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Target depth estimation using hull mounted active sonars

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Abstract

High false alarm rates are a problem in anti-submarine warfare in littoral waters using active broadband sonars. Automatic classification procedures may help combat this problem by filtering out detections due to non-threatening targets. An interesting feature for classification purposes is the depth of the target. Using sonars with vertical beamforming capabilities, the received signal from a target can be used to find a plausible estimate of the target's depth given an initial guess of the target's horizontal distance from the ship, the bottom profile and a profile for the speed of sound.

The estimation is done by an optimization procedure which varies the relevant parameters and models signals based on these parameters, then comparing the modelled signals with the received signal to find which parameters fit the received signal best. The modelling is based on a ray tracing procedure to find eigenrays for a proposed target depth, storing vertical arrival angles and arrival times for these eigenrays and synthesizing a signal based on the arrival angles and arrival times for comparison with the recorded signal. The ray tracing procedure is done numerically using Lybin, a platform developed by the Norwegian Defence Logistics Organization (NDLO). The validity of the eigenray finding procedure is confirmed, and results from testing the optimization procedure on synthetic data are presented.

Introduction

Sonar (SOund Navigation And Ranging) equipment is used, among other things, to detect underwater objects by propagation of sound waves. Active sonar systems are often able to obtain more information about their surroundings than passive systems, especially due to their ability to measure *travel times* (the time from a ping is emitted until an echo is heard), which is vital to distance estimation. They are therefore often used in anti-submarine warfare. When using active sonars in anti-submarine warfare, classification of targets is a key issue - if the ping elicits a response from the surroundings in the form of an echo from a target, is the target a shoal of fish, an oil pipeline, a submarine, rock formations or something entirely different? Such questions often arise when using active sonar in littoral waters. Alarms from non-threatening objects are a problem since they complicate the tactical picture, and automatic classification schemes that can identify the source of an echo and filter away such false alarms are needed in order to simplify the work of the operator [4].

Regular sonar processing focuses mainly on estimating the range and bearing of a target. A drawback with this approach is that large objects such as oil pipelines could be considered moving targets. A ship moving in parallel with the pipeline emitting pings at different points in time would place the pipeline at different points in space, thus creating the illusion of a moving target and causing a false alarm. Therefore, one important clue in automatic classification is estimating the target depth, as knowing the depth of the target could eliminate some options; if the target is located on the sea bottom, chances are that it is, in fact, a pipeline or some other large bottom litter object and thus not hostile, such that it may be deprioritized in favor of targets located closer to the surface.

Determining the directionality of the signals is usually done by the beamforming technique. This is usually done in the horizontal plane to determine the North-South-East-West directionality of the signal, but although horizontal beamforming is standard in most sonar systems, some sonars also have good *vertical* beamforming capabilities, meaning that one can obtain vertical directionality for the signal as well. Knowing the vertical directionality of a signal allows us to determine not only the range of the target, but also its depth [3]. After beamforming, the signal is matched filtered to increase the SNR of the recording.

The result of this processing is a data set containing digital acoustic data sampled over a period of time and distributed over several channels corresponding to the beamformed angles. Depth estimation of a target now becomes part of the inverse problem of determining the properties of the ship's surroundings from the recorded acoustic data. The proposed method for solving the inverse problem is by modelling and optimization. If a sufficiently accurate mathematical model is available, one may simulate the propagation of sound in the ocean and use this to synthesize signals. Working with the assumption that a modelled signal resembles the recorded signal if the model parameters resemble the real world parameters, one could try to fit the model parameters for the environment, including target depth, sound speed profile and bottom depth, to create a modelled signal that resembles the recorded signal as closely as possible. Here, we shall use a ray backpropagation scheme [1] for modelling signals. This entails using ray tracing to obtain eigenrays.

Calculating the path of sound rays in sea water is a non-trivial task. However, this problem has been thoroughly analyzed, leading to tools such as Lybin, a platform developed by the Norwegian Defence Logistics Organization (NDLO) which, among other things, can compute ray paths accurately and effectively [2]. Lybin is therefore used for this purpose here. Another point of interest is the determination of a proper objective function for use in the optimization procedure. The inverse problem should not be ill-posed, that is, there should exist a unique, stable solution to the problem. The choice of objective function will influence the problem's properties in this respect. In addition, the optimization procedure should not be too costly in terms of computational power, meaning the objective function should not be too time consuming to evaluate. Some objective functions will be presented which try to address these issues. The optimization procedure itself should be chosen so as to effectively and reliably produce satisfactory results. Since the objective function will have local minima, which should be avoided, there is also a need for an initialization procedure, in which a suitable starting point close to the global minimum is obtained with as little effort as possible.

Finally, the procedure's accuracy and stability in the presence of noise must be tested. The results presented here in this regard were obtained by use of synthesized data - signals created by means of the acoustic model. Hopefully, the synthesized signals are modelled with sufficient fidelity so as to be interchangeable with real signals when it comes to testing.

Theory

Problem formulation

Assume that a set of digital acoustic data is given, and that it is sampled at times $\{t_j\}_{j=1}^{N_t}$, beamformed in the directions $\{\theta_i\}_{i=1}^{N_\theta}$ and matched filtered to yield a set of measurements $\{S_{ij}\} = \{S(\theta_i, t_j)\}$ for the acoustic intensity S at the receiver. This set of measurements is what we consider as the *signal* emitted by the system. Thus, we have a signal recorded in the directions θ_i and at the times t_j. Given that the signal contains echoes from a ping, our problem now consists of finding the target depth z_t the ping was reflected from.

Complications arise due to the recorded intensity being dependent upon other quantities. In order to estimate the target depth, these quantities must either be known beforehand or estimated along with the target depth. Thus, our problem of target depth estimation is an inverse problem that is intractable unless certain other parameters can be determined simultaneously, such as the target range r_t and the depth of the sonar system, z_s. Another important factor, the sound speed profile $c(z)$, will generally be a function varying with depth. We shall consider only flat bottom profiles, and denote the bottom depth only as z_b henceforth.

Whereas z_t is completely unknown beforehand, some assumptions may be made about the remaining environmental parameters:

- r_t can be estimated by regular horizontal beamforming and processing, providing an initial guess.
- z_s may vary slightly according to sea activity; in rough seas, the ship will bob significantly up and down and so will the sonar system, as it is fixed onto the vessel. The sonar will, however, have a certain expected depth which may be used as an initial guess.
- The sound speed profile, $c(z)$, will depend on depth, and is not known exactly. However, sound speed profiles can be estimated satisfactorily from historical observations, and by representing

them as piecewise linear functions, we can optimize with respect to sound speed profiles by use of Empirical Orthogonal Functions (EOFs) [1].

