager himself was distinctly uneasy about people constantly addressing him as Dr Onsager. He had, admittedly, once tried to obtain a PhD. Convinced of the high standard of his research in irreversible thermodynamics, in 1932 he sent reprints of the previous year's two articles, thirty-seven pages in all, to his alma mater, NTH, for assessment as a doctoral thesis. To submit reprints of concisely worded articles as a doctoral thesis was unheard of at the time, as Onsager knew well. Writing in his accompanying letter, he says:

Should NTH prefer to wait for a detailed elaboration of the submitted problem, I believe a twelve-month postponement would suffice. I might have to delay work on other more urgent matters, and it goes without saying that I would prefer to have a free hand in that respect. The submitted thesis is, however, as far as I am able to judge it, my best work.<sup>33</sup>

Given the rigorous statutes of NTH pertaining to PhDs, the chemistry department had no alternative but to recommend the board of NTH to follow Onsager's own suggestion. But they were in no doubt as to the value of the thesis, rounding off their letter to the rector as follows:

The Chemistry Department would like finally to emphasise that in so far as we find it highly unlikely that the submitted work would be disallowed by any examination committee on the basis of substance, we recommend proceeding as set out above. We have procured opinions on the work, and they suggest that it fully meets the requirements of a doctoral thesis, and that further elaboration would result in a work of great distinction for the author and for NTH.

The board of NTH followed this advice, and voted to "request engineer Onsager to compose a more complete and detailed presentation of the problem and submit it for adjudication".<sup>34</sup>

Onsager had other, more pressing things on his mind, and never produced the

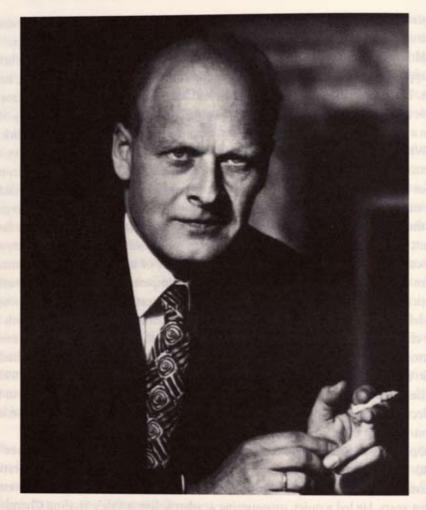
requested thesis. It is an appealing thought that had he taken time to relax the dense style of the articles, recognition in the scientific world might have come earlier.

Although Onsager had foreseen the difficulties, NTH's response was nevertheless disappointing. Relations between him and his alma mater were not harmed, however. He visited his old school regularly, and was awarded an honorary doctorate there in 1960. NTH decided soon after Norway's liberation in 1945 to formally offer him a chair, 35 but by then he was too settled at Yale to contemplate moving back.

When the problem of the missing PhD landed in Yale's lap, the chemistry department felt that any one of Onsager's recent publications would constitute a more than satisfactory PhD thesis. But according to Onsager, that would be taking the easy way out, and a few weeks later he submitted a major piece of work in pure mathematics. It concerned the properties of a certain class of mathematical functions, generally known as Mathieu functions.36 Assessing this work was beyond the capacity of Yale's chemists. The physicists were of no help either. Fortunately, there was one person at Yale with the requisite competence, the prominent mathematician Einar Hille.37 He studied the thesis with great interest, and said he would be delighted to offer Onsager a doctorate in mathematics. That was too much for the chemistry department, and they approved the thesis. Onsager thus became a doctor of chemistry in 1935, based on a work in mathematics. His thesis had to wait for publication until Onsager's complete works were released in 1996.38 Richard A. Askey, the renowned mathematician, writing in the commentary to the collected works, said that he read the thesis with interest and, indeed, pleasure, and remarked that there are not many theses in that category, after sixty years.<sup>39</sup>

Onsager was increasingly recognised at Yale, becoming "associate professor" in 1940 and being awarded the prestigious J. Willard Gibbs chair in theoretical chemistry in 1945. This position he retained for the rest of his career – twenty-seven years. He led a quiet, unassuming academic life at Yale's Sterling Chemistry Laboratory, free to concentrate on science. He was not interested in leading a team of scientists, unlike his colleague John Kirkwood, and he never held management responsibilities. For many years his office was not much more than a cell, windowless and telephoneless, with just enough space for a desk, on which lopsided piles of paper threatened to collapse at any moment. He But when he was appointed to the J. Willard Gibbs chair, the chemistry department felt his menial office would no longer do, and found a bigger and better one for him. Onsager was not interested, however. The department solved the problem by hosting a cocktail party at the laboratory and getting everybody to help Onsager relocate.

American universities during the war were less busy than usual; many students were off fighting, and scientists were doing military research. As an alien with an Austrian wife, Onsager was not part of this (he became a US citizen only in 1945)



Lars Onsager 1943.

and consequently had time to prepare the next jewel in the crown: the solution of the two-dimensional Ising model.

### SUDDEN FAME

Phase transitions are strange phenomena, of great interest to both physicists and chemists. Phase transitions occur when ice melts or water evaporates. But mag-

netic materials exhibit phase transition too: if we heat a magnetic compass needle, its magnetic phase will at some stage change into a non-magnetic phase, rendering the compass useless. The exact temperature at which this happens is called the magnet's critical point.

Phase transitions are subtle processes due to collective effects; this means that it is impossible to explain phase transitions simply by investigating what happens to a single atom or molecule. It is the concerted action resulting from the interactions between the basic constituents of the material that counts. Explaining phase transitions was therefore a major problem, especially the strange behaviour close to the critical point. By 1940 the understanding was still incomplete. The celebrated Soviet physicist Lev Landau, who later won the Nobel Prize in other aspects of his work, had formulated a simple macroscopic theory which yielded fascinating results. <sup>41</sup> But was Landau's theory correct?

