NTNU – Trondheim
Norwegian University of Science and Technology

Risk Assessment

Section 10.3

Importance of Basic Events

Stein Haugen stein.haugen@ntnu.no
Marvin Rausand marvin.rausand@ntnu.no

RAMS Group
Department of Production and Quality Engineering
NTNU

(Version 0.1)

Measures covered

The following component importance measures are defined and discussed in this chapter:

- Birnbaum's measure (and some variants)
- The improvement potential measure (and some variants)
- Risk achievement worth
- Risk reduction worth
- The criticality importance measure
- Fussell-Vesely's measure

Importance depends on

The various measures are based on slightly different interpretations of the concept *basic event importance*. Intuitively, the importance of a basic event should depend on two factors:

- Where in the fault tree the basic event is located
- The probability of the basic event in question

. . . and, perhaps, also the uncertainty in our estimate of the basic event probability.

Birnbaum's Measure (1)

Birnbaum's measure is named after the Hungarian - American professor Zygmund William Birnbaum (1903-2000).

Birnbaum proposed in 1969 the following measure of the importance of basic event i at time t:

$$I^{B}(i \mid t) = \frac{\partial Q_0(t)}{\partial q_i(t)}$$

where $Q_0(t)$ is the TOP event probability and $q_i(t)$ is the probability of basic event i at time t.

Birnbaum's Measure (2)

- Birnbaum's measure is seen to be obtained by differentiating the TOP event probability $Q_0(t)$ with respect to $q_i(t)$.
- When differentiating, we assume that all the other basic event probabilities are constant.
- The approach is well known from classical sensitivity analysis.
- If $I^{\mathrm{B}}(i \mid t)$ is large, a small change in the basic event probability $q_i(t)$ will result in a comparatively large change in the TOP event probability at time t.

Birnbaum's Measure (3)

Let $Q_0(1_i, t)$ denote the TOP event probability when we know that basic event i has occurred and let $Q_0(0_i, t)$ denote the TOP event probability when we know that basic event i has not occurred. By using the rule of total probability, we then have:

$$\begin{aligned} Q_0(t) &= q_i(t) \cdot Q_0(1_i, t) + (1 - q_i(t)) \cdot Q_0(0_i, t) \\ &= q_i(t) \cdot [Q_0(1_i, t) - Q_0(0_i, t)] - Q_0(0_i, t) \end{aligned}$$

Birnbaum's measure can therefore we written as

$$I^{\mathrm{B}}(i \mid t) = \frac{\partial Q_0(t)}{\partial q_i(t)} = Q_0(1_i, t) - Q_0(0_i, t)$$

Birnbaum's Measure (4)

- Birnbaum's measure of basic event i can therefore be calculated as the difference between the TOP event probability when we know that basic event i has occurred, and when basic event i has not occurred, respectively.
- Note that Birnbaum's measure of basic event i only depends on the "rest" of the fault tree and is independent of the actual probability $q_i(t)$ of basic event i. This may be regarded as a weakness of Birnbaum's measure

Birnbaum's Measure (5)

Assume that basic event i is a component failure with failure rate λ_i. In some situations it may be of interest to measure how much the TOP event probability $Q_0(t)$ will change if we make a small change to λ_i.

The sensitivity of $Q_0(t)$ with respect to changes in λ_i can obviously be measured by

$$\frac{\partial Q_0(t)}{\partial \lambda_i} = \frac{\partial Q_0(t)}{\partial q_i(t)} \cdot \frac{\partial q_i(t)}{\partial \lambda_i} = I^{\mathrm{B}}(i \mid t) \cdot \frac{\partial q_i(t)}{\partial \lambda_i}$$

Birnbaum's Measure (7)

In practical fault tree analysis of a complex system, one of the most time-consuming tasks is to find adequate estimates for the input parameters (failure rates, repair rates, etc.). In some cases, we may start with rather rough estimates, calculate Birnbaum's measure of importance for the various basic events, or the parameter sensitivities, and then spend most time finding high-quality data for the most important basic events. Basic events with a very low Birnbaum's measure will have a negligible effect on the TOP event probability, and extra efforts finding high-quality data for such basic events may be considered a waste of time

Improvement Potential

The improvement potential of basic event i at time t is

$$I^{\text{IP}}(i \mid t) = Q_0(t) - Q_0(0_i, t)$$

The improvement potential may be expressed as

$$I^{\text{IP}}(i \mid t) = I^{\text{B}}(i \mid t) \cdot q_i(t)$$

- $I^{\text{IP}}(i \mid t)$ tells how much the TOP event probability can be reduced by replacing basic event i with a basic event that can *never* occur.

