Chapter 5
Component Importance

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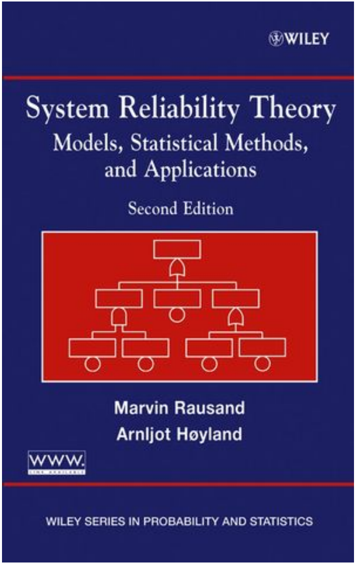
Slides related to the book

System Reliability Theory
Models, Statistical Methods,
and Applications

Wiley, 2004

Homepage of the book:
http://www.ntnu.edu/ross/books/srt



Purpose

The component importance may be used to:

- Provide a coarse ranking of the components with respect to their influence on the system reliability (or TOP event probability in a fault tree)
- Help focusing on the top contributors to system unreliability
- Help relax re for the lowest contributors to system unreliability
- Focus on improvements with the greatest reliability effect
- Indicate sensitivity for model parameters
- Help focusing reviews and sensitivity studies
- Give priorities for fault-finding in complex system

Measures covered

The following component importance measures are defined and discussed in this chapter:

- Birnbaum's measure
- The improvement potential measure
- Risk achievement worth
- Risk reduction worth
- The criticality importance measure
- Fussell-Vesely's measure

The importance measures are not always consistently defined in the literature.

Importance depends on

The various measures are based on slightly different interpretations of the concept *component importance*. Intuitively, the importance of a component should depend on two factors:

- The location of the component in the system
- The reliability of the component in question

and, perhaps, also the uncertainty in our estimate of the component reliability.

Birnbaum's Measure - 1

Birnbaum (1969) proposed the following measure of the reliability importance of component i at time t:

$$I^{\mathrm{B}}(i \mid t) = \frac{\partial h(\mathbf{p}(t))}{\partial p_i(t)}$$

Birnbaum's measure is therefore obtained as the partial derivative of the system reliability $h(\mathbf{p}(t))$ with respect to $p_i(t)$. This approach is well known from classical sensitivity analysis. If $I^{\mathrm{B}}(i \mid t)$ is large, a small change in the reliability of component i results in a comparatively large change in the system reliability at time t.

When taking this derivative, the reliabilities of the other components remain constant – only the effect of varying $p_i(t)$ is studied.

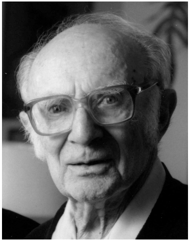
Birnbaum's Measure - 2

Birnbaum's measure measures the rate of change of the system reliability as a result of changes to the reliability of a single component.
By fault tree notation, Birnbaum's measure may be written as

$$I^{\mathrm{B}}(i \mid t) = \frac{\partial Q_0(t)}{\partial q_i(t)}$$

where $q_i(t) = 1 - p_i(t)$ and $Q_0(t) = 1 - p_S(t) = 1 - h(\mathbf{p}(t))$

Birnbaum's measure is named after the Hungarian-American professor Zygmund William Birnbaum (1903-2000).



Birnbaum's Measure - 3

In the definition of Birnbaum's measure, the system reliability is denoted $h(\mathbf{p}(t))$ and the system reliability is therefore a function of the component reliabilities only, i.e., of $\mathbf{p}(t) = (p_1(t), p_2(t, \ldots, p_n(t)$. This means that the all the n components must be independent.

This definition of Birnbaum's measure is therefore not useable when the components are dependent, e.g., when we have common-cause failures.