All macroscopic material properties can *in principle* be deduced from the properties of its microscopic building blocks, by application of the theory of statistical mechanics. When these building blocks (atoms, molecules, etc.) do not interact, the calculation is simple enough. Statistical mechanics had long since proved successful for systems of non-interacting particles. However, interaction is the crucial mechanism behind phase transitions, and no one before Onsager had been able to carry through the calculations required by statistical mechanics on a system with interacting building blocks.

During the autumn of 1940, a version of this problem caught Onsager's attention. The microscopic formulation is simplicity itself. Atoms are located on a plane (that is, in two spatial dimensions), creating an infinite, quadratic lattice. All atomic properties are ignored, expect their magnetic moment, or "spin". We can visualise a spin as a tiny magnetic needle, with a north and a south pole. Every spin points in one of two possible directions: north pole up or north pole down. A spin interacts only with its four nearest neighbours. From an energy viewpoint it is favourable for neighbouring spins to point in the same direction, both up or both down. Conversely, for two neighbouring spins to point in opposite directions (one up, one down), a given quantity of energy is needed. This constitutes an extremely simplified model of a ferromagnet.

Real ferromagnets can, as mentioned above, have phase transitions. The question is whether this simple model is sufficiently realistic to incorporate that subtle phenomenon. At first glance it appears promising: at low temperatures little energy is available. This implies that spins will tend to point in the same direction and, consequently, the material will tend to be magnetic at low temperatures. At higher temperatures, on the other hand, there is abundant energy available, and thermal motion makes it increasingly difficult to sustain order in the ranks. Thus,

as the temperature increases, the magnetism of the bulk material decreases. At a certain temperature – the critical point – the material ceases to be magnetic.

This rather simple lattice model of a ferromagnet is called the Ising model. In 1922, Hamburg professor Wilhelm Lenz asked one of his students, Ernst Ising, to use statistical mechanics to study the properties of this model of a ferromagnet. Ising only managed to solve the corresponding one-dimensional problem, in which all the spins are located on a line. Ising's result showed that the one-dimensional model has *no* phase transition, and was therefore uninteresting. Interest picked up, however, when Rudolf Peierls in 1936 proved that a *two-dimensional* Ising model must have a phase transition, and when it was rumoured in autumn 1940 that Hendrik Kramers and Gregory Wannier had found a transformation which gave the precise value of the model's critical temperature. Kramers and Wannier's result prompted Onsager's interest: he put his mind and formidable mathematical skills to the problem and eventually succeeded in coming up with the exact solution. He announced it in 1942 during a post-lecture discussion, and published it as a three-sentence abstract. The complete solution was sent to *Physical Review* in October 1943 and appeared in 1944.

His work astounded the physics community both because it was the first exact theory of a phase transition, directly based on statistical mechanics, and because Onsager's exact analysis, technically speaking, was so brilliant. It involved esoteric branches of mathematics, quaternion algebra and elliptic functions: unknown tools for physicists and chemists. However, more important than the analytical virtuosity needed was the physical substance of the results. And even if the mathematics was difficult, the physical results were easily understood. From Onsager's achievement flowed a number of interesting conclusions. It swept aside doubts about the ability of statistical mechanics to shed light on phase transitions. It showed that discontinuities in thermodynamic entities require an infinitely large system. It demonstrated that the heat capacity (the energy per degree required to heat the material) increases as a logarithmic function as the temperature approaches the critical point, and becomes infinite at the critical point. That kind of singular behaviour had never been seen in experiments, and flew in the face of the predictions produced by Landau's classic phase transition theory.

One episode illustrates the impact of these results: in 1945 the Dutch physicist Hendrik Casimir complained that he had been unable to keep track of developments in theoretical physics in the US during the five years of war. But Wolfgang Pauli, who won the Nobel Prize in Physics that year, assured him that he hadn't missed much since nothing of importance had been done, apart from Onsager's solution of the Ising model.<sup>44</sup>

Onsager would later simplify the solution, assisted by a young doctoral student

from Columbia University, Bruria Kaufman. She insisted on working on the Ising model despite Onsager's warnings. They soon announced another sensational finding: an exact equation describing how the spontaneous magnetisation decreases with rising temperature until it disappears entirely at the critical temperature. This was another result showing critical behaviour entirely different from all earlier predictions, including – again – Landau's classic theory. Onsager was never anxious to get results published quickly, and these new successes were never published through orthodox channels. They appeared on blackboards under discussions of papers presented by others at conferences in 1948–9. The first time the result appeared in print was once more in the form of a three-sentence abstract. But the full article was not forthcoming – Onsager never published his reasoning! Four years later, Chen Ning Yang, a 1957 Nobel laureate for work in high-energy physics, did manage to reconstruct Onsager's result. It was the longest calculation I ever did, Yang wrote. 45

Onsager's results generated a buzz of activity, theoretical and experimental. But why is the solution of such a recondite model so important? One reason is that by changing the language only slightly, the Ising model can be made to describe other physical systems: an antiferromagnet, an alloy, a binary compound, or a gas—liquid system. Another reason is this: nature has systems which are practically two-dimensional: atoms adsorbed on a plane surface of a crystal, or crystals comprising two-dimensional layers of magnetic atoms well separated by non-magnetic ones. <sup>46</sup> Onsager's results apply to these effectively two-dimensional systems, and can thus be verified experimentally. The third important reason is that classic Landau theory was shown to be unreliable. This prompted a flurry of activity, not least experimentally, to understand the details of phase transitions in various systems, three- as well as two-dimensional.