Solution approach

We shall determine z_t and the other parameters by a method based on comparisons of the recorded signal and signals obtained by use of a mathematical model. It is therefore important to obtain an accurate mathematical model of sound propagation and a good method of modelling signals based on this, so that the modelled signals closely resemble real signals. The modelling is based on finding arrival times and arrival angles for the reflected ping. With knowledge of these, a signal can be constructed. The arrival times and angles are calculated by use of eigenrays [5]. For a set of candidate parameters, including z_t, we want to find eigenrays for z_t at r_t; we know that the sound must follow the eigenray paths to reach the target, and once reflected from it, must follow eigenray paths back toward the receiver. Hence, all possible paths for the ping are given by combinations of eigenrays. Using the arrival angles and travel times we can synthesize signals by assuming that each arrival results in a Gaussian shaped signal centered at the arrival time and angle, and superpositioning these signals. The solution procedure can be separated into five subsections, visualized in figure 1:

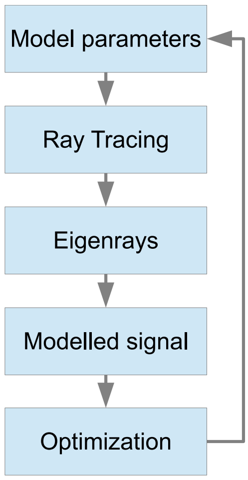


Figure 1: Flowchart of the solution process.

Finding eigenrays

An equation to model the propagation of sound in the ocean can be found by following [5], applying a high frequency approximation to the wave equation for pressure, and describing the paths taken by the ping by rays perpendicular to the wave fronts of the pressure wave by means of characteristics of the eikonal equation. The resulting *ray equation* for range independent sound speeds profiles c is as following:

$$\frac{\mathrm{d}^2 z}{\mathrm{d}r^2} = -\frac{1}{c}\frac{\partial c}{\partial z}\left[1 + \left(\frac{\mathrm{d}z}{\mathrm{d}r}\right)^2\right], \qquad z(0) = z_s, \quad z'(0) = \tan(\theta_0), \tag{1}$$

where r is the range from the vessel, z is the range dependent depth of the ray, z_s is the source depth and θ_0 is the initial angle of the ray [5].

By means of the ray equation, eigenrays can be found either analytically or numerically, depending on the complexity of the environment. For constant or linearly depth dependent sound speed profiles, solving the ray equation analytically is possible, and this is used to verify the numerical eigenray procedures. However, for general sound speed profiles, it is necessary to use numerical methods for obtaining eigenrays. We will used numerical schemes which depend on ray tracing procedures. These procedures have been studied extensively, and several good programs for this purpose are available, one of which is

Lybin, a platform developed by the Norwegian Defence Logistics Organization [2]. Lybin allows for two different approaches to calculating eigenrays numerically, which we will name *method 1* and *method 2*.

Method 1 relies heavily on Lybin, which through a built-in function can report all families of rays that enter a certain depth cell encompassing the target depth at the target range. The function passes information about these families; the rays' mean travel time, mean exit angles, transmission losses and certain other statistics, such as maximum and minimum angles within each family. The mean exit angle and mean travel time of a family is considered as the exit angle and travel time of the eigenray belonging to that family.

Method 2 relies on Lybin only for tracing the paths of a multitude of rays leaving the source at increasing exit angles. Each ray is then inspected to see whether it lands closer to the target depth at the target range than the preceding ray and the next ray. If this is the case, the travel times of the three rays under consideration are calculated numerically, and the exit angles and travel times of the three rays are interpolated to obtain an approximation to the exit angle and travel time of the eigenray they enclose. This leaves us more in control of the process, although the method is slower and may run into some issues with target depths that lie close to the surface or the bottom.

Modelling and pre-processing of signals

After N eigenrays have been found, we may construct a synthesized signal based on these, to use for comparison with the recorded signal. The ping will follow all possible combinations of eigenrays to the target and back to the ship, resulting in N^2 distinct arrivals, whose arrival times are the travel times along the eigenrays followed toward the target in addition to the travel times along the eigenrays followed back toward the source. The arrival angles are the exit angles of the eigenrays followed back to the source. Using this, all arrivals are assumed to result in a Gaussian signal given by

$$S_m(\theta_i, t_j) = A_m \exp\left(-\frac{1}{2}\left[\left(\frac{t_j - t_m}{\sigma_t}\right)^2 + \left(\frac{\theta_i - \theta_m}{\sigma_\theta}\right)^2\right]\right),$$

where the t_m are the arrival times, θ_m the arrival angles, σ_t the signal's standard deviation in time and σ_θ its standard deviation in angle. Typically, $\sigma_t = 1/B$, where B is the signal's bandwidth, and $\sigma_\theta = \theta_{BW}/2$, where θ_{BW} is the beamwidth of the receiver [4]. The amplitudes, A_m, are chosen depending on the desired signal-to-noise ratio if noise is present. Superpositioning all these signals into one yields the noiseless synthesized signal:

$$S = \sum_{m=1}^{N^2} S_m. \tag{2}$$

This is the basis for the model signals which we try to fit to the recorded signal. If we are to use the signal as a substitute for a real signal for testing the solution method, it can be subjected to additive white noise to create a more realistic signal:

$$S = n(\mu_n, \sigma_n) + \sum_{m=1}^{N^2} S_m,$$

where $n(\mu_n, \sigma_n)$ is a random Gaussian process with expectation μ_n and standard deviation σ_n.

After synthetization of a test signal, or after a real signal has been obtained, ambient noise is removed as the optimization procedure may become unstable in the presence of noise. This is done by employing a normalisation scheme such as cell-averaging constant false alarm rate filtering [6] before thresholding, leaving only signal entries with a higher SNR than a certain noise threhold. Here, the noise threshold used was 13 dB.