Birnbaum's Measure - 4

Example – Series Structure

Consider a series structure of two independent components, 1 and 2, with component reliabilities p_1 and p_2, respectively. Assume that $p_1 > p_2$, i.e., component 1 is the most reliable of the two.

The reliability of the series system is $h(\mathbf{p}) = p_1 p_2$.

1. Birnbaum's measure of component 1 is $I^{\mathrm{B}}(1) = \frac{\partial h(\mathbf{p})}{\partial p_1} = p_2$
2. Birnbaum's measure of component 2 is $I^{\mathrm{B}}(2) = \frac{\partial h(\mathbf{p})}{\partial p_2} = p_1$

This means that $I^{\mathrm{B}}(2) > I^{\mathrm{B}}(1)$ and we can conclude that when using Birnbaum's measure, the most important component in a series structure is the one with the lowest reliability.

To improve a series structure, we should therefore improve the "weakest" component, i.e., the component with the lowest reliability.

Birnbaum's Measure - 5

Example – Parallel Structure

Consider a parallel structure of two independent components, 1 and 2, with component reliabilities p_1 and p_2, respectively. Assume that $p_1 > p_2$, i.e., component 1 is the most reliable of the two.

The reliability of the parallel system is $h(\mathbf{p}) = p_1 + p_2 - p_1 p_2$.

1. Birnbaum's measure of component 1 is $I^{\mathrm{B}}(1) = \frac{\partial h(\mathbf{p})}{\partial p_1} = 1 - p_2$
2. Birnbaum's measure of component 2 is $I^{\mathrm{B}}(2) = \frac{\partial h(\mathbf{p})}{\partial p_2} = 1 - p_1$

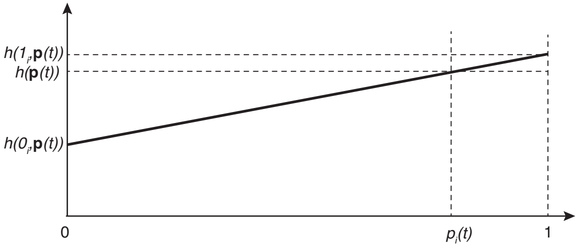
This means that $I^{\mathrm{B}}(1) > I^{\mathrm{B}}(2)$ and we can conclude that when using Birnbaum's measure, the most important component in a parallel structure is the one with the highest reliability.
To improve a parallel structure, we should therefore improve the "strongest" component, i.e., the component with the highest reliability.

Birnbaum's Measure - 6

By pivotal decomposition, we have

$$\begin{aligned} h(\mathbf{p}(t)) &= p_i(t) \cdot h(1_i, \mathbf{p}(t)) + (1 - p_i(t)) \cdot h(0_i, \mathbf{p}(t)) \\ &= p_i(t) \cdot [h(1_i, \mathbf{p}(t)) - h(0_i, \mathbf{p}(t))] + h(0_i, \mathbf{p}(t)) \end{aligned}$$

This shows that $h(\mathbf{p}(t))$ is a linear function of $p_i(t)$ (when all the other reliabilities are kept constant) as illustrated in the figure below.



Birnbaum's Measure - 7

Birnbaum's measure can therefore we written as

$$I^{\mathrm{B}}(i \mid t) = \frac{\partial h(\mathbf{p}(t))}{\partial p_i(t)} = h(1_i, \mathbf{p}(t)) - h(0_i, \mathbf{p}(t))$$

where $h(1_i, \mathbf{p}(t))$ is the system reliability when we know that component i is functioning and $h(0_i, \mathbf{p}(t))$ is the system reliability when we know that component i is not functioning. This leads to a very simple way of calculating $I^{\mathrm{B}}(i \mid t)$ – as illustrated by the example on the next slide.

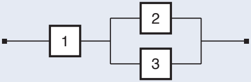
Most computer programs for fault tree analysis computes Birnbaum's measure by this approach.

The same approach is sometimes used to determine $I^{\mathrm{B}}(i \mid t)$ for systems exposed to common-cause failures.