In fact, the properties close to the critical point attracted interest for decades. Onsager's theoretical results always played a dual role: they furnished fresh ideas about general relationships, and they offered a reliable check point for quality testing of approximative theories. Work on phase transitions intensified in the 1960s and 70s, and culminated in Kenneth Wilson's Nobel Prize in Physics for "his theory of critical phenomena in connection with phase transitions". Wilson, unlike Onsager, did not find exact solutions for simplified models, but he formulated a coherent picture of phase transitions in general, one effect of which was actually to pinpoint the valuable aspects of Landau's theory. Wilson's perspective also explains why simple models like the Ising model can produce quantitatively accurate predictions for critical phenomena in substances that in detail are far more complicated. In the Nobel Committee's statement to the press on Wilson's work, they highlighted how Onsager's precise solution of the Ising model had

been instrumental in furthering the developments in the field that, ultimately, lead to Wilson's Nobel Prize work.

Contrary to what one would expect on general grounds, Onsager did not personally involve himself in the activity generated by his solution of the Ising model problem. This was typical of Onsager. After having understood the essentials of a problem, he proceeded to study a new one. He said, characteristically, "There are lots of folks, some quite talented, who arm themselves with methods and then go hunting for vulnerable problems; but to accept a problem on its own terms, and then forge your own weapons – now that's real class!" Class, thus defined, characterised Onsager probably more than anyone else. He was very good at cutting to the core of a problem, and if he needed sophisticated mathematical tools to solve it, he found and used those tools in masterly fashion. But his focus was the problem, not the tools. If easy mathematics sufficed, the solution was expounded with simple elegance.

Turning fifty, Onsager began receiving awards and commendations. Apart from prizes and medals, he was awarded nine honorary doctorates, the first by Harvard University in 1954, the second from NTH in 1960.

#### THE NOBEL PRIZE: BETTER LATE THAN NEVER

"It has occurred to me," Walter Kohn (who won the 1998 Nobel Prize in Chemistry) wrote in December 1967 to Maria Goeppert Mayer (winner of the 1963 Nobel Prize in Physics) and Harold Urey (winner in 1934 of the Nobel Prize in Chemistry), "that the Nobel Committee has made one terrible oversight, namely Lars Onsager. His work on irreversibility, reciprocity relations and the Ising model seems to me of a brilliance and significance which clearly merits a Nobel Prize."

The Nobel Foundation decided in 1974 to allow public scrutiny of archives at least fifty years old. We have therefore only had access to pre-1954 documents in the archives of the Nobel Committees for physics and chemistry. Where Onsager's nominations came from and how the committees dealt with them remain partly in the dark.

G. Stanley Rushbrooke at the University of Durham in Newcastle nominated Onsager for the Physics Prize back in 1952, sixteen years before he actually won it, for his work on irreversible thermodynamics. <sup>49</sup> Although it was already twenty years old, Rushbrooke argued that "the enormous amount of experimental and theoretical work" it gave rise to was of relatively recent date. In support of his claim he cites the new edition of de Groot's textbook:

A really systematic macroscopic and general thermodynamics of irreversible processes can be based on a theory published in 1931 by Onsager [...] Although Onsager's papers contained, in germ, many of the most important applications, most of them have only been formulated explicitly since about the beginning of the Second World War. A great number of results have been found in various fields of physics and chemistry ... many of them new.<sup>50</sup>

The Nobel Committee asked Oskar Klein, a Swedish theoretical physicist, to draft a report. Klein explores Onsager's general reasoning and offers examples of its application. He rounds off the evaluation like this:

Now as I hopefully demonstrated above, Onsager's brilliant proof has helped us understand the basis for the thermodynamics of transport phenomena, dispersing the darkness which used to surround these phenomena. He has also inspired the discovery of new such relationships in connection with more or less complicated transport phenomena.

As Onsager's theory is independent of special kinetic models and is based on general properties of physics, it is strongly reminiscent of the second law of thermodynamics, although its application may not be as universal.

That Onsager's account and proof of the reciprocal relations are major achievements of a highly original nature, no one should be in the slightest doubt. But whether they justify a Nobel Prize is perhaps, given current advances and activity in the field, too early to say.

The Nobel Committee adopted this wait-and-see policy.

Next year Onsager was nominated for the second time, somewhat surprisingly by the head of Yale's physics department, William W. Watson (who had worked with Onsager on isotope separation, and was the uncle, as it happened, of James Watson, one of the co-winners of the 1962 Nobel Prize for the discovery of DNA's double helix structure). According to Watson, Onsager deserved the Physics Prize for his theory of irreversible processes. The Nobel Committee was unswayed.

Since Nobel Committee business from then on is inaccessible, we do not know the date of Onsager's first Chemistry Prize nomination and, of course, nothing of the committee's discussions. However, from other sources we do know something about the post-1953 nominations.

There is evidence that George Uhlenbeck at the University of Michigan was behind an early nomination. <sup>51</sup> In 1961, Cyril Domb, professor at London's King's College, nominated him for the Physics Prize for advances in the field of irreversible processes. <sup>52</sup> Maria Goeppert Mayer and Harold Urey, recipients of Walter

Kohn's letter from which we quoted above, nominated Onsager for the Chemistry Prize in 1966, 1967 and 1968, largely for the irreversible thermodynamics work. John A.M. Cox at the Kamerlingh-Onnes Laboratory at Leiden, gave equal precedence to Onsager's work on irreversible thermodynamics and the Ising model to justify his 1968 Physics Prize nomination. Fourteen professors at Cornell's chemistry department nominated him in 1968 for the Chemistry Prize, for his solution of the Ising model problem. That was an extraordinarily prestigious nomination. The Cornell professors write that "it is no exaggeration to say that Onsager's contributions represent the most significant steps in our understanding [...] since the great work of J.D. van der Waals which was recognised with the award of the Nobel Prize in 1910." Walter Kohn nominated him for the Physics Prize in 1968, for the Ising model. Writing to congratulate Onsager on winning the Nobel Prize, Odd Hassel, old friend and Nobel laureate in chemistry the year after Onsager, writes, "I would have thought the prize in physics would have been more appropriate, and I myself nominated you for it."

