35th Scandinavian Symposium on Physical Acoustics, Geilo, Norway, January 29 - February 1, 2012

SONAR AND POWER BUDGET EQUATIONS FOR BACKSCATTERING OF FINITE AMPLITUDE SOUND WAVES, WITH IMPLICATIONS IN FISHERIES ACOUSTICS FOR ABUNDANCE ESTIMATION OF MARINE RESOURCES

Per Lunde[1,2,3,*] and Audun O. Pedersen[2,1,3]

[1)] *University of Bergen, Dept. of Physics and Technology, Allégaten 55, N-5007 Bergen, Norway*
[2)] *Christian Michelsen Research AS (CMR), P.O.Box 6031 Postterminalen, N-5892 Bergen, Norway*
[3)] *The Michelsen Centre for Industrial Measurement Science and Technology, Norway*

ABSTRACT

A sonar equation for finite amplitude (nonlinear) sound propagation has been derived, for backscattering from a single target in the far field. From this equation, power budget equations for finite amplitude sound propagation have been derived, for backscattering from single and multiple targets in the far field, giving expressions for measurement of the backscattering cross section (single target backscattering) and the volume backscattering coefficient (multiple target backscattering), using echosounders with source levels that cause finite amplitude propagation of the transmitted signals. These equations extend, and may replace, the conventional sonar and power budget equations applicable to small-amplitude (linear) sound propagation, used in underwater acoustics, sonar, fishery acoustics, etc. The results presented are considered to be valid under relatively general and relevant conditions for scattering from single and multiple targets located in the far field, subject to finite amplitude transmitted (incident) waves.

By applying these power budget equations to fishery acoustics, nonlinearity correction factors for finite amplitude sound propagation effects (nonlinear attenuation) in abundance estimation of marine resources have been derived. Two calibration scenarios are addressed: without and with finite amplitude effects during on-ship calibration of the echosounder transducer using e.g. an elastic sphere. The theory thus accounts for finite amplitude effects in field operation as well as possible finite amplitude effects in echosounder calibration. The nonlinearity correction factor is relevant in abundance estimation methods employing frequencies in the range of 100 kHz and above (and possibly below, depending on power level), such as e.g. species identification and zooplankton estimation. The factor may be used to correct measured volume backscattering coefficients used in abundance estimation, for cases in which the employed echosounder power levels are, or have been, so high that finite amplitude sound propagation effects are influent.

The expressions given here correspond to the *ad hoc* expressions proposed and used in [14] to correct for finite amplitude (nonlinear) sound propagation effects. Thus, a theoretical basis for these previously proposed *ad hoc* nonlinearity correction factors in fisheries acoustics has been established.

1. INTRODUCTION

Acoustic methods for estimating fish stock abundance have been in regular use for several decades [1-3], and constitute a key element in national and international regulations of marine resources, such as fish, zooplankton, etc. The abundance measurement is based on echo integration [4,5], supported by biological samples [2]. The methods rely on calibrated echosounders [6,7] operating at frequencies typically in the range 18 to 200 kHz, and even higher in some applications.

The methods used today rely on an assumption of small-amplitude (linear) sound propagation, i.e. that no finite amplitude (nonlinear) effects are influent in propagation of sound through seawater. For typical operational electrical power levels, say a few hundred Watts up to about 2 kW, the assumption of small-amplitude (linear) sound propagation seems to be reasonably good in the lower end of the operational frequency range, such as at 18 and 38 kHz. Finite amplitude effects increase however with increasing frequency, and for frequencies of 100 kHz and above, effects of nonlinearity seem to become significant even at electrical power levels in the range of 100 W and upwards [8-14].

* per.lunde@ift.uib.no

Over the last decade, new and promising methods for abundance estimation have been taken into use, such as species determination using multifrequency methods [15]. Echosounder frequencies up to 200 kHz and higher are used. The method relies on the relative frequency response of the volumetric backscattering coefficient, relative to e.g. the 38 kHz component. Large errors in one or several frequency components may cause erroneous interpretation, when comparing with the “relative frequency response signature” of candidate species. Zooplankton acoustics may also benefit from correction of data at the 120 and 200 kHz operational frequencies.

Recent work has shown that finite amplitude effects may cause errors in measurements of the volume backscattering coefficient at the high echosounder operational frequencies, such as 100 kHz and above, for typical operational power levels used in abundance estimation [8-14]. If not avoided or corrected for, finite amplitude effects may thus cause problems e.g. for species identification and zooplankton estimation.

A method to estimate such finite amplitude errors and correct for them, has been proposed in [14]. The method was based on (1) an *ad hoc* extension of the conventional power budget equation valid under conditions of small-amplitude (linear) sound propagation [16,17,14] (giving the volume backscattering coefficient for multiple-target backscattering) to conditions of finite amplitude (nonlinear) sound propagation, and (2) an *ad hoc* introduction and use of nonlinearity correction factors, for multiplication with the volume backscattering coefficient measured and calculated under assumptions of small-amplitude (linear) sound propagation. The correction method proposed in [14] and the calculations presented therein, were thus introduced *ad hoc*, and not derived from an acoustic model for finite amplitude sound propagation.

The objective of the present paper is twofold; (a) to derive expressions for acoustic backscattering from single and multiple targets (giving the backscattering cross section and the volume backscattering coefficient, respectively) under conditions of a finite amplitude transmitted sound pressure wave (hereafter referred to as the incident wave), and (b) use of these more general backscattering results to develop nonlinearity correction factors for applications in fisheries acoustics, such as for abundance estimation of fish and zooplankton, species identification, etc. This includes giving a theoretical basis for the *ad hoc* expressions proposed and used in [14] to correct for finite amplitude (nonlinear) sound propagation effects in fisheries acoustics.