Birnbaum's Measure - 8

Example

Consider the simple system:



We want to determine Birnbaum's measure for component 3.

1. When we know that component 3 is functioning (i.e., $X_3(t) = 1$), component 2 is *irrelevant* and the system reliability is $h(\mathbf{p}(t)) = p_1(t)$
2. When we know that component 3 is not functioning (i.e., $X_3(t) = 0$), the system reliability is $h(\mathbf{p}(t)) = p_1(t)p_2(t)$

Birnbaum's measure is therefore:

$$\begin{aligned} I^{\mathrm{B}}(3 \mid t) &= h(1_3, \mathbf{p}(t)) - h(0_3, \mathbf{p}(t)) \\ &= p_1(t) - p_1(t)p_2(t) = p_1(t)\left[1 - p_2(t)\right] \end{aligned}$$

Birnbaum's Measure - 9

Example – common-cause failures

Consider a parallel system of two independent and identical components with constant failure rate λ.

Birnbaum's Measure - 10

Note that Birnbaum's measure $I^{\mathrm{B}}(i \mid t)$ of component i only depends on the structure of the system and the reliabilities of the other components.

$I^{\mathrm{B}}(i \mid t)$ is independent of the actual reliability $p_i(t)$ of component i.

Birnbaum's Measure - 11

Since $h(\cdot_i, \mathbf{p}(t)) = E[\phi(\cdot_i, \mathbf{X}(t)]$, we can write

$$
\begin{aligned}
I^{\mathrm{B}}(i \mid t) &= E[\phi(1_i, \mathbf{X}(t)] - E[\phi(0_i, \mathbf{X}(t)] \\
&= E[\phi(1_i, \mathbf{X}(t) - \phi(0_i, \mathbf{X}(t)]
\end{aligned}
$$

When the structure is coherent $[\phi(1_i, \mathbf{X}(t)) - \phi(0_i, \mathbf{X}(t))]$ can only take on the values 0 and 1. Therefore

$$
I^{\mathrm{B}}(i \mid t) = \Pr\left[\phi(1_i, \mathbf{X}(t)) - \phi(0_i, \mathbf{X}(t)) = 1\right]
$$

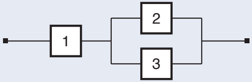
This is to say that $I^{\mathrm{B}}(i \mid t)$ is equal to the probability that $(1_i, \mathbf{X}(t))$ is a *critical path vector* for component i at time t.

Birnbaum's measure is therefore the probability that the system is in such a state at time t that component i is *critical* for the system.

Birnbaum's Measure - 12

Example

Again consider the simple system:



and assume that we want to determine Birnbaum's measure for component 3.

For component 3 to be *critical*, component 1 has to function and component 2 has to be failed. Because the probability that component 1 is functioning at time t is $p_1(t)$ and the probability that component 2 is failed is $1 - p_2(t)$, Birnbaum's measure is

$$I^{B}(3 \mid t) = p_1(t)\,[1 - p_2(t)]$$

Birnbaum's Measure - 13

Birnbaum's measure of reliability importance of component i at time t can therefore be determined by three different approaches:

1. By the partial derivative of $h(\mathbf{p}(t))$ with respect to $p_i(t)$, i.e.,

$$I^{\mathrm{B}}(i \mid t) = \frac{\partial h(\mathbf{p}(t))}{\partial p_i(t)}$$

2. By the difference between the system reliability when we know that component i is functioning and when we know that component i is not functioning:

$$I^{\mathrm{B}}(i \mid t) = h(1_i, \mathbf{p}(t)) - h(0_i, \mathbf{p}(t))$$

3. By the probability that component i is critical for the system:

$$I^{\mathrm{B}}(i \mid t) = \Pr(\text{Component } i \text{ is critical for the system at time } t)$$