Thirty-seven years elapsed between the prize-winning work and the prize. An overlong period of gestation, some may feel. But it is not unusual in Nobel Prize terms. Ten to twenty years is the norm, though Ernst Ruska can probably claim the record. He had to wait fifty-three years for his prize in physics for his 1933 work on the electron microscope. Thirty-seven years' delay is, therefore, not exceptional by any means, particularly in view of the late response – ten to fifteen years after the 1931 achievement.

Resolving the problem of whether to award a chemistry or a Physics Prize was not exactly uncharted territory for the Nobel Committee. In fact, it was almost routine, dating at least from 1903 and the case of Svante Arrhenius – nominated in both categories. Awarding half the prize in each one was suggested by the chemistry committee, but the physics committee rejected the idea, effectively creating a precedent of non-sharing between physics and chemistry (or chemistry and medicine). Many chemistry prizes have been awarded to scientists who by training and profession were physicists. Ernest Rutherford (Nobel Prize 1908) is probably the best-known example. And Svante Arrhenius (1903), Marie Curie (1911), Francis Ashton (1922), Peter Debye (1936) and Walter Kohn (1986) were all professors of physics.

The 1968 chemistry committee, whose recommendations were probably decisive, included Arne Tiselius (chairman), Arne Fredga, Karl Myrbäck, Gustaf Ölander, Gunnar Hägg and Arne Magnéli. Tiselius was professor of biochemistry at Uppsala and a 1948 Nobel laureate himself; Fredga was professor of organic chemistry, also at Uppsala, and an expert on optically active substances. Hägg, Myrbäck, Magnéli and Ölander were professors in Stockholm: Hägg in chemistry

- his main research interest was X-ray crystallography; Magnéli had a chair in inorganic chemistry, specialising in crystallography and solid-state chemistry; Myrbäck was a biochemist; Ölander a professor of inorganic chemistry who pursued research on reaction kinetics, X-ray crystallography and thermodynamics in metal alloys. Hägg and Magnéli were both new members in 1968.

When Maria Mayer and Harold Urey's 1966 and 1967 nominations failed to make headway, they began questioning which of the two categories it would pay to nominate Onsager in and asked Swedish-American oceanographer Gustaf Arrhenius for his opinion. In his 1967 reply, Arrhenius said that, naturally, much depended on the backgrounds and interests of the committee members. On balance he felt that the chemistry committee with the two new members Hägg and Magnéli would probably be more receptive than the physics committee to nominations that straddled physics and chemistry.<sup>59</sup> We should not forget the fierce competition Onsager faced in the 1950s and 60s in physics, where progress in high-energy and solid-state physics seemed more spectacular than advances in chemical physics, however fundamental. To take an example, in 1952, the year of Onsager's first Nobel Prize nomination, thirty-five scientists were nominated for the Physics Prize, among them Felix Bloch (who won), Frederik Zernike, Max Born, Walter Bothe, Willis Lamb, Pavel Cherenkov, Julian Schwinger, Sin-Itiro Tomonaga, Louis Néel and Nevill Mott. We get a sense of the competition from the fact that all nine won the prize in physics at some point or other!

The scientific community reacted enthusiastically to the Nobel Committee's decision to award the 1968 prize to Lars Onsager. Commentators highlighted his scientific brilliance, adding that it was difficult to explain in simple terms what the prize-winning work actually meant. Many felt that it was particularly fitting that the winner was a J. Willard Gibbs professor in theoretical chemistry. Gibbs, an outstanding theoretical chemist, had advanced knowledge of the thermodynamics of systems in equilibrium; Onsager's prize-winning work set the stage for the thermodynamics of systems in a non-equilibrium state. (The fact that the committee failed to award the 1901 or 1902 Nobel Prize in Chemistry to Gibbs – he died in 1903 – was a grave oversight.)

As far as the Norwegian media were concerned, one would expect only a moderate interest in Onsager's Nobel award. He was by then a US citizen and ties with Norway were weak. Nevertheless, they reacted with enthusiasm. For instance, Dagbladet printed a feature on Onsager followed by an interview headed "Mathematics virtuoso from Oslo and Trondheim". Both the announcement and the 11 December presentation were front-page news in Adresseavisen, which told their readers: "Professor Lars Onsager way ahead of his time". Drammens Tidende og Buskeruds Blad proudly announced, "Drammen family professor awarded Nobel

Prize in Chemistry", recalling that Onsager's grandfather, Lars Onsager, had resided in the town, where he worked as a master tanner and bank manager. Aftenposten took a historical view in its 31 October feature on Onsager, "Norwegian-born prize-winner got idea as 25-year-old", following up with an interview on 7 November. Verdens Gang entitled its interview with Per Onsager, Lars's brother: "Lars smitten by science as a boy". Reports from the ceremony and festivities followed, accompanied by a full-page photograph of the Onsager family and description of Mrs Onsager's gown! In December Onsager was interviewed by NRK, the Norwegian Broadcasting Corporation.

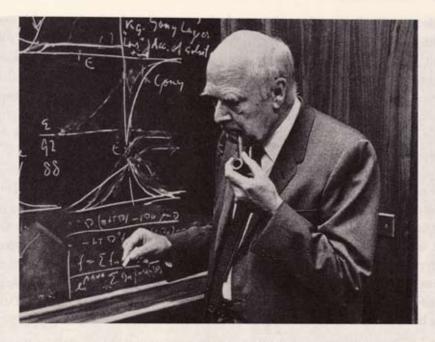
Onsager's university celebrated too – Yale was not over-represented in the Nobel Prize stakes; Onsager was, in fact, the first Yale professor to receive the prize.