Representing the sound speed profile by use of EOFs

There is some difficulty in optimizing with respect to the sound speed profile $c(z)$, the main challenge being that it is a function, implying the need for variational methods whereas standard numerical optimization methods optimize with respect to scalar quantities. We therefore want to represent the sound speed profile by means of scalars, preferably as few as possible, to simplify optimization. Extracting Empirical Orthogonal Functions (EOFs) from a data set of historical observations of sound speed profiles, we may represent the sound speed profile as [4]:

$$\mathbf{c} = \bar{\mathbf{c}} + \sum_{k=1}^{m} \gamma_k \mathbf{v_k}$$

where the $\mathbf{v_k}$ are the EOFs and γ_j their weighting coefficients. If the data is well correlated, the first few EOFs will account for most of the variation in the data, and we may therefore truncate the expansion of $\mathbf{c}$ after the first few EOFs; typically, three EOFs will account for $>95\%$ of the total variation in the data. This is an acceptable error, and we therefore let

$$\mathbf{c} = \bar{\mathbf{c}} + \gamma_1 \mathbf{v_1} + \gamma_2 \mathbf{v_2} + \gamma_3 \mathbf{v_3}.$$

Now, by varying γ_1, γ_2 and γ_3, we also vary $c(z)$ in an efficient manner which is susceptible to ordinary optimization methods.

Optimization

The third and final part of the solution procedure, optimization, is done by comparing the recorded signal to a modelled signal, then attempting to modify the optimization parameters in the modelled signal in order to obtain a better fit. The choice of which parameters to optimize with respect to is a matter of complexity and accuracy. Of course, z_t should be among the optimization parameters. Other suitable candidates for optimization parameters are r_t, z_b, z_s and c, as these parameters influence the eigenray paths used in the modelled signal.

In order to compare the two signals, we need an objective function. First, since the signal is sampled at discrete N_t discrete points in time and beamformed in N_θ discrete angles, it can be represented in a matrix $S = \{S_{ij}\}$, where each S_{ij} is the intensity sampled at the time t_i and in direction θ_j. Similarly, the modelled signal is given by $M = \{M_{ij}\}$. The most obvious objective function for comparing the two signals, which we will name the *full objective function*, is now given by

$$f(M;S) = \sqrt{\sum_{i=0}^{N_t} \sum_{j=0}^{N_\theta} |M_{ij} - S_{ij}|^2}, \tag{3}$$

effectively finding the root-mean-square distance between the two signals. A problem with the full objective function is that it is computationally expensive, since each evaluation requires the formation of a full model signal M. An approximation can be done by considering the signal in vector form. Let

$$\mathbf{s} = \begin{bmatrix} S_{11} \\ \vdots \\ S_{N_t 1} \\ S_{12} \\ \vdots \\ S_{1N\theta} \\ \vdots \\ S_{N_t N_\theta} \end{bmatrix} \quad \text{and} \quad \mathbf{m} = \begin{bmatrix} M_{11} \\ \vdots \\ M_{N_t 1} \\ M_{12} \\ \vdots \\ M_{1N\theta} \\ \vdots \\ M_{N_t N_\theta} \end{bmatrix}.$$

We now have

$$f(M;S) = ||\mathbf{s} - \mathbf{m}||_2^2 = \mathbf{s}^T\mathbf{s} - 2\mathbf{s}^T\mathbf{m} + \mathbf{m}^T\mathbf{m}.$$

Since the term $\mathbf{s}^T\mathbf{s}$ is independent of $\mathbf{m}$, it can be considered constant and therefore irrelevant to optimization. Moreover, the modelled signal M is a superposition of signals from the arrivals, as explained in (2), so we may write

$$\mathbf{m} = \sum_{k=1}^{N^2} \mathbf{m_k},$$

where each $\mathbf{m_k}$ corresponds to the partial signal resulting from the k'th arrival. From this, we see that by disregarding the $\mathbf{s}^T\mathbf{s}$ term, we can form the equivalent objective function

$$\bar{f}(M;S) = -2\mathbf{s}^T\mathbf{m} + \mathbf{m}^T\mathbf{m} = \mathbf{m}^T\mathbf{m} - 2\sum_{k=1}^{N^2} \mathbf{m_k}^T\mathbf{s}.$$

If the $\mathbf{m}^T\mathbf{m}$ term could now be disregarded, we would arrive at a much more computationally efficient objective function. Since each of the $\mathbf{m_k}$, due to the Gaussian shape of the signal they contain, are mostly zeroes, we can compute the sum term very quickly by simply truncating the $\mathbf{m_k}$ to a smaller size containing only nonzero entries and computing the inner product of the truncated vector with the corresponding entries in the recorded signal, essentially exploiting the sparsity of the $\mathbf{m_k}$ signals. We therefore introduce the objective function, which we will name the *simplified objective function*, given by

$$g(M;S) = -\mathbf{s}^T\mathbf{m} = -\sum_{k=1}^{N^2} \mathbf{m_k}^T\mathbf{s} \tag{4}$$

as an inferior, yet more efficient alternative to the full objective function.

The black box nature of Lybin makes partial derivatives of any objective function with respect to the problem parameters impossible to obtain, leaving us with the choice of either a derivative-free optimization algorithm or using numerical gradients in a more sophisticated algorithm. As the objective functions are generally computationally expensive to compute, we would like to limit the amount of evaluations needed. Calculating numerical gradients calls for several evaluations per approximation, thus favouring derivative-free algorithms. Due to its robustness and ease of implementation, the algorithm chosen here is the derivative-free Nelder-Mead algorithm [7].

To avoid local minima, the Nelder-Mead algorithm requires that the optimization start reasonably close to the global minimum. To find such an initial guess, an exhaustive search method is employed, computing the objective function values with different problem parameters and choosing the parameters that yield the lowest objective function value.

Test setup

Verification of numerical eigenray estimates

A test was done to check whether the eigenray candidates produced numerically in fact reach the specified depth at target range, and in which cases the eigenray estimates might fail. Five eigenrays were calculated by use of Lybin for each set of environment parameters (r_t, z_b and z_t). The target range was varied from 1000 m to 10 000 m in steps of 1000 m, the bottom depth from 100 m to 1000 m in steps of 100 m, and the target depth was varied from 50 m to 850 m in steps of 200 m. The source depth z_s was kept constant at 50 m throughout the test. A linear sound speed profile was used, in which sound speed varied from 1480 m/s at the surface to 1500 m/s at the bottom.