In Section 2 an expression is presented for backscattering from a single target in the far field, valid under conditions of finite amplitude incident wave. This expression can equivalently be formulated as a generalization of the classical sonar equation for small-amplitude (linear) sound propagation, to conditions of finite amplitude (nonlinear) sound propagation. On this basis, power budget equations are presented for backscattering from a single and multiple targets in the far field, under conditions of finite amplitude incident wave. In Section 3 these expressions are used to establish nonlinearity correction factors for abundance estimation in fisheries acoustics. A brief discussion is given in Section 4, and conclusions stated in Section 5.

Note that only an outline of the theory is given here, stating the main assumptions and the theoretical results. For the more detailed mathematical derivations, and further details on the interpretation of the results, including consistency with the conventional theory for small-amplitude (linear) sound propagation, it is referred to [18,19]. Examples of calculation results are given in [14], and are not repeated here.

2. SONAR AND POWER BUDGET EQUATIONS FOR FINITE AMPLITUDE SOUND

Expressions for backscattering from a single target under conditions of finite amplitude sound propagation are presented first (Section 2.1), serving as basis for volume backscattering results under such conditions (Section 2.2). The results presented are considered to be valid under relatively general and relevant conditions for scattering from single and multiple targets located in the far field, subject to finite amplitude incident waves. Such conditions are described below, and summarized in Section 5.

2.1. Single-target backscattering

First, consider a transmitting electro-acoustic transducer, radiating acoustic waves into a homogeneous fluid medium. The waves are scattered by a single non-moving object (target) of arbitrary shape, located at arbitrary position in the far field of the transducer, where far field here refers to small-amplitude (linear) sound propagation conditions. The backscattered waves are received by the same transducer. The transducer and the associated transmit and receive electronics are assumed to behave linearly.

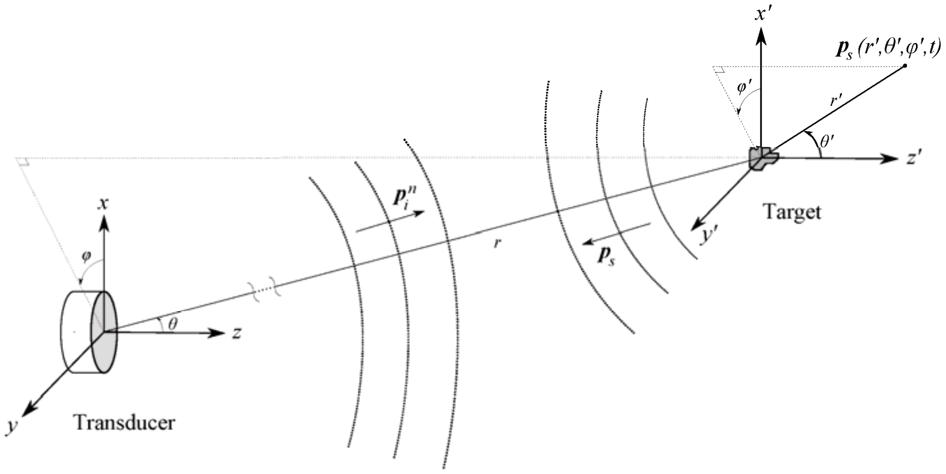

Fig. 1. Sketch of the acoustic system for single-target backscattering, with an electro-acoustic transducer operating as transmitter and receiver of ultrasound, acoustic backscattering from a single scattering object (target) at arbitrary location in the transducers's farfield in a homogeneous fluid medium, and the two spherical coordinate systems 1 and 2 used for the transmitted and scattered sound wave fields, respectively.

Two spherical coordinate systems are used to describe the electro-acoustic system, cf. Fig. 1. The origin of coordinate system no. 1 used for the transmitted wave field (with coordinates $\underline{r} = (r,\theta,\varphi)$) is located at the centre of the front face of the transducer, with $\theta \in [0,\pi]$, $\varphi \in [0,2\pi]$. The z axis ($\theta = \varphi = 0$) is taken along the transducer's acoustic axis. Coordinate system no. 2 used for the scattered wave field (with coordinates $\underline{r}' = (r',\theta',\varphi')$) is parallel to coordinated system no. 1, with origin located at the centre of the target, and $\theta' \in [0,\pi]$, $\varphi' \in [0,2\pi]$.

Finite amplitude sound propagation effects will mainly affect the transmitted, forward-radiated sound pressure wave (here referred to as the incident sound wave). For the backscattered wave the amplitude is, - without much loss of generality in the present application of fisheries acoustics, assumed to be so small that nonlinear attenuation can be neglected. Under these assumptions, and for ranges r in the far field of the transmitting transducer, the fundamental component of the finite-amplitude incident pressure wave (at angular frequency ω), $\boldsymbol{p}_i^n$, and the same frequency component of the scattered pressure wave, $\boldsymbol{p}_s$, can be modeled as (using bold-face letters for complex quantities, underline for vector quantities, subscripts "*i*" and "*s*" for the incident and scattered waves, respectively, and superscript "*n*" for quantities subject to finite amplitude (nonlinear) sound propagation effects)

$$\boldsymbol{p}_i^n(r,\theta,\varphi,t) = \boldsymbol{P}_i^n(r,\theta,\varphi)\cdot e^{i(\omega t - \underline{k}\cdot\underline{r})}, \qquad \boldsymbol{P}_i^n(r,\theta,\varphi) = \frac{\boldsymbol{A}_i}{r}\cdot e^{-\alpha r}\cdot \boldsymbol{C}_i^n(r)\cdot \boldsymbol{B}_i^n(r,\theta,\varphi), \tag{1}$$

$$\boldsymbol{p}_s(r',\theta',\varphi',t) = \boldsymbol{P}_s(r',\theta',\varphi')\cdot e^{i(\omega t - \underline{k}\cdot\underline{r}')}, \qquad \boldsymbol{P}_s(r',\theta',\varphi') = \frac{\boldsymbol{A}_s}{r'}\cdot e^{-\alpha r'}\cdot \boldsymbol{B}_s(\theta',\varphi'), \tag{2}$$