The prize money, amounting to 350,000 Swedish kronor, was of little consequence to Onsager, either financially or scientifically. He was sixty-five and, according to his daughter, enjoyed every minute of the Nobel festivities. Asked by the press if the prize made him feel free, he replied that "the prize was really designed for young men who had proven their worth and should open to them a productive life. It would anyway be disrespectful to quit research now." And, as expected, Onsager continued to research more or less as before, although leading as quiet a life as he would have liked, undisturbed by what he felt were mere trifles, was no longer that easy.

# "THE ONSAGER THEORY? THAT COULD MEAN ALL SORTS OF THINGS"

As mentioned above, nominations tended to highlight either one of Onsager's two main achievements: the theory of irreversible processes and the solution to the Ising model. But they also explored other "Onsager theories". We shall look at each of them briefly.

In 1925 Debye put forward a theory of the dielectric constant of gases and liquids. However, there were obvious disparities between his theory and experimental results in the case of liquids. Onsager improved the theory, once again as a correction (though admittedly not as ground-breaking as the first one) of a work by Debye. Onsager sent a German-language manuscript to Debye, editor at the time of *Physikalische Zeitschrift*. Debye rejected it on grounds of its being "unreadable". Onsager left it at that for a time. Encouraged, however, by experimental work and urged on by John Kirkwood, he published a revised version of



Onsager in his office at the Sterling Chemistry Laboratory at Yale University.

the theory in English.<sup>67</sup> This was in 1936, the year Debye won the Nobel Prize in Chemistry. While Debye in 1925 was more than willing to go along with Onsager's improvements of his electrolyte theory, he needed time to come to terms with yet another "Onsager correction" of a Debye theory.

Experiments in 1941 showed that when, in a water solution, the concentration of the tobacco mosaic virus – long, thin structures – is increased, the virus suddenly all "agree" to point in essentially the same direction. This is an example of a general state called the nematic phase, well known in liquid crystals, in which molecules are largely oriented in a particular direction. Onsager explained the phenomenon by showing that the simplest possible model, a system of hard, long rods completely without interaction, behaves like this.<sup>68</sup> This Onsager theory of liquid crystals is clearly an idealisation. All the same, scientists have used and developed it further.

All of these "Onsager theories" are based on classical physics. None of the problems we have described so far needed quantum mechanics. Was Onsager, therefore, a physicist of the purely classical mode? Werner Heisenberg and Erwin Schrödinger had formulated quantum mechanics by 1925–6, just after Onsager graduated from Trondheim. But Holtsmark, whose name is associated, in particular, with a standard method in quantum mechanical scattering theory, probably

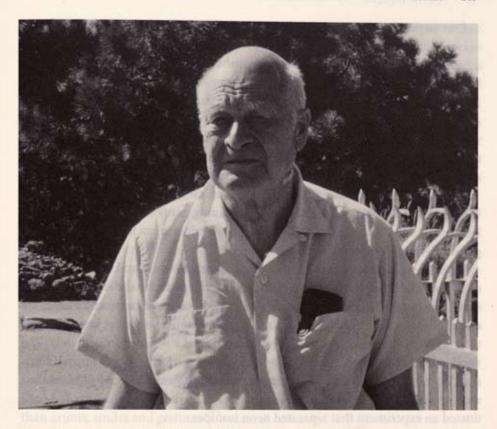
made sure that Onsager kept abreast of advances in atomic physics. In addition, we know that Onsager met the pioneers of quantum mechanics, Schrödinger and Pauli, when he was working in Zurich. And at the 1930 Ann Arbor summer school in theoretical physics mentioned above, Onsager attended a series of lectures on quantum mechanics given by eminent physicists like Enrico Fermi (Nobel Prize, 1938), Philip Morse, Paul Ehrenfest, Samuel Goudsmit and John Slater. And indeed, Onsager would demonstrate that he was no stranger to quantum mechanics.

According to Russell Donnelly, Onsager was telling students and colleagues at Yale as early as 1946 that vortices in superfluid helium are quantised. When Onsager mentioned this in connection with a discussion of a paper at a conference in Florence in 1949, he was following, for him, normal practice. Donnelly writes that "enormous ramifications of this single remark has come about, and it has been observed more than once that the ratio of scientific insight to the length of announcement must be a record in the history of science". Onsager never published an elaborated version. Montroll, Potts and Ward describe Onsager's Delphic habit of announcing discoveries like this:

In the days of Kepler and Galileo it was fashionable to announce a new scientific result through the circulation of a cryptogram which gave its author priority and his colleagues headaches. Onsager is one of the few moderns who operates in this tradition. <sup>70</sup>

He would return to the study of superfluid liquids later, and published together with Oliver Penrose a mathematical theory showing that liquids with quantum mechanical properties at sufficiently low temperatures, like helium, are superfluids.<sup>71</sup>

In 1951 Onsager moved to Cambridge for a year, bringing along his family, his domestic help, his doctoral student and his Ford. He shared an office with David Shoenberg, reader in physics. The experimentalist Shoenberg was interested in the de Haas-van Alphen effect, i.e. the strange oscillations that occur in a metal's magnetisation at sufficiently low temperatures when it is subjected to an increasing magnetic field. Just before Onsager was due to return to Yale, he handed Shoenberg a completely finished manuscript which explained the experimental results. There was no time for a discussion.<sup>72</sup> In the paper, Onsager went straight to the heart of the problem and showed that the oscillations are due to the metal's quantum mechanical electron structure.<sup>73</sup> Thus, on the basis of Onsager's insight, measurements of the de Haas-van Alphen effect can reveal aspects of precisely this structure. Onsager's explanation of this phenomenon is an integral part of solid state physics, and has long since become a section in all the textbooks.