The numerical eigenray procedure produced five exit angles $\{\theta_i\}_{i=1}^5$ for each set of parameters; these exit angles were used as initial conditions in an analytical ray tracing. The resulting analytical depth at target range given the numerical exit angles, $z(r_t;\theta_i)$, was compared with the desired target depth z_t, giving the mean error in eigenray depth at target range:

$$E = \frac{1}{5}\sum_{i=1}^{5} |z_t - z(r_t;\theta_i)|.$$

Optimization test on synthesized data

Due to a lack of real acoustic data to test the procedure on, it was necessary to test the procedure on synthesized data with added Gaussian noise. To obtain a more realistic signal and to test the method's sensitivity to disturbances, the arrival angles and arrival times used in synthesizing the received signal were considered Gaussian distributed random processes, as proposed in [4]:

$$\theta_i = n\left(\bar{\theta}_i, \frac{\theta_{BW}}{\sqrt{s}}\right), \quad t_i = n\left(\bar{t}_i, \frac{1}{B\sqrt{s}}\right).$$

Here, $n(\mu, \sigma)$ specifies a Gaussian process with expected value μ and standard deviation σ; $\bar{\theta}_i$ and $\bar{t}_i$ are the arrival angles and arrival times as found by the numerical eigenray scheme, θ_{BW} is the vertical beamwidth of the sonar, B is the bandwidth of the sonar and $s = 10^{SNR/10}$ is the linear signal-to-noise ratio of the echoes. The θ_i and t_i were obtained by first calculating $\bar{\theta}_i$ and $\bar{t}_i$ numerically, then adding Gaussian noise to these values. By varying the SNR values, s is also varied, allowing us to test the method's stability in the presence of different levels of noise in the signal and inaccuracies in measurements.

For all tests, the nonlinear sound speed profile shown in figure 2 was used. The parameters used in the tests were target ranges from 2000 m to 10 000 m in steps of 2000 m, bottom depths from 200 m to 1000 m in steps of 200 m, and target depths from 50 m to 50 m above bottom depth in steps of 100 m. The source depth was held constant at 5 m. In addition, all tests were done with use of three eigenrays, then redone with five eigenrays, in an attempt to determine how many eigenrays should be used in modeling signals to achieve a reasonable estimate of target depth. Due to time constraints, no tests were carried out in which the sound speed was varied as outlined in section **??**, and as such, these tests are a priority in future work.

For each set of parameters, five iterations were done in which a signal was synthesized by the method described above, and the optimization procedure applied to this signal in order to estimate the target depth. The mean error of these target depths estimates were then calculated. Both objective functions were used in the test, to see whether they yielded different results. While applying the simplified objective function (4), both methods of estimating eigenrays were used. Only method 1 was used while applying the full objective function (3). Again due to time constraints, the parameter range was shortened for the runs with full objective function; target ranges were varied from 2000 m to 10 000 m in steps of 4000 m, bottom depths from 200 m to 1000 m in steps of 400 m, and target depths from 50 m to 150 m above bottom depth in steps of 200 m.

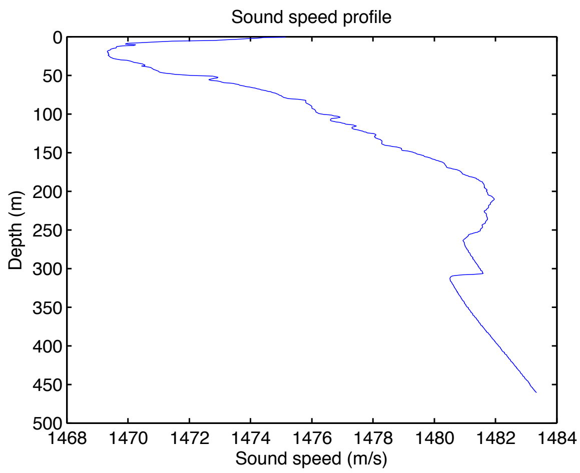


Figure 2: Sound speed profile used for testing.

Results and discussion

Verification of numerical eigenray estimates

Figure 3 shows the results of the verification test. Note that it is impossible that target depth is greater than bottom depth, and so the mean error in these cases is presented as 0 in the figures. From looking at the the figure, it is evident that method 1 is slightly less accurate than method 2. Note, however, that method 1 seldom produces a mean error larger than 5 m, which is acceptable.

Optimization test on synthesized data

Number of eigenrays for signal modelling

Figure 4 shows an example of the maximum, minimum and mean estimates of target depth obtained by use of the simplified cost function with numerical eigenray method 2. From the figure, we see that using five eigenrays for calculating arrivals on which to model signals provides more consistent and correct estimates for the target depth than using three eigenrays. For the sake of brevity, we shall henceforth consider only results obtained with the use of five eigenrays.

Estimation error as a function of SNR and target range

The plots in figure 5 show the error in the target depth estimation as a function of *SNR* and target range for four different target depths. First, we may observe that the errors are mostly within acceptable range for classification purposes; we only need an approximate estimate for target depth to say whether it is close to the bottom or not, and estimates with errors of less than 50 m are good enough for this purpose. Second, we may note the markedly better performance obtained by use of method 2, as compared to method 1, in nearly all cases but those with target depth 50 m and target range 2000 or 10 000 m, along with the case where the target depth is 350 m and the target range is 10 000 m. This may be attributable to the slightly more inaccurate eigenray estimates provided by method 1, as observed from figure 3. The sound speed profile chosen for this test is more irregular than a linear sound speed profile and as such, the differences in eigenray accuracy between method 1 and method 2 could be exacerbated in this case, leading to poor performance in estimating target depth.

Looking at the results from method 2, we see two irregular events with range 2000 m and target depths 50 and 150 m; with these parameters we recieve poor estimates of the target depths, whereas with all other ranges and the same target depths we find good estimates. This may be due to the sound speed profile being used which, due to its shape in the section 0-200 m, may make depth estimation in this depth range difficult. Rays will tend to curve toward areas of lower sound speed, meaning that in the channel between 0 and 200 m, there will be many eigenrays with small differences in exit angles and arrival times, making it hard to determine the exact depth of the target[5]. However, despite high errors we can conclude that a target is in this channel, giving important information for classification purposes.

Estimation error as a function of SNR and bottom depth

The plots in figure 6 show the error in the target depth estimation as a function of SNR and bottom depth for four different target ranges. Again we see that although most estimates using both methods are suitable for classification purposes, method 2 is superior to method 1 in most cases, with the exception of the cases where target range is 4000 m and bottom depth is 200 m or 800 m. As before, this may be attributable to the inaccuracy in eigenray calculation when using method 1.