respectively, where $\boldsymbol{P}_i^n$ and $\boldsymbol{P}_s$ are the sound pressure amplitudes, $\boldsymbol{A}_i$ and $\boldsymbol{A}_s$ are amplitude constants (in space), and

$$\boldsymbol{B}_i^n(r,\theta,\varphi) \equiv \frac{\boldsymbol{P}_i^n(r,\theta,\varphi)}{\boldsymbol{P}_i^n(r,0,0)} \;, \qquad\qquad \boldsymbol{B}_s(\theta',\varphi') \equiv \frac{\boldsymbol{P}_s(r',\theta',\varphi')}{\boldsymbol{P}_s(r',0,0)} \tag{3}$$

are the beam patterns of the incident and scattered sound pressure waves, respectively. $\boldsymbol{P}_i^n(r,0,0)$ is the axial sound pressure amplitude of the incident finite amplitude sound field (along the *z* axis). $\boldsymbol{P}_s(r',0,0)$ is the sound pressure amplitude along the *z'* axis, for the scattered sound field. $\underline{k} = k\underline{e}_k$ is the acoustic wave number vector, $k = \omega/c_0$ is the acoustic wavenumber, $\omega = 2\pi f$ is the angular frequency, f is the frequency, c_0 is the small-signal sound velocity of the fluid, and α is the acoustic absorption coefficient of the fluid.

The use of $\boldsymbol{P}_s(r',0,0)$ as the normalization pressure amplitude in the expression for $\boldsymbol{B}_s(\theta',\varphi')$ given by Eq. (3), may need a comment, since the *z'* axis (the normalization direction) is not necessarily the direction of maximum scattering. This approach has been chosen for convenience and without loss of generality, since the results presented in the following can be shown to be independent of the choice of normalization direction for $\boldsymbol{B}_s(\theta',\varphi')$ [18].

It is noted that $\boldsymbol{B}_i^n(r,\theta,\varphi)$ is range dependent, whereas $\boldsymbol{B}_s(\theta',\varphi')$ is not. This is due to the finite amplitude of the incident pressure wave, with flattening of the beam pattern relative to the corresponding small-amplitude (linear) sound wave, which changes with distance from the transducer [20,14]. Under conditions of small-amplitude (linear) sound propagation, $\boldsymbol{B}_i^n(r,\theta,\varphi)$ reduces to its range-independent linear counterpart, $\boldsymbol{B}_i(\theta,\varphi)$ [18].

In Eq. (1), the axial finite-amplitude factor of the incident pressure wave is defined as [18]

$$\boldsymbol{C}_i^n(r) \equiv \frac{\boldsymbol{P}_i^n(r,0,0)}{\boldsymbol{P}_i(r,0,0)} \;, \tag{4}$$

where $\boldsymbol{P}_i(r,0,0)$ is the incident axial sound pressure amplitude at range *r*, for small-amplitude (linear) sound propagation conditions. $\boldsymbol{C}_i^n(r)$ is a measure of axial nonlinear attenuation, where $\left|\boldsymbol{C}_i^n(r)\right| < 1$ for a finite-amplitude sound wave. $\boldsymbol{C}_i^n(r)$ depends on range, *r*, since the axial nonlinear attenuation increases with increasing range, until the wave amplitude eventually becomes so small that further nonlinear attenuation becomes negligible [14]. $\boldsymbol{C}_i^n(r)$ represents deviation from spherical spreading and absorption along the acoustic axis, due to finite-amplitude effects. For a small-amplitude incident wave (linear sound propagation), $\boldsymbol{C}_i^n(r)$ becomes independent of *r*, and reduces to 1.

Under these assumptions, and with target at position $\underline{r} = (r,\theta,\varphi)$ relative to coordinate system 1, it can be shown that the backscattered sound pressure is given as [18]

$$\left|\boldsymbol{P}_{bs}\right| = \left|\boldsymbol{P}_{i,0}\right| \cdot \left|\boldsymbol{C}_i^n(r)\right| \cdot \left|\boldsymbol{B}_i^n(r,\theta,\varphi)\right| \cdot \frac{r_0}{r^2} \cdot e^{-\alpha(2r-r_0)} \cdot \sqrt{\sigma_{bs}} \;, \tag{5}$$

where $\boldsymbol{P}_{bs}$ is the backscattered free-field sound pressure amplitude in the fluid at the centre of the transducer front, in absence of the transducer, $\boldsymbol{P}_{i,0}$ is the transmitted axial sound pressure amplitude at the axial reference range r_0 (e.g. 1 m) from the transducer under small-amplitude (linear) sound propagation conditions, extrapolated spherically from the far field, and σ_{bs} is the backscattering cross section for the target. In backscattering, *r* and *r'* refer to the same distance, and have been set equal.

For reference, it is noted that Eq. (5) can equivalently be written on a logarithmic (dB) sonar equation form, as [18]

$$EL_n = SL + NA_n + BP_n - TL_1 + TS - TL_2 - 20\log_{10}\left(\frac{r_0'}{r_1}\right) - \hat{\alpha} r_0' \;, \tag{6}$$