Onsager at a conference in Colorado, August 1971.

In 1954 Onsager calculated the energy spectrum of turbulence in liquids and gases, unaware of the work of Andrei Kolmogorov on the other side of the Iron Curtain, who had come to the same conclusion a few years previously.

As he advanced in years, Onsager turned his attention to the properties of ice, particularly how charge carriers move in solid state bodies. The final paragraphs of his Nobel speech described his view on the conduction mechanisms in ice. The issue was important to Onsager because he realised that similar conduction mechanisms would be of biological relevance, in connection for instance with ion migration through cell membranes. Onsager was highly interested in biophysics; he was associated with a neural research programme, and an editor of the journal *BioSystems*, H.C. Longuet-Higgins links these concerns with Onsager's career as a whole:

One might in retrospect see his early theory of electrolytes, his discovery of the reciprocal relations and his work on phase transitions as all fundamental of the cell membrane, which he regarded as the key to biological transport in general.<sup>74</sup>

In 1973, when Onsager was seventy, a scientific conference was held in his honour at Coral Gables in Florida. His own speech was called "Life in the Early Days", which might lead one to believe he would be musing over his youth. On the contrary, Onsager presented his ideas on the origin of life on earth, and on the necessary and likely chemical and biochemical conditions.<sup>75</sup>

Onsager's was a brilliant, penetrating theoretical mind. But he was no stickler for formalism for its own sake. Once, after listening to yet another proof of a well-known theorem, he exclaimed: "To be any more immaculate they will have to begin sterilising the paper as well as the theorem."

Experiments were an inspiration to his theoretical work, and his publications often include perceptive analyses of the relevant experimental data. Cyril Domb says, "When taken round a laboratory he would comment pertinently on every detail of the experimental set-up, to the great surprise of the experimentalists who knew of him only as an abstruse theoretician." We mentioned above the experiment Onsager planned at Brown but never performed, on thermal diffusion for isotope separation. He returned to this problem and published detailed computations. Together with two Yale colleagues, he built the apparatus and conducted an experiment that separated neon isotopes.

Why did Onsager publish so little during his most productive years? Almost certainly because of a deeply self-critical attitude: Whatever he wrote had to be perfect in every detail, not to mention strikingly original – the exact opposite of today's "publish or perish" syndrome.

#### MAN AND MYTH

In many ways, Onsager is your typical absent-minded professor. His lectures were virtually incomprehensible, and he was a hopeless letter-writer. For a period of his life he simply didn't collect his mail, and when somebody unloaded it onto his table, he swept it all into the wastepaper basket. Yale put a secretary in charge of his mail, if for nothing more, at least to save the cheques from extinction.<sup>78</sup>

He produced his most important work by working alone. Many seem to have been in virtual superstitious awe of his intellectual powers.<sup>79</sup>

However, this picture is not the whole story. At home, in the Austrian manner,

he was called "Vati", and there he demonstrated the talents of a jack of all trades: he repaired things, made things, understood electronics. He even built a stereo system for one of his sons. But gardening was his first love. The family owned a farm in Tilton, New Hampshire, and there Onsager planed and sawed, dug and planted and harvested. Visitors were amazed by Onsager's encyclopaedic knowledge of botany and zoology. He was able at the drop of a hat to launch forth on the lives of plant parasites, their life cycles and how to get rid of them. He was a wellspring of general knowledge too: Philip Lyons, a colleague at Yale, recounts how Onsager excitedly explained the relative merits of cherry and walnut wood in furniture making, or the impact of Old Norse on English cadences.<sup>80</sup>

The practical side of Onsager is illustrated in the following anecdote, supplied by Philip Anderson, winner of the Nobel Prize in Physics in 1977. At the first international conference on physics in Japan after the Second World War in 1953, the hosts wanted to to show people Japan's most spectacular landmarks. Everybody was therefore invited to take a trip to Nara. The muddy roads hampered the bus, which at one point ended up in a ditch. "Drivers, local farmers and physicists stood around jabbering in several languages until Onsager, with a sigh, firmly took charge. He organised a work crew of local farmers to dismantle a log bridge over the ditch, arranged a system of levers, and with the muscle of 20 or 30 physicists and Onsager's direction and encouragement, we, to our astonishment, put the bus back on the road: all of this without communication from Onsager other than grunts, smiles and gestures."

In his spare time Onsager played chess and Kriegspiel, 82 a sort of blindfold chess with a referee. He preferred to relax with a rubber of bridge, though. He enjoyed typical Norwegian sports like skiing and skating and long walks, often returning with many samples of the indigentous flora. Onsager even sent some rootstocks of the wild lily, Trillium undulatum, to Oslo's botanical museum, where they were planted in 1967. He was proud of his physical skills, and liked to reminisce about his rowing days in Swiss regattas.