Comparison of objective functions

Figures 7 and 8 show the error in target depth estimation as a function of SNR and bottom depth, and as a function of SNR and target range, respectively. In figure 7, we can see that the full objective function outperforms the simplified objective function. Note that the simplified objective function with eigenray method 1 actually performs better than the one evaluated by use of method 2 here, giving acceptable estimates of target depth in all cases, which would imply that the eigenray estimations work correctly for method 1 in this case. These results are to be expected, as the simplified objective function is an

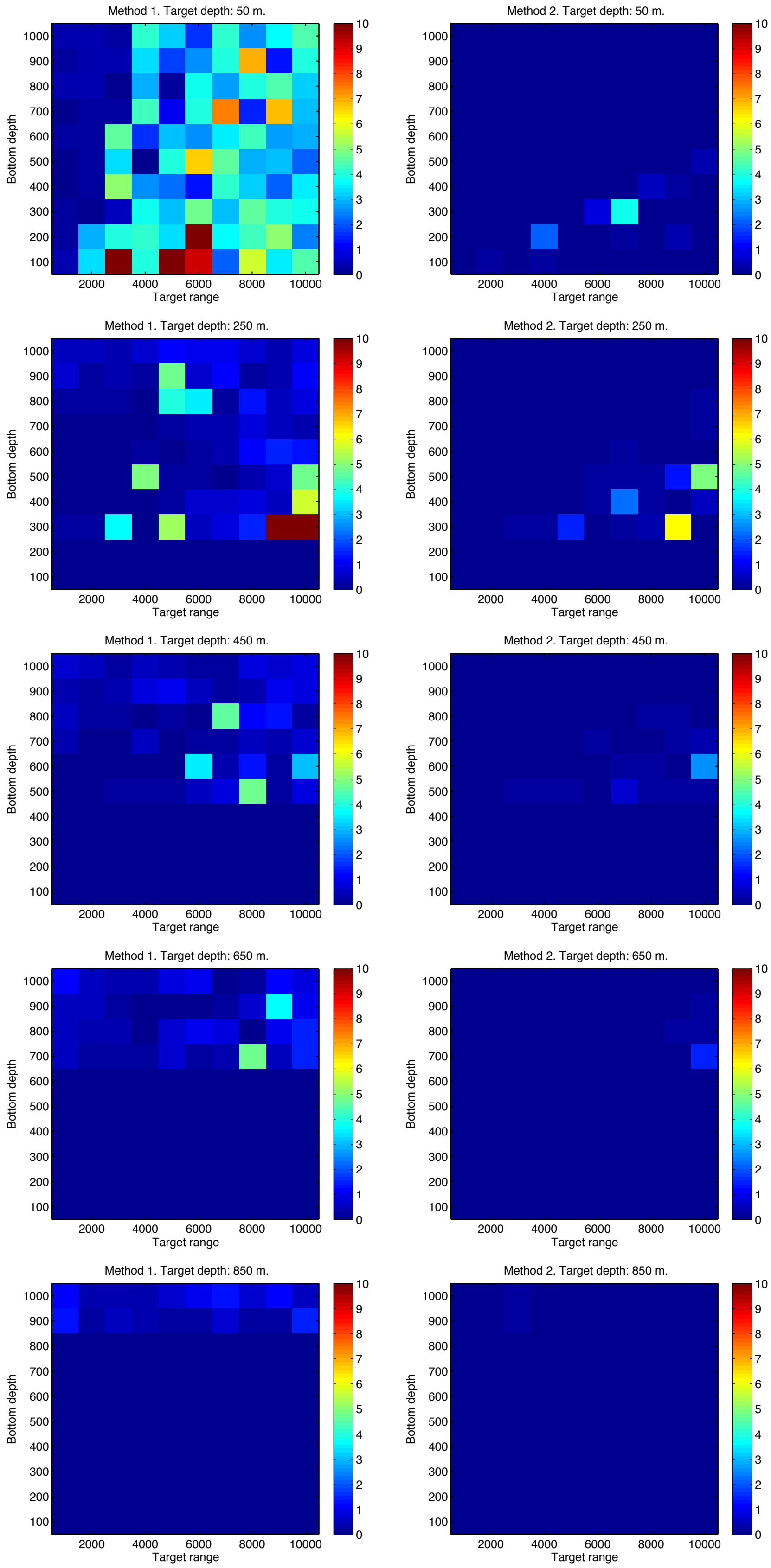
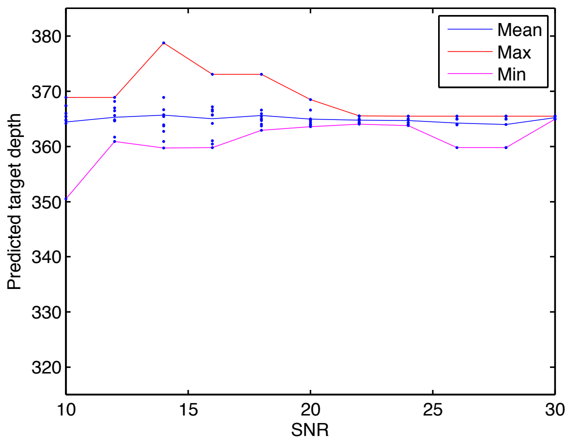


Figure 3: Mean error in eigenray depth at target range.
Left column: Method 1. Right column: Method 2.



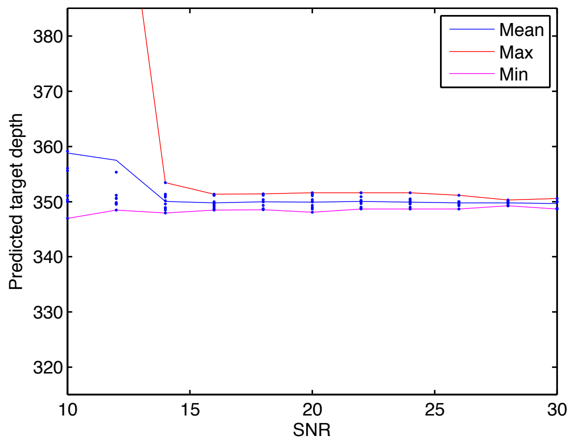


Figure 4: Estimates of target depth as a function of SNR. Target depth: 350 m. Bottom depth: 800 m. Target range: 6000 m. Left: 3 eigenrays. Right: 5 eigenrays. Obtained by use of simplified cost function with numerical eigenray method 2.

approximation to the full objective function, and we would expect the full objective function to perform better, given reliable eigenray estimates.