Birnbaum's Measure - 14

Assume that component i has failure rate λ_i. In some situations we may be interested in measuring how much the system reliability will change by making a small change to λ_i. The sensitivity of the system reliability with respect to changes in λ_i can obviously be measured by

$$\frac{\partial h(\mathbf{p}(t))}{\partial \lambda_i} = \frac{\partial h(\mathbf{p}(t))}{\partial p_i(t)} \cdot \frac{\partial p_i(t)}{\partial \lambda_i} = I^{\mathrm{B}}(i \mid t) \cdot \frac{\partial p_i(t)}{\partial \lambda_i}$$

A similar measure can be used for all parameters related to the component reliability $p_i(t)$, for $i = 1, 2, \ldots, n$. In some cases, several components in a system will have the same failure rate λ. To find the sensitivity of the system reliability with respect to changes in λ, we can still use $\partial h(\mathbf{p}(t))/\partial \lambda$

Birnbaum's Measure - 15

In a practical reliability study of a complex system, one of the most time-consuming tasks is to find adequate estimates for the input parameters (failure rates, repair rates, etc.). In some cases, we may start with rather rough estimates, calculate Birnbaum's measure of importance for the various components, or the parameter sensitivities, and then spending the most of the time finding high-quality data for the most important components. Components with a very low value of Birnbaum's measure will have a negligible effect on the system reliability, and extra efforts finding high-quality data for such components may be considered a waste of time

Improvement Potential - 1

The improvement potential of component i at time t is defined as:

$$I^{\mathrm{IP}}(i \mid t) = h(1_i, \mathbf{p}(t)) - h(\mathbf{p}(t))$$

$I^{\mathrm{IP}}(i \mid t)$ is hence the difference between the system reliability with a *perfect* component i, and the system reliability with the actual component i. It tells us how much it is possible to improve the current system reliability if we could replace the current component i with a perfect component.

Improvement Potential - 2

Example – Series Structure

Consider a series structure of two independent components, 1 and 2, with component reliabilities p_1 and p_2, respectively. Assume that $p_1 > p_2$, i.e., component 1 is the most reliable of the two.
The system reliability is therefore $h(\mathbf{p}) = p_1 p_2$.

1. $I^{\text{IP}}(1) = 1 \cdot p_2 - p_1 p_2 = p_2(1 - p_1)$
2. $I^{\text{IP}}(2) = p_1 \cdot 1 - p_1 p_2 = p_1(1 - p_2)$

This means that $I^{\text{IP}}(2) > I^{\text{IP}}(1)$ and we can conclude that when using the Improvement Potential measure, the most important component in a series structure is the one with the lowest reliability.
To improve a series structure, we should therefore improve the "weakest" component, i.e., the component with the lowest reliability.

Improvement Potential - 3

Example – Parallel Structure

Consider a parallel structure of two independent components, 1 and 2, with component reliabilities p_1 and p_2, respectively. Assume that $p_1 > p_2$, i.e., component 1 is the most reliable of the two.
The system reliability is therefore $h(\mathbf{p}) = p_1 + p_2 - p_1 p_2$.

1. $I^{\text{IP}}(1) = 1 - [p_1 + p_2 - p_1 p_2]$
2. $I^{\text{IP}}(2) = 1 - [p_1 + p_2 - p_1 p_2]$

This means that $I^{\text{IP}}(2) = I^{\text{IP}}(1)$ and we can conclude that when using the Improvement Potential measure, all the components in a parallel structure are equally important.