where $EL_n \equiv 20\log_{10}(P_{bs}^{rms}/P_{ref})$ is the echo level level under conditions of finite amplitude (nonlinear) incident wave, $SL \equiv 20\log_{10}(P_{i,0}^{rms}/P_{ref})$ is the source level, $NA_n \equiv 20\log_{10}\left|\boldsymbol{C}_i^n(r)\right|$ is the axial nonlinear attenuation, $BP_n \equiv 20\log_{10}\left|\boldsymbol{B}_i^n(r,\theta,\varphi)\right|$ is the beam pattern (expressed in dB) of the transmitted finite-amplitude pressure field, $TL_1 = 20\log_{10}(r/r_0) + \hat{\alpha}(r - r_0)$ is the small-amplitude (linear) transmission loss for the wave transmitted to the target, $TS \equiv 10\log_{10}(\sigma_{bs}/r_1^2)$ is the target strength of the scattering object, $TL_2 = 20\log_{10}(r/r_0') + \hat{\alpha}(r - r_0')$ is the small-amplitude (linear) transmission loss for the wave backscattered from the target, and $\hat{\alpha} \approx 8.686\alpha$ is the absorption coefficient expressed in dB/m. r_1 is a reference length (e.g. 1 m), and r_0' is a reference range from the target (e.g. 1 m). $P_{bs}^{rms} = \left|\boldsymbol{P}_{bs}\right|/\sqrt{2}$ and $P_{i,0}^{rms} = \left|\boldsymbol{P}_{i,0}\right|/\sqrt{2}$ are the effective (rms) sound pressures corresponding to the pressure amplitudes $\boldsymbol{P}_{bs}$ and $\boldsymbol{P}_{i,0}$, respectively, and P_{ref} is a reference sound pressure (e.g. 1 μPa). The latter two terms in Eq. (6) are necessary since TL_2 gives the transmission loss for the backscattered wave over the range $r - r_0'$ and not *r*, and since absorption has not been accounted for in *TS*.

Eq. (6) is the sonar equation for backscattering from a single target at arbitrary position in the far field, under conditions of finite-amplitude incident wave. It represents an extension (generalization) of the classical sonar equation for backscattering from a single target, from conditions of small-amplitude (linear) wave propagation [21], to conditions of finite-amplitude (nonlinear) wave propagation [18].

The derivation of Eq. (5) has been made along the lines described by Clay and Medwin [21] for plane wave incidence, small-signal (linear) sound propagation and neglection of absorption loss, with the exception that here (a) finite amplitude of the incident wave is accounted for, (b) there is no assumption of plane wave incidence, (c) the target can be located at arbitrary position in the far field of the transducer, and (d) absorption has been accounted for [18].

From Eq. (5), by accounting for the transmitting and receiving responses of the transducer, it can be shown [18] that the backscattering cross section for a single target located in the far field under conditions of finite-amplitude incident sound, is given as

$$\sigma_{bs} = \frac{16\pi^2 \cdot W_R \cdot r^4 \cdot e^{4\alpha r}}{W_T \cdot G^{T,n}(r,\theta,\varphi) \cdot G^R(\theta,\varphi) \cdot \lambda^2 \cdot F_Z} \ , \tag{7}$$

where W_T and W_R are the electrical power under transmission and reception, respectively, *r* is the target range relative to coordinate system no. 1, and $\lambda = c_0/f$ is the acoustic wavelength. It is here assumed that spherical reciprocity applies in the relationship between the transmitting and receiving responses of the transducer. The transducer intensity gain function for the finite-amplitude incident wave, $G^{T,n}(r,\theta,\varphi)$, the transducer intensity gain function for reception of the backscattered wave, $G^R(\theta,\varphi)$, and the electrical impedance factor, F_Z, have been defined as

$$G^{T,n}(r,\theta,\varphi) \equiv \eta \cdot D_i \cdot \left|\boldsymbol{B}_i^n(r,\theta,\varphi)\right|^2 \cdot \left|\boldsymbol{C}_i^n(r)\right|^2 \ , \tag{8}$$

$$G^R(\theta,\varphi) \equiv \eta \cdot D_i \cdot \left|\boldsymbol{B}_i(\theta,\varphi)\right|^2 \ , \tag{9}$$

$$F_Z \equiv \frac{4R_T R_E}{\left|\boldsymbol{Z}_R + \boldsymbol{Z}_E\right|^2} \ , \tag{10}$$

respectively, where η is the transducer's electro-acoustic efficiency, R_T is the real (resistance) part of the transducer's input electrical impedance when radiating into the fluid, $\boldsymbol{Z}_T = R_T + iX_T$, R_E is the real (resistance) part of the input electrical impedance of the receiving electrical network, $\boldsymbol{Z}_E = R_E + iX_E$, $\boldsymbol{Z}_R = R_R + iX_R$ is the output electrical impedance of the transducer at reception, and

$$D_i \equiv \frac{4\pi}{\int_{4\pi} \left|\boldsymbol{B}_i(\theta,\varphi)\right|^2 d\Omega} \tag{11}$$

is the directivity factor for a small-amplitude (linear) incident wave, where Ω is the solid angle and $d\Omega = \sin\theta\, d\theta\, d\varphi$, $\Omega \in [0,4\pi]$. It has here been assumed that under conditions of small-amplitude (linear) sound propagation, the transducer's beam patterns at transmission and reception are equal.

Eq. (7) represents an extension (generalization) of the power budget equation for backscattering from a single target, from conditions of small-amplitude (linear) wave propagation [16,17,14], to conditions of finite-amplitude (nonlinear) sound propagation. For a small-amplitude (linear) incident wave, Eq. (7) reduces to the corresponding expression for linear sound propagation [18], serving as a basis for the conventional abundance estimation in fishery acoustics [16,17,14].

2.2. Multiple-target (volume) backscattering

Now, consider backscattering from a volume, *V* (cf. Fig. 2), containing a multitude of scattering targets of different types, each type characterized with its own backscattering cross section, given by Eq. (7). It is assumed [18] that (a) the volume backscattering coefficient can be calculated as a sum (including a volume integration) over backscattering cross sections (i.e., intensities), and (b) excess attenuation due to volume scattering in the volume *V* can be neglected. These are the same assumptions as those underlying the derivation of the power budget equation used in conventional abundance estimation, for small-signal (linear) sound propagation [14] (cf. also [22]).