However, in figure 8 we see that although all three methods show mostly acceptable results, the simplified objective function with eigenray method 2 works best in all but two cases; with target range 2000 m and target depth either 250 m or 450 m. Also note that the full objective function and the simplified objective function with eigenray method 1 have similar patterns in where their estimates break down, which may be interpreted as further proof that numerical eigenray method 1 is somewhat unsound.

Execution time

An important part of the solution method is its execution time. Using the simplified objective function evaluated with method 2, the optimization procedure took 8 minutes on average in the worst cases (large target range), and 2.5 minutes in the best cases (small target range). This is mostly due to the need to call on Lybin many times to find the ray paths while modeling signals to evaluate the objective function. In contrast, evaluating with method 1, while inaccurate (as seen above), is 20-30 times faster than method 2 due to it needing only one call to Lybin. Therefore, if method 1 could be made more accurate, it would provide a faster alternative to method 2.

The optimization procedure took 90 minutes on average to finish when using the full objective function, making this option prohibitively slow. This is attributable to the need for a fully formed model signal for comparison with the received signal. In any case, an optimization procedure requiring less function evaluations would be useful. Specifically, the initial guess routine is quite slow due to the high amount of function evaluations involved. A more effective way of producing initial guesses would probably speed up the execution time considerably.

Conclusion

Estimation of target depth has been carried out on synthesized acoustic data using two different objective functions. The results obtained are acceptable for classification purposes, and the best results were obtained while using the simplified objective function evaluated using numerical eigenray method 2. It remains to be seen whether this method is successful when optimizing with respect to sound speed profile and source depth and if so, whether the success transferable to real scenarios. The anomalies encountered while employing numerical eigenray method 1 during estimation need to be investigated further, as alleviating these would result in a faster procedure.

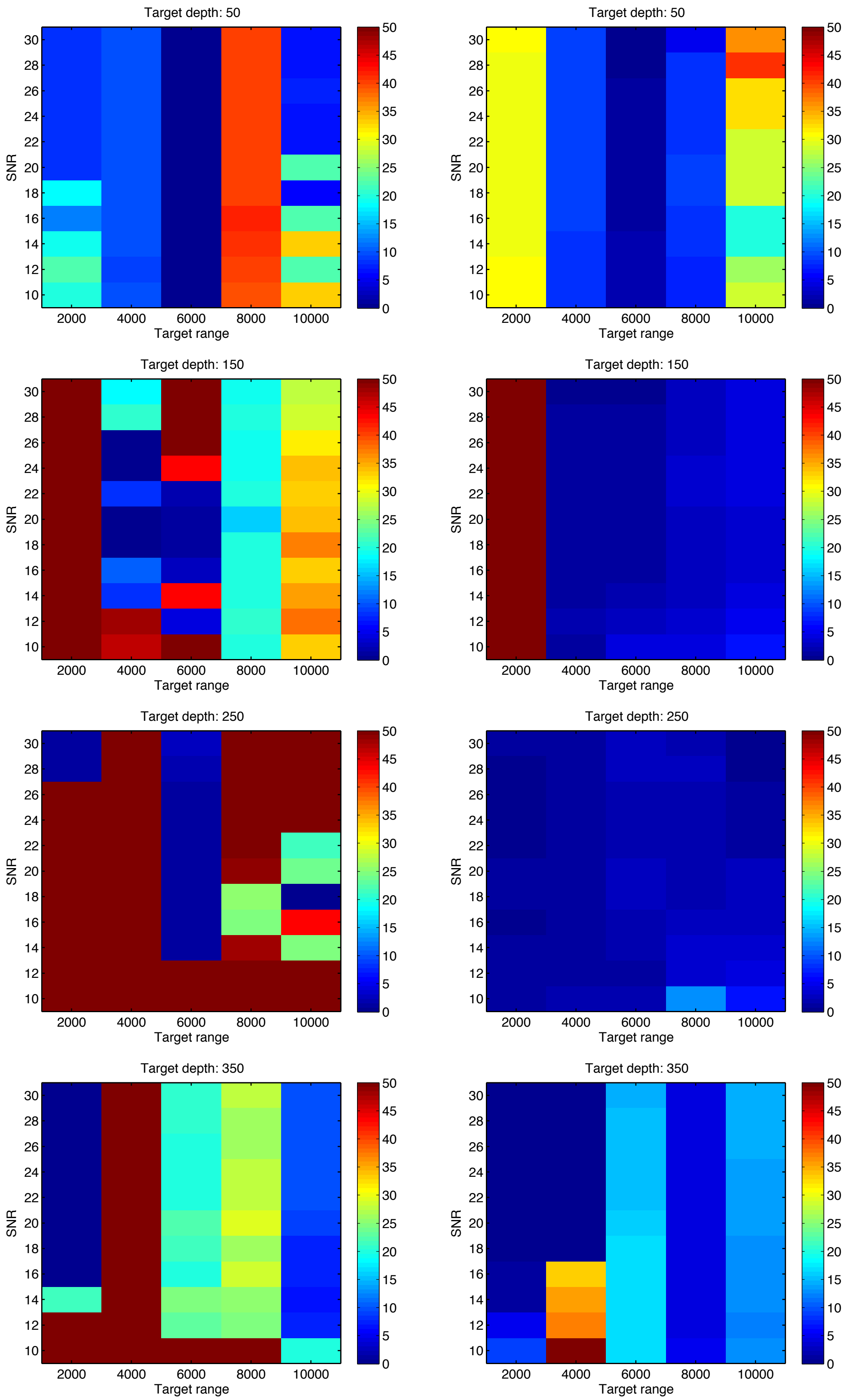


Figure 5: Mean error in estimates of target depth as a function of SNR and target range. Bottom depth: 400 m. Left column: Method 1. Right column: Method 2.