Improvement Potential - 4

From the figure on the slide "Birnbaum's Measure (3)", a simple geometrical comparison of triangles yields

$$\frac{I^{\mathrm{B}}(i \mid t)}{1} = \frac{I^{\mathrm{IP}}(i \mid t)}{1 - p_i(t)}$$

and the improvement potential can therefore be expressed as

$$I^{\mathrm{IP}}(i \mid t) = I^{\mathrm{B}}(i \mid t) \cdot (1 - p_i(t))$$

or, by using the fault tree notation

$$I^{\mathrm{IP}}(i \mid t) = I^{\mathrm{B}}(i \mid t) \cdot q_i(t)$$

Improvement Potential - 5

In practice, it is not possible to improve the reliability $p_i(t)$ of component i to 100% reliability. Let us assume that it is possible to improve $p_i(t)$ to new value $p_i^{(n)}(t)$ representing, for example, the state of the art for this type of components. We may then calculate the realistic or *credible improvement potential* (CIP) of component i at time t, defined by

$$I^{\text{CIP}}(i \mid t) = h(p_i^{(n)}(t), \mathbf{p}(t)) - h(\mathbf{p}(t))$$

where $h(p_i^{(n)}(t), \mathbf{p}(t))$ denotes the system reliability when component i is replaced by a new component with reliability $p_i^{(n)}(t)$.

Risk Achievement Worth - 1

The *risk achievement worth* (RAW) measures the relative increase in the system unreliability when it is known that component i is in a failed state.

The *nominal* system unreliability is $1 - h(\mathbf{p}(t))$, and when it is known that component i is failed, the system reliability is $1 - h(0_i, \mathbf{p}(t))$.

The relative increase in the system <u>un</u>reliability is hence

$$\frac{[1 - h(0_i, \mathbf{p}(t))] - [1 - h(\mathbf{p}(t))]}{1 - h(\mathbf{p}(t))} = \frac{1 - h(0_i, \mathbf{p}(t))}{1 - h(\mathbf{p}(t))} - 1$$

By using fault tree terminology, this is expressed as

$$\frac{Q_0(0_i, t) - Q_0(t)}{Q_0(t)} = \frac{Q_0(0_i, t)}{Q_0(t)} - 1$$

where $Q_0(0_i, t)$ is the TOP event probability when basic event i is occurred (i.e., component i is failed).

Risk Achievement Worth - 2

In most literature sources, however, RAW is defined without the "minus one" as

$$I^{\text{RAW}}(i \mid t) = \frac{1 - h(0_i, \mathbf{p}(t))}{1 - h(\mathbf{p}(t))} = \frac{Q_0(0_i, t)}{Q_0(t)}$$

which, for all coherent systems, fulfills $I^{\text{RAW}}(i \mid t) \geq 1$.

When $I^{\text{RAW}}(i \mid t) = 1$, failure of component i (or the presence of basic event i) has no effect.

Risk Achievement Worth - 3

Example

Assume that we have found that $1 - h(\mathbf{p}(t)) = Q_0(t) = 0.01$ with the nominal reliability of component (basic event) i, and that $1 - h(0_i, \mathbf{p}(t)) = Q_0(0_i, t) = 0.02$ when it is known that component i is failed (or basic event i has occurred). The RAW is then

$$I^{\mathrm{RAW}}(i \mid t) = \frac{Q_0(0_i, t)}{Q_0(t)} = \frac{0.02}{0.01} = 2$$

Risk Achievement Worth - 4

Example – Series Structure

Consider a series structure of two independent components, 1 and 2, with component reliabilities p_1 and p_2, respectively. Assume that $p_1 > p_2$, i.e., component 1 is the most reliable of the two.
The system reliability is therefore $h(\mathbf{p}) = p_1 p_2$.

1. $I^{\text{RAW}}(1) = \frac{1}{1-p_1 p_2}$
2. $I^{\text{RAW}}(2) = \frac{1}{1-p_1 p_2}$

This means that $I^{\text{RAW}}(2) = I^{\text{RAW}}(1)$ and we can conclude that when using the RAW measure, all the components in a series structure are equally important.

Risk Achievement Worth - 5

Example – Parallel Structure

Consider a parallel structure of two independent components, 1 and 2, with component reliabilities p_1 and p_2, respectively. Assume that $p_1 > p_2$, i.e., component 1 is the most reliable of the two.
The system reliability is therefore $h(\mathbf{p}) = p_1 + p_2 - p_1 p_2$.