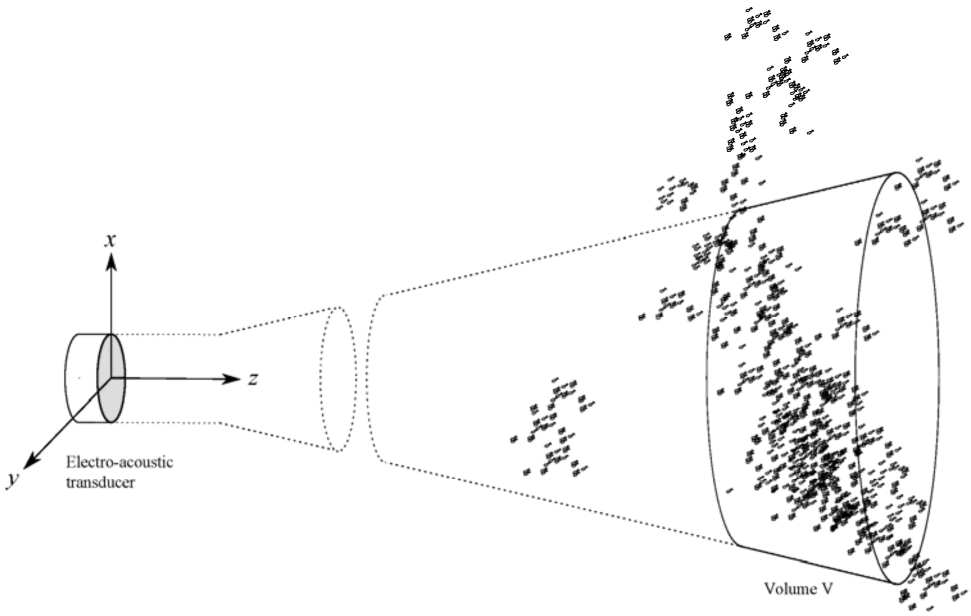

Fig. 2. Sketch of the acoustic system for multi-target (volume) backscattering, with an electro-acoustic transducer operating as transmitter and receiver of ultrasound, and backscattering from a spherical shell volume segment *V*, of thickness $dr = \frac{1}{2} c_0 \tau$, containing a distribution of scattering objects (targets).

For a finite-amplitude incident wave it can be shown from Eq. (7) that under these assumptions, the volume backscattering coefficient from a spherical shell of thickness $\frac{1}{2} c_0 \tau$, located in the far field, is given as [18]

$$s_v^n = \frac{32\pi^2 \cdot W_{\boldsymbol{R}} \cdot r^2 \cdot e^{4\alpha r}}{G_0^{T,n}(r) \cdot G_0^R \cdot \psi^n(r) \cdot W_{\boldsymbol{T}} \cdot \lambda^2 c_0 \tau \cdot F_Z} \,, \qquad (12)$$

Here, τ is the duration of the acoustic signal insonifying the volume *V*. The equivalent two-way solid beam angle for finite-amplitude incident wave is defined and given as

$$\psi^n(r) \equiv \frac{1}{G_0^{T,n}(r) G_0^R} \int_{4\pi} G^{T,n}(r,\theta,\varphi) G^R(\theta,\varphi) d\Omega = \int_{4\pi} \left| \boldsymbol{B}_i^n(r,\theta,\varphi) \right|^2 \left| \boldsymbol{B}_i(\theta,\varphi) \right|^2 d\Omega \,, \qquad (13)$$

respectively, where

$$G_0^{T,n}(r) \equiv G^{T,n}(r,0,0) = \eta \cdot D_i \cdot \left| \boldsymbol{C}_i^n(r) \right|^2 \,, \qquad (14)$$

$$G_0^R \equiv G^R(0,0) = \eta \cdot D_i \,, \qquad (15)$$

are the axial transducer intensity gains for the finite-amplitude incident wave and the scattered wave, respectively.

The assumptions mentioned above may need some comments. The volume integration (i.e., summation) of backscattering cross sections σ_{bs} (i.e., intensities) used to calculate the backscattering volume coefficient, s_v^n, as given by Eq. (12), implies that (i) the scatterers are homogeneously (uniformly) distributed in *V* [22], (ii) the scatterers have random phases (i.e. random spacing) [22], and (iii) single-scattering is assumed, so that possible multiple-scattering effects are neglected [18]. The assumption of homogeneous (uniform) distribution of scatterers in *V* can be questioned, cf. e.g. the situation illustrated in Fig. 2. This assumption is however used also to derive the power budget equation forming the basis for conventional abundance estimation, under the additional assumption of small-amplitude (linear) sound propagation [22,16,17,14]. It is expected to be more representative for smaller volumes *V*, such as a small volume thickness, $\frac{1}{2}c_0\tau$, and a more narrow transducer beam. The neglection of excess attenuation due to volume scattering in the volume *V* is expected to be a reasonable approximation when the thickness of the volume *V*, $\frac{1}{2}c_0\tau$, is very small relative to the distance from the transducer to *V*.

Eq. (12) represents an extension (generalization) of the power budget equation for backscattering from a volume of targets, from conditions of small-amplitude (linear) sound propagation [16,17,14], to conditions of finite-amplitude (nonlinear) sound propagation. For small-amplitude incident wave, Eq. (12) reduces to the corresponding expression for linear sound propagation [18], used in conventional abundance estimation in fishery acoustics [16,17,14] (cf. also [22], in which the "acoustic part" of the above electro-acoustic power budget equation is derived, i.e., addressing only the sound propagation in the water).

3. NONLINEARITY CORRECTION FACTOR FOR VOLUME BACKSCATTERING AND ABUNDANCE ESTIMATION IN FISHERIES ACOUSTICS

In the following, the results presented in Section 2 are applied to fishery acoustics, to derive nonlinearity correction factors for volume backscattering and abundance estimation, under conditions of finite amplitude incident wave. It is assumed that in field operation, finite-amplitude effects are influent. Two calibration scenarios are addressed: (A) without and (B) with finite amplitude effects (nonlinear attenuation) present during on-ship calibration of the echosounder transducer, cf. Sections 3.1 and 3.2, respectively.