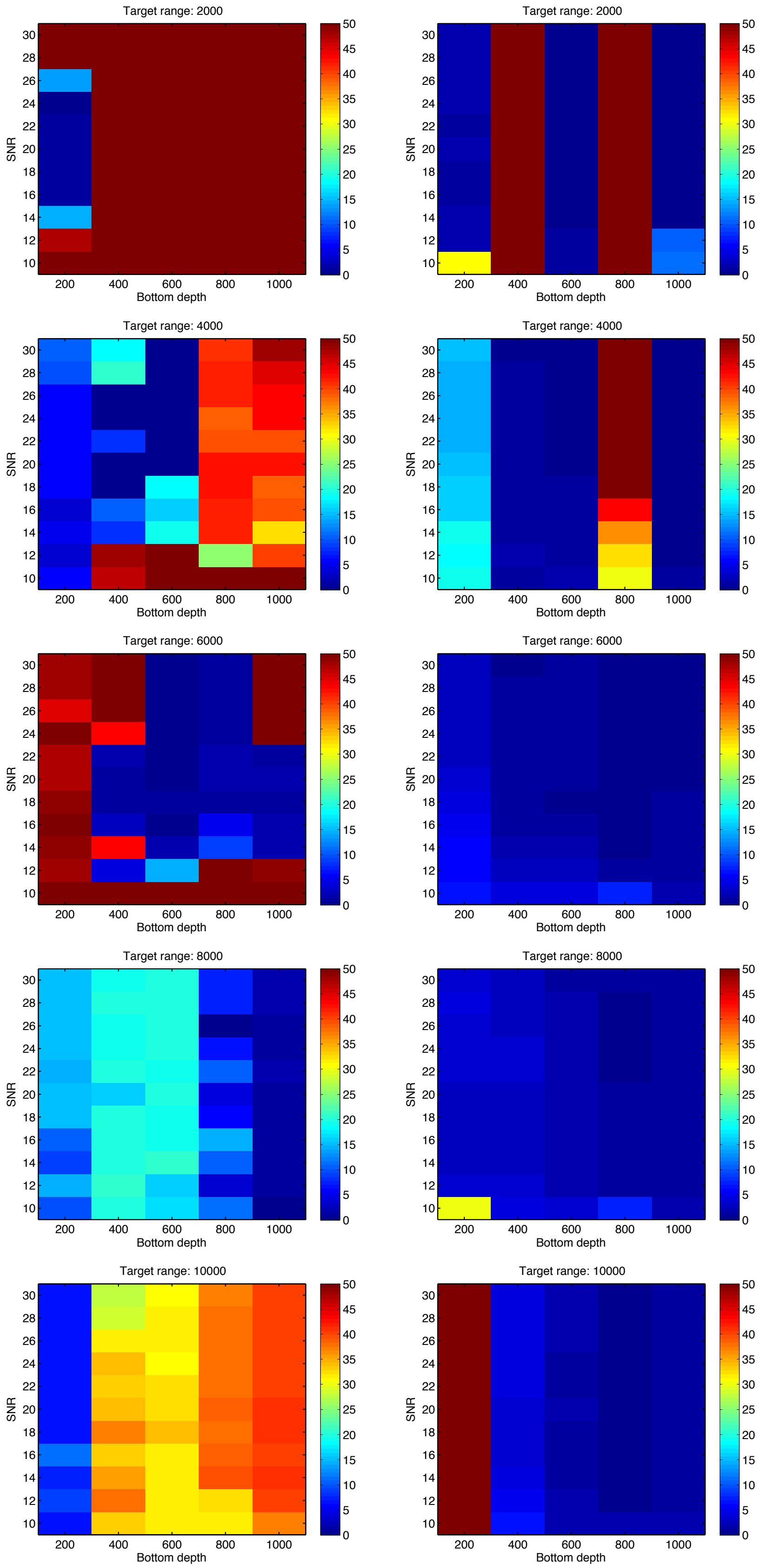


Figure 6: Mean error in estimates of target depth as a function of SNR and bottom depth. Target depth: 150 m. Left column: Method 1. Right column: Method 2.