Risk Achievement Worth

The importance measure *risk achievement worth* (RAW) of component i at time t is

$$I^{\text{RAW}}(i \mid t) = \frac{Q_0(1_i, t)}{Q_0(t)}$$

The RAW is the ratio of the TOP event probability when basic event i is known to have occurred with the actual TOP event probability.

RAW is a measure of the *worth* of basic event i in achieving the present level of the TOP event probability and indicates the importance of maintaining the current level of the basic event probability.

Risk Reduction Worth

The importance measure *risk reduction worth* (RRW) of component i at time t is

$$I^{\text{RRW}}(i \mid t) = \frac{Q_0(t)}{Q_0(0_i, t)}$$

The RRW is the ratio of the actual TOP event probability with the (conditional) TOP event probability if basic event i is known not to occur (or removed from the fault tree)

In some applications, the basic event' may be an operator error or some external event. If such basic events can be removed from the fault tree, for example, by canceling an operator intervention, this may be regarded as replacement with a basic event that cannot occur.

Criticality Importance (1)

The component importance measure *criticality importance* $I^{\text{CR}}(i \mid t)$ of component i at time t is the probability that component i is critical for the system and is failed at time t, when we know that the system is failed at time t.

$$I^{\text{CR}}(i \mid t) = \frac{I^{\text{B}}(i \mid t) \cdot (1 - p_i(t))}{1 - h(\boldsymbol{p}(t))}$$

By using the fault tree notation, $I^{\text{CR}}(i \mid t)$ may be written as

$$I^{\text{CR}}(i \mid t) = \frac{I^{\text{B}}(i \mid t) \cdot q_i(t)}{Q_0(t)}$$

Criticality Importance (2)

Let $C(1_i, \boldsymbol{X}(t))$ denote the event that the system at time t is in a state where component i is critical. We know that

$$\Pr(C(1_i, \boldsymbol{X}(t))) = I^{\mathsf{B}}(i \mid t)$$

The probability that component i is critical for the system and at the same time is failed at time t is

$$\Pr(C(1_i, \boldsymbol{X}(t)) \cap (X_i(t) = 0)) = I^{\mathsf{B}}(i \mid t) \cdot (1 - p_i(t))$$

When we know that the system is in a failed state at time t, then

$$\Pr(C(1_i, \boldsymbol{X}(t)) \cap (X_i(t) = 0) \mid \phi(\boldsymbol{X}(t)) = 0)$$

Criticality Importance (3)

Since the event $C(1_i, \boldsymbol{X}(t)) \cap (\boldsymbol{X}(t) = 0)$ implies that $\phi(\boldsymbol{X}(t)) = 0)$, we get

$$\frac{\Pr(C(1_i, \boldsymbol{X}(t)) \cap (X_i(t) = 0))}{\Pr(\phi(\boldsymbol{X}(t)) = 0)} = \frac{I^{\mathsf{B}}(i \mid t) \cdot (1 - p_i(t))}{1 - h(\boldsymbol{p}(t))}$$

$I^{\mathsf{CR}}(i \mid t)$ is therefore the probability that component i has *caused* system failure, when we know that the system is failed at time t. For component i to cause system failure, component i must be critical, and then fail.

When component i is repaired, the system will start functioning again. This is why the criticality importance measure may be used to prioritize maintenance actions in complex systems

Fussell-Vesely's Measure

Fussell-Vesely's measure of importance, $I^{\mathrm{FV}}(i \mid t)$ is the probability that at least one minimal cut set that contains basic event i is "failed" at time t, given that the TOP event occurs at time t.

Fussell-Vesely's measure can be approximated by

$$I^{\mathrm{FV}}(i \mid t) \approx \frac{1 - \prod_{j=1}^{m_i}(1 - (\check{Q}_j^i(t))}{Q_0(t)} \approx \frac{\sum_{j=1}^{m_i} \check{Q}_j^i(t)}{Q_0(t)}$$

where $\check{Q}_j^i(t))$ denotes the probability that minimal cut set j among those containing basic event i occurs at time t, and m_i is the number of basic events in minimal cut set i.