3.1. Case A: Echosounder calibration made under conditions of small-amplitude (linear) incident wave

First, consider the case for which finite amplitude (nonlinear) sound propagation effects are influent in field operation, while on-ship echosounder calibration has been made under conditions of small-amplitude (linear) sound propagation (here referred to as "Case A"). From Eqs. (7) and (12) a nonlinearity correction factor can then be derived, given as [19]

$$\left(\frac{s_v^n}{s_v^{used}} \right)_A = \frac{G_0^T \cdot \psi}{G_0^{T,n}(r) \cdot \psi^n(r)} \,, \qquad (16)$$

where subscript "A" is used to indicate "Case A". Here,

$$\psi \equiv \frac{1}{G_0^T G_0^R} \int_{4\pi} G^T(\theta,\varphi) G^R(\theta,\varphi) d\Omega = \int_{4\pi} \left| \boldsymbol{B}_i(\theta,\varphi) \right|^4 d\Omega \,, \tag{17}$$

is the equivalent two-way solid beam angle for small-amplitude (linear) sound propagation (by some authors referred to as the integrated beam width factor [22]), and

$$G^T(\theta,\varphi) \equiv \eta \cdot D_i \cdot \left| \boldsymbol{B}_i(\theta,\varphi) \right|^2 , \tag{18}$$

$$G_0^T \equiv G^T(0,0) = \eta \cdot D_i \ , \tag{19}$$

are the transducer intensity gain function and the axial transducer intensity gain for small-amplitude (linear) sound propagation, respectively. For numerical calculations, alternative and equivalent expressions may be more convenient, such as [19]

$$\begin{aligned} \left(\frac{s_v^n}{s_v^{used}} \right)_A &= \frac{\int_{4\pi} G^T(\theta,\varphi) G^R(\theta,\varphi) d\Omega}{\int_{4\pi} G^{T,n}(r,\theta,\varphi) G^R(\theta,\varphi) d\Omega} \\ &= \frac{\int_{4\pi} \left| \boldsymbol{B}_i(\theta,\varphi) \right|^4 d\Omega}{\left| \boldsymbol{C}_i^n(r) \right|^2 \cdot \int_{4\pi} \left| \boldsymbol{B}_i^n(r,\theta,\varphi) \right|^2 \left| \boldsymbol{B}_i(\theta,\varphi) \right|^2 d\Omega} = \frac{\int_{4\pi} \left| \boldsymbol{P}_i(r,\theta,\varphi) \right|^4 d\Omega}{\int_{4\pi} \left| \boldsymbol{P}_i^n(r,\theta,\varphi) \right|^2 \left| \boldsymbol{P}_i(r,\theta,\varphi) \right|^2 d\Omega} \ . \end{aligned} \tag{20}$$

where $\boldsymbol{P}_i(r,\theta,\varphi)$ is the incident sound pressure amplitude at small-amplitude (linear) sound propagation conditions.

3.2. Case B: Echosounder calibration made under conditions of finite amplitude incident wave

Next, consider the case for which finite amplitude (nonlinear) sound propagation effects are influent in field operation as well as during on-ship echosounder calibration (here referred to as "Case B"). From Eqs. (7) and (12) a nonlinearity correction factor can be derived for this case, given as [19]

$$\left(\frac{s_v^n}{s_v^{used}} \right)_B = \frac{G_{0,cal}^{T,n}(r_{cal}) \cdot \psi}{G_0^{T,n}(r) \cdot \psi^n(r)} \ , \tag{21}$$

where subscript "B" is used to indicate "Case B". Here,

$$G_{0,cal}^{T,n}(r_{cal}) = \eta \cdot D_i \cdot \left| \boldsymbol{C}_{i,cal}^n(r_{cal}) \right|^2 , \tag{22}$$

and

$$\boldsymbol{C}_{i,cal}^n(r_{cal}) = \frac{\boldsymbol{P}_{i,cal}^n(r_{cal},0,0)}{\boldsymbol{P}_i(r_{cal},0,0)} \tag{23}$$

is the axial finite-amplitude factor evaluated at the calibration range, r_{cal}, for the electrical power level used during calibration, $W_{T,cal}$. $\boldsymbol{P}_{i,cal}^n(r_{cal},0,0)$ is the incident finite-amplitude axial sound pressure amplitude for the electrical power level used during calibration, and $\boldsymbol{P}_i(r_{cal},0,0)$ is the incident axial sound pressure amplitude for small-amplitude (linear) sound propagation conditions, both evaluated on the acoustic axis at the calibration range, r_{cal}.

For numerical calculations, alternative and equivalent expressions may be more convenient, such as [19]

$$\left(\frac{s_v^n}{s_v^{used}}\right)_B = \left|\boldsymbol{C}_{i,cal}^n(r_{cal})\right|^2 \cdot \frac{\int_{4\pi} G^T(\theta,\varphi)G^R(\theta,\varphi)d\Omega}{\int_{4\pi} G^{T,n}(r,\theta,\varphi)G^R(\theta,\varphi)d\Omega}$$
$$= \left|\frac{\boldsymbol{C}_{i,cal}^n(r_{cal})}{\boldsymbol{C}_i^n(r)}\right|^2 \cdot \frac{\int_{4\pi}\left|\boldsymbol{B}_i(\theta,\varphi)\right|^4 d\Omega}{\int_{4\pi}\left|\boldsymbol{B}_i^n(r,\theta,\varphi)\right|^2\left|\boldsymbol{B}_i(\theta,\varphi)\right|^2 d\Omega} = \left|\boldsymbol{C}_{i,cal}^n(r_{cal})\right|^2 \cdot \frac{\int_{4\pi}\left|\boldsymbol{P}_i(r,\theta,\varphi)\right|^4 d\Omega}{\int_{4\pi}\left|\boldsymbol{P}_i^n(r,\theta,\varphi)\right|^2\left|\boldsymbol{P}_i(r,\theta,\varphi)\right|^2 d\Omega} \tag{24}$$

In case of small-amplitude (linear) sound propagation conditions during calibration, one has $\left|\boldsymbol{C}_{i,cal}^n(r_{cal})\right| = 1$ and thus $G_{0,cal}^{T,n}(r_{cal}) = G_0^T$, and Eqs. (21) and (24) reduce to Eqs. (16) and (20), resprectively. The expressions given for Case B are thus consistent with and include Case A, as a special case.