1. $I^{\text{RAW}}(1) = \frac{1-p_2}{1-(p_1+p_2-p_1p_2)}$
2. $I^{\text{RAW}}(2) = \frac{1-p_1}{1-(p_1+p_2-p_1p_2)}$

This means that $I^{\text{RAW}}(1) > I^{\text{RAW}}(2)$ and we can conclude that when using the RAW measure, the strongest (i.e., most reliable) component in a parallel structure is the most important.

Risk Achievement Worth - 6

Basic event i may sometimes be related to a *feature* that has been installed to achieve the present TOP event probability $Q_0(t)$. If this feature is missing or has failed, $Q_0(0_i, t) - Q_0(t)$ tells us how much the TOP event probability (and the risk) will increase.

The RAW is seen to be useful for estimating the risk significance of components (or features) that are removed from the system.

Risk Reduction Worth - 1

The *risk reduction worth* (RRW) measures the relative reduction in the system unreliability when it is known that component i is functioning. The relative reduction in the system unreliability is

$$\frac{[1 - h(\mathbf{p}(t))] - [1 - h(1_i, \mathbf{p}(t))]}{1 - h(1_i, \mathbf{p}(t))} = \frac{1 - h(\mathbf{p}(t))}{1 - h(1_i, \mathbf{p}(t))} - 1$$

where the reduction is measured related to "best situation" when component i is functioning.

With fault tree terminology, this relative reduction is expressed as

$$\frac{Q_0(t) - Q_0(1_i, t)}{Q_0(1_i, t)} = \frac{Q_0(t)}{Q_0(1_i, t)} - 1$$

where $Q_0(1_i, t)$ is the TOP event probability when it is known that component i is functioning (i.e., that basic event i has not occurred).

Risk Reduction Worth - 2

In most literature sources, however, RRW is defined without the "minus one" as

$$I^{\mathrm{RRW}}(i \mid t) = \frac{1 - h(\mathbf{p}(t))}{1 - h(1_i, \mathbf{p}(t))} = \frac{Q_0(t)}{Q_0(1_i, t)}$$

which, for all coherent systems, fulfills $I^{\mathrm{RRW}}(i \mid t) \geq 1$.

When $I^{\mathrm{RRW}}(i \mid t) = 1$, to improve component i to a be always functioning has no effect.

Risk Reduction Worth - 3

Example

Assume that we have found that $1 - h(\mathbf{p}(t)) = Q_0(t) = 0.01$ with the nominal reliability of component (basic event) i, and that $1 - h(1_i, \mathbf{p}(t)) = Q_0(1_i, t) = 0.005$ when it is known that component i is functioning (or basic event i has not occurred). The RRW is then

$$I^{\text{RRW}}(i \mid t) = \frac{Q_0(t)}{Q_0(1_i, t)} = \frac{0.01}{0.005} = 2$$

Risk Reduction Worth - 4

Example – Series Structure

Consider a series structure of two independent components, 1 and 2, with component reliabilities p_1 and p_2, respectively. Assume that $p_1 > p_2$, i.e., component 1 is the most reliable of the two.
The system reliability is therefore $h(\mathbf{p}) = p_1 p_2$.

1. $I^{\text{RRW}}(1) = \frac{1-p_1 p_2}{1-p_2}$
2. $I^{\text{RRW}}(2) = \frac{1-p_1 p_2}{1-p_1}$

This means that $I^{\text{RRW}}(2) > I^{\text{RRW}}(1)$ and we can conclude that when using the RRW measure, the most important component in a series structure is the one with the lowest reliability.

Risk Reduction Worth - 5

Example – Parallel Structure

Consider a parallel structure of two independent components, 1 and 2, with component reliabilities p_1 and p_2, respectively. Assume that $p_1 > p_2$, i.e., component 1 is the most reliable of the two.