Onsager is generally characterised as a congenial companion who never had a harsh word to say about anyone. Lyons says he was generous and honest at the same time, this combination often causing conflicts which Onsager tended to resolve by silence. He was, however, known for his ability to cut self-important lecturers down to size. One outstanding British professor of chemistry, renowned for his acrimony, especially towards younger colleagues, experienced this at first hand. There are several different versions of the story, but in Longuet-Higgins and Fisher's account Onsager gave every appearance of taking a nap while the said professor discoursed and filled the blackboard with formulas. When the chairman opened the floor for questions and comments, Onsager, as if aroused from

his slumbers, raised his hand. Receiving the chairman's nod of assent, he strode up to the blackboard, picked up the rubber and erased every single equation and figure, turned to the audience, gave a Cheshire grin and sat down. The chairman, feeling that justice had been done, brought the meeting to a rapid conclusion.<sup>84</sup>

Onsager's Yale years did not end graciously. At sixty-eight he reached Yale retirement age. That led to difficulties with his research council contract. Yale statutes, as one Yale middle manager pointed out, did not allow contracts with retired professors. Onsager took umbrage at what he felt was a particularly inflexible interpretation of the rules. When it was clear that his protests were getting him nowhere, he accepted a job offer from the Center for Theoretical Studies at the University of Miami, and moved lock, stock and barrel to Coral Gables in Florida. Yale's president, Kingman Brewster Jr., was on a sabbatical during these bureaucratic calamities, and was deeply grieved at losing a man of Onsager's capacity. He tried to persuade Onsager to return, but without success.

In Florida Onsager enjoyed excellent working conditions, allowing him to finish the research programme originally intended for Yale. He worked with several young assistants and published regularly. His scientific work proceeded at the same speed until the day in October 1976 when he collapsed and died after his normal morning swim.

#### LEGACY

Science is a constant search for new theories and new knowledge, and many discoveries are therefore soon forgotten. Even a Nobel Prize cannot confer eternal life on a scientist or his achievements.

Onsager was a retiring scientist, best remembered by those who knew him personally. Science students at the Norwegian University of Science and Technology (NTNU) in Trondheim are reminded of him by a portrait relief near the lecture halls. A stamp bearing Lars Onsager's portrait was issued in 2003 by the Norwegian Post Office and has to some extent jogged our collective memory. 85

But within the international scientific community Onsager is not likely to be quickly forgotten. In 1993 NTNU started awarding the Onsager medal to outstanding scientists, recipients of which deliver the annual Lars Onsager Lecture. And in 1994 the American Physical Society inaugurated an annual Lars Onsager Prize. This strongly suggests that Onsager's scientific reputation is alive and well.

But the real question is whether his scientific achievements stand the test of time. His Nobel work on irreversible thermodynamics revealed a law of nature which governs all coupled linear transport processes. Natural laws do not wither away. And the technically brilliant solution of the Ising model will stand as a magnificent achievement which dramatically and decisively increased our understanding of phase transitions. Onsager's greatest achievements also represent, each in their individual field, ground-breaking steps forward. They will always be embedded in further progress in the field, and be visible to the informed eye. His unique combination of breadth and depth gives Lars Onsager a secure place among the most outstanding scientists of the twentieth century.

Challenging Authority: Saga, Gossip, Ballad and Legend as Narrative Voices in Sigrid Undset's Kristin Lavransdatter, Doctoral dissertation, University of Washington, Seattle, 2003.

#### LARS ONSAGER

- 1 Nature vol. 220, p. 533 (1968).
- 2 Svenska Dagbladet, 31 October, 1968.
- 3 Cited in Barry (1976).
- 4 Source: Professor Jørn Sandnes, Norwegian University of Science and Technology, Trondheim.
- 5 Letter of 16 September 2003 from Erling Onsager (Lars' nephew) to PCH.
- 6 Murray (1995), p. 1507. According to the biographical note on Onsager posted on the Nobel Academy's e-museum (http://www.nobel.se/chemistry/laureates), his private tutors over a three-year period were Inga and Anna Platou, Oslo.
- 7 Wergeland (1976). Wergeland quotes Per Onsager, Lars' brother.
- 8 Interview with Onsager 14 May 1976 by Donald G. Miller. It was recorded on tape, and a complete transcript is held at the Onsager Archive at the Gunnerus Library, Norwegian University of Science and Technology, Trondheim.
- 9 Whittaker (1902). Onsager had solved all of the exercises. Onsager's completely frayed copy of the book, full of notes, additions, corrections and comments, is held in the Onsager Archive, Gunnerus Library, Norwegian University of Science and Technology, Trondheim.
- 10 Aftenposten, 4 September, 1946.
- 11 Riiber (1922, 1923).
- 12 Wergeland (1976). Holtsmark had worked as an assistant under Peter Debye at Göttingen in 1917–18, so it is hardly surprising he had copies of Debye's recent work.
- 13 Debye (1923).
- 14 See Ebeling (1996).
- 15 Onsager's diploma thesis was entitled "En undersøkelse av absorbtions-spektra ved lave temperaturer" ("A study of adsorption spectra at low temperatures").
- 16 Hückel (1975).
- 17 Onsager (1926).
- 18 Onsager (1927).
- 19 The academic council at Johns Hopkins University resolved on 3 October 1927 "that Lars Onsager be appointed Associate in Chemistry January 1, 1928 – June 1, 1928 for \$1500".
- 20 Hanisch (1985), pp. 102 and 295.
- 21 Domb (1976).
- 22 Thomson (1854).
- 23 With a magnetic field present, it is only when the magnetic field points in opposite directions in the two situations that a past–future symmetry in the atoms' mechanics exists.
- 24 Onsager (1929). An English translation can be found in Collected Works p. 71.
- 25 Bjerrum was in "the final heat" twice for the Nobel Prize in chemistry. Nielsen (2001), p. 216.
- 26 Wergeland (1976).
- 27 Onsager (1931).
- Onsager notes in the 1931 papers that the symmetry relations must be modified in a simple way when a magnetic field is present. (See note 23.)
- 29 Onsager said as much to J. M. J. van Leeuwen in 1970. (Personal communication from van Leeuwen.)