4. DISCUSSION

Eqs. (16) and (11) correspond to the *ad hoc* expressions proposed and used in [14] to correct for finite amplitude (nonlinear) sound propagation effects in the Case A scenario. Similarly, Eqs. (21) and (24) corresponds to the *ad hoc* expressions proposed and used in [14] to correct for finite amplitude (nonlinear) sound propagation effects in the Case B scenario. Examples of calculation results for the nonlinearity correction factors $\left(s_v^n/s_v^{used}\right)_A$ and $\left(s_v^n/s_v^{used}\right)_B$ based on these expressions are given in [14], using the *Bergen Code* numerical solution [23] of the Khokhlov-Zabolotsksaya-Kuznetsov (KZK) nonlinear parabolic equation [20]. Since a main objective of the present work is to provide a theoretical basis for the expressions on which these calculations are based, the calculations themselves are not repeated here.

Correction for finite amplitude effects on fish stock and zooplankton abundance measurements in the sea may be made as follows. Assume the volume backscattering coefficient, here denoted s_v^{used}, has been calculated from measurements in the sea according to conventional methods, using the small-amplitude (linear) theory for volume backscattering as given e.g. in [16,17,14] (cf. also [22]). The corrected volume backscattering coefficient (i.e., corrected for errors due to finite amplitude effects) is then calculated by multiplying s_v^{used} with $\left(s_v^n/s_v^{used}\right)_A$ in the case A scenario, or with $\left(s_v^n/s_v^{used}\right)_B$ in the case B scenario.

Calculation of $\left(s_v^n/s_v^{used}\right)_A$ or $\left(s_v^n/s_v^{used}\right)_B$ to enable correction for finite amplitude sound propagation effects according to this method, requires use of a simulation model capable to describe such finite amplitude effects, such as e.g. the *Bergen Code* [23]. Parameters involved in such calculations are described and discussed in [14].

5. CONCLUSIONS

In the present paper, errors in acoustic fish abundance estimation caused by finite amplitude (nonlinear) sound propagation effects are investigated theoretically, in order to achieve a theoretical fundament for numerical calculation and correction of such errors [14], as well as a quantitative understanding of the errors. The finite amplitude effects and errors of the type discussed here are due to properties of the propagation fluid medium (in this case, sea water), and apply to sonars and echosounders in general, irrespective of manufacturer.

A sonar equation for finite amplitude sound propagation has been derived, for backscattering from a single target located in the far field. Based on this equation, power budget equations for finite amplitude sound propagation have been derived, for backscattering from single and multiple targets in the far field. Expressions are given for the backscattering cross section (single target backscattering) and the volume backscattering coefficient (multiple target backscattering) for finite amplitude incident wave, in terms of the transmitted and received electrical powers measured at the transducer electrical port, properties of the transmitted and scattered sound fields, properties of the fluid medium, and electrical impedances at transmission and reception. These equations extend, and may replace, the conventional sonar and power budget equations for small-amplitude (linear) sound backscattering, used in underwater acoustics, fishery acoustics, etc. For small-amplitude (linear) incident wave, the sonar and power budget equations for finite amplitude sound propagation reduce to the conventional expressions for linear sound propagation known from the literature.

The results presented are considered to be valid under relatively general and relevant conditions of scattering from single and multiple targets located in the far field, subject to finite amplitude incident waves. Assumptions made include: (a) the electronics and the electro-acoustic transducer behave linearly at transmission and reception (linear responses), (b) the scattering objects are located at arbitrary positions in the far field of the homogeneous medium (where far field refers to small-amplitude (linear) sound propagation conditions), (c) spherical reciprocity applies in the relationship between the transmitting and receiving responses of the transducer, (d) for the backscattered waves, the amplitudes are so small that finite amplitude effects can be neglected, (e) the volume backscattering coefficient can be calculated as a sum (volume integration) of backscattering cross sections, (f) several types of scatterers may be present, each with its own backscattering cross section, (g) the scatterers are homogeneously (uniformly) distributed in V, with (h) random phases (i.e., random spacing), (i) possible multiple-scattering effects are neglected, (j) excess attenuation due to volume scattering is neglected, and (k) under conditions of small-amplitude (linear) sound propagation the transducer's beam patterns at transmission and reception are equal. The assumptions (a) - (k) are the same as those used to derive the power budget equation used in conventional acoustic fish abundance estimation, for small-amplitude (linear) sound propagation [16,17,14] (cf. also [22]).

By applying these power budget equations to fishery acoustics, nonlinearity correction factors for finite amplitude sound propagation effects (nonlinear attenuation) in abundance estimation of marine resources have been derived. Two calibration scenarios are addressed: without (Case A) and with (Case B) finite amplitude effects present during on-ship calibration of the echosounder transducer using e.g. an elastic sphere. The theory thus accounts for finite amplitude effects in field operation as well as possible finite amplitude effects experienced during echosounder calibration.

The nonlinearity correction factor is relevant in abundance estimation methods employing frequencies in the range of 100 kHz and above (and possibly below, depending on power level), such as e.g. species determination and zooplankton acoustics. The factor may be used to correct measured volume backscattering coefficients used in abundance estimation, for cases in which the employed echosounder power levels are so high that finite amplitude sound propagation effects are influent [14]. Besides of possible applications in current and future surveys subject to such conditions, the factor may be used to correct historical data originating from earlier surveys subject to such conditions, provided sufficient information on operational echosounder parameters are known. The nonlinearity correction factor may also be used to establish updated and possibly improved recommendations for echosounder operating power levels in abundance estimation, relative to earlier proposed levels [9].

The expressions given here correspond to the *ad hoc* expressions proposed and used in [14] to correct for finite amplitude (nonlinear) sound propagation effects in fish stock abundance estimation. Thus, a theoretical fundament for these previously proposed *ad hoc* nonlinearity correction factors in fisheries acoustics has been established.