The system reliability is therefore $h(\mathbf{p}) = p_1 + p_2 - p_1 p_2$.

1. $I^{\text{RRW}}(1) = \text{NA}$
2. $I^{\text{RRW}}(2) = \text{NA}$

When we know that component i is functioning in a parallel structure, the system is also functioning and the denominator in the expression for $I^{\text{RRW}}(i)$ is always equal to zero.

The reduction in the system unreliability for the parallel system is $1 - (p_1 + p_2 - p_1 p_2)$, but the relative reduction is not defined since the denominator is zero.

Risk Reduction Worth - 6

Let the TOP event probability be $Q_0(t)$ when the probability of basic event i has its nominal value. If we are able to secure that basic event will not occur, $Q_0(t) - Q_0(1_i, t)$ tells us how much the TOP event probability (and the risk) will be reduced.

In some applications, the basic event i may be an operator error or some external event. If such basic events can be removed from the system, for example, by canceling an operator intervention, the basic cannot occur.

The RRW is seen to be useful for bounding the risk benefits from proposed improvements.

Criticality Importance - 1

The Criticality Importance measure $I^{\text{CR}}(i \mid t)$ of component i at time t is the probability that component i is critical for the system and is failed at time t, when we know that the system is failed at time t.

We will soon show that we may write

$$I^{\text{CR}}(i \mid t) = \frac{I^{\text{B}}(i \mid t) \cdot (1 - p_i(t))}{1 - h(\mathbf{p}(t))}$$

Remember that when component i is critical, the other components are in such states that the system will fail if and only if component i fails. This also means that the system will start functioning again if component i is repaired.

Criticality Importance - 2

By using the fault tree notation, $I^{\mathrm{CR}}(i \mid t)$ may be written as

$$I^{\mathrm{CR}}(i \mid t) = \frac{I^{\mathrm{B}}(i \mid t) \cdot q_i(t)}{Q_0(t)}$$

Criticality Importance - 3

Let $C(1_i, \mathbf{X}(t))$ denote the event that the system at time t is in a state where component i is critical. We know that

$$\Pr(C(1_i, \mathbf{X}(t))) = I^{\mathrm{B}}(i \mid t)$$

The probability that component i is critical for the system and at the same time is failed at time t is

$$\Pr(C(1_i, \mathbf{X}(t)) \cap (X_i(t) = 0)) = I^{\mathrm{B}}(i \mid t) \cdot (1 - p_i(t))$$

When we know that the system is in a failed state at time t, then

$$\Pr(C(1_i, \mathbf{X}(t)) \cap (X_i(t) = 0) \mid \phi(\mathbf{X}(t)) = 0)$$

Criticality Importance - 4

Because the event $[C(1_i, \mathbf{X}(t)) \cap (\mathbf{X}(t) = 0)]$ implies that $\phi(\mathbf{X}(t)) = 0$, we get

$$\frac{\Pr(C(1_i, \mathbf{X}(t)) \cap (X_i(t) = 0))}{\Pr(\phi(\mathbf{X}(t)) = 0)} = \frac{I^{\mathrm{B}}(i \mid t) \cdot (1 - p_i(t))}{1 - h(\mathbf{p}(t))}$$

$I^{\mathrm{CR}}(i \mid t)$ is therefore the probability that component i has *caused* system failure, when we know that the system is failed at time t. For component i to cause system failure, component i must be critical, and then fail.

When component i is repaired, the system will start functioning again. This is why the criticality importance measure may be used to prioritize maintenance actions in complex systems.

Criticality Importance - 5

Example – Series Structure

Consider a series structure of two independent components, 1 and 2, with component reliabilities p_1 and p_2, respectively. Assume that $p_1 > p_2$, i.e., component 1 is the most reliable of the two.
The system reliability is therefore $h(\mathbf{p}) = p_1 p_2$.