- 30 The first reference to Onsager's reciprocal relations was made by Josef Meixner (1941). Some years later the Dutch physicists Hendrik Kramers, Hendrik Casimir and Sybren de Groot called attention to Onsager's work.
- 31 Onsager (1932).
- 32 Lyons (1982).
- 33 Onsager's letter is dated 17 December 1932. A copy of the letter is held in the Onsager Archive, Gunnerus Library, Norwegian University of Science and Technology, Trondheim.
- 34 Copy of the correspondence is held in the Onsager Archive, Gunnerus Library, Norwegian University of Science and Technology, Trondheim.
- Decision of the NTH Council, 25 October 1946. 35
- The PhD thesis has the title "Solutions of the Mathieu equation of period  $4\pi$  and certain re-36 lated functions".
- 37 Hille (1977). Bertil Hille is the son of professor Einar Hille of Yale.
- 38 The PhD thesis, originally written on a typewriter, is included in Collected Works, pp. 923-987.
- 39 Askey (1996).
- 40 Letter of 17 October 2003 from professor Michael McBride, Yale University, to PCH.
- 41 Landau (1937).
- 42 Ising (1925). The model could more properly be called the Lenz-Ising Model.
- 43 Onsager (1944).
- 44 Cited in Montroll (1977).
- 45 Yang (1952) and (1996).
- 46 The first experimental verification of the Onsager equation depicting how magnetisation varies with temperature came from the Kjeller Research Institute for Energy and Nuclear Technology. The "two-dimensional crystal" was dirubidium cobalt tetrafluoride. See Samuelsen (1975).
- 47 Longuet-Higgins (1978).
- 48 Kohn's letter is held by The Harold Clayton Urey Papers, at Mandeville Special Collections, Geisel Library, University of California at San Diego. We would like to take this opportunity to extend our gratitude to director Lynda Corey Claassen at the Mandeville Special Collection for her generous assistance in locating this letter, along with correspondence from Harold Urey, Maria Goeppert Mayer, H.N.W. Temperley and Gustaf Arrhenius to which we refer in ensuing notes. Professor Steve Moszowski, at the University of California at Los Angeles, kindly facilitated contact with Lynda Claassen.
- 49 Nobel Archives, Royal Swedish Academy of Sciences, Stockholm. We owe Karl Grandin at the Nobel Archives a debt of gratitude for his kind assistance in locating relevant material on 1952 and 1952 prizes for chemistry and physics.
- 50 de Groot (1951).
- 51 A letter dated 19 April 1965 from H.N.V. Temperley to Maria Goeppert Mayer suggests Uhlenbeck's involvement in Onsager's nomination. Temperley writes that Uhlenbeck "is in full agreement with us about Onsager, and says that he has in fact made several recommendations already". Temperley's letter is held in The Maria Goeppert Mayer Papers, Mandeville Special Collections, Geisel Library, San Diego.
- 52 Letter of 27 November, 2003, from Cyril Domb to PCH.
- 53 Nomination letters are in The Maria Goeppert Mayer Papers and The Harold Clayton Urey Papers, both at the Mandeville Special Collections, Geisel Library, San Diego.
- 54 This nomination document is held in the Onsager Archive, Gunnerus Library, Norwegian University of Science and Technology, Trondheim.
- 55 This nomination letter was signed by Andreas C. Albrecht, Simon H. Bauer, W. Donald Cooke, Michael E. Fisher, Gordon G. Hammes, James L. Hoard, Robert E. Hughes, Franklin

- A. Long, George H. Morrison, Robert A. Plane, Richard F. Porter, Harold A. Scheraga, Michell J. Sienko, and Benjamin Widom. Letter of 24 September, 2003 from Benjamin Widom to PCH.
- 56 Letter of 28 October, 2003 from Walter Kohn to PCH.
- 57 Undated letter from Odd Hassel, held in the Onsager Archive, Gunnerus Library, Norwegian University of Science and Technology, Trondheim.
- 58 Malmström (2001), p. 76.
- 59 Letter from Gustaf Arrhenius, dated 14 December, 1967, held in the Mandeville Special Collections, Geisel Library, San Diego.
- 60 Dagbladet, 31 October, 1968.
- 61 Drammens Tidende og Buskeruds Blad, 31 October, 1968.
- 62 Verdens Gang, 31 October, 1968.
- 63 Verdens Gang, 12 December, 1968.
- 64 Private communication from Inger Woerheide.
- 65 Interview with Onsager in The New Haven Register, 1 November, 1968.
- 66 Longuet-Higgins (1978).
- 67 Onsager (1936).
- 68 Onsager (1949).
- 69 Donnelly (1996).
- 70 Montroll (1963).
- 71 Penrose (1956).
- 72 Pippard (1996).
- 73 Onsager (1952).
- 74 Longuet-Higgins (1977).
- 75 Onsager (1974).
- 76 Longuet-Higgins (1978).
- 77 Domb (1976).
- 78 George Uhlenbeck visited Yale in the spring of 1970, and recounted the story to PCH.
- 79 Anderson (1996).
- 80 Philip Lyons in an unpublished obituary1978. This manuscript, held in the Onsager Archive, Gunnerus Library, Norwegian University of Science and Technology, Trondheim, is signed "A Colleague, Sterling Chemistry Laboratory".
- 81 Anderson (1996).
- 82 Shedlovsky (1963).
- 83 Letter of thanks from Rolf Y. Berg at Botanical Museum, University of Oslo. Dated 26 July, 1967. It is held in the Onsager Archive, Gunnerus Library, Norwegian University of Science and Technology, Trondheim.
- 84 Longuet-Higgins (1978).
- 85 The stamp alludes to both of the works emphasised in the Nobel Prize nomination letters. The solution of the two-dimensional Ising model is symbolized by a quadratic lattice of magnetic moments, and the theory on irreversible processes by Onsager's reciprocity relation L<sub>ij</sub> = L<sub>ji</sub>.