ACKNOWLEDGEMENTS

The authors wish to acknowledge Magne Vestrheim, University of Bergen, Dept. of Physics and Technology, Rolf J. Korneliussen, Institute of Marine Research, and Frank Tichy, Kongsberg Maritime AS, for useful discussions related to finite amplitude sound propagation effects in fisheries acoustics.

REFERENCES

[1] O. Nakken and Ø. Ulltang, "A comparison of the reliability of acoustic estimates for fish stock abundance and estimates obtained by other assessment methods in Northeast Atlantic", FAO Fish. Rep. **300**, 249-260 (1983).

[2] D. N. MacLennan, "Acoustical measurements of fish abundance", J. Acoust. Soc. Am. **87**(1), 1-15 (1990).

[3] J. Simmonds and D. N. MacLennan, *Fisheries acoustics*, 2nd ed. (Blackwell Science Ltd., Oxford, UK, 2005).

[4] O. Dragesund and S. Olsen, "On the possibility of estimating year-class strength by measuring echo-abundance of 0-group fish", Fiskeridirektoratets Skrifter, Serie Havundersøkelser, **13**(8), 48-75 (1965).

[5] J. Dalen and O. Nakken, "On the application of the echo integration method", ICES Document CM 1983/B:19 (1983), 30 p.

[6] K. G. Foote, "Optimizing copper spheres for precision calibration of hydro acoustic equipment", J. Acoust. Soc. Am. **71**, 742–747 (1982).

[7] K. G. Foote, H. P. Knudsen, G. Vestnes, D. N. MacLennan, and E. J. Simmonds, "Calibration of acoustic instruments for fish density estimation: A practical guide", ICES Cooperative Research Report No. 144 (1987).

[8] F. E. Tichy, H. Solli, and H. Klaveness, "Nonlinear effects in a 200-kHz sound beam and consequences for target strength measurement", ICES Journal of Marine Science, **60**, 571-574 (2003).

[9] "Nonlinear effects: Recommendation for fishery research investigations", on URL: http://www.simrad.com/www/01/nokbg0240.nsf/AllWeb/F176A291CB3E64CBC12578E800491241?OpenDocument (date last viewed August 21, 2011). Originally published as Simrad News Bulletin, Simrad AS (now Kongsberg Maritime AS), Horten, Norway (March 2002), 2 p.

[10] A. C. Baker and P. Lunde, "Nonlinear propagation from circular echo-sounder transducers. Numerical simulation results", CMR Technical Note CMR-TN01-A10010-rev-01, Christian Michelsen Research AS, Bergen, Norway (August 2011), 23 p. De-classified revision of CMR Technical Note CMR-TN01-F10010 (February 2001) (confidential), 22 p.

[11] A. C. Baker and P. Lunde, "Nonlinear effects in sound propagation from echo-sounders used in fish abundance estimation. Numerical simulation results", CMR Technical Note CMR-TN02-A10008-rev-01, Christian Michelsen Research AS, Bergen, Norway (August 2011), 27 p. De-classified revision of CMR Technical Note CMR-TN02-F10008 (April 2002) (confidential), 26 p.

[12] A. Pedersen, P. Lunde, and M. Vestrheim, "Nonlinear sound propagation effects in fisheries research echosounders, - Measurements and simulations in freshwater", in Proc. of 28th Scandinavian Symposium on Physical Acoustics, Ustaoset, Norway, 23-26 January 2005, Kristiansen, U. R. (ed.), The Norwegian Physical Society (June 2005) (CD ROM only, ISBN 82-8123-000-2).

[13] A. Pedersen, M. Vestrheim, and P. Lunde, "Quantification of nonlinear sound propagation effects in fisheries research echosounders", in Proc. of Underwater Acoustic Measurements, Technologies & Results, Heraklion, Crete, June 28 – July 1, 2005, Papadakis, J.S. and Bjørnø, L. (eds.), Foundation for Research and Technology – Greece (June 2005) (ISBN 960-88702-08, 960-88702-2-4), Vol. II, pp. 751-756.

[14] A. Pedersen, *Effects of nonlinear sound propagation in fisheries research*, PhD thesis, University of Bergen, Dept. of Physics and Technology, Bergen, Norway, 2006, 307 p.

[15] R. J. Korneliussen, "The acoustic identification of Atlantic mackerel", ICES Journal of Marine Science, **67**, 1749–1758 (2010).

[16] "Instruction manual, SIMRAD EK500 Scienific echosounder. Operator manual, P2170", Simrad AS (now Kongsberg Maritime AS), Horten, Norway (1990-1997).

[17] R. J. Korneliussen, *Analysis and presentation of multifrequency echograms*, Dr. Scient. thesis, University of Bergen, Department of Physics, Bergen, Norway, 2002.

[18] P. Lunde, "Sonar and power budget equations for single-target and volume backscattering, under conditions of finite-amplitude incident sound", manuscript to be submitted to a scientific journal.

[19] P. Lunde and A. O. Pedersen, "Nonlinearity correction factor for volume backscattering and abundance estimation in fisheries acoustics, under conditions of finite-amplitude incident sound", manuscript to be submitted to a scientific journal.

[20] M. F. Hamilton, "Sound Beams", in *Nonlinear Acoustics*, edited by M. F. Hamilton and D. T. Blackstock (Academic Press, San Diego, CA, 1998), Chapter 8.

[21] C. S. Clay and H. Medwin, *Acoustical oceanography: Principles and applications* (J. Wiley & Sons, New York, 1977), pp. 180-182.

[22] Reference 21, p. 220, 229-234.

[23] J. Naze Tjøtta, S. Tjøtta, and E. H. Vefring, "Effects of focusing on the nonlinear interaction between two collinear finite amplitude sound beams", J. Acoust. Soc. Am. **89**(3), 1017-1027 (1991).