1. $I^{\text{CR}}(1) = \frac{I^{\text{B}}(1) \cdot (1-p_1)}{1-h(\mathbf{p})} = \frac{p_2(1-p_1)}{1-p_1 p_2}$
2. $I^{\text{CR}}(2) = \frac{I^{\text{B}}(1) \cdot (1-p_1)}{1-h(\mathbf{p})} = \frac{p_1(1-p_2)}{1-p_1 p_2}$

This means that $I^{\text{CR}}(2) > I^{\text{CR}}(1)$ and we can conclude that when using the Criticality Importance measure, the most important component in a series structure is the one with the lowest reliability.

Criticality Importance - 6

Example – Parallel Structure

Consider a parallel structure of two independent components, 1 and 2, with component reliabilities p_1 and p_2, respectively. Assume that $p_1 > p_2$, i.e., component 1 is the most reliable of the two.

The system reliability is therefore $h(\mathbf{p}) = p_1 + p_2 - p_1 p_2$.

1. $I^{\mathrm{CR}}(1) = \frac{I^{\mathrm{B}}(1) \cdot (1-p_1)}{1-h(\mathbf{p})} = \frac{(1-p_2)(1-p_1)}{1-(p_1+p_2-p_1 p_2)}$
2. $I^{\mathrm{CR}}(2) = \frac{I^{\mathrm{B}}(2) \cdot (1-p_2)}{1-h(\mathbf{p})} = \frac{(1-p_1)(1-p_2)}{1-(p_1+p_2-p_1 p_2)}$

This means that $I^{\mathrm{CR}}(2) = I^{\mathrm{CR}}(1)$ and we can conclude that when using the Criticality Importance measure, all the components in a parallel structure are of equal importance.

Fussell-Vesely's measure - 1

Fussell-Vesely's measure of importance, $I^{\mathrm{FV}}(i \mid t)$ is the probability that at least one minimal cut set that contains component i is failed at time t, given that the system is failed at time t.

Fussell-Vesely's measure can be approximated by

$$I^{\mathrm{FV}}(i \mid t) \approx \frac{1 - \prod_{j=1}^{m_i}(1 - (\check{Q}_j^i(t))}{Q_0(t)} \approx \frac{\sum_{j=1}^{m_i} \check{Q}_j^i(t)}{Q_0(t)}$$

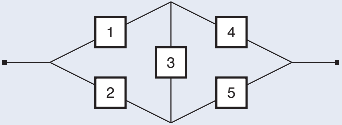
where $\check{Q}_j^i(t))$ denotes the probability that minimal cut set j among those containing component i is failed at time t.

Fussell-Vesely's measure is easy to determine by hand-calculation and gives a similar result as the Criticality Importance measure.

Fussell-Vesely's measure - 2

Example – Bridge structure

Consider a bridge structure of five independent components, with unreliabilities q_i, for $i = 1, 2, \ldots, 5$.



The structure has four minimal cut sets: $C_1 = \{1, 2\}$, $C_2 = \{4, 5\}$, $C_3 = \{1, 3, 5\}$, and $C_4 = \{2, 3, 4\}$. We hence obtain $\check{Q}_1 = q_1 q_2$, $\check{Q}_2 = q_1 q_2$, $\check{Q}_3 = q_1 q_3 q_5$, and $\check{Q}_4 = q_2 q_3 q_4$. The system unavailability may be determined by the upper bound approximation formula $Q_0 \approx 1 - \prod_{j=1}^{4}(1 - \check{Q}_1)$. To determine Fussell-Vesely's measure of, for example, component 3, we have to find the minimal cut sets where component 3 is a member, which are C3 and C_4. Hence

$$I^{\text{FV}}(3) \approx \frac{\check{Q}_3 + \check{Q}_4}{Q_0} = \frac{q_1 q_3 q_5 + q_2 q_3 q_4}{Q_0}$$