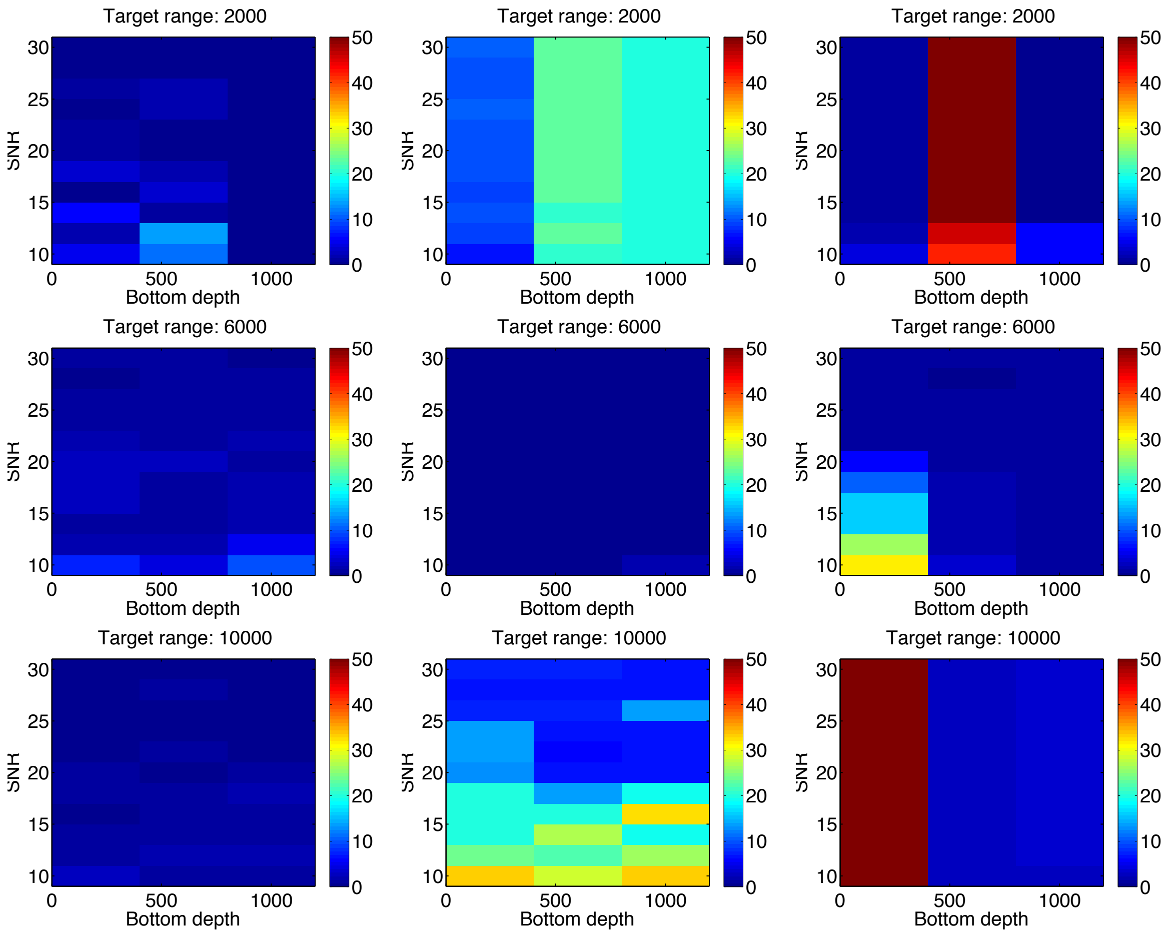


Figure 7: Mean error in estimates of target depth as a function of SNR and bottom depth. Target depth: 50 m. Left: Full objective function. Middle: Simplified objective function, method 1. Right: Simplified objective function, method 2.

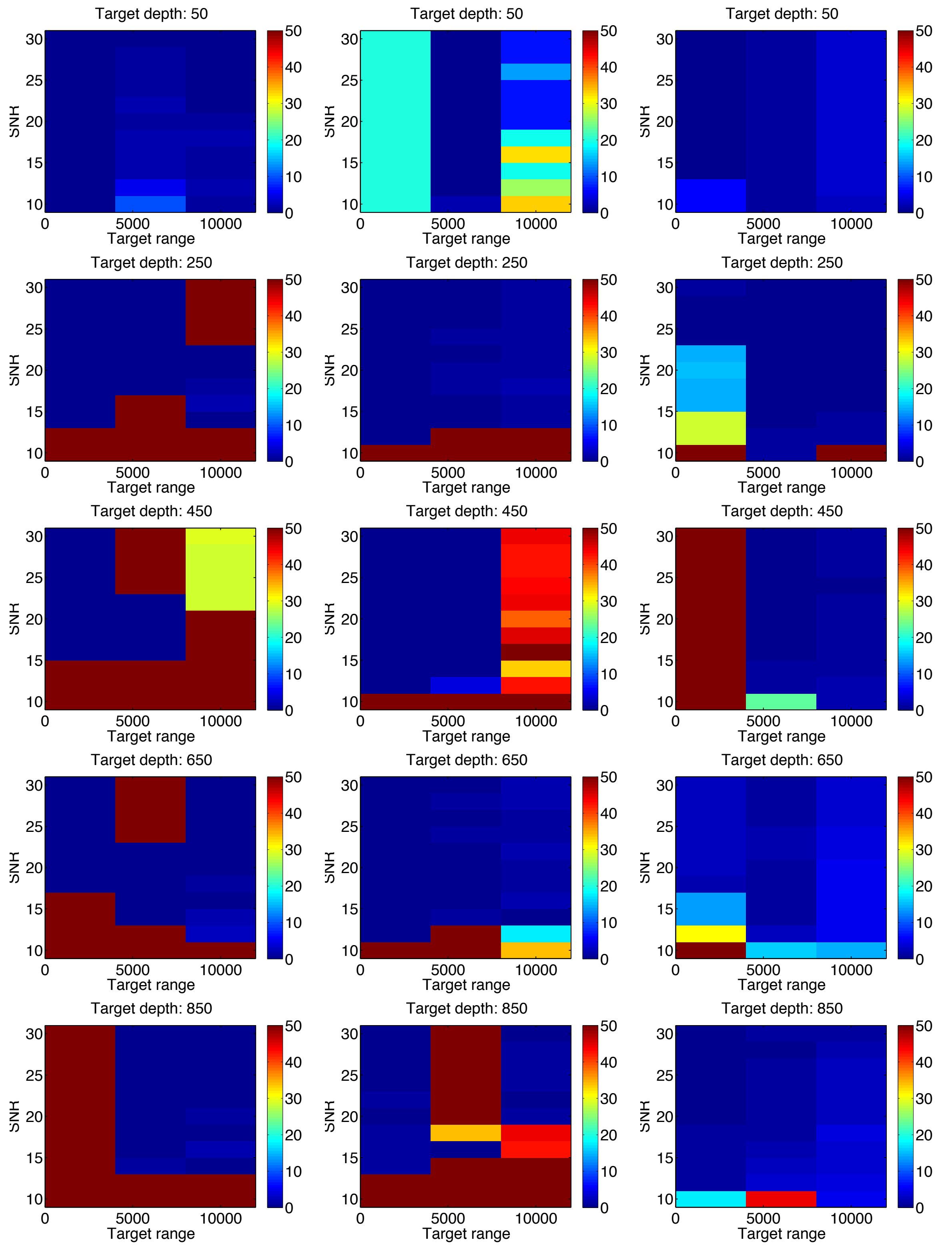


Figure 8: Mean error in estimates of target depth as a function of SNR and target range. Bottom depth: 1000 m. Left: Full objective function. Middle: Simplified objective function, method 1. Right: Simplified objective function, method 2.

References

[1] M.D. Collins and W.A. Kuperman. Environmental focusing and source localization. *Journal of the Acoustic Society of America*, 90(3), 1991.

[2] Elin Dombestein and Trond Jenserud. Improving underwater surveillance: Lybin sonar performance prediction. In *Maritime Systems and Technologies Conference*, 2010.

[3] Karl-Thomas Hjelmervik. Target depth estimation using a ray backpropagation scheme on mid-frequency, active sonar data. In *Proceedings of European Conference in Underwater Acoustics*, 2010.

[4] Karl-Thomas Hjelmervik. *Sonar false alarm rate suppression using classification methods based on acoustic modelling*. PhD thesis, NTNU, 2011.

[5] Finn B. Jensen, W.A. Kuperman, M.B. Porter, and H. Schmidt. *Computational Ocean Acoustics*. Springer-Verlag, New York, 2000.

[6] Bassem R. Mahafza. *Radar Systems Analysis and Design Using MATLAB*. Chapman & Hall/CRC, Huntsville, Alabama, 2000.

[7] Jorge Nocedal and Stephen J. Wright. *Numerical Optimization*. Springer Verlag, New York, 2006.